Introduction to Mineral Processing
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Lecture - 37
Movement of Solids in Fluids (Contd.)

Hello welcome back. So, we are discussing about movement of solids in fluids. We have discussed so far about the how do we calculate the presetting velocity of spherical particles of different Reynolds number region it was stagnant fluid and we have also discussed about the complexities associated with the say in predicting the presetting velocities of non spherical particles into a stagnant fluid medium and what are the equations available and what are the limitations of all those equations.

Now, in today's lecture I will try to discuss about it is known as hindered settling of spherical particles. Hindered means normally in a mineral processing operation I have mentioned it quite a few times that we deal with large volume of particles. So, when you have a very large amount of particle coming into fluid medium and there how the individual particles they try to get settled into that fluid medium because it has to overcome some other forces then what are the forces we have discussed for a free settling condition, because free settling condition you have got the of forces acting in the opposite direction of the particle movement that is your drag force and buoyancy force.

But in the hindered settling conditions we have got some other additional forces which are known as that is hindrance.

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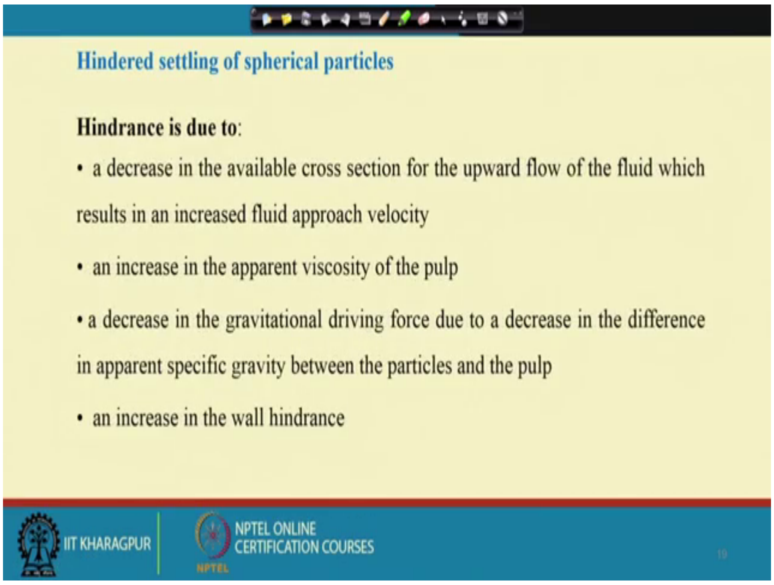

Hindrance means there is a resistance; that means, the hindered settling velocity that is when the particles are crowded they in the settling velocity of individual particle class into the fluid medium will be decreased than the free settling condition. So that means, in essence what we try to do that is we want to incorporate a correction factor for the particle crowding effect into the free settling velocity of the particles.

So, what is the hindrances because of? That is a decrease in the available cross section for the upward flow of the fluid which results in an increased fluid an increased fluid approach velocity. So, it is a basically a decrease in the available cross section for the upward flow of the fluid which results in an a decrease fluid approach velocity it should be your no sorry it should be increased fluid approach velocity.

Now, what does it mean? It means that that is when you have your say suppose if you I have a 100 litre of water into a column of fluid and if I 400 kg of a material having wide size distribution starting from your 1 millimetre down to 1 micrometer, then if I want to calculate the settling velocity of 10 micrometer particle in that they know what will happen the particle the fluid approach velocity in the upward direction will be a net result of the total displaced fluid velocity based on the total volume of the particle you affect that is because of the 100 kg of the particle. It is not that single particle how much it has displaced. So that means, the fluid will have more approach velocity in the upward direction because of the displaced fluid than the particle downward velocity.

Then number 2 is an increase in apparent viscosity of the pulp what is the definition of pulp, pulp is a solid liquid mixture. So, now, what will happen? If I have large number of particles of huge concentration, so the viscosity of the fluid is no longer dependent on the fluids individual viscosity even the viscosity nothing, but the resistance to flow even the pine particles which are in suspension that will also increase the apparent viscosity of the pulp. So, the particle has to displace that viscous fluid not the original fluid.

Then a decrease in the gravitational driving force due to a decrease in the difference in apparent specific gravity between the particles and the pulp because you remember that your original equation it is shown that it is rho f minus your rho p minus rho f. So, rho p minus rho f as it is becomes larger, then the more the value of the downward settling velocity of the particles.

But when the rho p that is the particle density and the minus fluid density that when the fluid density because the fluid density is now the suspension density is no longer the original fluid density. So, because the fluid density has increased, so the difference between rho p minus rho f will be less. So, naturally your the velocity of the particle in the downward direction will be lesser. And increase in the wall hindrance; that means, it is the wall effect if the surrounding wall also will try to have some effect on to the settling velocity of the particle.

Now, this is a little bit of complicated your subject, but you will try to understand this if I give this example that is your hindered settling velocity like is like having a similarity like when you are driving a vehicle into a crowded traffic condition. So, what happens even though you are not hitting the your another vehicle which is in front of you or maybe in the side, but your speed of your vehicle has to be reduced significantly otherwise you may heat the party, you may heat the vehicle which is ahead of you or maybe you can heat the particle heat the vehicle which is beside you. So, automatically your speed decreases, reduces.

