Mineral Resources: Geology, Exploration, Economics and Environment

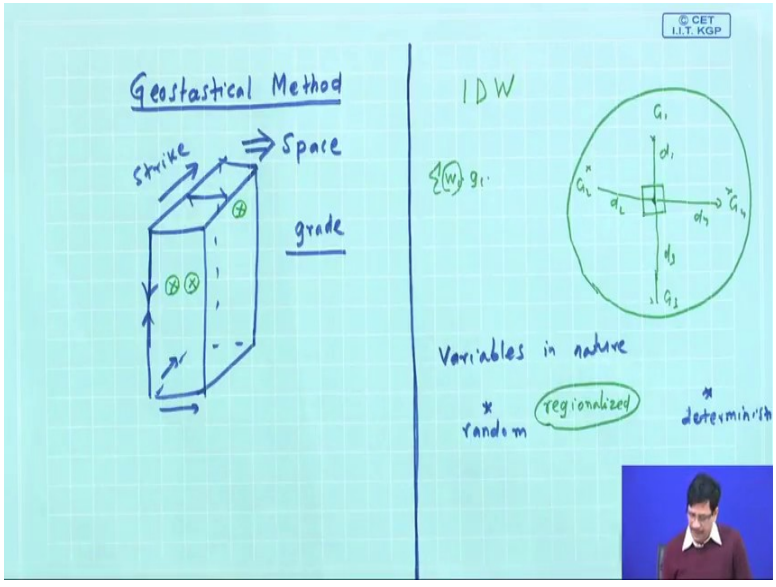
Prof. M. K. Panigrahi
Department of Geology and Geophysics
Indian Institute of Technology, Kharagpur

Lecture – 47
Mineral Inventory Estimation (Contd.)

Welcome to today's lecture. We have discussed the traditional methods of mineral inventory estimation, including the one which is the inverse distance weighting method. So, for a few of our discussions; for the few of the coming lectures, we shall be discussing or just getting introduced to the method of the geo statistical ore reserve estimation or the geo statistical method which is employed in the mineral inventory estimation.

So, essentially before getting into the geo statistical method, let us first try to know what exactly, what kind of variable we are actually dealing with. By this time we got a very fair idea, that the kind of parameters which we are trying to understand in relation to the ore deposit, the evaluation of the ore body in terms of its quality and quantity are the attribute values of the variables in space. So, certainly we are talking about variables and in variable in space.

(Refer Slide Time: 00:49)



So, whenever we are talking about suppose, I could just draw a very simplified diagram, this cuboid, let us say it represents an ore body. So, it has got a dimension of length,

breadth and height and this represents an ore body. So, it has got its dimension, it extends in three dimensions X, Y and Z. And we are talking about the parameter, which is the grade of the ore body and we have already got some idea, that this grade that we are talking about in an ore body, is not the one which is going to be of a uniform value, same value all throughout its dimension in length, breadth and width. It is a rule rather than exception, that the grade is not actually same everywhere.

There are domains in which the grade might decrease in one direction, or the grade might vary more in one direction and less in the other direction. Suppose, this is a strike of the ore body, if this is the strike of the ore body and this is the width of the ore body and this is the depth of the ore body. So, what basically might happen that the variation of the grade might be different along strike or across strike or along depth, which is very usual because it is the result of a very complicated ore forming process, which has resulted in bearing very few exceptions like a banded iron formation where an un-enriched, unaltered primary banded iron ore can possibly have a very uniform content of iron in an infinite strike direction, but in most of the types of deposits, hydrothermal sulfide deposits that we generally try to do this exercise. They will always have such kind variation. So, they vary in space. So, they are variables in space.

So, now, the other point that we could possibly remember about this is that, the grade which is calculated, suppose the grade calculated any point in space over here and the grade which is calculated at a point, which is infinitesimally or, is a very close in space to it or a grade which is calculated at a point, which is far off from the point that we are referring to it at this point here.

