Introduction to Mineral Processing
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Lecture - 36
Movement of Solids in Fluids (Contd.)

Now, we are discussing about the free settling velocity at the terminal condition that is a terminal free settling velocity of a particle. We discussed about stokes, actually the Stokesian particle settling velocity. There is another scientist who is well known to everybody that is known as Newton's equation.

Now, this region that is Stokesian region is also known as laminar region; that means the it has hardly disturbed that your flow of the fluid surrounding the particle.

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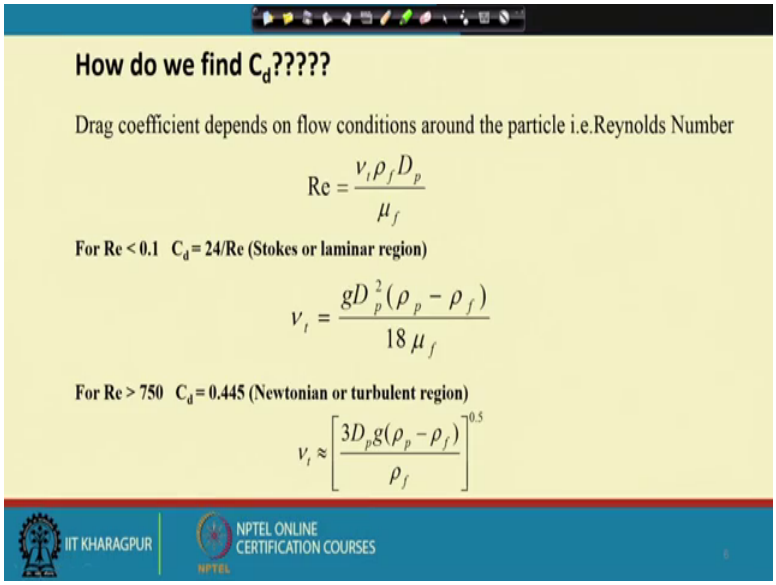

Now, this is applicable for your bigger particles when the particle mass that is the inertia dominates over the viscous forces or the viscous stresses offered by the fluid itself. So, that is known as Newtonian region or we call it the turbulent region. That means, it has given the your ad formation and all this happens and it is the disturbances to utter disturbances in the fluid medium it is created.

So, Newton has proposed that when the Re is greater than 750 look at Stokesian region less than 0.1 and this is Re greater than 750 the C d is no longer dependent on Reynolds

number it has got a constant value. So, beyond a certain particle size it does not matter how much of disturbances it has created, the C d the drag coefficient remains constant it has got a value of 0.445 although many literature they debate on the this third decimal point value and that third decimal point value difference makes lot of a difference in your calculations.

But let us stick to this that is C d is equal to 0.445 or we can assume that C d is equal to 0.5 approximately equal to approximately close to 0.5. So, if I put this value of 0.5 into the basic equation of the settling velocity that is a general expression what we had already discussed then the equation final equation becomes that v t is equivalent to I am not writing equal because I am not sure that whether you can really approximate it to be 0.5, but generally we do it for simplification of the expression that is v t is equivalent to 3 D p g rho p minus rho f divided by rho f to the power 0.5.

So, we got the expression. And now, what could be the particle size of this I again give this reference of quartz density particles that is particles who are having a density of 2650 kg per meter cube and is close to particle size more than 5 millimeter. So, this is a basically. So, we are in the range of. So, below 50 micron we have got your Stokesian region above 5 millimeter size we can use this equation for quartz density particles for Stokesian for Newtonian particles.

But what will happen to those particles whose Reynolds number is in between this and then that is another condition is that how do we find the Reynolds number because here also that terminal velocity term is there. So, these are the two questions we have to answer because neither we know the v t nor we know the Reynolds number and then in between Reynolds number, in between Reynolds number what would be the settling velocity equation even at free settling conditions for spherical particles we do not know.

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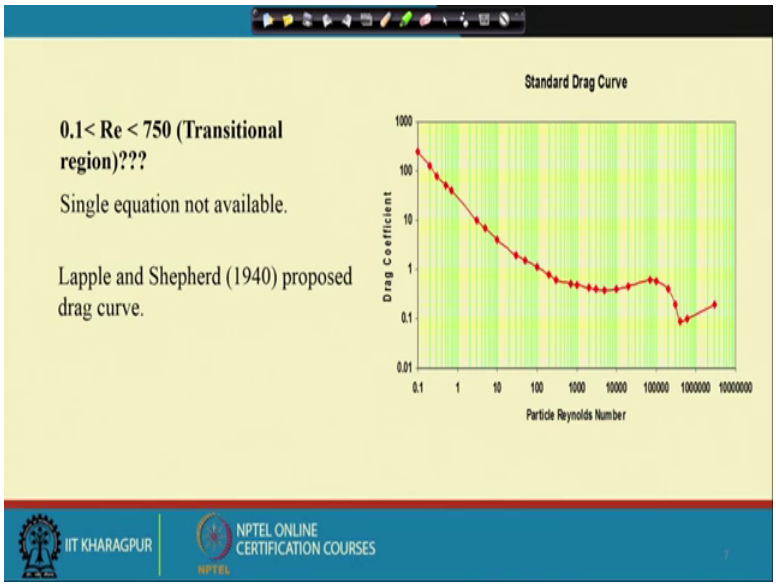