So, you can drive the vehicle at a speed of say suppose your 100 kilometre per hour when you have less traffic density. Same road same vehicle same driver you have to reduce it the, your speed if you see that there is a traffic as the traffic congestion increases. You may reach a certain point that when it you may reach a stand still condition that is called the traffic jam or something like that, but here it is a similar condition only exception is that there if you heat another vehicle that is called an accident, but here even the particle particle collisions should be there. So, and then, although it is not mentioned here. So, the particle particle collision also can come into this. So, there are so many factors that is which influences it in a different way that is it reduces your free settling velocity of the particles when the crowding of the particle becomes higher and higher.

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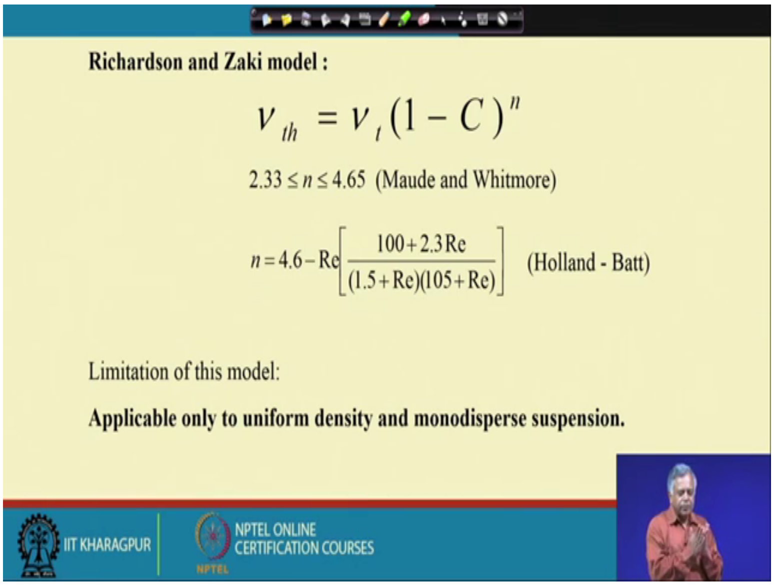

So, how do I calculate that into a hindered settling condition what will be the settling velocity of my particles. That means, what would be my correction factor for the particle crowding on the your say free settling velocity of my particles. This is a very popular model use frequently by the chemical engineers that is called now v th, h stands for hindered settling hindered, t stands for terminal. So, hinder terminal settling velocity for the spherical particle it is proposed by Richardson and Zaki, that is your v t is the your free terminal setting velocity and the correction factor is 1 minus capital C to the power n.

So, what is the C? C is the volumetric concentration of your particle because you do not have to consider the what is the mass percent concentration because the fluid is getting displaced based on the your what is the volumetric concentration of your solid particles. So, you have to consider the volumetric concentration of your total particle and n, n is a constant. And then what would be the value for n? That is different researchers they have given different values for n, but these two gentlemen Maude and Whitmore they propose that n is a greater than is equal to 2.33 and less than equal to 4.65, but for your calculation for your system what would be the value of n it is very you have to do the test and you have to find out that.

So, here in mineral processing literature for application in mineral processing the Richardson and Zaki model this gentleman Holland Batt he proposed 8 equation for n while in relation to Re and years this is an empirical equation, but it works well for mineral system in many instances. So, this is the equation proposed by Holland Batt where you do not have to worry about what is the value of your n because if you know the Re and how do you know the Re, because if you go for iterative technique and you can get to know that what is the Reynolds number of your particle and then based on that you can get the value of n you know the volumetric concentration of your solids. And you have already calculated the terminal settling velocity of your particle based on the equations we have proposed if it is a stock seeing you can use the stock seeing condition or out suggest again that you better go for this iterative technique, based on that you can get the value of hinder terminal settling velocity of the particle.

But there is a caution before using this equation. What is that caution? That this model is applicable only for uniform density and monodisperse suspension; that means, when the particle density and the particle size they are fixed that they are identical. That is suppose I have got a particle size within 100 micrometer average size means size of 100 micrometers and the density is fixed; that means, you do not have any other different density classes. So, then if you are crowding it that means, if you increase the relative volumetric concentration of these particles, you get the value of your say you can use safely this equation. So, what it is saying? That if after beyond a certain particle concentration this value will be 0; that means, when the your entire fluid has become totally viscous then there wont be any movement of the particles. So, again I repeat with caution you should use this equation that is it is applicable only for uniform density and monodisperse suspension.

But in mineral processing field we hardly encounter this situation because the mind materials they neither have uniform density or not they are mono dispersed; that means, you have got a size distribution you have got a density distribution. So, in that case even we assume that they are spherical particles what equation we should use.

