

**Soil Mechanics/Geotechnical Engineering I**  
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**Lecture – 50**  
**Earth Pressure (Contd.)**

Ok let me continue with these earth pressure and this is ah; obviously, we are doing active earth pressure, passive earth pressure and then how they are different from whatever we have learnt at the very beginning that effective stress at any depth. So, effective stress at any depth if you know then a lateral earth pressure at active condition or lateral earth pressure at passive in passive condition what way they are different.

So, you have to find out the way we did at the beginning, effective stress, effective vertical stress and then we multiply simply by active earth pressure coefficient, if it is a active condition to get the active earth pressure and multiplied by active earth pressure earth coefficient, passive earth pressure coefficient if you want to get passive earth pressure and how to get coefficient of active earth pressure. If you know the value of  $\phi$ , then  $K_a$  will be equal to  $1 - \sin \phi$  by  $1 + \sin \phi$  and similarly if you know the  $\phi$  value  $K_p$  will be equal to  $1 + \sin \phi$  by  $1 - \sin \phi$ . So,  $K_p$  will be equal to  $1 + \sin \phi$ ,  $1 - \sin \phi$  by  $1 - \sin \phi$ .

So, those are the things are known. So, you know the  $K$ , expression for active earth pressure coefficient, passive earth pressure coefficient and we know the procedure how to find out the vertical effective stress at any point. So, if you know the vertical effective stress at any point, simply multiply by coefficient of earth pressure at sorry active earth pressure, if you want to get a active earth pressure at any point or we multiply by a coefficient of passive earth pressure if you want to find out the passive earth pressure at any point ok.

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The image shows a presentation slide from NPTEL. At the top, there are logos for IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES. The main title is "SOIL MECHANICS/GEOTECHNICAL ENGINEERING I" and the subtitle is "EARTH PRESSURE". The presenter's name is "DILIP KUMAR BAIDYA" from the "DEPARTMENT OF CIVIL ENGINEERING, IIT KHARAGPUR". Handwritten in orange ink are the formulas for active and passive earth pressure coefficients:  $K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2(45^\circ - \frac{\phi}{2})$  and  $K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$ . A small video inset in the bottom right corner shows a man in a suit speaking.

And this active earth pressure coefficient and passive earth pressure coefficient, they are known that is  $K_a$  equal to  $1 - \sin \phi$  by  $1 + \sin \phi$  or equal to  $\tan^2 45^\circ - \phi/2$  and similarly  $K_p$  equal to  $1 + \sin \phi$  by  $1 - \sin \phi$  and corresponding this value also similar. So, these are the things already we have know we have discussed and how to find out the thrust behind the wall some we have given.

But so far we have not taken the problem. Problem will be taken later on now. So far whatever we have done the Rankin case we have considered and that to that to, we have considered level backfill Rankins theory means what there is a assumption that there is a no friction between the wall and the backfill and the wall is vertical and there is a level surface back filled, but here now I will consider that the inclined back fill.

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**EARTH PRESSURE**

Cohesionless back fill and inclined surface: Let us consider a cohesionless mass with a sloping surface behind the smooth vertical retaining wall. The lateral stress acting on the vertical faces of the element are parallel to the inclined surface. Thus any such planes experience not only normal but also shear stresses. Needless to say, they are no longer principal planes as was the case for horizontal surfaces.

The corresponding resultant pressure on the wall could be determined with the aid of Mohr's circle. The magnitude of the vertical stress is depicted by the distance OC, the lateral stress, acting parallel to the sloped surface is represented by the distance OA. Hence  $\sigma_h = \sigma_v \tan^2 \alpha$