So, that is what we have to discuss. So, if you look at the literature although I have written here the single equation not available, but this is not correct statement. What I try to mean with this that there is no reliable single equation which available, reliable means there are so many equations, but available in the literature, but all these equations they have got their own limitations. So, there is no universally acceptable equation.

So, please read this statement carefully that no universally acceptable single equation is available for predicting the settling velocity of particles in the transitional region. Like we do not have any controversy about the Stokesian condition we do not have any controversy with the Newtonian region, but we have many equations proposed by many stalwarts in this field, but still all individual equations they have got certain limitations.

Now, the question is how do I select them and when we have many equations or what basis I will select that which equation is the most suitable or most appropriate. So, to solve this problem in nineteen forty the two scientists Lapple and Shepherd they proposed a curve. What is this curve? This curve is known as standard drag curve. So, what they did they collected they reviewed all the proposed equations by many scientists and they have and they have reviewed it very carefully and they have selected the different relationship proposed by different researchers for different Reynolds number region.

Because someone proposed that the C d value as C d is having a relationship with Reynolds number which is applicable from this Reynolds number range to that Reynolds number range, but actually you have to cover the entire range of the your Reynolds number. So, when they have proposed this curve that is if you plot the particle Reynolds number versus the drag coefficient proposed by different researchers you will find a typical shape of this curve.

Now, this curve is known as the standard drag curve. Why it is called standard drag curve? Now, the chemical engineering fraternity in particular and fluid mechanics community at large they have accepted that this curve is the let us consider that this curve is a standard one based on which we will compare the different models. That is what is if I propose a new model for this settling velocity I have to say that what is that C d and Re correlation I have used and I have to prove that that C d and Re correlation appropriately fits into these or gets superimposed on top of this drag curve otherwise the predicted values may be questionable.

So, now many attempts were made after that that is to propose a single equation to predict this standard drag curve, but because of this typical shape of this curve. There is no single equation available which can predict this curve perfectly and that is why I say that that a single equation may be proposed by someone they may be is a quite good when you are dealing with the Reynolds number range within these region.

That means they work well if your particle mass varies from this to this; that means, if your size and density relationship brings the particle Reynolds number range within that range. So that means, my C d and Re should be having this type of trend. But that equation I cannot apply it when I am dealing with much bigger particles which are having a Reynolds number of this range or maybe for very smaller particle very finer particle very fine particles which follow this range because and that is the limitation what I was mentioning that for different equations they have got these limitations.

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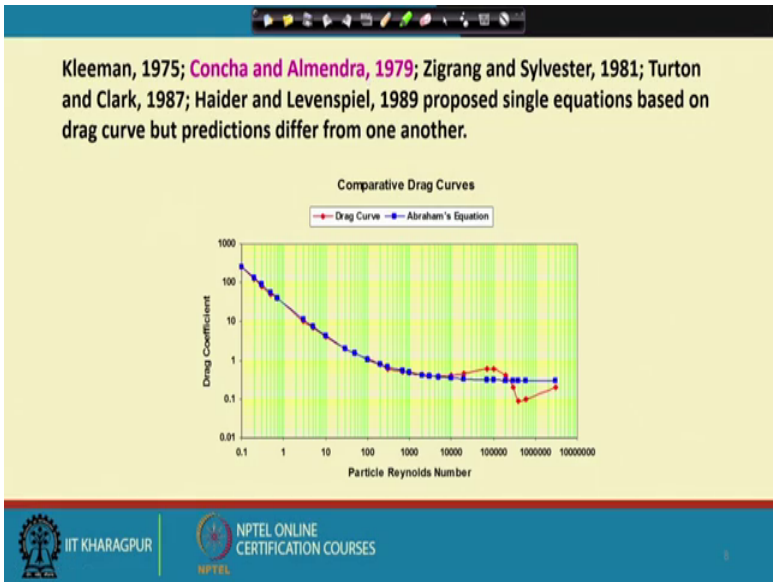

Even there are equations I will name some of them that is they are very famous equation they are will referred into that that is Kleeman, Concha and Almendra. Concha and Almendra's equation probably are frequently used by the mineral processing community then Zigrang, Sylvester, Turton, Clark even Haider Levenspiel they all proposed single equations based on drag curve that is the entire drag curve.

