Introduction to Mineral Processing
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Lecture – 35
Movement of Solids in Fluids

Hello. Welcome back. so, we are starting a new topic which I have given the name as movement of solids in fluids. Why this topic? As I said many times in my previous lectures that most of the mineral processing operations starting from grinding to screening and to the mineral separation processes, we use a fluid medium most of the cases it is water and sometimes in the dry processes we also use air, but majority of the cases we use water.

So, it is essential to know how the particles behave in a fluid medium because without this knowledge, it is very difficult to have fundamental understanding of the mineral processing operations mostly their separation processes.

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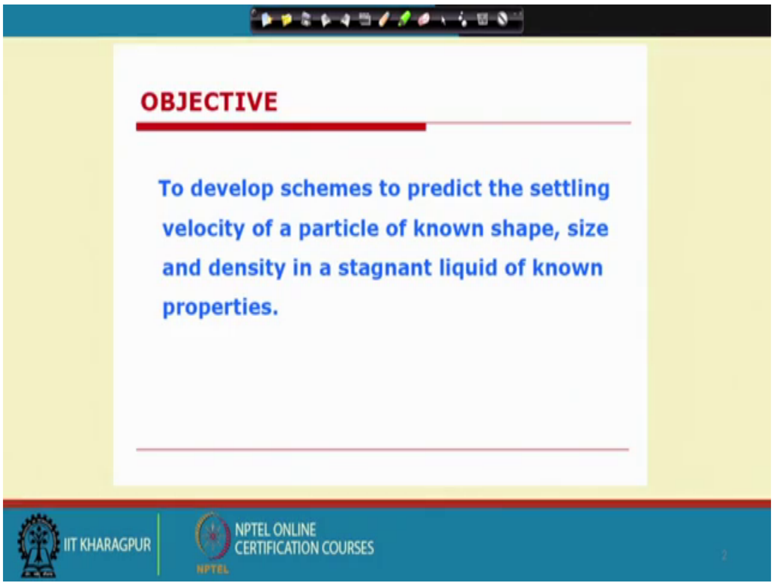

So, the subject itself is a very big topic is a very big subject and actually we can have a separate course on this subject, but I will try to focus, I will try to emphasize only on those aspects of the movement of solids in fluids, which are very fundamental in nature and to a mineral processing engineer and with this background knowledge, you can understand how the particles are separated and then based on that understanding, you should be able to improve the efficiencies of the existing processes or even with this understanding you can select the right kind of equipment for your a specific material.

So, I will confine this talk having this objective that is to develop schemes to predict the settling velocity of a particle of known shape, size and density in a stagnant liquid of known properties that is fluid is not flowing, but in reality you have the fluid flow also. So, when you have a fluid flowing the particle behaviour in that fluid medium will be different than what we will be discussing here.

But even to predict that event to model that particle behaviour in a flowing fluid, the very fundamental criteria is to know about the settling velocity of the particle; that means, settling velocity means if I have a column of liquid of say suppose its water if I drop a particle of his specified size density and shape. How much time it takes to travel through that fluid medium, to cover a certain distance that is called the your settling velocity.

So, I will try to show you that what are the parameters what are the fluid properties and what are the particle properties you must consider for predicting the settling velocity of the particles in the fluid medium and what are the physics of that and then what has been done so, far by different researchers and then I will try to help you that is how do I identify my basic models for a specific application.

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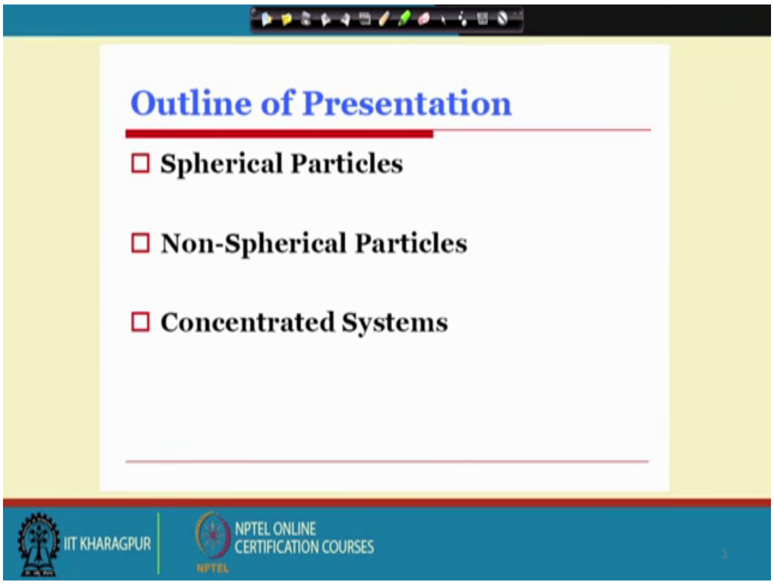