What we see is that, they will generally not be just, be very different from each other. In one of our procedures of the estimation through conventional method, which you use for the IDW, the Inverse Distance Weighting method, where we were discussing about having a block, which is defined here and we wanted to compute the grade, the center of this particular block with certain points where the grades are known and these points were denoted as G1, G2, G3, G4 and so on and they were at distance of d1, d2, d3, d4 likewise and we saw a calculation procedure, in which if we are trying to find out a weight as Wi for finding out the grade here by using a simple formula of $\Sigma WiGi$ and this W being varying from 0 to 1 and sigma Wi is equal to 1 and in this we discussed that the

value at any point in space, will be influenced by its nearby neighborhood points and shorter is the distance, greater will be the influence.

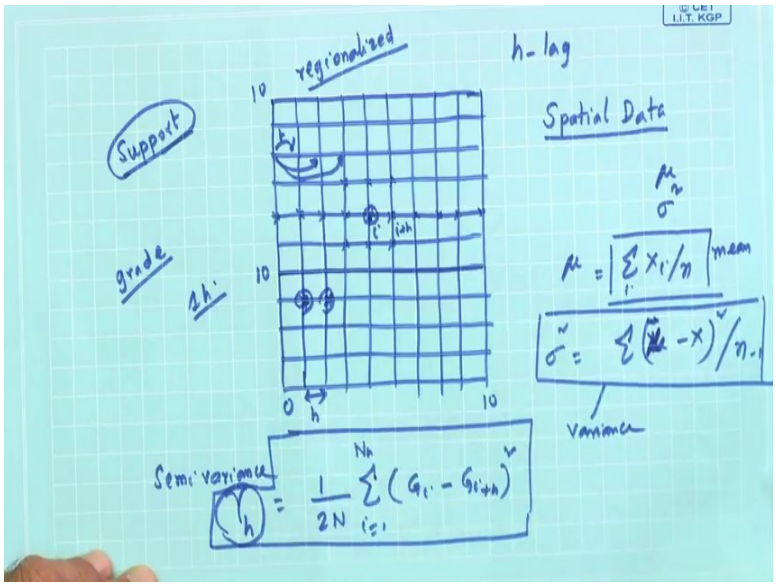
So, we would expect that this Wi will have a higher value for points which are closely spaced compared to the points which are far away and in that, we also raised the question that what is the kind of the special limit, and that we can use this IDW. Now, if we try to understand this geo statistical method that possibly would give answer to this question that we raised before.

So, before we get into all this discussion, let us be reminded of one very basic premise, that whenever we are talking about variables in nature, so they generally could be between two extremes. They either are random or they are deterministic. In case of random, any two successive observations either in time, very closely spaced or succession in space, which is very infinitesimally closely spaced or even any space interval. So, no two successive observations are dependent on each other. So, they are just independent of each other and treat them as random variables.

So, the value of a particular variable, that is occurring at a point here and a point which is away from it in space will be absolutely independent of each other and behaving as completely as random. The other one is that in the deterministic case, every successive value will have a very well defined relationship with the previous one, like say, talk about, when a body is moving with a certain force applied, its velocity with time it follows a very well defined deterministic relationship.

Now, can we think of a situation where a variable will behave exactly something intermediate between the truly random or truly deterministic? That we will call as regionalized variable. So, this regionalized variable means that is a representation of something in space and each node are our point of observation. This is being drawn in both the directions X and Y and also, we can think of the diagonal directions as well. Suppose, we are talking about an observation at any point here, then the value which are the values of the variable.

(Refer Slide Time: 09:06)



So, here in the, as far as the spatial data are concerned, we are talking about. So, these are these are variables in space and we are calling them as spatial data. So, this spacial determines that if we are defining a coordinate, if I am representing this as 0 and 10 and 10. So, this is a grid where it is 10 cross 10, 100 points and each one coordinate is known and that is the data in space. So, we can think in terms of any of the attribute, whose value to be determined here, at these points and that is the attribute data and we are talking about the variation of that attribute data and let us, that attribute be a be the grade and let us say that, this variable is our grade.

