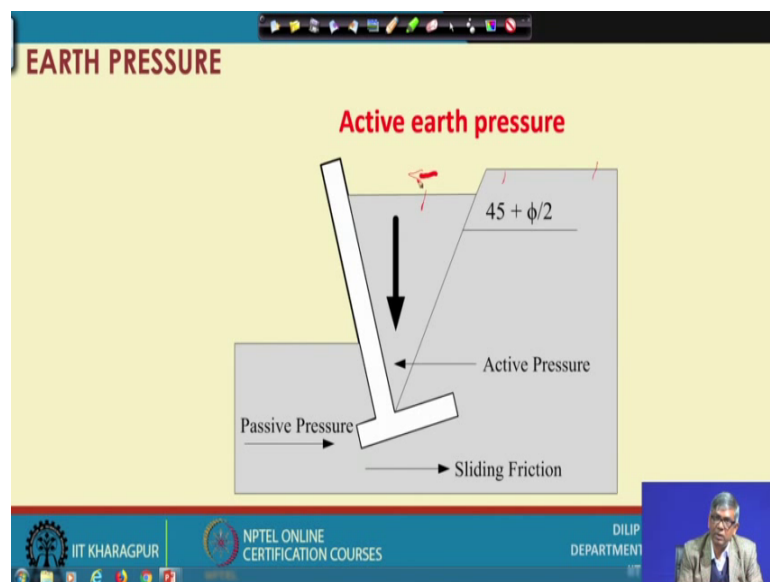


Soil Mechanics/ Geotechnical Engineering I
Prof. Dilip Kumar Baidya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture – 48
Earth Pressure (Contd.)

Let me continue with active and passive case this after completeness though I have been I have shown the before the active, how it is happening address to active, but I will take again that part and then passive then I will try to show active earth pressure coefficient passive earth pressure coefficient how to find out.

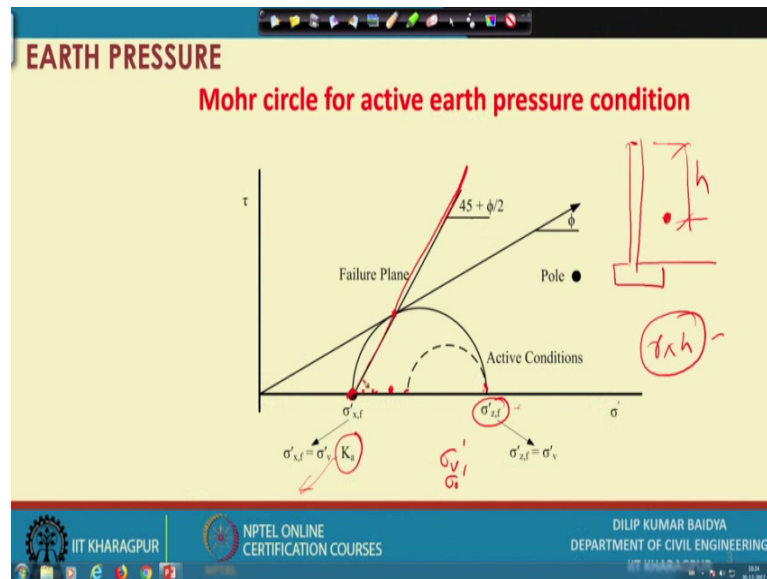
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So, in see this already I have shown once and you can see that that when there is a active earth pressure; that means, the entire soil will not move actually when this because of the weight this wall will move this direction and then how much soil will go, whether soil up from here will go, from here will go from here you go. So, that actually there is a limit.

So, that limit is actually from here if you draw a line with 45 degree plus phi by 2 this is a line. So, beyond that soil will not will be intact and this soil mass will move along with the movement of the wall and this is active case and when the movement will be there slowly and at certain movement it will leads to the minimum value and beyond that it will be considered as a failure, that minimum value is called active earth pressure.

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And that one if you if you see in the Mohr circle you can see that this was either sigma z dash or sigma v dash anything you can draw sigma naught dash anything you can write all are same. So, the how to find out this one suppose this is the wall and. So, this is the wall and I have find out at this point. So, initially I have to find out gamma times. So, gamma times h that will be your. So, if there is a water table etcetera you have to subtract gamma into gamma w into h.

