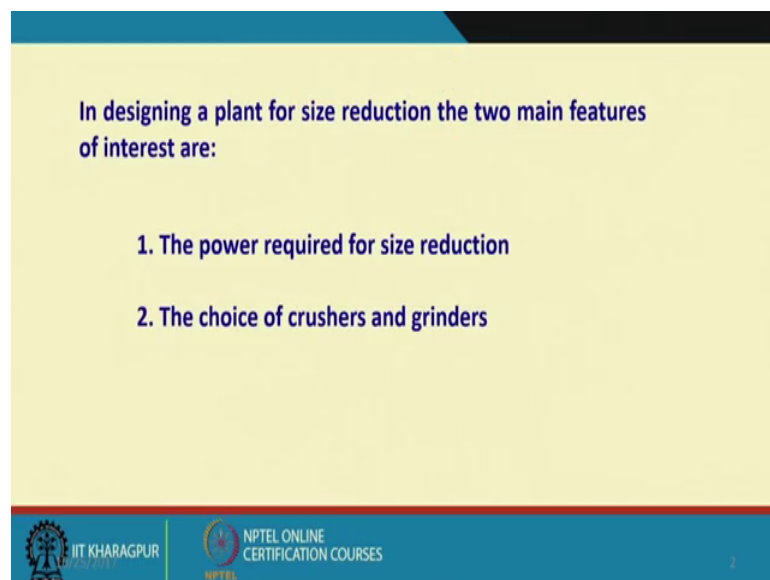


Introduction to Mineral Processing
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Lecture – 18
Comminution Fundamentals

Hello, welcome everyone, to this week 4 of this lecture series. So, last week we have started discussing about the fundamentals of comminution processes. We had discussed that what is the basic difference between the comminution and blasting and then what are the factors that affect comminution processes. So, in this first lecture we will extend that discussion on comminution fundamentals.

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In designing a plant for size reduction the two main features of interest are:

1. The power required for size reduction
2. The choice of crushers and grinders

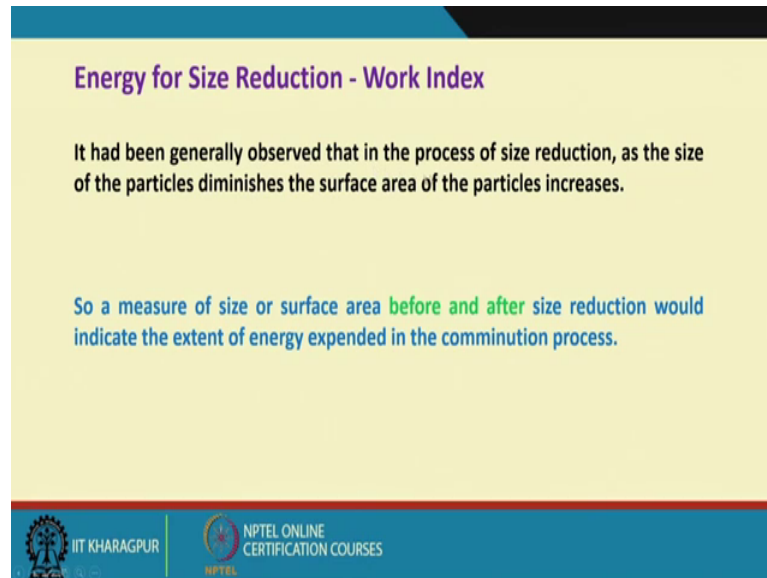
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You know in designing a plant for size reduction, the 2 main features of interest are one is the power required for size reduction; that means, if I want to break or say if I want to have a fragment from a particular feed size distribution to a product size distribution, how much of energy is required. Because the comminution is essentially a energy intensive process. So that is one of the jobs of the mineral processes; that is, how do I minimize the energy consumption into a comminution process?

And the second one as because there are varieties of comminution devices available, we call them crushers or relatively coarser sizes when we want the fragmentation and the grinding grinders, that is, when we need to break the particles to very finer sizes.

