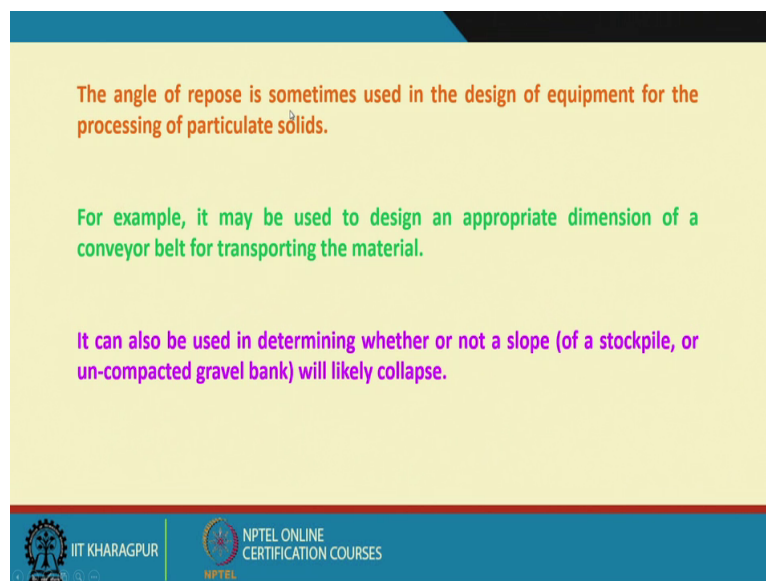


**Introduction to Mineral Processing**  
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**Lecture – 11**  
**Particle Characterization (Contd.)**

Hello. So, we will try to complete the topic particle characterization in this lecture. We were discussing about angle of repose in my previous lecture.

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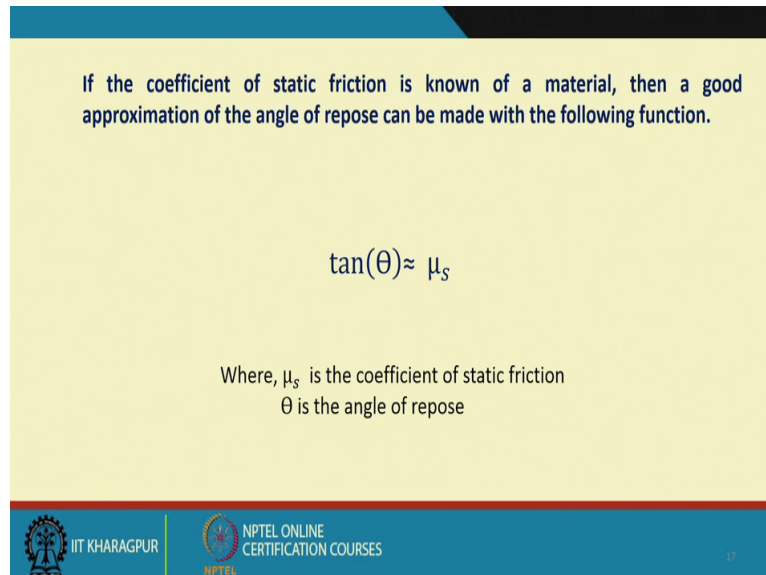


We were talking about the application of angle of repose. Angle of repose sometimes used in the design of equipment for the processing of particulate solids that is if I am using a as I said that your say conveyor belt that is we can appropriately design or say decide about the dimension of a conveyor belt if we know the angle of repose of that material.

It can also be used in determining whether or not a slope of a stockpile or uncompacted gravel bag will likely collapse that is very very important; know. Suppose I have got a stack of particles we call it stockpile having 50,000 tons of materials must be very confident about the stability of that because of that shape collapses and if you do not have additional space to accommodate these particles, what will happen? Now, if you have structures. So, this particles we try to have your induce the stresses on the structures and eventually the structures may collapse. So, the stability of the stockpile is basically decided by the angle of repose of those particles.

There is another way indirect way of knowing the angle of repose if we know the coefficient of static friction of a material.