But if you look at the basic correlation they have used on C d and Re they are not able to get superimpose exactly on top of that standard drag curve. And possibly that is the reason that if I have a synthetic particle size or density and if I try to predict the presetting velocity of those particles at different sizes or different density combinations by using these models ideally if they are fundamentally right all the equations. Then ideally the predictions irrespective of the models they should have the as identical predictions, but if you try to predict them you will find that there is a large difference between the predicted values when you use the equation of different equations.

So, again the question comes which equations should I use? I personally believe that I must use that equation which can predict the standard drug up much accurately. So, for that if you see that that this Concha and Almendra's equation as I said that it is frequently used and it is a very, it is a really a robust equation and the basic equation the basic model they have used while formulating while formulating this model by Concha,

Professor Concha and the student Almendra they used the Abrahams equation for C d and Re correlation.

And if I plot that C d and Re correlation I see that we can see that that the blue lines the blue lines they are showing the Abrahams equation and red line is a your actual standard drag curve that they get almost superimposed up to this region, but they start getting deviating even when the Reynolds number is going in a much higher side. Even with the naked eye we may not see the differences between this the blue and the red values, but I will show you that even with this how the even with this small differences how your predicted values can differ to each other.

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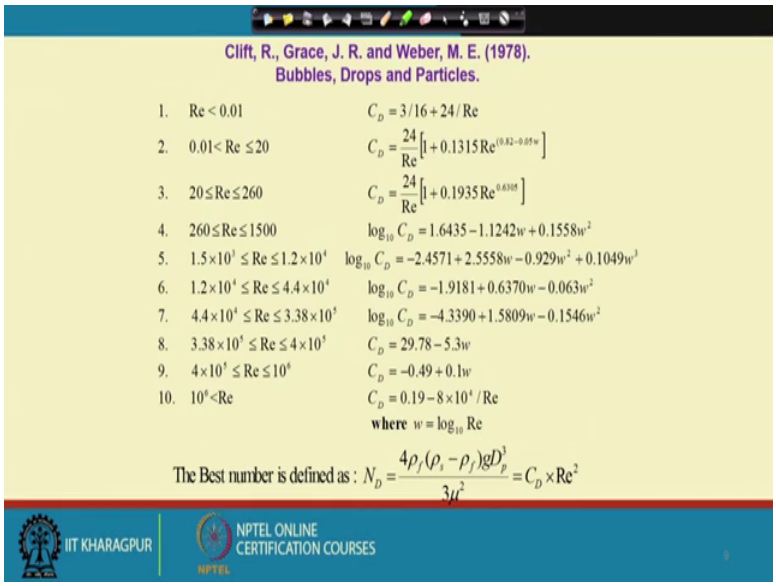

So, before I show you that let me show you that another it is not a single equation it is there is a classic book on bubbles drops and particles written by Clift Grace and Weber. There you will find that that they are proposed a set of equation, not a single equation a set of equation where you have got the C d and Re correlations at different innards number range they have even modified the stokes equation they have modified the Newton's equation based on the findings of many researchers.

So, you see that it starts from less than 0.01 Reynolds number and it considers up to 10 to the power 6 Reynolds number 10 to the power 6 and they are all having this type of relationship they may look little bit complicated, but they are very simple when you look

at closely because the w is equal to the low octane Re. So, you have to replace the values here. So, you get a correlation of C d into Re, C d and Re.

Now, the question is can we not use this set of equation because this if you try this try plotting this C d and Re correlation with this set of equations you will see that probably it gives the best feat so far to the standard drag curve than any other equation. So, what we were trying that is can we not use these set of equations for predicting the settling velocity of terminal settling velocity of the spherical particles into a stagnant fluid medium.

So, for that you see that everywhere your C d and Re is there and I said that for Re also there is a v t term. So, you have got your two unknowns, you have got say two unknowns and a single equation. So, it is impossible to find a solution. So, what do you have to do? We have to go for an iterative technique to use this for finding a solution for v t.

Now, for any iterative technique you need some convergence criteria. So, what could be that convergence criteria? You may be thinking that this is an introductory course why I am discussing this topic at that at that depth. The reason is that is there will be many participants who are coming from the industry and to educate themselves or to fine tune their knowledge so that they can make some differences after this course to their respective plates.