But the problem is which one, which crusher or which grinders or grinding means we should select and as we said that it is a progressive breakage. So, what are the combinations of crushers and grinders, we should have with an aim to minimize the energy consumption. But, we need to have that desired fragmentation in the product from a particular feed size distribution.

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Energy for Size Reduction - Work Index

It had been generally observed that in the process of size reduction, as the size of the particles diminishes the surface area of the particles increases.

So a measure of size or surface area before and after size reduction would indicate the extent of energy expended in the comminution process.

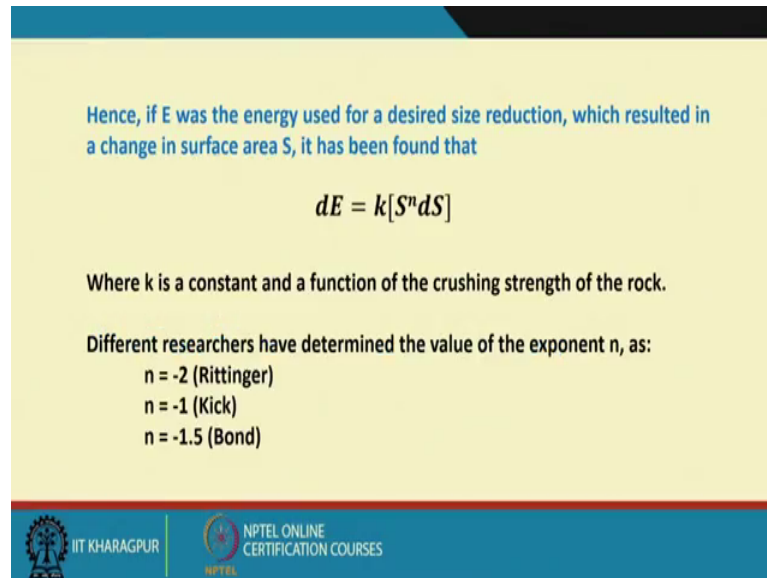
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So question is how do I measure the energy consumption in the size reduction processes? It is one very popular way of measuring the energy consumption in size reduction is called the Work Index and if we look at that in a it is a general observation, that in the process of size reduction, as the size of the particles diminishes, the surface area of the particles increases; that means, when we are breaking the particles, we are getting a fragmentation, that means, we are increasing the surface area of point particles.

So, if I want to if I have some means to measure the size or the surface area of the particles, before and after size reduction, it should indicate the extend of energy expended in that comminution process. So, what is that we are going to do? Now the essential principle is that, that is what we are following that, when you break the particles from a particular feed size to a particular product size, then, we are basically increasing the surface area. So, if we know the particle size at the feed and at the product or if we know the surface area of my feed particle and my product particles then, probably we

can calculate that, how much of energy has been expended to do that job or to create those new surfaces.

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Hence, if E was the energy used for a desired size reduction, which resulted in a change in surface area S, it has been found that

$$dE = k[S^n dS]$$

Where k is a constant and a function of the crushing strength of the rock.

Different researchers have determined the value of the exponent n, as:

- n = -2 (Rittinger)
- n = -1 (Kick)
- n = -1.5 (Bond)

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So, if E was the energy used for a desired size reduction, which resulted in a change in the surface area S; that means, there is a change in the surface area.

Then, it has been found that, dE that is the change in the energy; that means, your input energy, how much is required and how much it is consumed and is equal to k S to the power n d S. So where, k is a constant and a function of the crushing strength of the rock; that means, how amenable your rock is for crushing; that means, it is dependent on the rock characteristics. There is how hard it is? What type of orientations is there and then, whether it has got internal cracks or flaws or faults? All these basically getting quantified, in that value of k.

Question comes that is how what should be the value of n? The different researchers have done very important work. But out of that the only 3 researchers work are normally cited and accepted a literature. They are that is one is Rittinger. He proposed a value of n is equal to minus 2. Another scientist is a kick. He proposed a value of n is equal to minus 1 and other scientists Fret Bon. He proposed a value of n is equal to minus 1.5. Now let us see that, what are the advantages and disadvantages associated, when we incorporate the different values of n into that basic equation dE is equal to k S to the power n dS.