So, that; that means, you have to by this way you calculate sigma v dash at this point and then slowly the sigma v dash will be reduced and it will become finally, in general sigma h will be is the smaller that how to find out that initially rest coordination it is cannot into cannot multiplied by this, but slowly because of this movement this value will be reduces. So, like this as I have mentioned before. So, this value will be coming to this, this was sigma v this sigma h initially it comes here then comes here then comes in like that at failure suppose this is this here.

So, this is minimum value this is maximum value. So, from here I can and they are principal place principal stress. So, through this; that means, I can draw a Mohr circle and since this is a failure minimum value and this is a point of tangent. So, joining these two point that gives you the failure plane and if you geometrically.

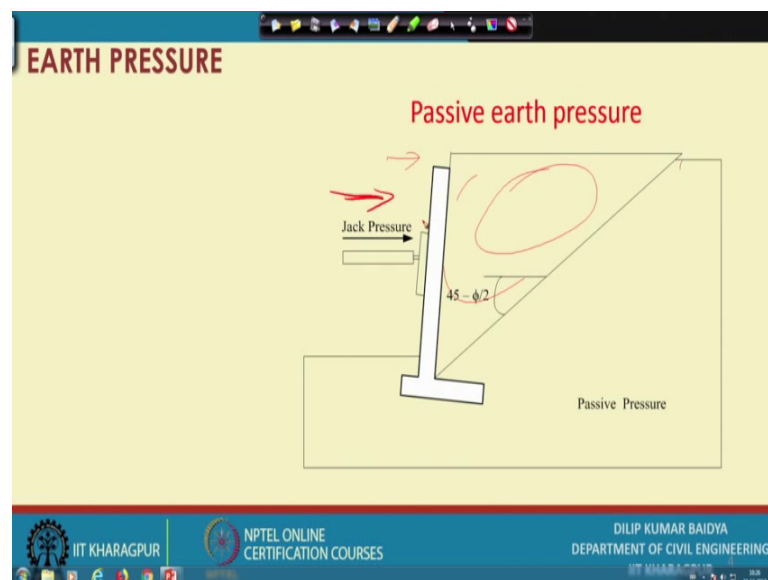
If you see that this will come 45 degree plus phi by 2 I will show also that one later on. So, this value will be how much it will be whatever sigma v dash we had there sigma v dash multiplied by k times k at multiplied by ka multiplied by ka, ka is actually active

earth pressure coefficient. So, active earth pressure coefficient like earth pressure raised there was a k naught, similarly active earth pressure coefficient k is there. So, you have to find out that so.

How to find out we will discuss that for the time being I will just mention here that this σ_h are lateral pressure on the wall will be k times σ_v whatever σ_v was there if you multiply by k then you will get a lateral stress that is active earth pressure. So, at this point and this is the failure plane.

Similarly, if you have passive suppose this is a wall.

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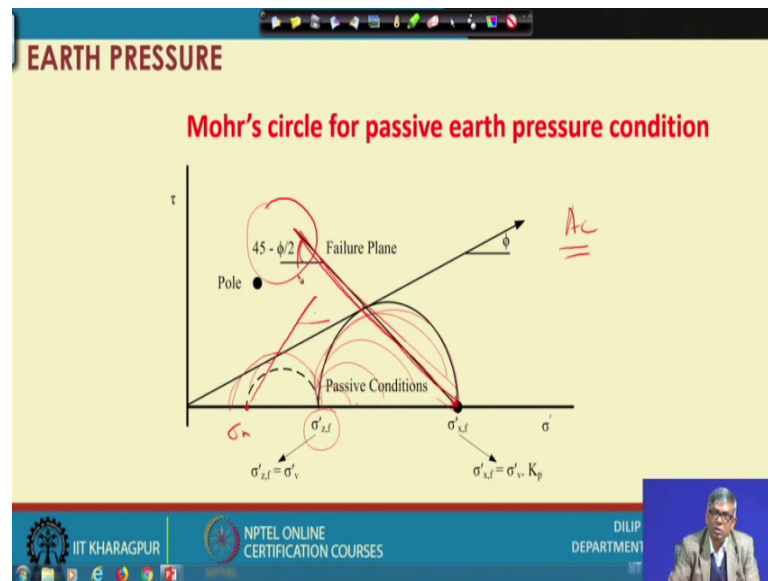
And because of by some mechanism suppose we could push the wall sorry, we can push the wall this direction or you can push the wall this direction by that arrangement and you can see again when if you push the wall like this like previous case whether entire zone of soil will be effected or whether particular zone of soil will be effected.

