Mineral Resources: Geology, Exploration, Economics and Environment
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Lecture – 49
Geostatistical Method of Mineral Inventory Estimation

Welcome to today's lecture. So far we have been discussing about the Geo Statistical Method for estimation of the quality parameter in ore bodies. And we have got a brief idea about the nature of the variability of this particular variable, which we are considering is a regionalized variable behaving intermediate between random and deterministic. And the behavior can be quantified with respect to the spatial domain or, spatial influence of the variability of the data in the form of the semi variance, where we saw the nature of the several variance and the different types of mathematical models, that could be fitted to the semi variance in terms of a spherical model, parabolic, exponential a linear and so, on.

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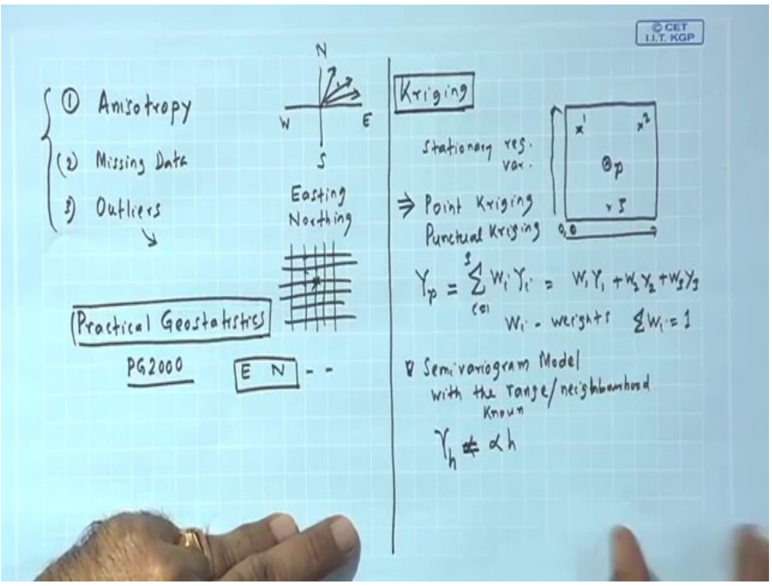

And also we just made an introduction. So, this exercise of the variogram modeling, at least one in this particular context forms the background for the estimation procedure, which is basically known as kriging.

Here, we go for the estimation of the regionalized variable at any unknown location and go ahead with the calculation procedure. We will just see briefly how the calculation goes on. Let me tell you the beginning that this procedure that I am giving the bit a bit of an idea or introducing this subject to you. It is actually pretty much elaborate and it involves quite a lot of exercise as far as the modeling of the variogram is concerned, because we have to take into account many of the situations such as the anisotropy.

For example, when the semi variance nature of the semi variance is different in different directions for example whenever we see a spatial data; we always put them in the form it is north, south and east, west. So, we always represent in the form of whenever we have any point particular point in space we express the spatial data in terms of easting and northing and when we have discussed the data in the form of grid the x and y direction.

So, we know that the semi variance could be different in different directions and that is one of the reasons and we have to have the methodology to have something like average semivariogram or to have the model, in spite of this kind of anisotropy. There could be missing data, there could be outliers. Outliers are the ones where we can have an array of points, which are basically represented on these nodes.

We might get a situation, where a particular value, certain value becomes unusually very high or very low compared to the values in the surrounding. In such kind of situation we call them as outliers. So, there are many elaborate methods which are there to take care of these situations like anisotropy, missing data, outliers.

So, what is being presented here is an overall idea that one can have to deal with the spatial data and do the calculations at this particular level to have an idea as to how a quality parameter for an ore body can be estimated. Once the nature of variation of the quality parameter like gray is known in a in a data matrix. So, before we go to the procedure just that I will be discussing about the kriging process it would be worthwhile just to note that there are many commercial software which are available for processing of such kind of spatial data.

The real life data like, data that is acquired from any ore body or otherwise any kind of a spatial data like depth of water table, surface generation, all these type of data we define in the beginning is regionalized variable. So, there are commercial software which are available, but there are to my knowledge there is one teaching software, that is available to do geo-statistics; that is the practical geo-statistics.