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It has been found that Rittinger's expression,

$n = - 2$, is more applicable for coarse size reduction while that of Kick, $n = - 1$, is more appropriate for finer size reductions in the region where large surface areas of particles are exposed as in the case of grinding operations.

Bond's intermediate value of 1.5 covers almost the entire range of particles.

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It has been found that treating just expression; that means, when we put n is equal to minus 2, that works well for predicting the size reduction in the coarser size ranges; that means, for a crusher product, this equation works well; that means, we can use this model, that is dE is equal to $k S$ to the power minus 2 dS , for predicting the energy consumption in a crushing operation. Whereas, for n is equal to minus 1 that was proposed by Kick is more appropriate for finest size reductions like in grinding mills.

Where, basically large surface areas of particles are exposed. Because when you are grinding it, you are generating a large surface area, because we want a very fine particle size distribution out of that. So that means, the Rittingers works well for crusher product; Kicks works well for grinding mill product. But many a times we need to know, the energy consumption of the entire comminution processes; that means, we have got a r o more starting from 1 meter and then, we need a product of 40 micron. So, what is that total energy consumption? For that, the bonds intermediate value of 1.5 covers reasonably well, the entire range of particles.


Now, let us see what happens, if we incorporate this value, your minus 1.5 into the basic equation.

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Substituting $n = -1.5$ in the above equation and integrating between feed particle size, F , and product particle size, P , yields

$$E = 2k \left[\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}} \right]$$

where k is a constant and a function of ore characteristics



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So, if you substitute n is equal to minus 1.5, that dE is equal to $k s$ to the power $n dS$ and if we integrate between the feed particle size F and product particle size P . I repeat it, that is if I incorporate a value of n is equal to minus 1.5, in that basic equation of dE is equal to $k S$ to the power $n dS$ and if we integrate that equation between feed particle size F and product particle size P , it would yield E is equal to $2k \left[\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}} \right]$ within bracket.

Where, k is again a constant at a function of ore characteristics; that means, it is the basically depending on the nature of the particle, you are trying to break. Now question comes that, are we basically breaking a single particle size? No, because when we are getting rom ore that is run of mine, so that means, the ore which is directly coming from the mine site, it has got a size distribution and when it is being broken, you are not producing a single size, you are also producing a particle size distribution.

So for that we can, what is that, we can measure in the pit party tube to determine the feed particle size and the particle size. That is, if you remember the size distribution curve, what we discussed, that is normally the convention is the 80 percent passing size. Why 80 percent passing size? Because, this is simpler to find out in a laboratory scale operation at means what we have to do now, we have the rom ore. We need to do the size analysis we plot the size distribution curve and then, from there we need to get the 80

percent passing size, in the feed and as well as in the product. So the product 80 percent passing size.

We denote it as P 80 and the feed 80 percent passing size we denote it as F 80.

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The final form of Bond's equation for size reduction of a mass of feed, M_f , in closed circuit grinding is now written as:

$$E_G = 10WI \left[\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right] M_f \text{ kWh}$$

Where

- F_{80} = 80% passing size of the feed in microns
- P_{80} = 80% passing size of the circuit product, in microns
- WI = A constant for the ore
- WI is known as Bond work index and represents the work required to reduce the ore from an infinite size to 100 μm .

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So the final form of Bonds equation for size reduction of a mass of feed say M F. Now this is the M F is the total quantity of material. In closed circuit grinding, like where, it is being recycled back. The particles which are not yet given a size what we want. So, you remember that, we have discussed about the closed circuit crushing operation, that means, you need to have a mill and then you need to have a size separated and say suppose, my initial feed size is say, your say a 2 centimeter or maybe 5 centimeter and I want a product of say below 40 micron.