So, soil within these zone only will be effected and this zone will be how to determine you can draw a line from here 45 degrees minus ϕ by 2 with horizontal and then this complete in this zone this is actually passive zone this soil will only move along with the wall and this you can see and because of these the pressure on the wall will be since it is coming this direction pressure on the wall will be increasing slowly and at some time it

will reach to the maximum if you apply further more than that then the wall will may will wall may fail or collapse.

So, that is the end where actually reaching the maximum value that is actually called passive pressure if I see corresponding Mohr circle for this you can see here.

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You can see initially your sigma v was here and sigma this is sigma h was there cannot condition and when it was active case the circle actually moved in the direction and finally, it become tangents is like this.

Whereas whether this is a passive case this pressure will be increasing ok. So, it will be now it will become this. So, like this, like this, like this, like this, like this finally, it reach here. So, this is the failure point actually. So, and this is actually in the passive case horizontal pressure is a major principle plane major principle planes active case horizontal pressure is a minor pressure principles says.

Whereas in passive case horizontal pressure is the major principle say because this is larger, you can see from here to here this is larger. So, it become. So, now, from these if I join envelop tangent of in envelop to the circle and that gives you the failure plane and geometrically again you can find out that this angle is 45 degrees minus phi by 2. So, this is the thing to be known ah; that means, addressed condition active condition or passive condition how with respect to wall movement how it is and again with respect to Mohr

circle how it is. Mohr circle for active condition will be Mohr circle much below the envelop.

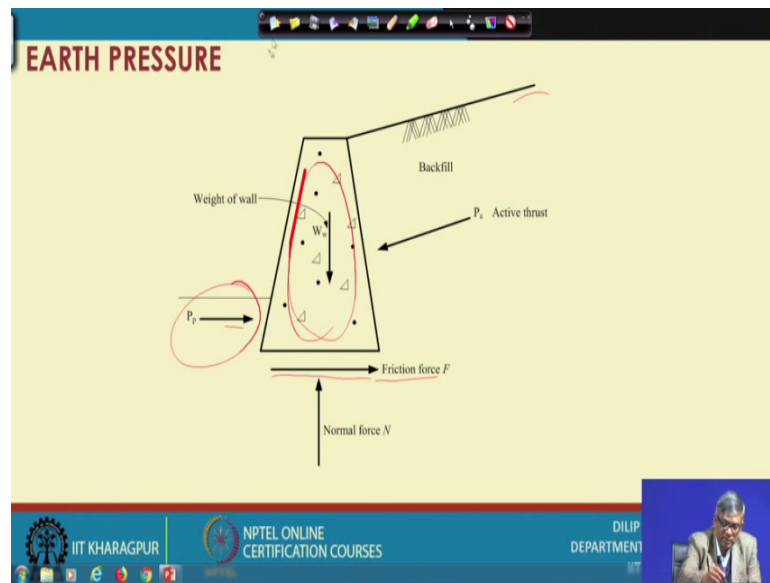
Whereas, in active case Mohr circle will be just tangent to the envelop and originally whatever vertical stress was there horizontal stress was there that will be reduced and finally, it will reduced to a minimum value and that is because equal to active pressure and that that is Mohr circle will be tangential to them that mean circle originally that addressed condition what was the circle position that will move towards left.

And in passive condition the wall actually with respect to wall movement wall will be moving towards the backfill and then with this your pressure on the wall will be increasing and as a result your horizontal pressure or lateral pressure will become larger than the vertical pressure and your lateral pressure will become the measure principle stress.