And for the students also who are taking this you may join the meander processing industry in your future and this aspect that is the your moment of solids in fluids in conventional literature in mineral processing in standard textbook they are not that given priority or they are not given that much of emphasis because of the complexity associated with this.

So, but as I said that we are dealing with water based processes, so this is a fundamental prerequisite that we must know how to calculate the settling velocity of the particle into a fluid medium. And we must be aware that what are the limitations of these equations and what are the how much of error we can have because of your say selection of a particular equation and this will not help in optimizing our process conditions quickly.

But also in better designing our equipment in better selecting our equipment for a particular process that is why I am giving so much of importance to this topic. So, you

may not you may not have to remember all these equations yeah, I may not ask you very difficult questions based on this, but I thought that it is my responsibility to tell you that what are the difficulties associated and where we stand and how we can utilize this knowledge for fine tuning our processes.

So, for this iteration process we are looking for a convergence criteria and there is one number that is called the best, b e s t is the name of scientist it, it is available in fluid mechanics literature I am not getting into that detail what is the number is, but I have used it we can use it as a convergence criteria, why. Now, the best number is defined as 4 rho f rho s minus a rho s is the density of the solid rho f is the fluid density g is a gravitation acceleration D p is the particle size and mu is the fluid density here mu f it should be. So, that is how it is written in the base number is defined as and if you convert this equation in terms of C d and Re it can you can have it is this is equal to C d into Re square.

Now, you see that this are the input these are the prior information we can easily have about my particle and about the fluid because if I know the fluid. And if I know the temperature I can easily get to know the rho f and mu f and particle size I can measure particle density I can measure and I know we all know the gravitational acceleration number. So, this is equal to C d into Re square.

So, how you can use these set of equations?

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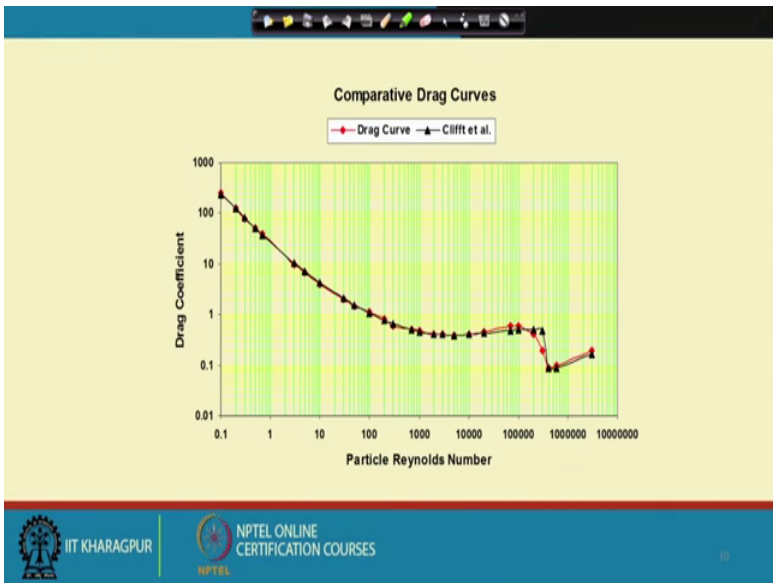

Now, before I go to that that how you can use it. So, if you plot that set of equations you see that that it can get superimpose on top of the standard drag curve that is your red one is the standard drag curve and this is what I have said that this is the clipped at all correlations. And you see that at least it is taking that shape, but it is not yet getting superimposed on top of this. So, maybe someone in your future from you will come up with a better equation to predict this standard drag curve. But as far as I am concerned this is the best available relationship which can predict the standard drag curve.

Now, what we can do? If you have a little bit exposure in writing computer codes, you can write this in MATLAB or in any other language what you have to do that is your first input values will be that particle properties. Like your size density and the operating temperature because if I know the temperature if I know the fluid I can get to know the fluid density and viscosity.