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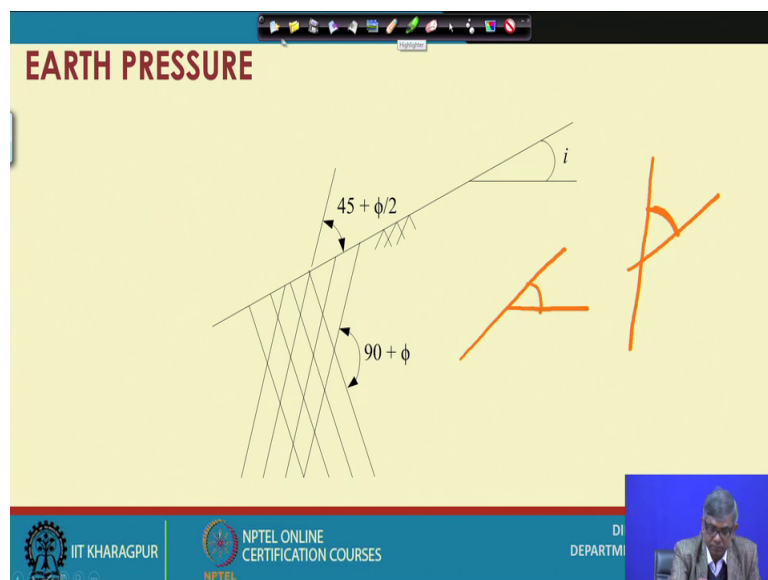
So, cohesion less back fill and inclined surface. So, if you consider that then cohesion less mass with sloping surface behind the smooth vertical retaining wall. So, smooth vertical wall means thin friction you are ignoring vertical wall we also we are considering, then only instead of, if this is the wall this is the earlier consideration, now the back fill is slope with the angle some I suppose. The lateral stress acting on the vertical faces of the element are parallel to the inclined surface; that means, if I consider the elements. So, always this the pressure also will be parallel to this inclined surface, if I go any point, if I go any point is around this always direction of the pressure will be parallel to the inclined plane.

Ah and then thus any such planes experience not only normal, but also shear stress. So, when there is a active case we have consider at any point we have shown this and this, but now in this when there is a inclined surface on the element not only this normal stress will be there, not only this normal stress will be there, that will be shear so; that means, needless to say they are no longer principal planes; that means, whatever plane you have got or the plane you have got they are actually principal plane. But now when you consider the inclined back fill if I draw any at anyway that any depth they those anyway it will not have a principal plane and on that plane normal shear both will be there. The corresponding resultant pressure on the wall could be determined with the aid of Mohr circle.

So, Mohr circle I will show the next case the magnitude of the vertical stress is depicted by the distance OC. I will show this plane and the lateral stress acting parallel to the sloped surface is represented by the distance OA and hence  $\sigma_h$  equal to OA. OC and OA what is this I will show in this diagram.

You can see initially whatever you have mention that this element now, I will drop parallel to this and parallel to this and profitably this element. If I draw on that element, these element this planes no longer a principal plane, this plane no longer principal plane as a result 1 thing that is a normal and that is a shear what is the value that can be estimated and this thing first thing you have explained and next thing actually what I have mentioned that a Mohr circle.

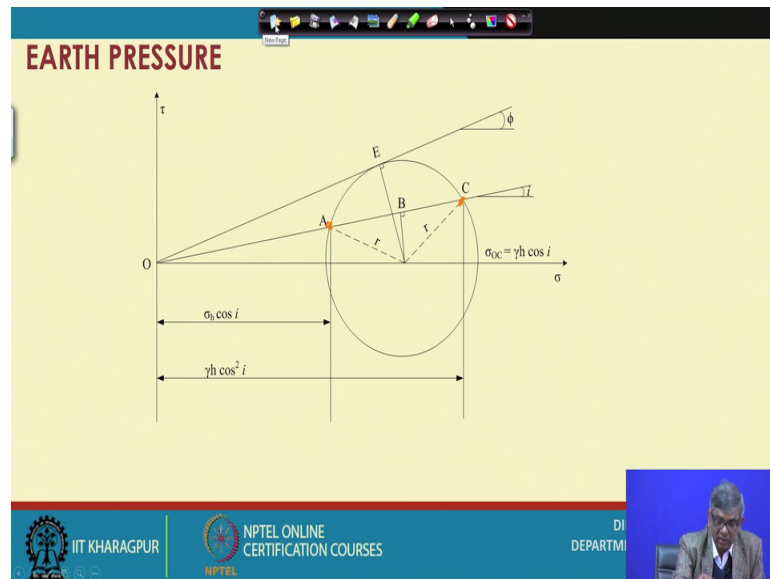
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And it is of course, failure planes failure planes will be for the such thing the same 45 degree plus phi by 2 with respect to this.