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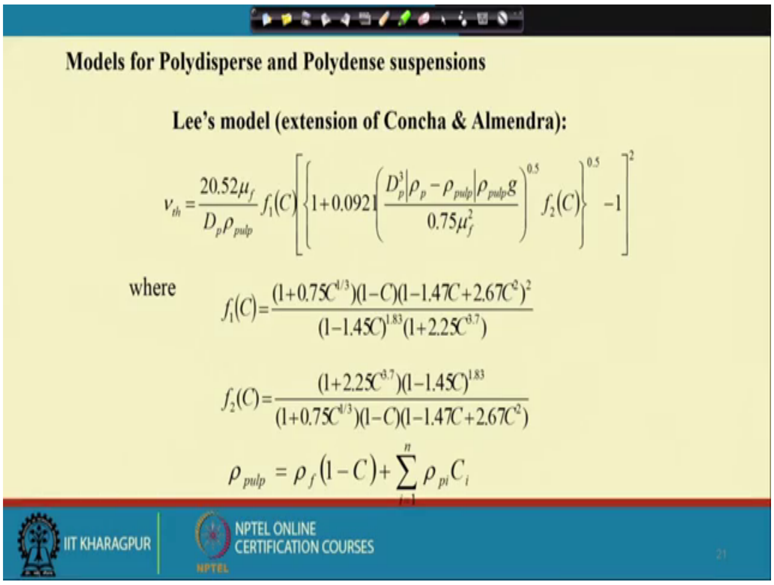

What I propose this is a again a model proposed by say Professor Concha's group that is Lee's model and there is an extension of Concha Almendra model initial model what we have discussed.

So, this is an empirical model again looks little bit complicated, but if you look at it closely it is not that difficult to calculate based on this. So, there are two functions a f one C that is they are all function of concentration f 2 to C they are all function of concentration and here are only change you are not considering the row fluid you are taking the row pulp. So, this is the absolute value of that and if you use this equation you use a part density and in place of this and you use this function that is you know the volumetric concentration of solids. Then it is not saying that if I have a size distribution and density distribution how do I use it, but this is taking also the total solid concentration. So, then the v th is you can calculate based on that equation.

Now, again this has got a limitation because it does not take into consideration of the particle density distribution as it is evident from this and the size distribution. Another thing what I try to highlight here as I said that that when I have said suppose I am coming back to that previous example and as I have got 100 kg of material I have poured into a liquid column of your 100 litre volume.

So, what will happen? I have particle classes from your 1 millimetre down to 1 micrometer size. Now when I am pouring this 100 kg material, the dispersed fluid will have some upward velocity now imagine I have got a 5 micrometer particle or particles below 5 micrometers they are downward velocity that m g may be much less than the upward velocity of the fluid. So, in that case those particles may not even settle at all. So, that is called the counter flow factor that is whether we have considered the upward increased upward velocity of the fluid. So, that is why in many cases you will find that the in a wide size distribution of particle some particles they never settle into that fluid.

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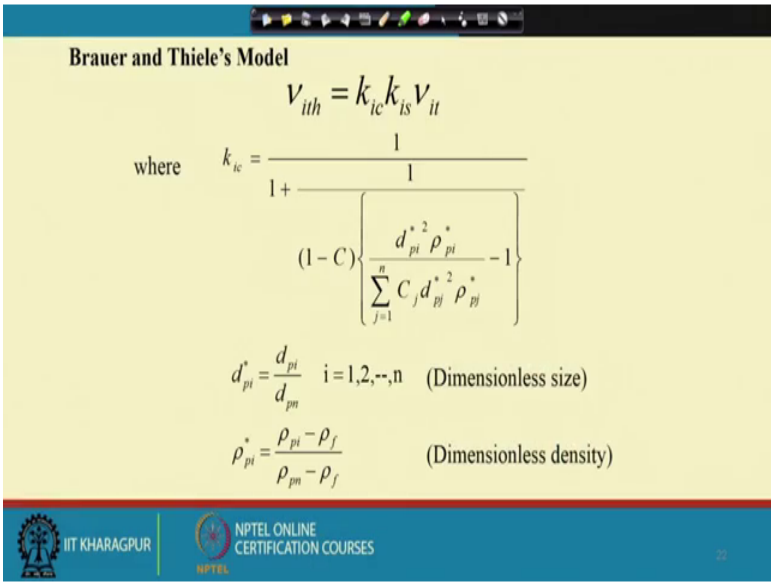

And when I was looking at the literature I found that there is a your literature written in a German language that is called Brauer and Thiele’s model which considers these two that is its written that for a ith class particle that is when you have an assembles of different particle classes varying in size and density.

So, as a hindered terminal settling velocity of the ith class particle in that assembles is equal to k ic it is called the counter flow factor of the ith class particle what it is experiencing and these s stands for swarm that is the your swarm velocity of the ith class particle in that mixture and it is the terminal settling velocity of this particle. So, this is bit complicated do not have to worry about it. Just remember the reference of these when do you really require to apply adequate model you can try this model. So, this is known as Brauer and Thiele’s model.

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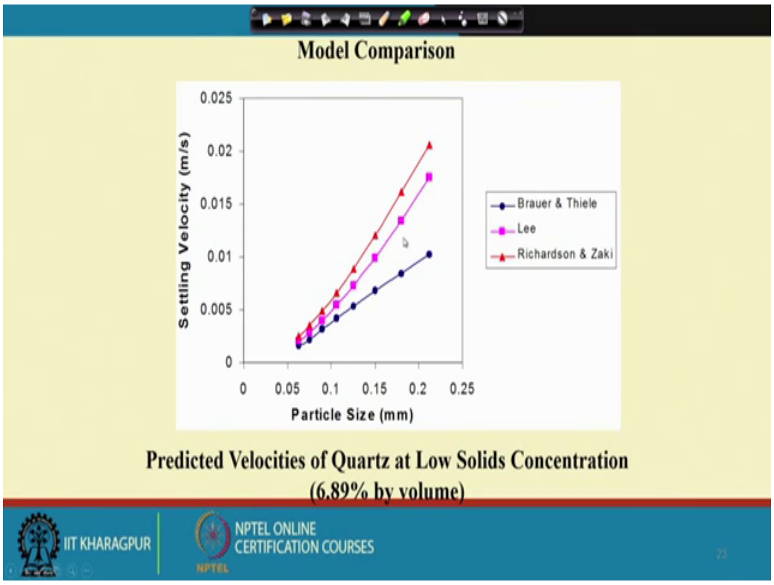