So, we try to separate that product, coming out from that mill and at a 40 micrometer size and the particles which are coarser than 40 micro meter, we try to send it back for again further grinding. So that is for closed circuit grinding operation, if we have a total mass of feed M F. So the, basically, we can write the, the grinding energy, that is required or that is consumed is equal to 10 W I 1 by root P 80 minus 1 by root search is square root of F 80 multiplied by Mf kilowatt hour.

Where, we have replaced that k value with the W I. This W I is called the Work Index. And is known as Bond Work Index and represents the work required to reduce the ore

from an infinite size to a product size of 100 micrometer. I repeat it, that W I is a constant for the ore and it is known as a Bond Work Index, because it is Fret Bond who proposed it, and it presents the work that means, how much of work it has to do, to reduce the ore from an infinite size to 100 micrometer; that means, from any size to 100 micrometer size F 80, I have already discussed that it is 80 percent passing size of the feed in micrometers.

P 80 is the 80 percent passing size of the circuit product in micrometers and Mf I already discussed that, it is the how much of the feed, what is the quantity of the feet we are using?

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By definition the specific grinding energy is the energy required per unit mass of the rock.

The specific grinding energy of a particular mineral is written as:

$$E_G = 10WI \left[\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right] \text{ kWh/t}$$

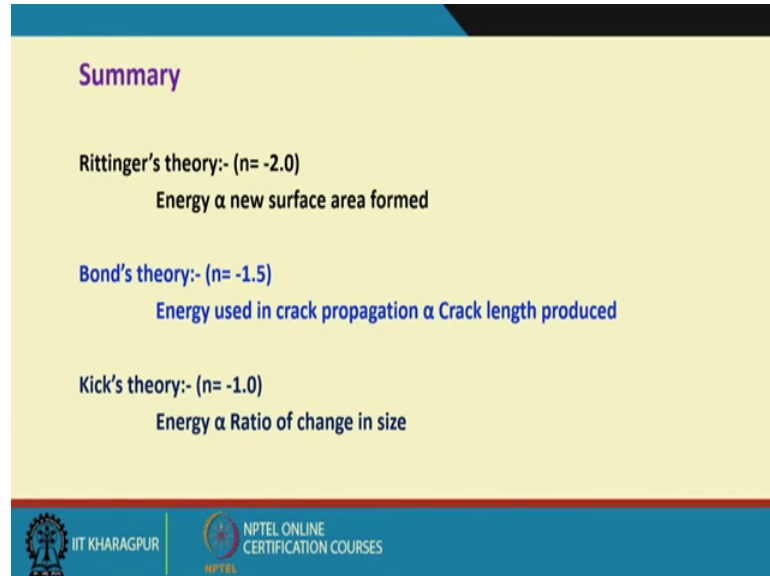
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So, this E G is called the Specific Grinding Energy. So that is defined as the energy required per unit mass of the work. So we can convert that equation, the previous equation in the form of specific grinding energy, represented as E G required per unit mass of the rock. So, what you try to do? We try to take out that Mf from that equation. So, that specific grinding energy of a particular mineral is written as E G is equal to 10 W I and where from the 10 is coming? 10 is basically, as coming from, because the, we are saying that 100 micron that product size has to be below 100 micron. So, square root of 100 becomes 10. So 10 is coming from there.

So E G is equal to 10 W I 1 by root P 80 minus 1 by root F 80 kilo watt hour per ton. This is very important equation for calculating the energy consumed in the size reduction

processes. Now, what is unknown here? Unknown here is that $W \propto I$, that is what is the Bond Work Index. So now, there is a test that is called Bond Test.

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Summary

Rittinger's theory:- (n= -2.0)
Energy \propto new surface area formed

Bond's theory:- (n= -1.5)
Energy used in crack propagation \propto Crack length produced

Kick's theory:- (n= -1.0)
Energy \propto Ratio of change in size

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So that I will be discussing that before, we go to that that is how the Work Index is calculated.