Earlier what you have done it was like the sorry it was like this and failure plane was this is the angle. Now your this one is there and this one is there. So, this is the angle. So, this is the shown here.

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Now, that we are talking about the Mohr's circle this is the Mohr circle a normal case, horizontal when it is a horizontal backfill, this is the Mohr's circle and now if there is a sloping surface at an angle  $i$  and you can draw a line through the origin to at an angle  $i$  and then it will intersect this 2 point on the circle then we will be representing actually vertical stress and lateral stress.

So, this is vertical stress and this is lateral stress and this lateral stress  $OA$  actually is given and I have shown the expression and now taking this geometrically and considering this, you can establish the earth pressure coefficient at active condition or passive condition, when there is a sloping surface; that means, by drawing the with the help of Mohr's circle, we have already established what is the expression for coefficient of earth pressure for active case and coefficient of earth pressure for passive case.

Similarly, when there is a sloping backfill, again we can take the help of Mohr circle and we can find out the expression for coefficient of earth pressure at sloping surface coefficient of earth pressure active condition and coefficient of earth pressure for passive condition. So, that is the 1. So, this is actually when there is sloping backfill, this is actually your lateral stress and this is actually vertical stress.

Now, taking this geometry, you can establish other things.

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### EARTH PRESSURE

$$OA = \frac{(OB - OA)}{(OB + AB)} OC = \frac{(OB - AB)}{(OB + AB)} \gamma h \cos i$$

$$OB = OD \cos i$$

$$r = OD \sin \phi$$

$$BD = OD \sin i$$

$$AB = \sqrt{(r^2 - BD^2)} = \sqrt{(OD \sin \phi)^2 - (OD \sin i)^2}$$



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You can see if I will if I will look back to that diagram we can express OA by this, geometrically and again and OC is nothing, but  $\gamma h \cos i$ , OC is actually vertical stress at that point and  $\gamma h \cos i$ . So,  $\gamma h \cos i$  actually was vertical here, but if I want to make inclined. So, that component will be with  $\cos i$ .

And then ob if I go back to the diagram ob equal to OD cos i and r equal to od sin i and BD equal to OD sin i so, then AB equal to this ok. So, these are the things all we can get from the geometry and once you get this and then substitute all in this way.

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### EARTH PRESSURE

$$OA = \left( \frac{OD \cos i - OD \sqrt{\sin^2 \phi - \sin^2 i}}{OD \cos i + OD \sqrt{\sin^2 \phi - \sin^2 i}} \right) \gamma h \cos i$$



$$OA = \left( \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right) \gamma h \cos i$$

$$\sigma_h = OA = \gamma h k_a$$

$$P_a = \frac{1}{2} \gamma h^2 k_a = \frac{1}{2} \gamma h^2 \left( \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right) \cos i$$

$$k_a = \left( \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right) \cos i$$

$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$   
 $i, \phi, K_a, \frac{1}{2} \gamma h K_a x h = \frac{1}{2} \gamma h^2 K_a$

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So, originally whatever expression I have written, that one I can substitute here and then it will get this type of expression and OD get cancel everywhere you have see. So, it will become  $\cos i$  by  $\cos I$ ,  $\cos^2 i$  minus  $\cos^2 \phi$ ,  $\cos^2 i$  minus  $\cos^2 \phi$  into  $\gamma h \cos i$  ok.