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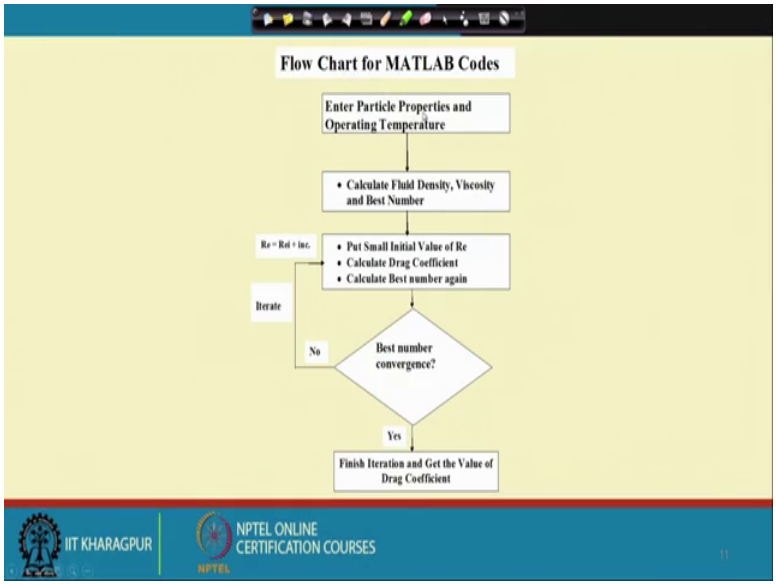

And once I know these parameters from that best number correlation I can get to know that what is the best number for that solid and fluid when you have the interactions. So, now, what we can do. We can put a small initial value of Re because we do not know the Re we do not know the Reynolds number of the particle neither we know the drag coefficient we try to find out that which correlation fits well because we have got 10 different sets of equations.

So, you put a very small Re value. So, if it is a very small Re value. So, Re say suppose less than 0.001, then we can calculate the drag coefficient value based on the past set of equations.

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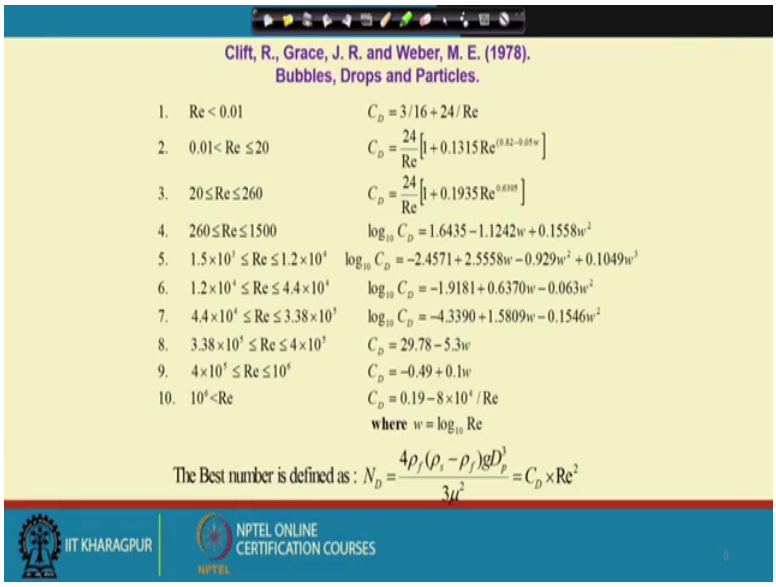

So that means, if my Re is very very small I can use this equation to calculate the value of C d and once I know the value of C d I can go back to the original equation or the general expression for terminal settling velocity of a spherical particle and I can easily get to know the settling velocity of that particle that is the v t.

Now, after putting that your very small value I get to know the C d value from this set of equation. Now, once I know the C d value. So, I know the Re says suppose Re is 0.0001 and I know the C d which is being calculated based on this equation. So now, I can calculate what is the value of C d into Re square, if that C d into Re square, that is is equal to best number. So, from your assumed value that is your initial value what is the C d into Re square you are getting, and if your approximated value is accurate then it should be equal to the value of best number which you have calculated based on these input values of particle properties that is size density and the fluid density and viscosity.

So, if the difference between this that is your best number which you have calculated based on the your assumed value and is, and the calculated value of based number based on the particle size and density and the fluid density and viscosity if they tends to 0 that

is the difference is negligible then we say that we conclude that my approximated value of Re that is Reynolds number is correct.

And if not, then what we can do we can go back that is how we try to do a your basically iterative program that is we call it iteration and you can practice it if you have exposure in computer programming. So, then what we can do we can put an incremental value of that initial say suppose my initial approximated value of Re was 0.00001 we can. Now, increment it with the value of let us say 0.00002.

And then again we will re do this calculation and we will try to see that whether my predicted value and the calculated value of best number the difference between that is close to 0. So, we keep on doing this unless and until they converge the convergence criteria you can set that is how much of what is that value the difference you will consider that that is your convergence criteria. So, once it gets converged so that means, that is the correct value of your particles Reynolds number into that fluid medium. So, once I know that. So, we can finish the iteration and we can get the value of drag coefficient and then based on that drag coefficient we can calculate the free settling velocity of the particles.

So, you see that what are the difficulties even in predicting the pre terminal settling velocity of a spherical particle in a stagnant fluid medium.