What I try to highlight here that again if I compare with some synthetic particles that is with quartz particles having various sizes and having a very low concentration that is 6.89 percent by volume. And if we compare the predicted values proposed your superlative values based on these 3 different models what we had discussed that is Richardson Zaki, Lee's model, that is your extension of Concha and Almendra model, and Brauer and Thiele's model. You see that all three your predicted values they are entirely different.

So, again the question comes which one I should use. Again my personal suggestion would be that we may use the Brauer and Thiele's model because fundamentally it looks much better than the other two models. The difficulty in this type of your hindered setting model development linear velocity your model development for hindered settling velocities in a large concentration of particles the problem is with the measurement that is how do I measure the individual particles settling velocities in that condition. And it is a very difficult domain and many people are doing are active or active in this research field, but as far as I am concerned this is the scenario as of today there may be some better models published in other journals, but what I have seen that this Brauer and Thiele's model works well for our minted processing systems.

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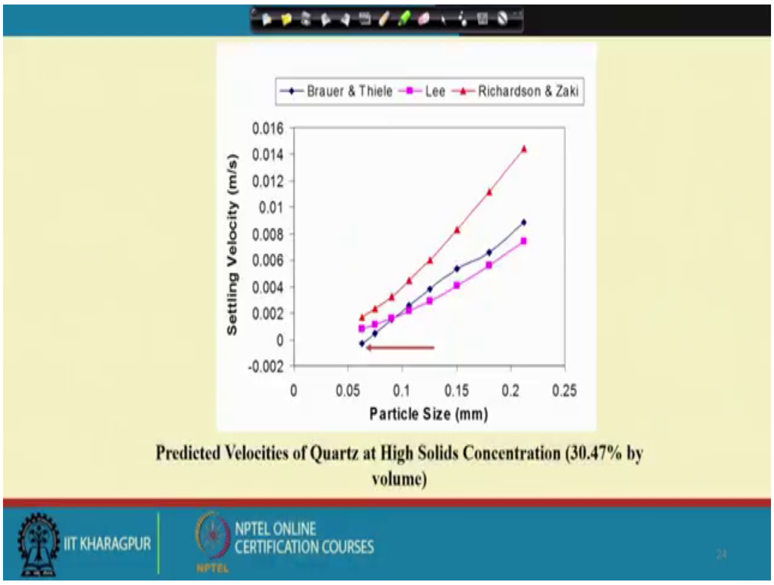

This model, this comparison if we extend it at a much higher concentration we will find that that is at thirty point four seven percent concentration we find that only the Brauer and Thiele's model is predicting that some particles that is at this size range they have got 0 settling velocity; that means, they are not settling at all. And this is what we commonly observed in our mineral processing field in a dewatering system we call it thickness that is where we try to d water our, that is we try to get rid of our water so that we can recycle it back. And we want to have a solid liquid separation we find that the ultrafine particles the never settled. It does not matter how much of your chemicals that is we call it flocculants you are adding to that. Because what happens when you add more flocculent the bigger particle becomes much more bigger and the smaller particles although they become bigger, but they are your ratio of their sizes remains the same that is the largest to smallest sizes. So, I think, I personally believe that it is because of the counter flow factor the your disperse fluid they have provided to these finer particles.

So, what is my suggestion in that case that you do it in stages, do not try to do it in one go. You may send this particles along with the water into a next thickener and there you may try.

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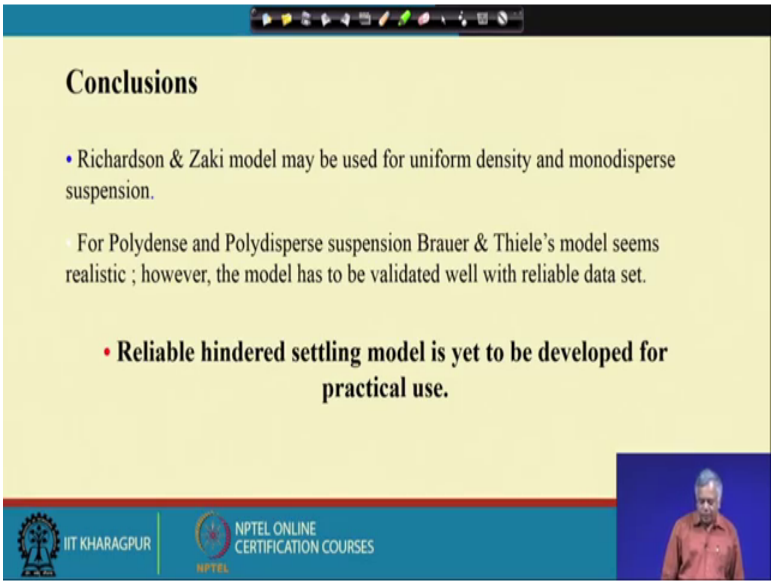

So, what we can conclude based on that? These are typically my personal opinion that Richardson and Zaki model may be used for uniform density and mono dispersed suspension for polydense and polydispersed suspension. Brauer and Thiele's model seems realistic; however, the model has to be validated well whether reliable data set nobody has validated it because the intrinsic difficulty in measuring the hindered settling velocities in a crowding in a crowded condition.