So, the outline of this presentation is we will start dealing with first spherical particles, even with the spherical particles I will try to show you that what are the difficulties then non spherical particles what we as a mineral processing engineers normally encounter that is your non spherical particles.

So, what has already been done and what is the scenario and what are the some of the popular models and how do I select them then third one is the concentrated systems because we are not dealing with a single particle, we are dealing with large number of particles because we have to process large volume of material per unit time. So, when we increase the concentration of the particles how do they behave in a fluid medium so, this is the outline of this presentation or the lecture.

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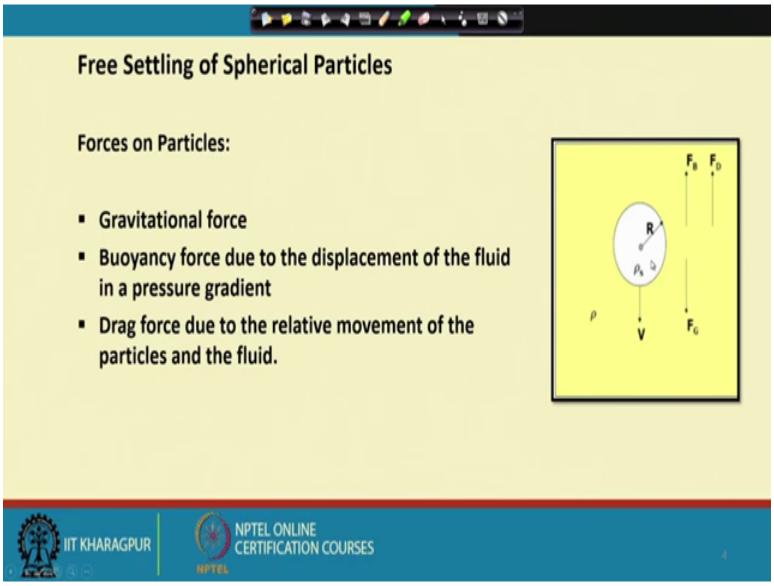

Now, if we look at this called a free settling of spherical particles into a fluid medium please remember that free settling does not mean a single particle always. It is free settling means the when the relative percentage of fluid is much more in comparison to the relative volume fraction occupied by the solid particles. Some people they say that we can apply the free settling equations when the total percentage of solid by volume is less than 5 percent by volume so; that means, up to 5 percent by volume of solid concentration in to fluid medium.

We can apply free settling equations, but some other people they advocate that even more than 2 percent of volume concentration of solids up to 2 percent volume concentration of solids into fluid medium, we can apply free settling conditions. So, that means, free settling means it is only the interaction of the particle and the fluid we are ignoring the interaction between the particle and particle collisions and other things.

So, and we are also saying that it is a spherical particle; that means, we are not dealing with arbitrary shaped particle which we normally encounter in our while dealing with mine samples. So, if you look at this suppose this is a particle having a radius of R and density rho s. So, when it is trying to settle through a fluid medium; that means, if I drop a particle of radius R and density rho s into a stagnant fluid column or say water column then what is that downward velocity that what is that settling velocity at what rate it will travel through that fluid medium that is what we try to calculate here.

So, if the fluid density is rho so, then you look at that when the particle tries to travel through that you have got three different forces acting on that particle. What are those three different forces? One is gravitational force which will try to pull the particle downward that is which is helping the downward movement of the particle, but that downward movement of the particle through a fluid medium will be opposed by two other forces which acts in the opposite direction of the your say particle say movement that is you have in this direction two opposite forces one is called the buoyancy force.