I will not discuss in detail. But let me summarize it again, what we have discussed so far. Because this is very important that, Rittingers theory is basically, you are putting n is equal to minus 2 into the dE is equal to $k S$ to the power $n dS$, that is the basic equation for size reduction, where it is assumed. Where the Rittingers is assumed that or he proposed that the energy consumed is proportional to the new surface area formed, that is how much of energy you have utilized to form a new surface area. Bonds theory is n is equal to minus 1.5.

Here he proposed that, the or he proved that the energy used in crack propagation, that is when you try to break a particle inside, you have to, have a crack first to be generated and then, when the crack length basically reaches a maximum value. Then, the particle breaks. So, that is energy used in crack propagation, is proportional to the crack length produced and Kicks theory is based on this hypothesis; that is energy consumed is proportional to the ratio of change in size as because this is an Introductory Course. I do not want to trouble you to give, to get into the detailed analysis of this. So, this much of knowledge is enough, I think to begin with.

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Energy utilization

One of the first important investigations into the distribution of the energy fed into a crusher was carried out by OWENS who concluded that energy was utilized as follows:

- In producing elastic deformation of the particles before fracture occurs.
- In producing inelastic deformation which results in size reduction.
- In causing elastic distortion of the equipment.
- In friction between particles, and between particles and the machine
- In noise, heat and vibration in the plant, and
- In friction losses in the plant itself

Owens estimated that only about 10 per cent of the total power is usefully employed.

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Now, if I look at that in a comminution process, how the energy is being consumed? Is it only being consumed by the only for the breakage of the particles or for generating the new surface area? Probably not; so, one of the first important investigations on this was carried out by Doctor Owens, who concluded that energy in the comminution process. It was utilized as follows, like first in producing elastic deformation of the particles before fracture occurs; that means, even before when you are putting some kind of your force into a particle, even because the your crack gets initiated or the fracture occurs there will be some kind of your elastic deformation in many cases. So, you need to, you need energy for that also.

Second in producing inelastic deformation, which results in size reduction also; that means, if your material is Brittle. So in that case, you will have inelastic deformation, which results in size reduction in causing elastic distortion of the equipment, even, the equipment also will consume energy because of his elastic distortion. Then, there only friction between particles and between so, it is particle-particle friction and the particle and machine surface friction. So, these frictions would result into heat and that will be basically consumed as the energy a vast amount of energy will be consumed and then, it is only to produce heat.

So it is not helping the particle to be broken and also a noise that is the energy input will be transmitted as a noise. In the form of noise, in the form of heat and even the vibration

in the plant and in friction losses in the plant itself; that means, the structure, the entire structure of my comminution devices, they will be also having some kind of a vibration with the ground and with the floor. So all these are basically saying that, whatever the input energy is there. So, essentially the major part of the energy will be consumed in doing all this, in having all these actions and these are proposed by Doctor Web.

And once estimation was that, only about 10 percent of the total power is usefully employed. What does it mean? That means, whatever the input energy we are giving, only 10 percent of that input energy is effectively being utilized for generating the new surface area of the particles. So, if we want to maximize the energy utilization, we have to take into consideration of all these sources of losses and if we can minimize the losses in the form of all this, the ways how the energy is getting wasted, then, probably we will save some more energy for effective breakage of the particles.

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Estimation of Work Index for crushers and grinding mills

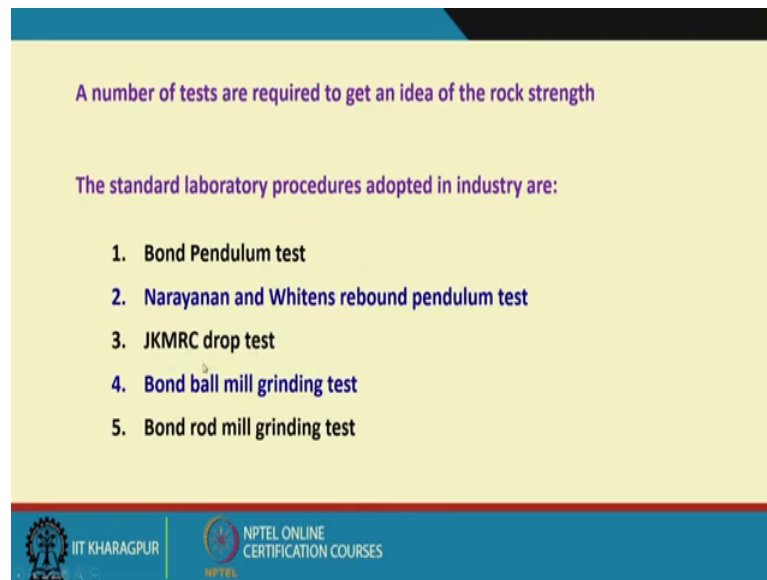
The standard laboratory procedures for estimating work index can be divided into two categories.