So, reliable hindered setting model is yet to be developed for practical use. So, this is a strong message that when you are designing your mineral processing equipment or you are trying to fine tune it be careful that what settling velocity model you have used and what are the limitations of those models and how much of error you can induce into your entire simulation package or in the your optimization package because that you must be very careful.

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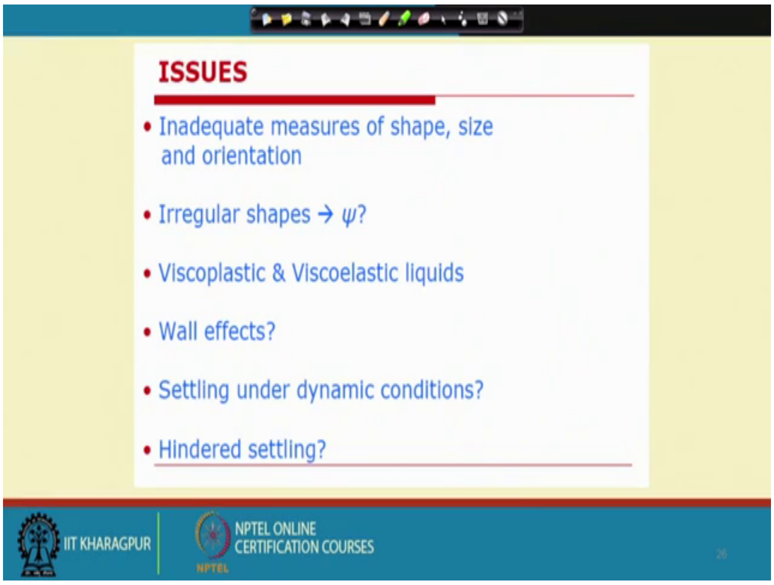

So, what are the issues? That in adequate measures of shape size and orientation we do not know of the very fundamental questions. Irregular shapes how do psi the appropriate is the sphericity is the appropriate criteria may be may not be, in viscoplastic and viscoelastic liquids what will happen in that case. So, these are wall effects how do I quantify it a settling under dynamic conditions that is when fluid is also moving then your hindered setting conditions.

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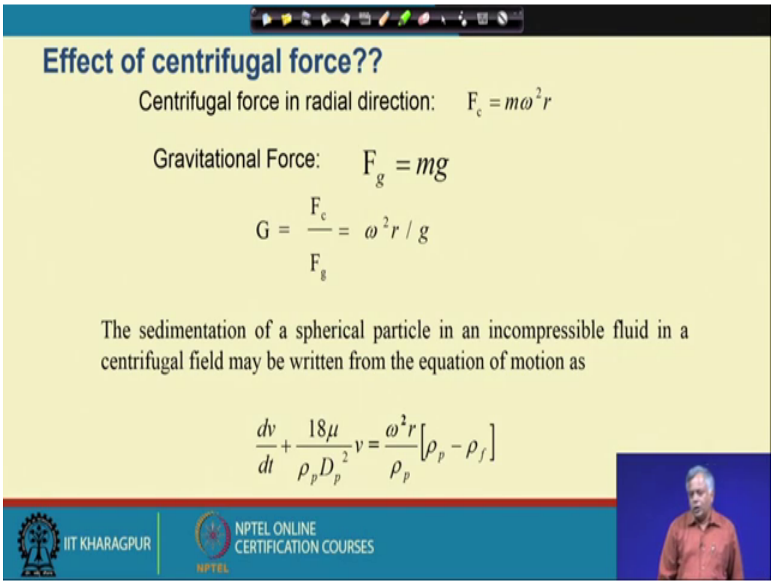

Now, in many instances in our mineral processing system or mineral processing operations what we try to do while dealing with very fine particle sizes where it is almost impossible to separate particles at that size because mostly we try to separate the particles in a fluid medium based on the principle of relative settling velocity. That means, if I drop two particles into a fluid medium after a delta t time they should have a your say different distance travelled. So, based on the differences in the travelling distance mechanically if we assume that there is some separator you can separate the your finer particle from coarser particle or maybe lighter particle from your heavier particles. So, in that case suppose imagine that we want to separate two particles based on their density at a size of 40 micrometers.

Now what will happen there your differences in the settling velocity may be very less. So, in that case can we not increase their settling velocities by some other means? Yes, we can do it how we can do it. Now if we can incorporate a centrifugal force on that. So, what will happen? When we incorporate a centrifugal force so that is called that is defined as G force, we popularly known as capital G, capital G force which is nothing, but a basically your in terms of your say calculation force calculations it can be written as omega square r by g if the fluid is basically you are rotating it. But actually g has got a similarity with that that how many times of the gravitational acceleration you have enhanced is it 100 g means 100 times of the your acceleration due to gravity. So, you have increased that.

