Mineral Resources: Geology, Exploration, Economics and Environment
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Lecture - 50
Geostatistical Method of Mineral Inventory Estimation (Contd.)

Welcome to the to today's lecture. We have been discussing the geo statistical method for estimation of the quality parameters of ore body, different types of ore bodies and we know that the diverse nature in terms of their extension, their strike and then their length, breadth and width and their quality and the quality parameters such as grade are varying in space.

And we have some idea about how they vary in space and we just saw how we could go for an estimation of the quality parameter in an ore body by using the method of the ordinary Kriging which essentially assumes the variogram based on stationary regionalize variable where the variogram model has been fit. So, just for the sake of a bit of completeness we can extend these calculations to regionalize variable which behave in a non stationary way.

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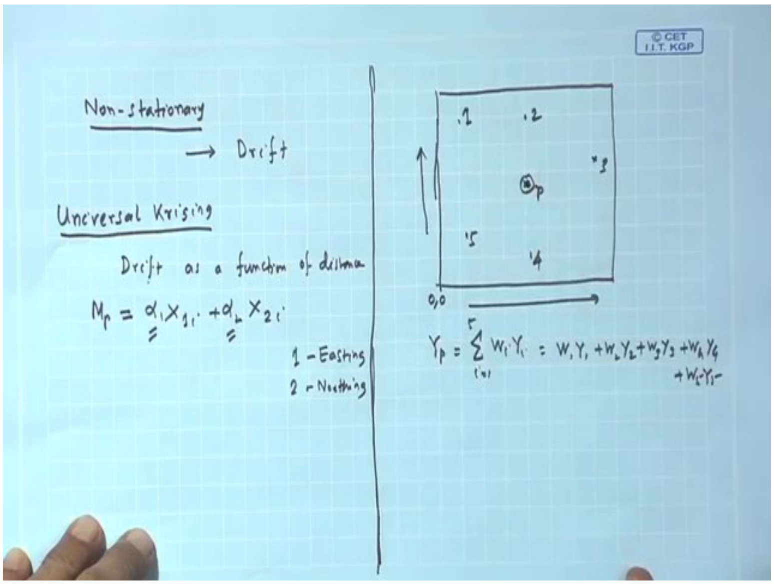

So, we know that the non stationary regionalized variable can be described in a very simple way that this non stationary regionalized variable has something which we call as a drift.

That means the average that is computed at any point within the speatial domain does not remain fixed, it varies either increases or decreases. And we know that if we can somehow estimate or somehow approximate this drift we can subtract the drift and get the residual and from the residual we can compute the semivariogram.

And we discuss something about the structural analysis which is done this elaborate process of modeling of non stationary regionalized variable to arrive at a situation of convergence of a assume semivariogram and a conversed semivariogram. So, that remains at the background of it. So, if we just see with that we want to do the Kriging estimate of a non stationary regionalized variable which is in many of the text books you will find them mentioned by the name as Universal Kriging. So, in the case of universal Kriging what we have? We have to approximate the drift as a function of distance.

So, that means, say for example, if our drift is the denoted as M_P and the most simple way of approximating the drift by considering a linear model. Let us say that is our M P is alpha 1 X 1 i plus alpha 2 X 2 I; here 1 represents you may say easting and 2 as northing and this alpha 1 and alpha 2 are the unknowns.

So, if we have to extend the calculation procedure of ordinary Kriging to universal Kriging; then we will see how we can do that. So, if the extension becomes very simple logically. We have an unknown point here as P. If we have an unknown point here as P and then we already we are starting with two unknowns which is alpha 1 and alpha 2. And then if we have to base our estimation on the value of the regionalized variable at at known points then how many points we should have?

So, if we take 3, then 3 will lead to 3 unknowns and plus alpha 1 and alpha 2 it becomes 5 unknowns. So, if it becomes 5 unknowns; that means, we need to have 5 points for the 5 neighboring points within the validity of the special influence which is decided by the model semivariogram.

So, now let us say this is 1, this is 2, now this is 3, 4 and 5. So, now we are going to do the calculation or extend this Kriging method to universal Kriging and our unknowns are now 5 will also proceed on the same way we did before for the punctual Kriging. So, we will write; gamma h 1 1; here also we will do the same things.