The first category involves tests on individual particles of rock and the second category deals with bulk rock material.

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Now, how do we estimate the work index for my machines? That is the crushers and the grinding mills. There are some standard laboratory procedures for estimating Work Index. They can be divided into 2 categories, that is first category involves tests on individual particles of rock, that is single particle breakage and the second category deals with bulk rock material, lot of investigation has been carried out.

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A number of tests are required to get an idea of the rock strength

The standard laboratory procedures adopted in industry are:

1. Bond Pendulum test
2. Narayanan and Whitens rebound pendulum test
3. JKMRC drop test
4. Bond ball mill grinding test
5. Bond rod mill grinding test

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Some of them are very popular in the industry, even out of that one is the Bond Pendulum test and is basically based on a single particle breakage.

Then, is the Narayanan and Whitens rebound pendulum test proposed in JKMRC Julius Kruttschnitt Mineral Research Center, In University of Queensland, Australia. This one is very popular these days that is called a JKMRC drop test. That is you are breaking the particles individually, but you are fast having a different sizes different distinct say size groups and then, within that group you have a statistical tool to tell you that how many particles of that particle size, you should take and then, you break it from a particular height and then, when you are.

So, breaking says suppose you are 30 particles, you have broken for a discrete particle size, you collect them all and then do the size analysis and you know the initial size, you know the final size distribution and you know the load, you have given that is your how much of your energy you have given or how much of force you have applied based on your mgh , that is what is the height of that and then, there are some calculations and based on that, you can predict that, what is the crushing say energy being consumed for that.

So, that has become a guiding tool these days in many industrial practices and it is an essential input you require, when you are using some simulation softwares developed by JKMRC, that is one very popular software is JK Smite, where the input you require from



this JKMRC drop test. Another one is called Bond ball mill grinding test that means, here it is not a single particle you are breaking, you are basically trying to break a number of particles of different size distributions that means, you know the F 80 of that.

And then, you have a basically tumbling mill of a specified your dimensions and the bond has also given the specified say quantity and the sizes of the box also steel balls. So, this is a standard test, you have to perform in a standard equipment which is available readily, available in the market and the principle is like your Locked Cycle Test. If you are familiar with the statistical tool called Lock Cycle Test, that is you keep on doing it unless and until all the particles have broken to a particular size distribution. So, based on that, you calculate the bond work index. Similar test was proposed in a rod mill also by again that bond. So, that is called Bond rod mill grinding test and now, for some common minerals, as because many people have used them.

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Bond Work Indices for some Common Minerals

Material	Work Index	Material	Work Index
Barite	4.73	Fluorspar	8.91
Bauxite	8.78	Granite	15.13
Coal	13.00	Graphite	43.56
Dolomite	11.27	Limestone	12.74
Emery	56.70	Quartzite	9.58
Ferro silicon	10.01	Quartz	13.57

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So, these are available in literature. So, that is the Bond Work Indexes, distance for some common minerals are given, which are easily readable, that is you see that Barite, the work index is 4.73. Bauxite is 8.78. Coal is 13, Dolomite 11.27, like that. So, the next class, the next lecture I would like to show you that is, how this Bond Work Index, we can apply it for my decision making process, like your plant optimization or for other purposes also. So, till then.

Thank you very much.