Now, what will happen that is your particle setting velocity in an incompressible fluid in a centrifugal force field may be written like this again this is a difficult domain. But just for your information sake that we can write that again it is related to your Reynolds number range that is what size range of particle you are discuss you are dealing with.

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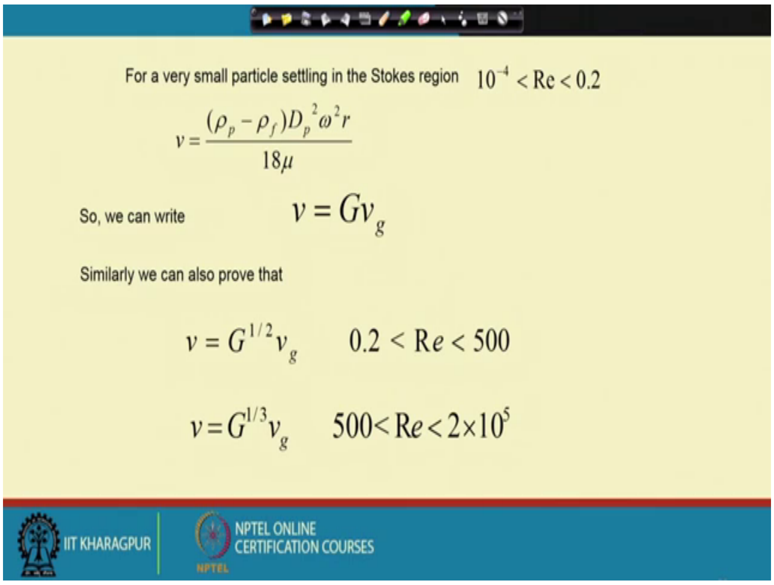

That is when the Reynolds number is greater than 10 to the power minus 4, because if you go beyond that you may be reaching a Brownian motion, there is your different behaviour of the particle. So, there the body forces are not at all dominant it is the basically the surface forces which dominate.

An Reynolds number less than 0.2; that means, in between 0.2 to 10 to the power minus four we can write v that is your settling velocity in a centrifugal force field is g that is how many times that centrifugal force field you have applied multiplied by what is the gravity what is the settling velocity of that particle into a normal gravitational force field. That means, what is the free settling velocity of that particle multiplied by how many times the gravitational force field you have increased. And if it is in a crowded condition you can replace it with a your say hindered settling velocity and multiplied by that capital G, how many times you have.

This equation is will be different when the Reynolds number is in between 500 to 0.2. It becomes square root of g into v g and when it is more than that beyond 500 it becomes cube root of g into v g; that means, it does not increase you are proportionally with the increase in your centrifugal force or the intensity of your centrifugal force. Anyway these are all required for your fine tuning your processes.

Now, the question comes that is as a mineral processor why should I be worried about all these models and complexity of the problem. I will try to show you with a simple example that is if I am very clear about the models or if I am familiar the models then we can solve certain problems even we can better design our equipment.

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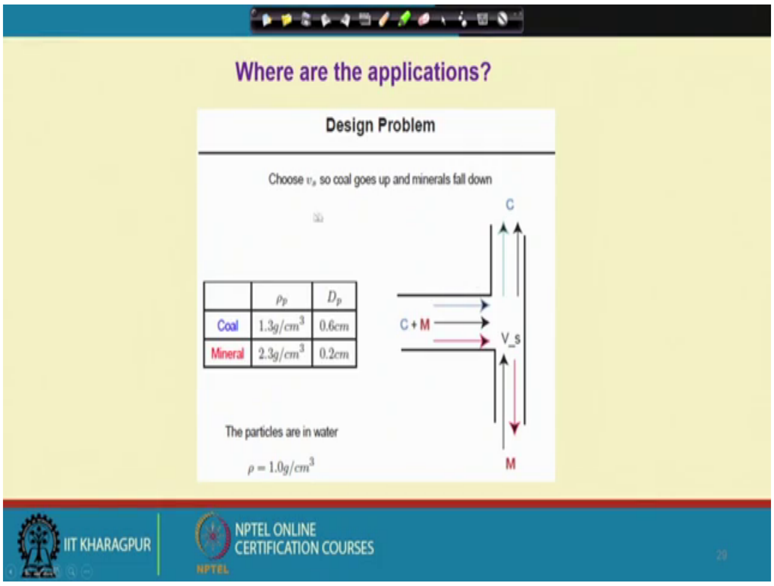

Now, this is the example I am given that is suppose if you have a coal and mineral mixture and this coal suppose this coal particle density is 1.3 gram per centimetre cube and its size is 0.6 centimetre. So, you have got a bigger size and these density and you have got a mineral particle whose density is 2.3 gram per centimetre cube and size is 0.2 centimetre.

Now, if I ask you that I have a mixture of this and I want to have a system like this that is it will be fade at the middle of a liquid column and where the liquid will be, liquid will be say sent through the bottom in the upper direction. That means, what should be the rising velocity of my fluid in the upward direction to have a separation in between coal and mineral. That means, why I should have upward velocity of this because this upward velocity of water should be able to take out my coal particle from this mixture and your mineral particles should be collected at the bottom separately. So, that is what I want to design, that is I have to fix that what should be the rising velocity of my water.