So, the buoyancy force is because of the because we know from Archimedes principle that the particle, when it travels through a fluid medium it will displace the fluid of equal volume in the opposite direction so; that means, the it is buoyancy force due to the displacement of the fluid in a pressure gradient. So, that means when the particle is trying to go down through a fluid medium, it will displace the equivalent volume of fluid in the opposite direction and there will be a pressure gradient because of that so, that force we call it buoyancy force or that is known as buoyancy force.

There is another force that is called the drag force is because of the particle is trying to travel in a downward direction and the displaced fluid is trying to go in the upward direction. So, there will be shear in between two layers of this movement because of the opposite directions the both the things are moving. So, because of that it is a relative movement of the particles and the fluid there will be a frictional force that is called a drag force. The drag force has got different nature, but we are not going into that detail.

So, if I want to calculate that what is the settling velocity of the particle in the fluid medium I need to know the mass of the fluid, the mass of the particle and where from I will get to the mass of the particle that is if I know the size and the density of that particle because size will give me the volume and the density if I multiply it with the volume, I will get to know the mass and I need to know the how much is the buoyancy force and how much is the drag force. So, the net result if the F B plus F D is more than the F G, then the particle will not move downward, it will remain stagnant on top of the fluid.

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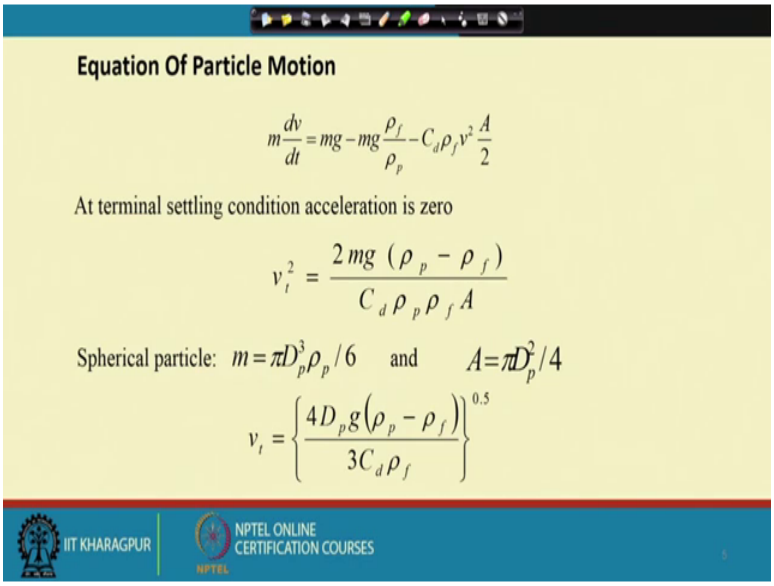

So, let us show that let us see that so, if I write if we write the equation of particle motion into a fluid medium when the fluid is stagnant. So, initially the particle will try to accelerate so; that means, it will have an accelerating velocity and base with time its velocity will keep on increasing and after a certain point beyond a certain point it will travel through a constant velocity that is called the terminal settling velocity. So, when the fluid is accelerated the when the particle is having accelerating velocity.

So, the equation of motion as the mass it is from your conservation of mass so, that is m d v by d t that is your m is the mass of the particle, d v by d t is the acceleration of that particle in downward direction is equal to m g that is your how much is the net gravitational force. What is the gravitational force? That is the m g minus the buoyancy force that is m g rho f by rho p, rho f is the fluid density rho p is the particle density and minus C d, I will come to this point later on minus C d rho f is the fluid density.

Then v square that is the velocity of the particle multiplied by A by 2, what is A? A is the projected surface area. What does it mean? So, projected surface area means that for a spherical particle, the projected surface area does not change because the surface area remains identical, but if I have say suppose a cylindrical particle. So, if the cylindrical particle is travelling like this so, the projected surface area is different; that means, what is that surface area of that particle is in contact with the fluid medium and if I change the orientation then your surface area reduced gets reduced so that your projected surface area changes.