We are assigned with the duty to understand how the grade is varying in particular, taking only the two dimension as the example, even we can first start with this one dimensional case. So here, what basically had it been a random variable, then it would have been totally independent. Any of the values, which are spatially close, would have been independent of the value that is measured here and the behavior of that variable would have been easily depicted by using some kind of distribution following a normal distribution

or any kind of a statistical distribution where taking the very simple example of a normal statistical distribution, where we represent the characteristics of a normal distribution by its mean, which is 'μ' and the variance is σ square and we know what are the formula for calculation. For example, when we are talking about mean, we know that this will be

sigma Xi divided by n. So, this gives us the mean and when we talk about sigma square, we talk about the situation where this will be mu minus X square divided by n minus 1, kind of formula. This is for the variance and this is for the Mean. And here, this gives us a central tendency of the data in terms of its dispersion and it is expressed as sigma square, which is the variance. So, this particular variable which is, our regionalized variable, is expected to behave in a way which is intermediate between truely random and truly deterministic. Then we see exactly what it does. So, what happens is that the value, , which is the most closely spaced, influences the value or has some relations, even the strongest relationship with the value here and that relation actually dies out, as we move away from the point.

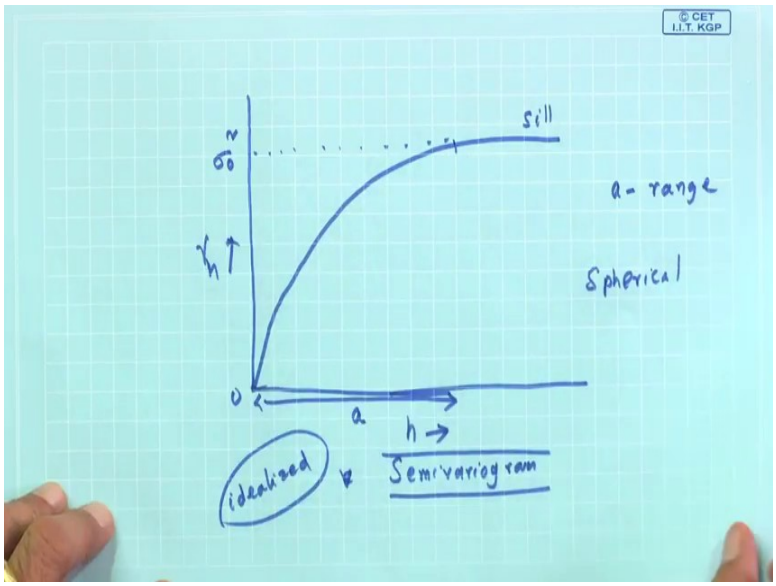
So, this relationship is quantified in terms of a quantity, which is like variance, which is actually the semi variance, which is given by a formula. I will first write out the formula. The semi variance is represented as a quantity as gamma h and that is actually 1 by 2n sigma i equals to 1 to Nh, if I say I am using the regionalized variable and my attribute is grade, let me represent it as value as G. So, I would say that Gi minus Gi plus h squared. So, this is a quantity, this is a thing gamma h, which is a semi variance, which can be estimated in this way. One can see the relationship which looks kind of a little bit, not that very far off here also. So, the connotation here is Gi.

Let us say, for example, this is the point I and this is the point which is i plus h and in this grid, that is shown here, let us say that it is separation of the grid B represented by h and this representation of this h. So, that means, it can be thought of in terms of multiples of h, for example, this point is away from this point by a distance h and this point is away by a distance 2h, this point as 3h. For example, if I take the distance here. So, this distance is a distance h, this distance is 2h; this distance is 3h and so on. So, here this h is basically called as the lag and then, what exactly happens here is that, if there are 10 points, suppose consider any particular one line on which there are 10 data.

So, this 10 data are separated each by a distance of h. So, if I have to calculate this kind of a formula, of taking the value at any point of determination, with respect to a value that is determined just h distance away, then I will get n minus 1 number of pairsSo, like that I will be getting. If there are 10 points with 1 h separation, I will be getting nine pairs of such points, for which I can calculate this value by taking Gi minus Gi plus h squared divided by this 2N.

Which is exactly the value as the total number of points as n minus h and it will be summation, will be over I is equal to 1 to Nh, means when the lag is 1, there will be 9 points. When the lag is 2, there will be 8 points; the lag is 3, there will be 7 points and so on. Now, what happens is that if I am plotting, if I do have to plot.