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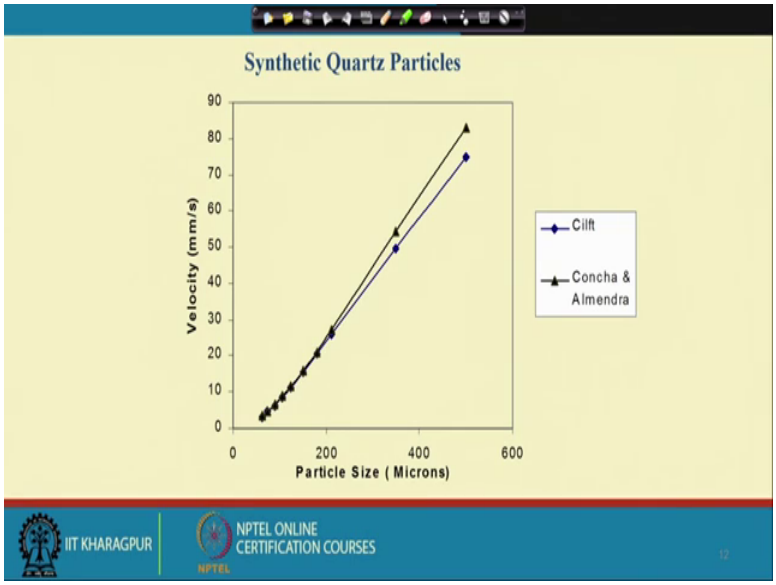

Now, this is what I have tried personally you can also give it a try that suppose you assume that there are some synthetic particles of different sizes and they are quartz particles up. That means, the density we know 2650 kg per meter cube and if I try to predict their settling velocities at different sizes using the Concha and Almendra's equation and Clift's and the equation based on this your say set of equations proposed by Clift at all. So, I have named it as Clift's your model and based on that iterative technique you will find that up to a very fine size range; that means, up to 200 micrometer size they are say matching with each other they are agreeing with each other, but as the size of the particle increases they are value differs they starts differing. So, now you see that even at this size around 500 micrometer the predicted values are having a difference of almost 10 millimeter per second.

So now, the question is which equation I should use I leave it to you. But my personal recommendation would be that if you have some knowledge in computational skill in computational you are say coding your coding then you can write an algorithm and based on that Clift's set of equation and that would be ith my personal recommendation you may differ different people may have different opinion, but I would say that I will go for that because it can better fit, it can better predict, the standard drag curve.

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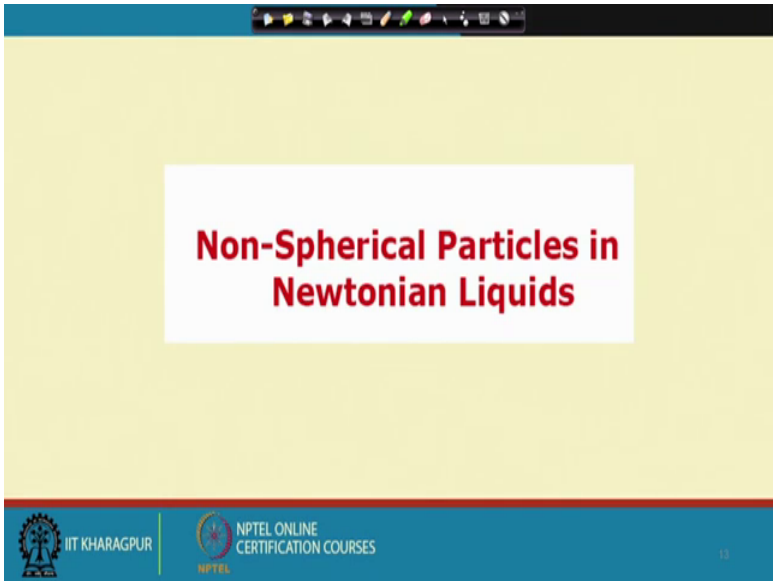

Now, this is about spherical particles. Now, in reality we said that we hardly get spherical particles. So, what will happen with the non spherical particles in Newtonian liquids? I

am not going into Newtonian non Newtonian fluids, I say that non spherical particle in a fluid stagnant fluid medium.