So that means, the rising velocity of my water should be in between the settling velocity of these two particles. That means, if my coal particles are having, is having a velocity says suppose your 1 millimetre per second and my mineral particle is having a settling velocity of 2 millimetre per second. So, in a velocity in between 1 to 2 millimeter per second of this fluid you should be able to take out my coal particle from this mixture so that means, I can have a separation of coal and mineral by this simple principle.

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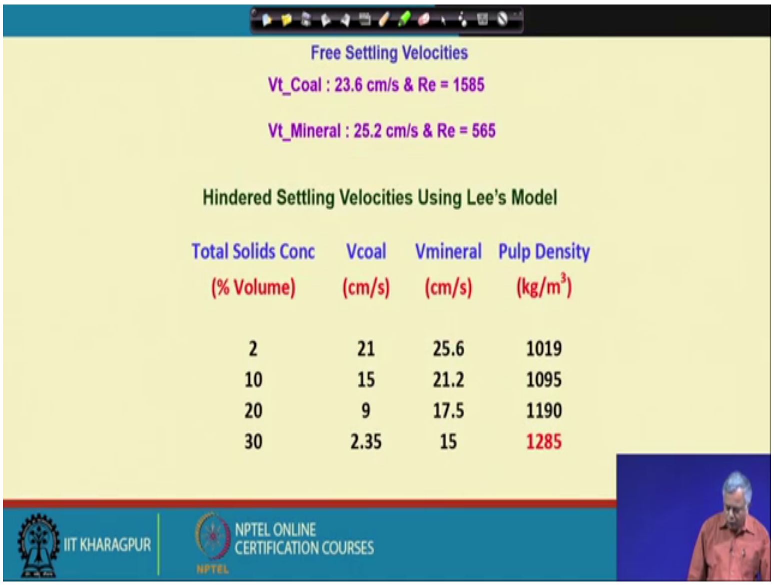

So, to do that what I have to do? I have to first calculate the free settling velocities of these two particles based on their size and density. So, we have done that and we have found that the terminal settling velocity of coal particle is 23.6 centimeter per second and its Reynolds number is 1585 and your mineral particle terminal settling velocity is 25.2 centimetre per second and its Reynolds number is 565.

Now, in this case you see that the difference between the settling velocity is only 1.6 centimeter per second so that means, my water should have a rising velocity more than 23.6 centimeter per second, but below 25.2 centimeter per second. Theoretically it may be possible, but practically it may not be that possible that to precisely maintain by upward water velocity within that range. So, I need a wider range of my I need to have more flexibility of my rising water velocity to have a better separation.

So, now, let us see that if I use a heated settling condition; that means, we are not talking about two particles we are having many particles we are having a crowded condition and if we use the Lee's model if you remember the Lee's model. So, if we use the Lee's model and we keep on increasing the percentage volume concentration of the solids having equal proportion that is your 50, 50 mixture of coal and mineral particles and we increase the volumetric concentration of the total particle class in that medium.

If we if it has got 2 percent by volume they in the naturally you are setting velocity of your coal and mineral particle will be less. So, your coal particle now setting velocity becomes 21 centimeter per second whereas, your mineral particles settling velocity is 25.6 centimeter per second. There is a difference it should be ideally less than that, but that is the error what is being induced based on these differences in the Lee's model and the model you have used for free settling velocities. Anyway, and the pulp density will be 1019 kg per meter cube.

If you increase it to 10 percent by volume you see the difference becomes 15 and 21.2, if you keep on increasing that up to 30 percent by volume you see that difference is significant 2.35 and 15. So, does not matter how accurate these numbers are. So, now, if you use a perfect model the difference here is around 12.65, but that may come down to maybe 11.

So, in this case this demonstration do not worry about the accuracy of the predicted values, but what I try to show that how much you can go that is your particle your coal particle density was 1.3; that means, it is 1300 kg per meter cube. So, if you have more density than this, so that is 1300 kg per meter cube, so the mineral particle the coal particle will not will be only floating. So, that is a different separation that is called a density separation you do not need even upward velocity of the fluid in that condition, your coal particle will simply float and the mineral particle will settle. So, that is a different technique.

But in that case the viscosity of the fluid will increase and you have a different problem, but you see that here when we have more flexibility with the based on the differences in their settling velocities we can have a better design of my equipment. So, what I try to show here that is even why the particle crowded condition that is your hindered sending condition the separation between the two particles it not always becomes difficult it may be easier also in certain conditions. But what is the essence of this discussion that if we know how to calculate the settling velocity of the particles and if we know that what are the errors we can have based on those models. So, when you are not sure about the errors you can have your as a wide variation in there why do you can choose a condition where you have got more flexibility in your settling velocity differences. So, that is my final advice.