This is mostly available as a group of programs, which are available from the site by two scientists Isabel and Clark from their practical geo statistics webpage; where these programs can be obtained and there are numerous set of data small and big data sets, which data set could be observed for example, the data set could be having in terms of easting, northing and any attribute you have value that we could talk about in terms of the value of the metals one or more than one metal grade, which will be given at different points.

And such kind of example data set can be taken and one can always get a feel of plotting these semivariograms, and also to feed them into different type of model and also move on the further exercise. So, now, coming to the the estimation procedure, which is known as a kriging by the famous the person who pioneered this exercise.

So, let us let us look at what this exercise could be for estimation of the regionalized variable at any unknown point. Let us first consider, this is the spatial domain, in which the semivariogram is valid whichever model has been obtained by doing the exercise. And first let us assume that the data or this first particular procedure that we are discussing right now is on stationary regionalized variable and such a kriging exercise, which is done on stationary regionalized variable, once the semi variance is known the model is fit.

We will first look at the method which is known by the name as point kriging or punctual kriging essentially telling us that the value of the regionalized variable at any particular point can be determined by using this procedure.

Let us think that we want to estimate an unknown point where, p the value of the reasonable variable, let us say it is our grade, where we want to and this is the spatial limit within which the semi variogram model is valid. And now if we do that, then we have to do it with respect to the some points some known points on which the value of the regionalized variable is known.

So, if you want to do that; suppose the value of the regionalized variable at Y is denoted as Y p. So, this Y p can be estimated based on some known points. So, how many known points we could possibly have? So, our common sense will tell that ok, in order to have any kind of an estimate for an unknown point we should have minimum three points in the neighborhood, where you should have the value known from which we will be able to calculate.

And here; so the first thing that is available to us is a variogram. It is a semivariogram model with the range or neighborhood known. Let us take it for the timing that it is a linear model, where gamma h is equal to alpha h, where alpha is a constant quantity and this is h and here is the range is defined.

So, then if we want to have 3 points, let us take three points point 1, point 2 and point 3. And the coordinates of this in the easting and northing are known; let us say this is the origin. So, the coordinates of point 1, point 2 and point 3 are all known to us as well as point p. So, we will use a simple formula for calculation of this particular variable at p as sigma i equals to 1 to 3, we can put n is equal to 3, 1 to 3 W i and Yi; that means, it will be Y 1 W 1 plus W 2, Y 2 plus W 3, Y 3, and this W s are the W i are the weights.

And then remember the inverse with distance weighting method we used, let us say method which were using here is more or less analogous to that and here the constraint is that sigma W i will be equal to 1. Now, how we can do that. So, here the value of the regionalized variable at p is unknown and the value at 1, 2 and 3 are known, and the only thing that we have here is the semi variance, the model semivariogram.

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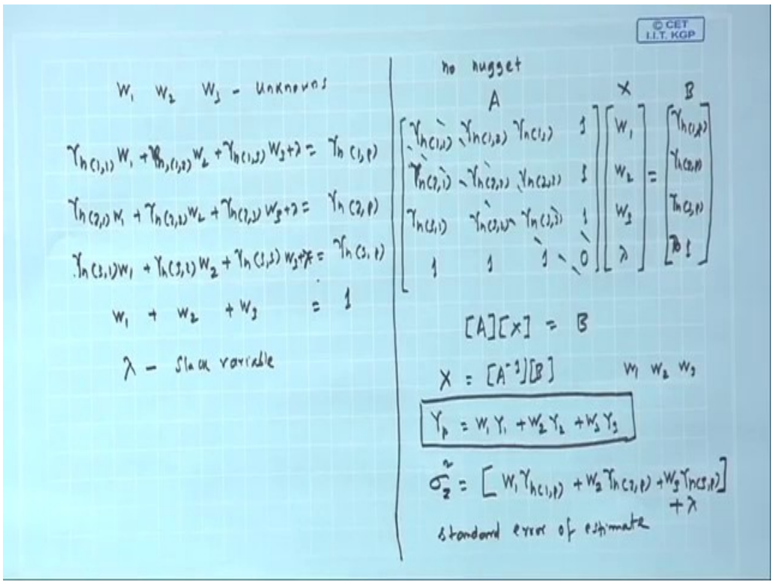

So, here W 1, W 2, W 3 are the unknowns, which we have to solve. So, if you have to solve then we have to have 3 equations. So, let us try to formulate it.So these three equations when you formulate we take help of the semi variance. So, here it is gamma h 1, 1 into W 1 plus gamma h 1, 2 W 2 plus gamma h 1, 3 W 3 will be equal to gamma h 1 p .