Now, we have considered the spherical particle and say at we are interested in knowing at terminal settling condition because the accelerating velocity, it is very difficult to calculate the accelerating velocity and normally the residence time whatever we give for our particles for separation, except from couple of equipment. Most of the equipment we give sufficient residence time where the time for a to reach the terminal settling velocity is minimal with respect to the total travelling time given. So, what I try to mean that this accelerating velocity is a very small fraction of the total time taken for travelling a certain distance by the particle.

So, we can safely ignore for simplifying our calculations we ideally we should not ignore it, but for simplifying our calculations we can ignore, we can assume that the particle has reached your steady state that is your at least the terminal settling condition very soon when it has got say having dropped into the fluid medium. So, at terminal settling condition we said that there the acceleration is zero; that means, it has got a constant velocity. So, that they had a d v by d t term is equal to 0 and when d v by d t is equal to 0 because we are interested in knowing v and this is the terminal settling velocity v t square and this C d is drag coefficient. It is called a drag coefficient or the friction coefficient, it is analogous to friction coefficient

So, in that case if I rearrange this equation setting d v by d t is equal to 0 so; that means, this term is 0 am I, if I rearrange this equation and if we write the v square is in terms of v t square t stands for terminal. So, we can write v t square is equal to 2 m g rho p minus rho f divided by C d rho p rho f a. So, now for a spherical particle the m, what is m? m is equal to as I said that volume into density so, for a spherical particle so the volume is pi D p cube by 6 that is D p is the diameter of the particle. So, it is pi D p cube by 6 multiplied by mass is equal to volume into density.

So, we have multiplied the volume with the density rho p so, we get the m of the particle in terms of D p and rho and the A the projected surface area for a spherical particle is pi D p square by 4, we all know that what is the surface area. So, if I replace this 2 that is the value of A and m and this equation we can further simplify this equation with, in this form and we are interested in knowing the v t. So, we can write v t is equal to 4 D p g multiplied by rho p minus rho f divide by 3 C d rho f to the power 0.5 so, this is a general expression for the terminal settling velocity calculation in a free settling condition of a particle, having spherical shape in a stagnant fluid medium. So, many clauses are there be careful about that.

So, there is a general expression for a spherical particle in a stagnant fluid medium, in a free settling condition this is the general expression for settling velocity of that particle. Here also you see that D p we can easily know if we know the size of the particle, g we know the gravitational acceleration, rho p particle density I can easily get to know that is what we said that the characterization we have to do so, What is the density of my particle? What is the size of that particle? rho f that is the fluid density, here there is a question that the fluid density it varies with the temperature.

So, you must note down that what is the temperature of that fluid and that density we should take, normally we use for simplification of our calculations the water density as equal to 1000 kg per meter cube and that is at 4 degrees centigrade temperature.

So, here rho f is the fluid density the question is how do I get to know the value of C d that is the drag coefficient. So, you see that even or this so, many your assumed conditions that you have a spherical particle, we are saying it is a free settling condition in a stagnant fluid, when water is at 4 degrees centigrade. Even the free settling the expression we have got for free settling equation, we still do not know; what is the value of C d. So, the challenge lies here that what would be the drag coefficient in that condition.

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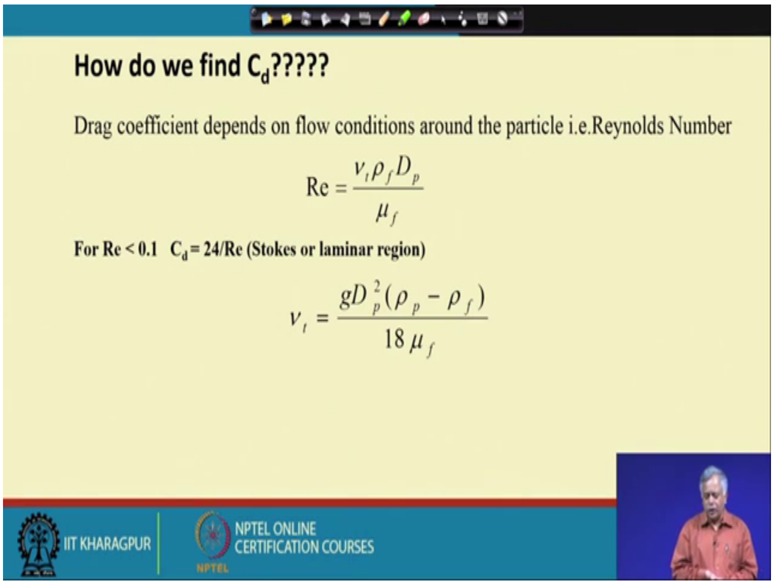