So, this is the expression we are getting. So,  $\sigma_h$  lateral pressure equal to  $OA$  equal to  $\gamma h K_a$  ok. So,  $\gamma h$  was there and then we have multiplied by  $K_a$ . So, normally  $\gamma h$  and  $K_a$  that; that means,  $K_a$  is expression is what,  $K_a$  is actually will be ultimately we can say this  $K$  is nothing, but this  $\cos i$  minus under root  $\cos^2 i$  minus  $\cos^2 \phi$  and  $\cos i$  plus under root  $\cos^2 i$  minus  $\cos^2 \phi$ . So, this is actually your multiplied by  $\cos i$ . So, this is actually your coefficient of earth pressure for active condition, when there is a sloping backfill vertical wall, but sloping backfill then we get the  $K_a$  is this.

So,  $\phi$  to find out to find out find out the the lateral pressure what I will have to do, I need to know  $I$ ; that means, slope angle  $\phi$  then I will get the  $K_a$ , once you get the  $K_a$  the value  $K_a$  and then I will know the vertical stress  $\gamma h$ . So,  $\gamma h$  into  $K_a$  will be there and then the there will be acting parallel to this. So, I would draw a line like this. So, this is the way you have to do.

So, and then what is the thrust total of this. So, total actually we can see  $\gamma h k$ . So, you can draw like this diagram  $\gamma h k$ . So, half this is half multiplied by  $\gamma h K$  multiplied by  $h$ . So, half  $\gamma h^2 K_a$ . So,  $K_a$  is nothing, but this you can see here,  $p_a$  equal to half  $\gamma h^2 K_a$  or half  $\gamma h^2$  multiplied by this sorry this and this is nothing, but  $K_a$ .

So, this is actually we are we are getting when there is sloping surface this is the active earth pressure coefficient expression. So, this is little bigger expression because of the  $I$ , but interestingly if you put  $I$  equal to 0 here, then definitely we will get the same expression of  $K_a$  equal to  $1 - \sin \phi$  by  $1 + \sin \phi$ . You can put 0 here, this become 1, this become 1 and  $1 - \cos^2 \phi$ , then it will  $\sin^2 \phi$  under root; that means,  $\sin$ .

So,  $1 - \sin \phi$  (Refer Time: 12:58) you have come, then this will be again 1, this is plus and this will come again  $\sin \phi$ . So,  $1 - \sin$ . So, this, will again 1. So, this is the case

we are getting when the phi equal to when there is a inclination, but in this expression if you put I equal to 0 you will get the same value.

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**EARTH PRESSURE**

$$P_p = \frac{1}{2} \gamma h^2 K_p = \frac{1}{2} \gamma h^2 \left( \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}} \right) \cos i$$

$$K_p = \left( \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}} \right) = \frac{1 + \sin \phi}{1 - \sin \phi} \rightarrow K_p \text{ for level backfill.}$$

$i=0$

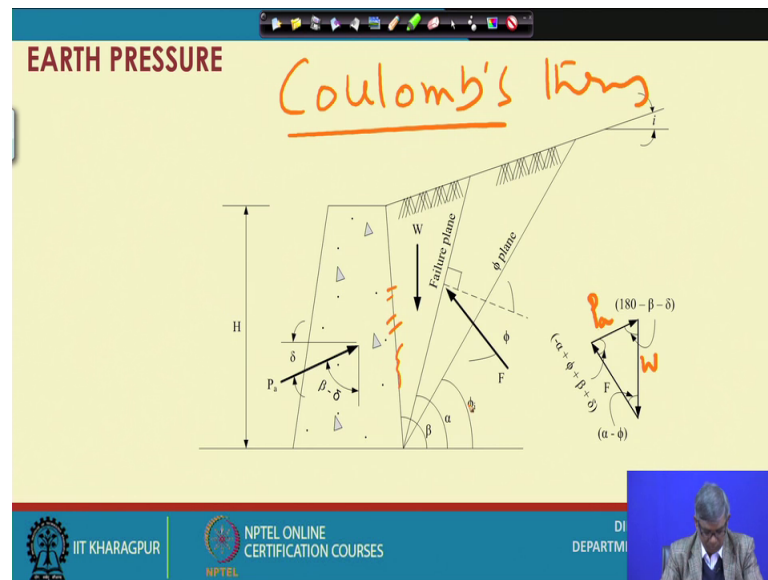
Similarly, for passive case also, we can find out  $p_p$ . Geometrically again we can do similarly you can find out the different component and then substitute and then the get the expression, but I am not doing this because that is not very important for at this level. So, what we need to know the active earth pressure coefficient or passive earth pressure coefficient by which we can do the design ok.