Here it is very well understood gamma h (1, 1) is the semi variance of 1 with respect to 1 itself; 1 with respect to 2, 1 with respect to 3 and these values are all can be calculated once we have the gamma h as a function of distance. Because we know the distance between the point 1 and 2, 1 and 3 and 2 and 3 respectively, so, here again gamma h 2, 1 W 1 plus gamma h 2, 2 W 2 plus gamma h 2, 3 W 3 is equal to gamma h 2 p, 3 p.

But, we have another equation here where W 1 plus W 2 plus W 3 is equal to 1. So, what we have could; so this is an over determined system. So, what we have to do? We have to get a unique solution for a situation, where there is the number of equations are unknowns; we will introduce a slack variable as lambda and we will put here as lambda plus lambda.

So, that the numbers of unknowns are 4 and the numbers of equations are 4. So, if we now put them put this in the form of a matrix. So, here we put it in this way. So, this is the way in which you can write it in a matrix form, this is gamma h 1 p, gamma h 2 p, gamma h 3 P and this is one the last equation W 1 plus W 2 plus W 3 is equal to a 1 and all these.

So, as it can be seen here, that this is the matrix A and this is the matrix X, which we are going to solve and this is the matrix B. Now the interesting thing you should observed here is that; this matrix has these diagonal elements; in this case we are considering it stationary generalized variable and we take that there is no nugget effect.

So, if there is no nugget effect, then the gamma h (1, 1), gamma h (2, 2), gamma h (3, 3) are 0. So, we will have this diagonal element as all 0s, and this is the form of the matrix that we could write. So, we can use our simple knowledge of matrix algebra and you can solve this and in case there is a nugget effect, this nugget effect is known and this diagonal element will all have the same value and we can solve this.

So, if this forms AX is equal to B. So, X would be A inverse B, because this with 3 is to 3 and this is 3 is to 1; so it will be 3 is to 1. So, we get the column vector, which is our X; so which will be the value of W 1, W 2 and W 3 can be solved in this way. We will be posting in a few example exercises with this kind of a data, taking some real data, synthetic data from any data set.

And so finally, we will be able to get our value of the regionalized variable calculated at a point P as W 1 Y 1 plus W 2 Y 2 plus W 3 Y 3 and as we have said before, these values that we get here will essentially be a value on which the true value for the regionalized variable at point p is actually not known. In spite of that we still could possibly approximate, what kind of error we might have introduced in this calculation. So, that we know that it could be calculated, if I represented a sigma z square as W 1 gamma h (1, p) plus W 2 gamma h (2, p) plus W 3 gamma h (3, p) plus lambda.

So, this is the kind of error, the standard error of estimate, which is by taking the square root of it, which you can calculate.

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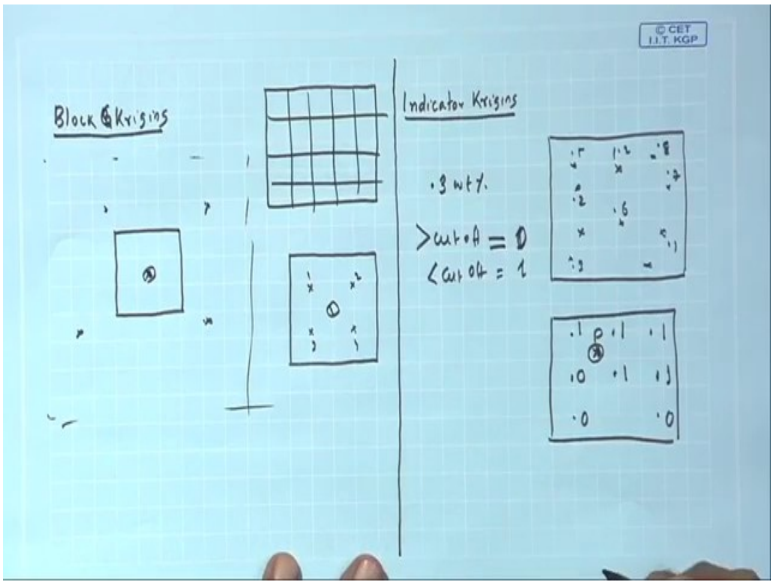

So, this particular exercise is the punctual kriging or the point kriging. As we discussed before, we sometimes in terms of our selective mining unit of the block, which we have discussed before. If this is the body that we divide it into different blocks, and we need to report and average grade for a particular block. Although, we know that in ore deposit we come up with a value, which is the average grade for the over deposit with uncertainty and sometimes with reporting the cutoff grade.