(Refer Slide Time: 16:56)



h versus gamma h, this is why the conventional plot is done as it will be understood from this formula, that any particular point on which we are doing the measurement, if we done on this very same point, then we will get a value of zero and as h will increase to h2, h3 and h4 and so on. This value is likely to increase. So, this is the general behavior of a regionalize variable. What exactly, what type of a regionalized variable, we will discuss.

Now, where if this value of gamma h starts from a value of zero at the origin and then, goes on increasing and attains a certain value after which, it becomes flat or parallel to the X axis at which point it is called the sill and the value corresponding to the sill, is actually the sigma zero square, which should be same as the so that means, if the value, the variable is behaving like a random variable after a certain intervals. So, that particular interval, exactly at which the value has become parallel, is something which can be represented as a. So, here it is the sill where the variable become the parameter of interest, when it is calculated, the semi variance has actually taken a constant value, and become flat, that is seal and this a will be called as the range of this semi variance and

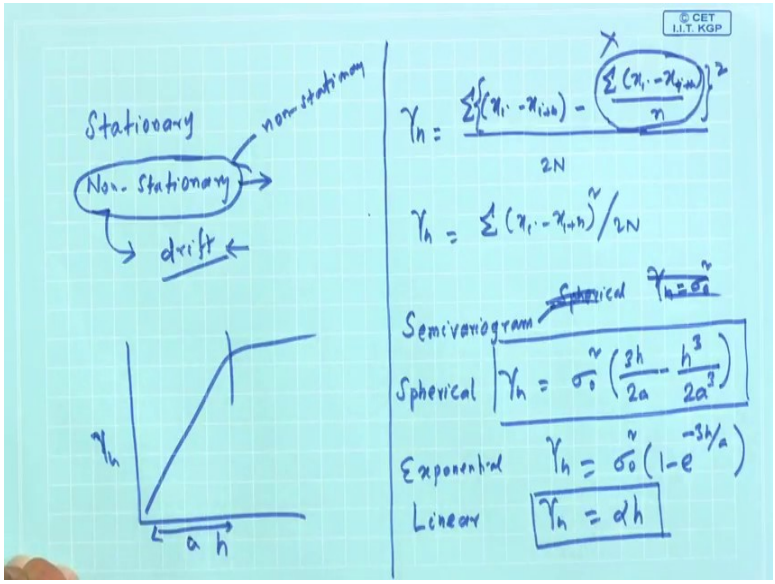
this will be called as a variogram or the semi – variogram. It may be different in different books, will have different terminologies.

So, let us call it is a semi -variogram. So, this is essentially is an idealized semi – variogram. So, this is a very basic idea about the behavior of regionalized variable, which behave something intermediate between truly random and truly deterministic and the value of the variable has a strong dependence on values, which are measured at very closely spaced or infinitesimal small intervals. Now, there are two other things which can be talked about this regionalized variable is that, we are considering the regionalized variable in convert with respect to this particular space and we have put it on a two dimension with third dimension can always be there and we talk about a variable.

Let us say, which is grade. Now, this regional is variable, is always referred to in connection with something which called a support. Examples of regional variables as many there even a surface elevation, that is measured at different points on the earth's surface can be regionalized variable. Depth of water table can be a regionalized variable and there are many such examples. So, when we talk about a regionalized variable behavior in different directions in X, Y and Z directions, depending on the volume of that, because when we are talking about an ore body, which is very finite in its dimensions, measuring in whatever say in the kilometer long or several tens of meters wide or even going to 700 800 meter deep. So, when we say that this variation will be different in different kind of directions. So, what we basically mean by is, that is a support of this regionalized variable and each type of regionalized variable will have a different support.

For example, the surface elevation, the elevation on the earth's surface can be thought of varying only in terms of kilometers, whereas, an ore body grade can be visualized varying within just tens of centimeters or tens of meters and that also might vary in different directions like, along strike, across strike and so on, and then what is the other part, other aspect of this regional variable is that a regionalized variable, the way we are visualizing it could be either stationary or it could be a non stationary.