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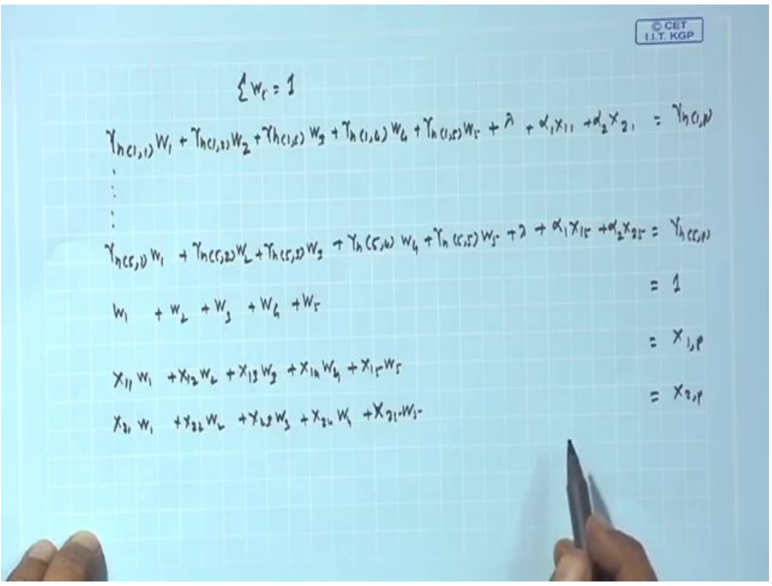

So here our Y P will be equal to sigma i is equal 1; 5 W i Y i means W 1 Y 1 plus W 2 Y 2 plus W 3 Y 3 Y 5 and we also have the uncertainty which is inherent; so, gamma h 1 1; W 1 plus gamma h 1 2 W 2 plus gamma h 1 3; W 3 plus gamma h 1 4; W 4 plus gamma h 1 5, W 5.

Similarly we will also since sigma W i is equal to 1. So, we know that; so, here even if we take as 5 unknowns as W 1, W 2, W 3, W 4 and W 5. So, the number of equations what we are getting is more than 5. So, we are involving one slag variable like before as lambda; so, here to be lambda.

And then we will have something as alpha 1 X 1 1 plus alpha 2; X 2 1. Now similarly we can go on doing it up to the gamma h 5 1, W 1 plus gamma h 5 2 W 2 plus gamma h 5 3 W 3 plus gamma h 5 4 W 4 plus gamma h 5 5; W 5 plus lambda plus alpha 1 X 1 5 plus alpha 2 X 2 5.

And then we will have our W 1 plus W 2 plus W 3 plus W 4 plus W 5. So, here this will be is equal to gamma h 1 P, similarly; here it is gamma h 5 P, here it will be equal to 1. Now here this again the same formula with same principle we can use here X 1 1, W 1 plus X 1 2 W 2 plus X 1 3 W 3 plus X 1 4; W 4 plus X 1 5 W 5 will be equal to X 1 P.

Remember as done before, the distances all are known because we are taking the easting and northing are known.

Ah we can for convenience take the value of the origin as 0 0 and this is X 1 P. Similarly X 2 1 W 1 plus X 2 2 W 2 plus X 2 3 W 3 plus X 2 4 W 4 plus X 2 5 W 5 will be equals to X 2 P. So, now, we will see how the matrix looks like and also as we did in case of the ordinary Kriging, the punctual Kriging you would have noticed that this matrix that we are solving in addition to the fact that the diagonal elements are all 0; since it is a symmetric matrix in the sense that the elements which are above the diagonal and below the diagonal are also same.