But, for the continuous mining operations and to know exactly the quality parameter, because this over body is actually mined discretely in the form of the selective mining unit or, block; so the kind of kriging estimate, which is essentially required for this is known as block kriging. But this block kriging is actually nothing a very different from the method. Also here we would again go by our assumption of stationary regionalized variable; the grade is behaving like a stationary manner. And in that case suppose now we need to have a block of this size whose average grade has to be determined and this in the context of the larger the ore body that we have; maybe we just need to have the central point here for the estimate of the block average.

So, this could also be done in a similar way taking this number in one of the ways, that it could be done is that; this particular point will be just be treated as the similar way in which we have treated the previous example as estimating the unknown at any particular point.

So, in this case the center point of the block becomes the point at which we want to do the kriging. Here also we can do the same way by taking some points within the closed neighborhood and if the spatial influence is not violated and we can take a points anywhere either within the block itself or just a little outside taking some points you can still calculate the point, and this kind of formulation of the matrix that we have shown here would be exactly the same way that it could proceed, and could assign a particular value is the average for this particular block.

Alternatively, we can also do any that case our matrix method will be the exactly the same that we have seen here or we could possibly take some four unknown points within the block itself and estimate the value for this grade at all those 4 unknown points. And then take a point average of this 4 and assign it to the central point. In this case, it can be done individually for this point 1, 2, 3 or 4 or it could be done even together by using the same kind of a matrix. Just one point to recapitulate here, that if we could see that when our numbers of unknowns were three W 1, W 2 and W 3 and we introduced one slack variable and the size of the matrix becomes number of unknowns plus 1, 4 cross 4 matrix.

So, in this case if we can use the same 4 cross 4 matrix for these individually these 4 points, or can do them simultaneously by taking another 3 points here and in the outside in the region in the closed spatial influence and can solve the 8 cross 8 matrix. To solve for the weights for each of these 4 unknown points and get the an average, which will be applicable for the central point for this block.

So, block kriging is essentially nothing very different from the normal point kriging methods. See other kind of kriging, which is sometimes used and is used in mining industries often, is the indicator kriging. Without getting much into details of that I can just briefly explain. Suppose this is the spatial domain in which the semi variogram is fit, and then we are trying to actually do the geo statistical exercise. There could be many points within this.

And now each point here is assigned with the value of the grade of some finite value. Let us say any finite values is 0.5, 1.2, 0.8, 0.7, 0.6, 0.3, 0.2, 0.1 something like that. And now suppose in this particular ore deposit from the calculations that we did earlier for the calculation of the cutoff grade. Let say that we have came out to the cutoff rate of 0.3 weight percent of the metal.

So, here out of these points, the points which will be above 0.3 can be marked as 1. For example, which we have seen here say this will be pointed as 1, this will be as 1, this will be as 1, this will be as 0, this will be as again 1, this would be as 1 and this would be as 0 and this would be is also 0.

So; that means, the points where the grade is greater than cutoff is taken as 1 and where it is less than cutoff it is 0 and greater than cutoff is 1. So, now, here it almost boils down to the situation that, if we want to do the kriging operation for any particular unknown point p here, then we would only be asking their question to ourselves that whether the grade here would be above the cutoff or below the cutoff. So, that means, almost it will be something like, if finding out the probability for the grade to be greater than cut off or less than cut off.

So, we will be doing the kriging taking the value is 1 or 0 with this kind of binary number and the value that will come out for this particular unknown point p will not be exactly the value of the grade, but in terms of the probability of the point having greater than cut off or less than cut off. So, with this I would just conclude with this ordinary kind of kriging methods and in a very simplistic way and within the limits of this particular lecture series; so we will continue the next class.

Thank you.