So,  $p_p$  you also equal to similarly  $\gamma h^2 K_p$ . So, half  $\gamma h^2$  multiplied by this expression and this is nothing, but  $K_p$  and these  $K_p$  you can see you have got a expression here again and this  $K_p$  we have now additionally  $i$  and  $\phi$  both are there and if I put  $i$  equal to 0 again here.  $i$  equal to 0 here again, you will get this expression will reduce to  $1 + \sin \phi$  by  $1 - \sin \phi$ , which is actually  $K_p$  for level backfill ok. So, this way we do it so; that means, we have got now Rankine's method by level surface, now you have got inclined surface, both active pressure and passive earth pressure.

Now, we will look at some other thing that is actually your, you can see this is actually Coulomb.



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We assume Rankine theory this is coulomb theory. Actually in Rankines theory before. So, for you have discussed Rankines theory, Rankines theory actually there are set of assumptions assumption are wall is vertical and there is no friction between the wall and the backfill and level backfill all those things are there, but coulomb theory is very much general ; that means, why very much general.

You can see the wall need not be a vertical. Most of the time in actual designing wall particularly there will be involve will little (Refer Time: 15:53) be vertical it will be inclined. So, inclined back trace of the wall can be considered. And friction between the wall and the back fill also can be on consider and any slope of the back fill can be consider. So, this is the generalized coulomb theory based on coulomb theory this is a generalized force diagram ok. What are the, this is the wall and what are the forces acting here and based on that a there are different force and then based on that you can draw the force polygon and this is suppose this is  $W$  actually and this is actually  $P_a$ , this is actually  $P_a$  on written here this is  $F$  and this is  $P_a$  and this is  $W$ .

Now, based on this force polygon the way we have done Rankines theory initially you can find out the  $P_a$  in terms of this angle, different angle and then this angle for any angle we can we have drawn this and then we can differentiate that that active earth pressure with respect to the angle  $\beta$  and set to 0 to find out the angle at which it is become it become optimum.

So, all those thing process process can be done, but they are very lengthy steps and I have just not do that ah. So, that is the procedure; that means, similar way the way we have done for Rankines case force for based on force polygon we will explains the  $P_a$  in terms of angle theta, beta, phi, delta, alpha all those thing and here actually the failure plane is important the failure plane angle is here actually alpha and these alpha will be changing. So, alpha can be optimized. So, the for a particle value of alpha we can found out, what is the value of active earth pressure.

So, that way if you do and then finally, we will end up with a end up with a value you can see now next slide.

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**EARTH PRESSURE**

$$P_a = \frac{1}{2} \gamma h^2 \left[ \frac{\csc \beta \sin(\beta - \phi)}{\sqrt{\sin(\beta + \delta)} + \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - i)}{\sin(\beta - i)}}} \right]^2 = \frac{1}{2} \gamma h^2 K_a$$

$$P_p = \frac{1}{2} \gamma h^2 \left[ \frac{\csc \beta \sin(\beta + \phi)}{\sqrt{\sin(\beta + \delta)} - \sqrt{\frac{\sin(\phi + \delta) \sin(\phi + i)}{\sin(\beta - i)}}} \right]^2 = \frac{1}{2} \gamma h^2 K_p$$

We can see that we will get a expression for  $P_a$  equal to half gamma h square equal to cosec beta sin beta minus phi under root these and these whole square equal to half gamma h square  $K_a$ . So, now, what is delta that is angle between the because of the friction the back fill and the wall because of that that delta and beta is the angle from the back fill inclination.