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If the coefficient of static friction is known of a material, then a good approximation of the angle of repose can be made with the following function.

$$\tan(\theta) \approx \mu_s$$

Where,  $\mu_s$  is the coefficient of static friction  
 $\theta$  is the angle of repose

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We can only approximate the angle of repose that is why I have written it is equivalent to not always true there are some exceptions, that is your coefficient of static friction of the material is equivalent to  $\tan \theta$  where  $\theta$  is the angle of repose. There many ways we can measure the coefficient of friction of materials and then we can get to know the angle of repose or vice versa. If we know the angle of repose we can have some kind of a approximated at idea about the coefficient of static friction.


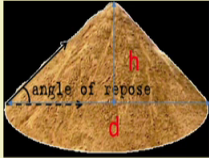
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**Tutorial**

Find the angle of repose for a granular material which forms a cone-shaped pile with a height of 13 feet and a base diameter of 28.3 feet

**Solution** Input  $h = 13$  feet  
 $r = 14.15$  feet

Angle of repose ( $\theta$ ):

$$\theta = \tan^{-1} \frac{h}{r}$$
$$\theta = \tan^{-1} \frac{13}{14.15} = 42.57$$


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There is a small example through which I would like to show you that how to calculate it. Suppose I say that find the angle of repose from a granular material which forms a cone shape pile with a height of 13 feet; that means, you abstract this material into that it is a granular material and the height is basically 13 feet and the base that diameter is 28.3 feet. So, what will the angle of repose; that means, what is that angle? It is very easy to do that. So, it is nothing but your theta,  $\tan \theta$  is equal to  $h$  by  $r$ . So,  $\theta$  is equal to  $\tan^{-1} h$  by  $r$ . So, that is  $\tan^{-1} 13$  by  $14.15$  that will give you a value of  $42.57$ .



Now, what do I do with this number? Suppose I have got another grain that gives me a  $\theta$  of another particle that gives me a  $\theta$  of  $30$  degree and another particle which  $\theta$  is  $60$  degree. So, I know that what should be the base of that is required; that means, if I know the angle of repose vice versa and if I know the height I know this dimension. So, we can find out even that what is the height or what is the base area I require to accommodate that particles.

So, and now I will try to show you that how to calculate your say cumulative size distribution I will show you an example that with a data because it is very important. So, we have discussed about the particle size distribution I mentioned about the cumulative weight percent plot.

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**Sieving Data Representation – An Example**

(1) Sieve size range ( $\mu\text{m}$ )	(2) Sieve functions wt (g)	(3) Wt. %	(4) Normal aperture size ( $\mu\text{m}$ )	(5) Cumulative % undersize	(6) Cumulative % oversize
+250	0.02	0.1	250	99.9	0.1
-250 to +180	1.32	2.9	180	97.0	3.0
-180 to +125	4.23	9.5	125	87.5	12.5
-125 to +90	9.44	21.2	90	66.3	33.7
-90 to +63	13.10	29.4	63	36.9	63.1
-63 to +45	11.56	26.0	45	10.9	89.1
-45	4.87	10.9			

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Suppose I have performed a size analysis using sieves in my laboratory for a specific material and the sieve data whenever I have got, that is I started with a 250 micrometer of a sieve that is the coarsest one and the finest one was 45 micrometer. What is this plus sign says? It is a convention in mineral processing that plus means that whatever particle reports here whatever weight corresponding weight I write here; that means, it is the retained one; that means, this 0.02 gram of material retained onto the sieve of aperture having an aperture of 250 micrometer.

Next one is very interesting, minus 250 to plus 180 it is not that minus 250 plus 180 is equal to minus 70 please do not get confused with that. It is minus 250 means that you have got a 250 micron sieve and you have got a 180 micron sieve. So, the particles which are retained on top of on the 180 micrometer sieve and the it is and the particles which are finer than 250 micron micrometers sieve they will only be retained here. That means, this sizes we are telling that this particles are finer than 250 micrometer, but coarser than 180 micrometer. We are not telling that individually how many particles are having 182 micrometers size or 230 micrometer size.