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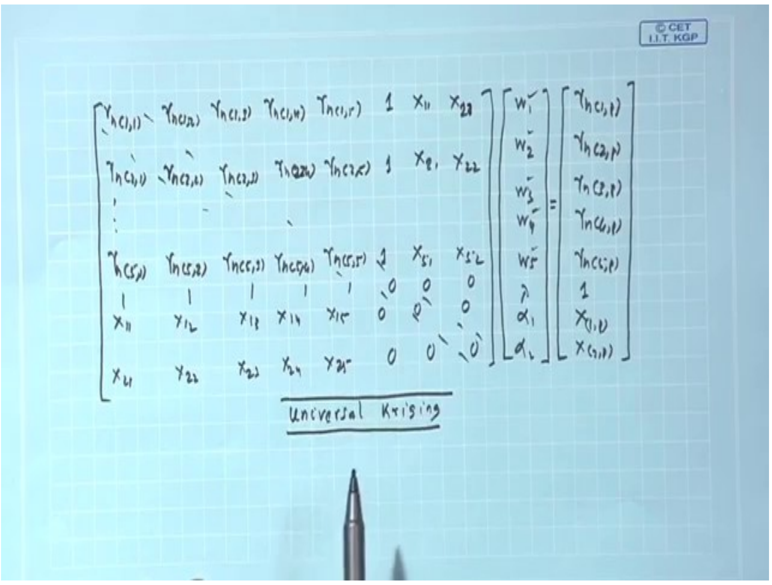

Let us see how this matrix look like; gamma h 2 2, gamma h 2 3, gamma h 2 4, gamma h 2 5, 1 X 2 1 X 2 2. So, we will write the matrix up to gamma h 5 1, gamma h 5 2, gamma h 5 3, gamma h 5 4 5 5 1, X 5 1, X 5 2.

And here we will have X 1 1, X 1 2, X 1 3, X 1 4, X 1 5 then we will have 0 0 0. Similarly, X 2 1, X 2 2 sorry we will have 1 here as 1 1 1 1 1 0 0 0 and this will be X 2 1, X 2 2, X 2 3, X 2 4, X 2 5 0 0 and 0. So, you could see here that the diagonal elements are all coming to be 0 here. Since it is a symmetric matrix gamma h 1 2, gamma h 2 1; since gamma h 1 2 will be equal to gamma h 2 1, gamma h 1 3 is same as gamma h 3 1.

And here also X 1 1, X 1 1, X 2 2, X 2 2 2 are 2 1 and then here we will have W 1, W 2, W 3, W 4, W 5 lambda and alpha 1 and alpha 2. This is equal to gamma h 1 P gamma h 2 P, gamma h 3 P, gamma h 4 P, gamma h 5 P 1; X 1 P, X 2 P. So, essentially this actually forms the structure the matrix for universal Kriging.

Here we see that the order of the matrix is 8 when our numbers of unknowns are 5 plus 2 that is 7; the number of the order of the matrix is 8 cross 8 matrixes and it can be solved by whichever method we may choose. There are many methods available for any standard matrix solution we can use to solve this matrix and we will get these varied values W 1, W 2, W 3, W 4, W 5 lambda and alpha 1 and alpha 2.

As a little bit of for our convenience what we can do that like previously shown on this diagram that if we the unknown point P here. So, we are essentially going with origin is 0 and this was the easting and this was the northing that we are using. Now what we could do we could shift this (0, 0) to the origin to P. So, if we shift the origin (0, 0) to P then what we can get here is a situation where these values this X 1 P and X 2 P will become 0. And there will be some of the parameters in this matrix entry will be different, but the value will be remaining the same.

And in that case we can calculate the value of the regionalized variable at unknown point P by getting these solving for these Ws. So, this is the method of universal Kriging in a very simple way. There are elaborate ways that we could solve for the for calculate the drift. And also this method could be extended in by model by doing by fitting a non-linear model to the drift variation. So, incorporating more and more number of variables like we discussed before taking a second order or a third order equation for the variation of the approximating the drift.

So, within these the basic purpose of introducing this here is that even though the many of the part like the modeling of the semivariogram remains a very involved exercise. But beginner can always by using his very simple ability of handling with this set of linear equations, can get a feel of how the geo statistical exercise are done. And can always work out with some example data set and you can do this exercise of getting the feel of estimating the quality parameters into ore body.

So, within this frame work here we conclude the discussion on the geo statistical methods of mineral inventory estimation. This can be dealt in much more detailed and elaborate way at higher levels, but one can always make a beginning by understanding this things.

And within this process there are many other tricky, many other affairs, many other points which are also very important as per as the quality estimation or estimation of the quality parameter in the ore bodies are concerned. Out of which the most important part is the sampling which we just briefly discussed.

So, with this I conclude the discussion on the mineral deposit inventory estimation by use of conventional and the geo statistical methods which you can always do this exercise as examples which will be posted and can get a feel of it and can pursue this at a high level.

Thank you.