So, if this is a back fill is like this inclined, then indirect sorry back fills this surface of the wall and this is not 90 degree. So, what is the angle from horizontal it can be 90 degrees or it can be more than 90 degree that is shown here perhaps in the previous diagram. I will show you once again..

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**EARTH PRESSURE**

$$P_a = \frac{1}{2} \gamma h^2 \left[ \frac{\operatorname{cosec} \beta \sin(\beta - \phi)}{\sqrt{\sin(\beta + \delta) + \frac{\sin(\phi + \delta) \sin(\phi - i)}{\sin(\beta - i)}}} \right]^2 = \frac{1}{2} \gamma h^2 K_a$$

$$P_p = \frac{1}{2} \gamma h^2 \left[ \frac{\operatorname{cosec} \beta \sin(\beta + \phi)}{\sqrt{\sin(\beta + \delta) - \frac{\sin(\phi + \delta) \sin(\phi + i)}{\sin(\beta - i)}}} \right]^2 = \frac{1}{2} \gamma h^2 K_p$$

Handwritten notes:  $K_a = 1/K_p$ ,  $\frac{1}{2} \gamma h^2$  (circled), and a diagram of a soil wedge with height  $h$  and angle  $\delta$ .

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You can see beta is shown from here to here and phi is the angle of and the delta is the by friction between the back fill and the or in the wall and I is the inclination of the back fill and all those thing you can see with based on that we have got a expression for Pa and now half gamma h square, a normal vertical this is a half gamma h square. This is actually area of this equal to half gamma h square and to find out. So, if I this one expressed this problem will be that is a. So, I can explain this one by K.

So, K ultimately become, this K is nothing, but this expression. So, these expression is this K equal to entire bracket portion ok. So, similarly PP also can be obtained half gamma h square in multiplied by this. Only you can see it was plus here, it will be minus here. Everything is same, like previously you have got active earth passive earth pressure it will be inverse only, Ka equal to 1 by Kp similar thing here also. So, now this Kp. So, here actually Kp is equal to entire expression in the bracket.

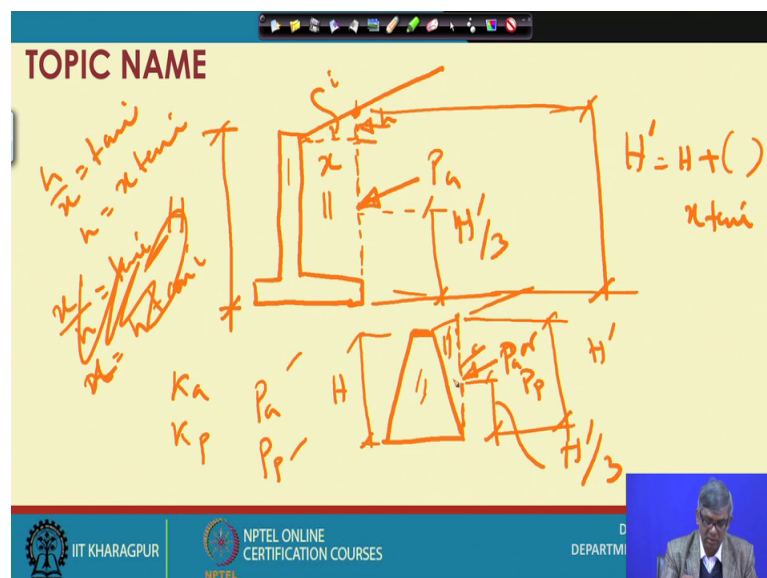
So, this is a very generalized expression and these expression little lengthy, but it is function of all angle of the wall face and the angle of internal friction and a friction angle between the back fill and the wall face and the back fill inclination So, this coulombs law, a coulombs theory actually, though the expression a little lengthy, but since it is it is a more practical; that means, all possible difficulties or possibilities in the field we experiment all are consider.