So, likewise when we come to here we say minus 45 micrometer; that means, how much of that material they are finer than 45 micrometers. So, we have taken a sieve and you see that that we have got a 250 to 180, 180 – 125, they are in a series. And the raw data whatever we got that is we have taken a total sample if we sum them up that will give you a rough that will

give you an idea that what is the initial weight of the sample. So, now I have discretize it that out of the total population 0.02 gram of material retained on a sieve of 250 micrometer 1.32 gram of material retain on 180 micrometer; that means, they are finer than 250 micrometer, but coarser than 180 micrometer like that I have tabulated all this. So, the only these information you will be generated from a laboratory size analysis device that is a sieve shaker.

Now how do I get to know all other information that is your size distribution and related information? So, what are we try to do first we try to convert them into weight percent how do I do it now we have to add the total weight. So, sum of all these individual weights will give you the initial weight of the sample you may ask me that why we should not why the initial amount I would as biased not to do. So, because during the sieving operation you may lose some particles some particles may get stuck inside the sieve apertures you may not be able to recover them 100 percent. So, suppose if I add them together it gives you 60 grams, but your initial weight could have been 65 grams, when you are converting them into weight percentages that may be different. But you can weigh it initially to have an idea that how much of particle we are losing if we are losing too many particles then the technique is not appropriate. So, if I add them together I get the total weight and then convert this data into weight percent like this. So, 0.1 to 0.9 like that we get this.

Now, when we try to plot it I cannot plot because your x axis is always size whether it is log scale or linear scale that is different, log scale or natural scale. So, I cannot plot minus 250 to plus 180 I have to take a decision that what is that size we use. There many ways of doing it, but in this example I have shown I have taken a normal aperture size normal aperture size means that is the finest one. So, here that is the 250 I have got no other alternative. So, I use 250 in this case I use 180, this is 125, like this here it is 90 here it is 63, here it is 45. But I can always use the different means also like I can take a mean of this minus 250 plus 180; that means, 250 plus 180 is 430 divided by 2 its 215 also I can write, but here then I have to mention that what do I mean by the size although it is sieve size, but then you should say that this is the mean size.

I can take even the geometrical mean that is square root of 250 multiplied by 180, but I must mention here that it is a geometric mean size, but there must be some reason for using that. So, normal aperture size why I am using it because maybe I will try to use this information

for designing my or replicating the similar say size separation into a large scale operation like we call it screening operation.

Now, this is the weight percent what that is a plus 250; that means, this now let us say discuss only the weight percent that out of the total population only 0.1 weight percent of material was retained at 250 micron sieve. So, how much was passed through that 250 microns sieve? It is 100 minus 0.1 that is 99.9 percent. So, that is what I am writing here that that is your undersized percentage if I have another column that what is they this is basically the weight percent retained we should right here and if I have another column like say weight percent passing. So, here in this case it will be 99.9 percent and in this case it will be 2.9; that means, how much would have been passed that is your 97.1 percent like that we should have read. So, that is the passing sizes individual passing sizes.

But cumulative manner that if I have only a 250 micron sieve, if I have a single 250 micron sieve how much of that material will be retained and how much will be passing. So, 0.1 percent of the material will be retained and 99.99 percent material will be passing through that sieve. But say suppose out of all these sieves if I have only a your 125 micron sieve, if I have taken a 125 micron sieve how much of material what percent of total feed material would have been retained or would have been passed.