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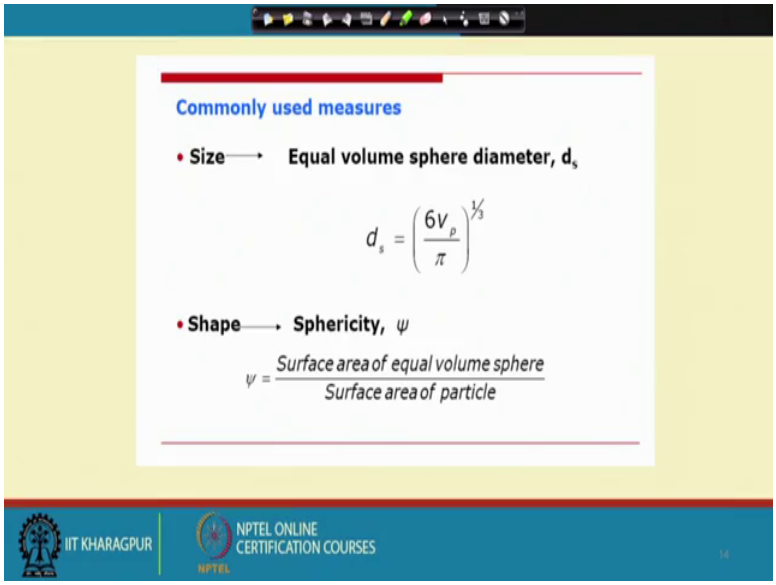

So, we have already discussed, when we discussed about particle shape that what are the intrinsic difficulties in defining the particle shape and ultimately we had concluded that the particles shape can be defined in terms of sphericity, where sphericity is equal to the defined as surface area of equal volume sphere divided by surface area of particle we had a detailed discussions on this. So, I do not want to repeat that.

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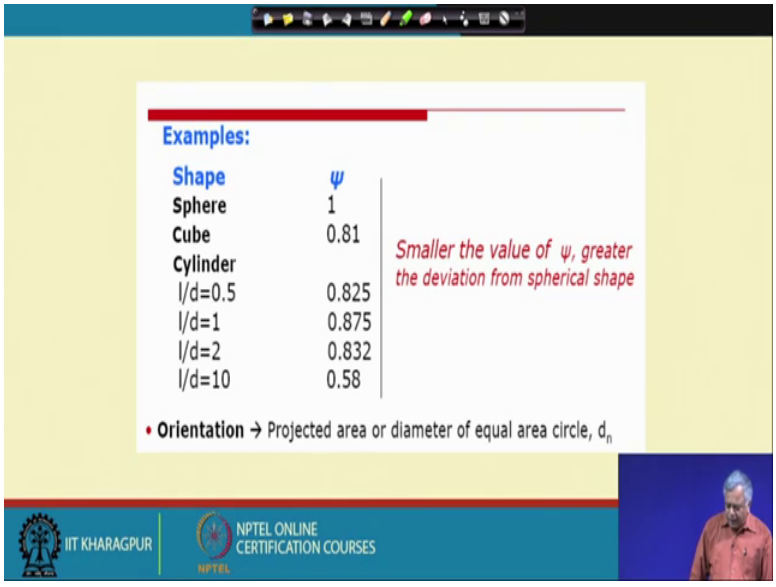

So, if you look at and these examples also there I have shown that your sphere has sphericity of 1 and any particle deviating from spherical shape the it will have sphericity your much lesser than 1. So, for cube it is 0.81 and cylinder based on its cylindrical particle based on its orientation they have got different numbers and these numbers I have got it from literature and smaller the value of size greater the deviation from spherical shape; that means, they are more flatter particles.

Now, there are some popular correlations for predicting the settling velocity of your non spherical particles into a stagnant fluid medium, some of them are highlighted here.

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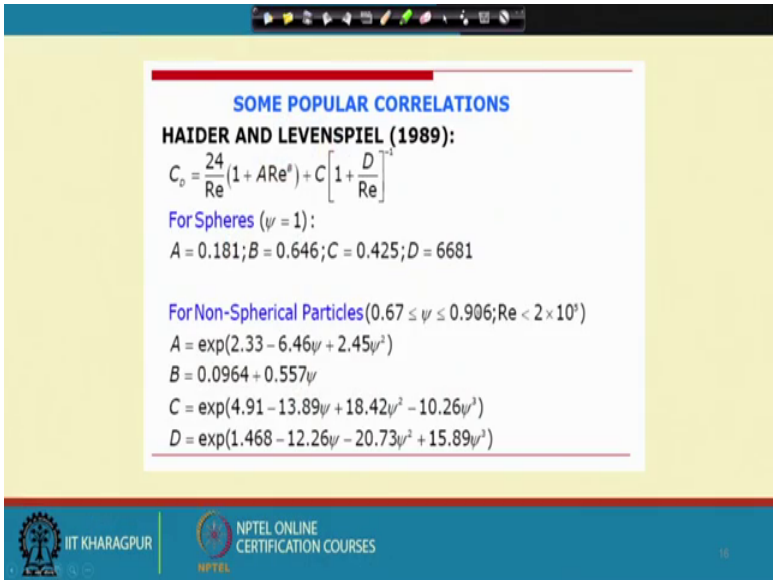

One is Haider and Levenspiel I do not want to trouble you with the complexities of these equations, but I would like to highlight only one thing that you look at that they are simply empirical in nature. So that means, these values you see A is equal to 0.181, B is equal to 0.646.