It that is friction between the back fill and surface is considered, the back fill can be inclined that is consider, wall face can be inclined that is considered. So, this 3 additional things actually considered here ah. So, because of that this is more actually actual actually, actual to the or close to the practice; however, most of the time because of this lengthy nature of the expression people do not use this. Most of the time people prefer to use the Rankines theory and in by this process actually when you calculate the value based on Rankines theory and by coulombs theory we will get a different approximately 10 to 15 percent.

So, sometimes to take that effect in consideration or in the design sometime they can they can use the Rankines theory and the pressure will be reduce by 10 to 15 percent actually, more accurate value you get from the coulombs theory that is lesser value we get whereas, in Rankines theory you get more value. So, 2 knowing this fact to make the design more appropriate. People use Rankines theory with some reduction 10 to 15 percent reduction. Otherwise one can do directly using coulombs theory.

So, these are actually by and large various aspect earth pressure calculation and you need to know.

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Next actually we have I just show 1 or 2 thing, that is suppose we have as I have mentioned that Rankines theory and coulombs theory, a Rankines theory suppose this is 1 wall and back fill is something like this. Though some modification will do in

calculation, what modification will do though wall height is this is  $h$  and that is by Rankine theory we will do calculation, what I what we do generally, we will do a imaginary line from here extend intersecting this and then we will consider these as a wall height. You consider these ends and the height of the wall and then we calculate  $K_a$  or  $K_p$  and then we find out active earth pressure  $P_a$  or  $P_p$  and then we will apply those here this is  $h$  and this is suppose  $h'$ .

So, ultimately we will apply  $P_a$  suppose somewhere here.  $P_a$  and this height will be equal to  $h'$  by 3, not  $h$  by 3. So, this is a simple modification we will do, while this type of problem will be there in the calculation we do this type of modification can be done.

Similarly, if there is a wall something like this. Actually then what will be the height of this ok. So, this height this  $h'$  will be  $h$  plus,  $h'$  will be  $h$  plus and if this angle is  $i$  and then this if you know these distance, then you can find out this ok. So, plus something, suppose this is  $\tan i$ . So, these  $X$  by  $h$ ,  $\tan i$ . So,  $h$  equal to  $x$  this is this is  $x$  this this is  $\tan i$ . So,  $x$  will be,  $x$  will be. So,  $x$   $x$  will be sorry  $x$  will be  $h \tan i$ . So, this will be  $h$  plus sorry  $ah$ . So, this is suppose this distance is  $x$  this distance is  $h$ . So,  $h$  by  $x$  equal to  $\tan i$ . So, I need to find out  $h$  equal to  $x \tan i$ .

So, if I know this distance from this  $h$  to this. So, this will be  $h$  plus  $x \tan i$ . So,  $x$  is here mention. So, so; that means,  $h'$  will be modified and based on that I can calculate  $P_a$   $P_a$  and  $P_p$  at that point of application will be 1 third height from this.

Similarly, wall gravity, (Refer Time: 27:53) sometimes line will be like this in that case, how to do this? Though this is for this case coulomb theory is more applicable. So, instead of using coulombs theory, what we can do we can do the vertical line from here we can draw a vertical line from here and then whatever height was here,  $h$  and these height can be obtain  $h'$ .

And then the force what the resultant  $P_a$  or  $P_p$  can be applied like this from  $h'$  by 3 distance and here additionally what we have to do while doing analysis these weight has to be taken here also these weight has to be. So, when we do the stability analysis and all, which actually is not in this course here actually our job is to find out only the pressure or thrust.

But when you do the stability analysis you have to do this type of modification. By doing this modification, when you are imagining the wall is here. Then the weight of this concrete weight of the soil, soil will be to be considered. So, this we have to consider similarly when you consider this one weight of this wall will be consider again weight of this portion also will be consider for the stability analysis. So, the stability analysis is out of the scope of this course. So, we will not discuss that. Only we will show how to find out active earth pressure, passive earth pressure by different methods.

So, this much here, let me stop here and, let me proceed for next.

Thank you.