(Refer Slide Time: 21:48)



For example, in this case, within this space, let us try to explain or let us try to understand what you mean by that. So, there are many of these points on which the regionalized variable of interest. Let us say, a grade is estimated and let us say, that we are calculating kind of taking these nine points, we calculate an average and assign that average to the central point and then, let us move this window in any direction, which we want left or right or top or bottom and go on calculating the average of that value, for each such central point, with respect to these nine neighboring points.

So, the first case, where the regional is variable, is stationary, we will find that the mean computed at different points, all within this special domain or it might be just a small representation of that. This mean will maintain a uniform value or the same value. So, it is a situation where the mean is not changing. The other case, suppose a mean is computed over here and a mean is computed over here and the mean is computed in all other successive nodes here, they are going to be different from each other. So, they computed mean on this, either will go on increasing or decreasing either of these two things might happen.

So, if we look at the actual formula for calculation of the semi variance, then it would look something like this. I am just representing, just as n and this will be sigma. Let us represent our variable as X for the time being. So, it is xi minus xi plus h minus divided by n. Now, this expression is something which is to be looked at. Now, the second part of

it is actually representing the mean, because this is xi minus xi plus h computed over this n is nothing but the mean of it. So, if it happens to be a non stationary regionalized variable, then this value is not going to be the same and it will not because this will have a significant value if the value is non – stationary. If the value is stationary, where the mean that is computed at each point is same, then this will reduce to zero and this formula will reduce to the earlier formula, which we derived or we just stated, that this is because this particular term will vanish for stationary regionalized variable, where is this term will be significant, if the situation is a non stationary regionalized variable.

So, the non stationary regionalized variable is actually characterized by something which is called as the drift; that means, as we have seen here with respect to this particular diagram, that the regionalized variable if non – stationary, it will have drift means that the mean values, which are computed at successive points are going to be either in an increasing order or a decreasing order and that will be actually designated, as the drift in the regionalized variable.

So, where as the non stationary is quite more usual for variables to be non stationary rather than stationary, but the fact is that we can always construct something which is our experimental semi variogram or the calculated semi – variogram, which can be done only if the variable is stationary, because it will have a range which is designated by that we did in our previous graph and it will behave in certain manner that we have only shown it by an idealized diagram, which we call this particular diagram, where we have discussed, that is an idealized semi variogram. So, it essentially is a spherical shape, because it is almost up to this point, it can be thought of as a radius of a circle.

So, this semi variograms of many different types of regionalized variables can be thought of being approximated by such kind of models, say for example, the model could be spherical, we will discuss about this point a little later about the stationary and non stationary and how to deal with them, may not be up to that much of a detailed, not that much of rigorous exercise, but just for our simple understanding at this level. So, a semi variogram would be spherical, where it could be represented as gamma h, will be equal to sigma 0 square. I will just write it here, spherical is gamma h is equal to sigma 0 square. Sigma 0 is the seal 3h by 2a minus h cube by 2a cube. So, where h is our lag and ‘a’ is the neighborhood; that means, the range is also a, is also called the range or the neighborhood within which the relationship is valid. So, exactly what we are trying to

understand here is that, the regionalized variable characteristic, that they have a special restrictions or the special dependence that is actually quantified by the parameter which is ‘a’ or, the neighborhood or the range.

So, that means we can expect our variable to be behaving like a regionalized variable with any particular point, the measurement to be influenced by its nearby point of the neighborhood points, within the spatial limit of this range or the neighborhood. So the spherical model, we are not going to get into the derivation or we are also not just getting into any other exercise on that. A semivariogram could also be an exponential semi – variogram, where gamma, it could be sigma 0 square into 1 minus e to the power minus 3h by a. It is also in relationship to this h and a or we could also think of a very simple type of semivariogram.

Where gamma h will be simply alpha h; that means, it is just a linear situation like this, if it is h, this is gamma h. It will be something like this, where for the most, the part within the part which is our a here. For this part, we can always think of that is behaving very much like a straight line, where gamma h could be computed given the value of h at any point within that particular special restriction. So, this is a beginning or the very introductory step towards making a very beginning or the very preliminary understanding on the principle based on which this geo-statistical ore reserve estimation is based. And we will continue discussing on this.

Thank you.