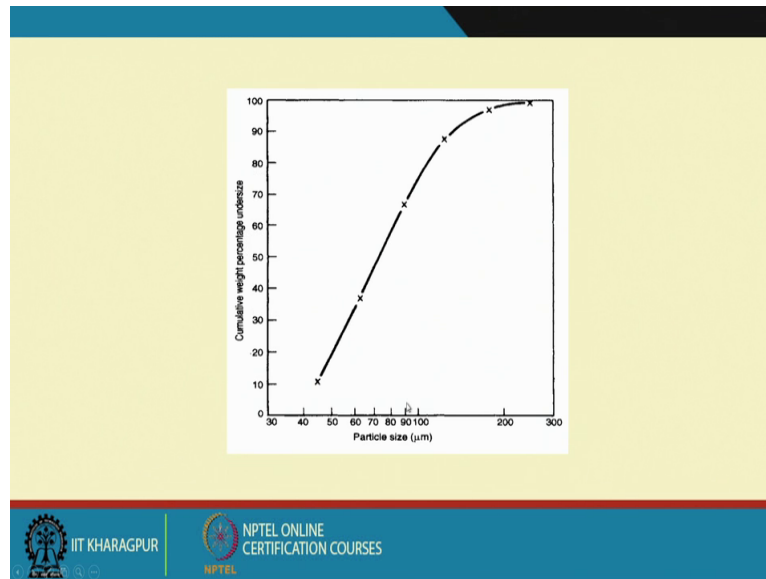
So, when I am having a 125 micron meter sieve. So, the particle which are already retained at 180 and 250 they will also be retained on this 125 micrometer size, is not it. So, what will happen? So, to calculate that, I have to do that that its I have to add that 0.1 plus 2.9 it is 3 plus 9.5 that is 12.5 percent of the total material that will be retained on a 125 micro meter sieve. I repeat it again, that cumulative means that if I have a single sieve of 125 micro meter and if I do the sieving operation for that sieve I will have 12.5 weight percent of the total population which will be retained on that, it is not 9.5 we have to add them together because I have not discretized them on top of this size and once 12.5 percent is retained how much will be passed, how much will be the passing percentage passing size percentage, it will be 100 minus 12.5, so that is 87.5percent.

Likewise, I can also calculate it for 63 that if I have a single sieve of 63 micrometer. So, what percentage of material will be retained that is I have to add 0.1 plus 2.9 that is 3 percent 9.5 is 12.5 plus 21.2 that is 33.7 plus 29.4 is 63.1. So, that is exactly the 63.1 percent of the material which will be retained. And how much will be passed? As 63 it is 100 minus 63.1 that is 36.9

percent. So, we get, if you add them together in any row that should be exactly 100. So, we are getting an information of cumulative either passing size or cumulative oversize particle.

Now, if I plot this size and the x axis and y axis if we plot this und either cumulative undersize or cumulative oversize, then we get a plot like this.

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So, when I have a plot like this now. So, if I have the cumulative undersize and oversize plotted together they will be just the mirror image of each other and if I have a mirror image of this, there will be another plot like this and they will intersect at a point where it is equal to 50 percent passing size, 50 percent size. So, this is frequently used in mineral processing literature we will also be using will also use this throughout this course that is the 50 percent size we call it d<sub>50</sub> size.

Now, what is the d<sub>50</sub> size for this? That you have to go to this and you have to see what is the corresponding size it may be 73 micron or 74 micrometer. So, although I have not used a sieve of 74 micrometers, but still I can generate this information from my size analysis data that is if I have an imaginary sieve size of 74 micrometer my entire population will be equally distributed; that means, it will have 50 percent detained particle, 50 percent passing particles; that means, the 50 percent of the entire population will report to the oversize fraction and 50 percent will report to the undersize fraction.

Now, say suppose I want to have an information that if I want to sieve at a size of say suppose 90 micrometers. So, what will be the corresponding, how much of material I will get it into the undersize and how much of material I will get into the oversize. That means, now if I am processing 500 tons per hour of that material and I if I am using a 90 micrometer screens, 90 as a screen having an aperture of 90 micrometer how much of material I will be getting into oversize and undersize, I said 500 tons. So, 90 corresponding I am look at here, so this will be close to 70 percent; that means, the seventy percent of the material will report to undersize and remaining thirty percent material we report to oversize.

So, like that it is very common also to report the size distribution data in the form of d 80 size or d 75, there is 75 percent passing size, 75 percent passing size means what is the corresponding size. So, 75 percent passing size is around 95 micrometers. So, something like that you have to look at you have to plot it. What is the 25 percent passing size? It is like this, so it is basically 50 micrometer close to that. So, all this information throughout the course I will show you that how do we apply them. Till then goodbye.

Thank you.