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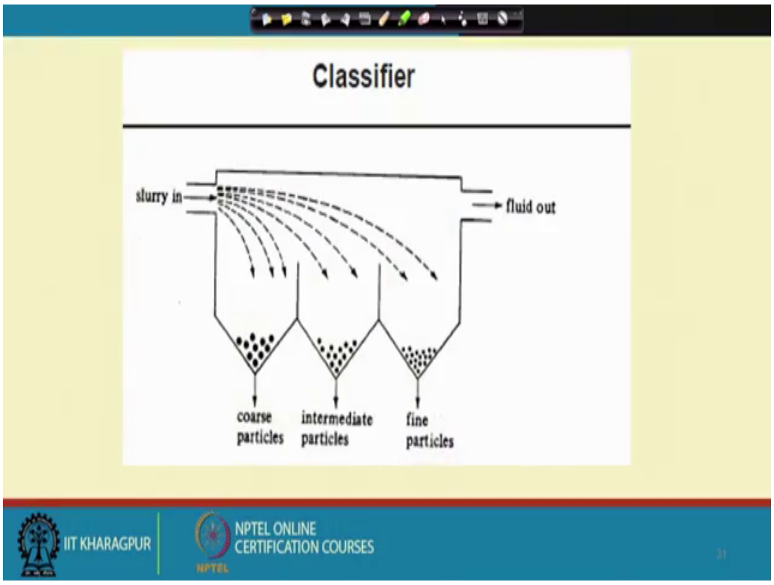

So, based on that even if we know the settling velocity of the particles we can have your as we can design, we can think of a design of a classifier that is a size separation device is like this that is we can have it your size separation. Suppose if I have the 3 different yours separate vessels with a partitioning in between and if we have a slurry and we let them fall freely on to this. So, what will happen? The coarses particle will settle faster and if we know the setting velocity and if we know the flow path of these. So, we can calculate the trajectories of that that particle and we can separate this coarse particle here, intermediate particles here and fine particles here. So, the first thing I should know what is the settling velocity of that, then we have to take into consideration the different features of my flow and then we can have a fluid out here.

So, we can have easily a classifier to be designed or may be the existing classifier size separation devices you can modify them based on this knowledge gained that is on setting velocity models.

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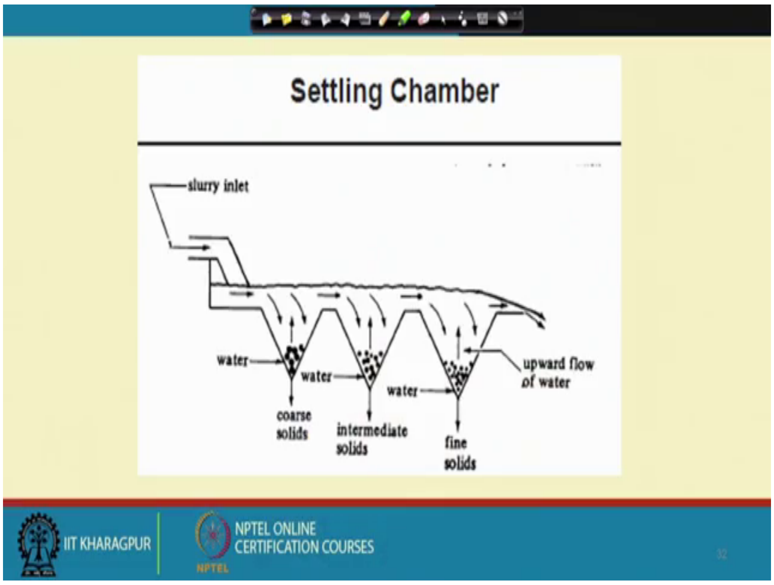

Similarly, we can design we can think of a settling chamber based on your solid liquid separation or maybe we can have coarse solids, intermediate solids and fine solids, you can have rising water here and you know that what should be the rising water velocity and you can have this type of design. So, this is how the mineral processing systems or mineral new equipment you can design or maybe you can optimize the process parameters or the design variables of your mineral processing equipment for performance improvement in your plant or maybe the students who will be joining in your future the mineral processing industry they can contribute immensely by say having updated knowledge in this area of solid fluid interactions.

Thank you very much, we will.

So, this is another example that is what is the particle size versus velocity relationship for a thickening that is your dewatering purposes. This is what we have already discussed.

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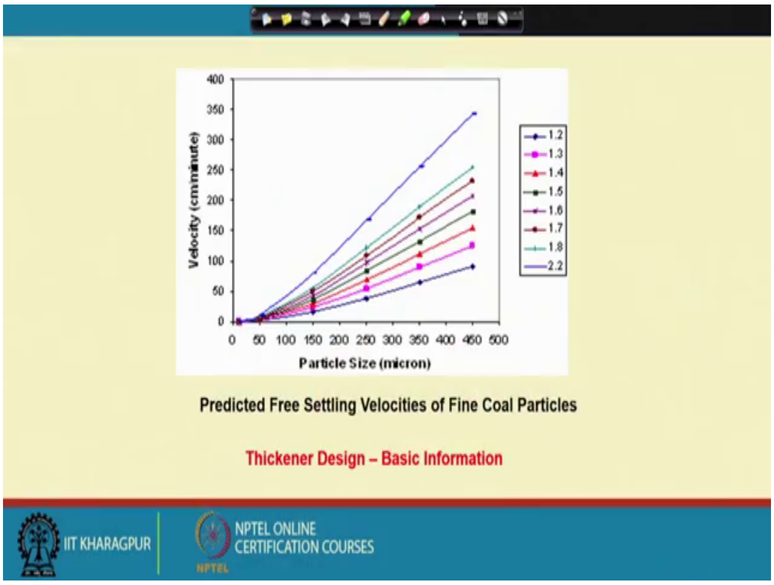

So, there are many examples what we can give and when we discuss about different mineral processing equipment I will show you how your basic knowledge of the setting velocity of the particles will help you in better understanding the separation mechanism or particles in different mineral processing equipment.

Thank you very much.