So, when you try to use these equations you must look at that those original publications that what are the limitations of this for what are the particle types they have used while proposing these equations because that is the main limitation of any empirical model.

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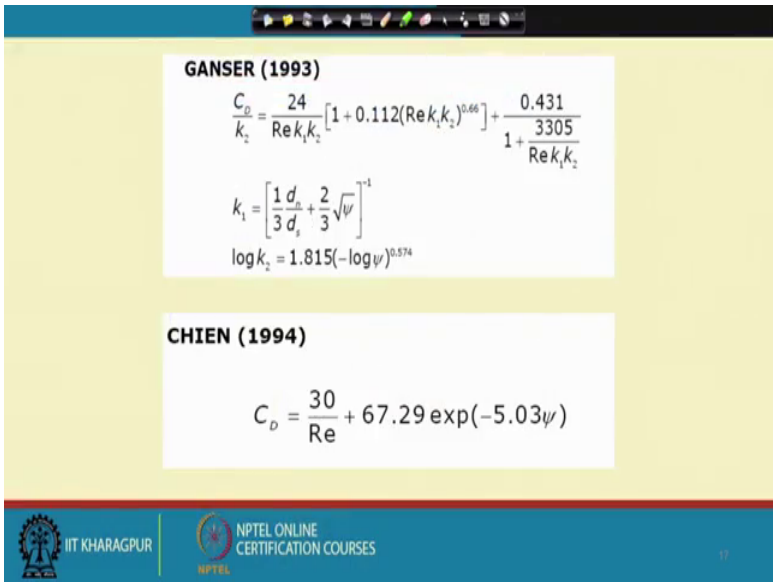

There are certain other models which are very popular that is one is Ganser. Again you see that this is again a empirical equation a very complex one a Chien that is also an empirical equation.

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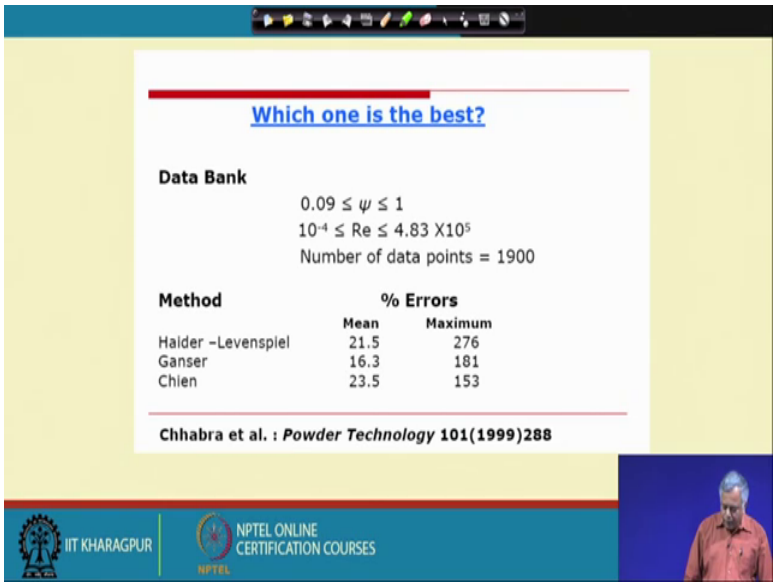

Now, which one is the best? So, this question was asked by a Professor Chhabra of Indian Institute of Technology, Kanpur and in one of his publications in the late 1999, he published this in a powder technology journal and he had shown that with the large databases he has used 1900 databases based on some experimental data these are well,

very carefully measured data set and he has tried to use this Haider and Levenspiel model, Ganser model and Chien model.

He has shown that although the mean error are within 20 percent in most of the cases 21.5, 16.3 and 23.5, but what he has highlighted that look at the maximum errors it can go up to 276 percent. So, now this is what is telling us that we should be very careful about using these equations for non spherical particles because you never know that whether you are within that maximum error zone or not. So, you should try to verify it with some kind of your major data set. So, that is my suggestion to all of you.

So, we will continue this discussion for the concentrated systems in the next lecture.

Till then, thank you very much.