So, how do we find C d? The drag coefficient, this is a biggest question in this field and I will show you that; what are the difficulties, even with in knowing the value of C d. So, the drag coefficient basically depends on the flow conditions around the particle and what does it mean I give this example that if I have a lake and in that lake suppose the water is standstill and you drop a very small particle into that will you notice any disturbance into the water. No, it will just go down maybe you cannot hear the noise also, the sound also.

So, the what is the flow conditions in that case or when you drop a very small particle into that that is entirely different than if I have a very big boulder and if I throw it into the lake you see that entire water is getting disturbed and there would be splashes and all this. So, the flow conditions around the particle they are different when the particle is very big or very large in comparison to a very small particle. How do I quantify those flow conditions? How do I say basically say measure or say estimate the flow conditions around the particle.

For that there is a number it was proposed by Reynolds and that is known as Reynolds number, which is the very key your very fundamental your characteristic through which the entire fluid mechanics is basically developed so, this is one of the key factors through based on which the fluid mechanics the science of fluid mechanics or the your entire fluid mechanical analysis has been developed. So, this drag coefficient depends on that because what we said the drag coefficient means it is how the particle is travelling so, in the downward direction and the displaced fluid how it is moving. So, what type of disturbances that movement of that particle has created into the fluid medium that will determine that, what is the drag coefficient? What kind of frictional forces you have that depends on the nature of the disturbances that fluid that material that particle has created into the fluid medium.

So, what is the Reynolds number? So, Reynolds number is a basically a ratio between your inertial stresses and viscous stresses. So, what will happen when the viscous stresses are dominant; that means, the viscous stresses are very large in relation to the inertial stresses then the Reynolds number will approached towards 0; that means, the it is the denominator is much higher than your numerator. So, what will happen so, it is it will reach towards 0.

So; that means, there the viscous stresses are more dominant, when the viscous stresses will be more dominant. When the particle has created little disturbance into the fluid medium because the fluid itself has absorbed all those basically disturbances and it could not displace the fluid much, but when the inertia is more that is the when the particle is very big so, inertial stresses will be very high in relation to my viscous stresses. So, it will disturb the entire water body and that is why that is when the Reynolds number is high that is more than one and so, more than certain value which is beyond a certain value then we say that it is inertial stresses are more dominant.

So, based on that concept that is one formula that is proposed by Reynolds it is called v t rho f D p by mu f. What is the mu f? Mu f is the viscosity of the fluid which is again dependent on the temperature of a fluid.

So, with the increase in temperature the viscosity changes so, or with the change in temperature the viscosity changes and these values can be easily found out for your very common fluids in a standard textbook of fluid mechanics or many other books, v t is the terminal settling velocity rho f we have already discussed and D p we already know the size of the particles rho f is the fluid density. So, what it says that that if I know the Reynolds number then it is another famous scientist in the field of fluid mechanics Stokes he propose that when the Reynolds number of the particle is less than 0.1, the c d is equal to the drag coefficient is equal to 24 by Re and this is called the Stokesian condition. It applies for very fine particle for a quasi density particle that is particle who has got a density of 2650 kg per meter cube, this sizes could be approximately equal to less than 50 micrometers.

So, there the C d is equal to 24 by Re and we can replace this equation the value for equation for Re into that and now if we put this value of Re that is 24 by this into the original equation, into the previous equation. We get the final equation for settling velocity of a particle v t is equal to g D p square rho p minus rho f divided by 18 mu f, but please remember that this is for particles which follows the Stokesian condition. What is that Stokesian condition?

Now, when the Reynolds number is less than 0.1, that means when the viscous stresses are more dominant. So, it is applicable for a very fine particle and I have given you an example that how fine it could be, that is for quasi density particle it is around 50 micro up to 50 micrometer particle size. You can apply this formula and this is well known formula well documented formula in many textbooks and in many literature, we will continue this discussion in the next lecture.

Thank you very much.