And if you draw the finally, Mohr circle for corresponding to passive case that again will be tangential to the envelop and from the major principle 0.2 the tangent to the envelop of the circle; if draw connect that line give you the failure plane and that failure plane inclined to the horizontal is 45° minus ϕ by 2. So, that is the thing to be remember.

So, when you I have done for this 1. So, you have done this way that was 45° plus ϕ by 2 what I am doing 45° minus ϕ by 2. So, these two things very very important to be remember and many frequently it will be required in solving problem or sometimes some small short time short answer type question it would be important.

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Now, suppose I have mentioned that gravity retaining wall and this is suppose a gravity retaining wall and you can see the different forces acting on the wall the gravity retaining wall major part, there is a gravity weight will be there that will be acting w here and in addition to that it will have backfill is this side. So, from backfill there will be active pressure thrust will be there on this and this two side since wall is tendency to move this direction.

So, it is towards going this, so it will give you this portion the passive pressure to make a you can say conservative design sometime this can be ignored, but if you want to do accurate design or calculation then this should be there this is active this is passive.

And since the there is a huge backfill here. So, there will be tendency on the wall if you also two slide along this. So, if it is slide then the friction will develop between the soil and the base of the wall. So, that is a frictional force to be shown how much to be frictional force that can be calculated we will discuss later on and then; obviously, there will be reacting force the entire thing there will be vertical reaction to this. So, all forces are there once you know these forces do some analysis then one can find out what is active what is passive and all.

So, I will do that one by one suppose you can see.

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EARTH PRESSURE

Assumptions:

- Frictional force between backfill and retaining wall is assumed to be negligible
- The wall is straight and the surface of the backfill is horizontal
- The backfill is a homogeneous granular material
- The failure surface is assumed to be plane

Movement of wall is away from backfill

a

I will consider the wall suppose moving this way and you can see we can take a few make a few assumptions that frictional force between the backfill and retaining wall are assumed to be negligible backfill and retaining wall backfill and retaining wall. this is the backfill and retaining wall the frictional force to be to be reduce this ignored.

The wall is straight and the surface of the backfill is the d is not there is horizontal; that means, your wall can be like this wall can be like this and backfill can be like this. So, this is not the case you are considering wall suppose like this. So, backfill is like this.

The backfill is homogeneous granular material; that means, again here we have consider granular material. If it is granular material all the ϕ will be there and if it is an not granular soil then it will have c also. So, if it is c is there calculation everything will be different. So, we will see later on and right now we are considering only granular material and homogeneous.

And failure surface is assumed to be the plane; that means, this plane you planed taken as a plane it is generally little come with the research you could spend but for approximate calculation we do the straight.

If you do this and you see if this is the wall some imaginary wall is there then p_a will be here w will be here r will be here τ will be here this will be angle β and. So, at, so, this angle β basically at active case it will 45° plus ϕ by 2 that is there, but for

the timing I take I took a any beta and I consider these weights and then I got the force diagram force polygon here ok.

So, I for a particular beta I got this and now I can optimize for beta to find out at what value of beta the force active force or active thrust become minimum, if you can find out that that is the work actually we went.

So, initially what I done what we have done we have taken a beta value any value and if it is beta that this one $H \cot \beta$ this is w this is p_a these are all shown and based of these this is the force polygon if you have this force polygon and then by doing earth pressure will be P_a will be sorry, at present P_a will be $w \tan \beta - \phi$.

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EARTH PRESSURE

$$P_a = W \tan(\beta - \phi)$$

$$W = \frac{1}{2} \gamma H (H \cot \beta) = \frac{1}{2} \gamma H^2 \cot \beta$$

$$P_a = \frac{1}{2} \gamma H^2 \cot \beta \tan(\beta - \phi)$$

$$\frac{\partial P_a}{\partial \beta} = 0 \quad \boxed{2\beta - \phi = 90}$$

$$\beta_{cr} = 45 + \frac{\phi}{2}$$

$$P_a = \frac{1}{2} \gamma H^2 \tan^2 \left(45 - \frac{\phi}{2} \right)$$

$$P_a = \frac{1}{2} \gamma H^2 K_a$$

$$K_a = \tan^2 \left(45 - \frac{\phi}{2} \right) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Diagram of a soil wedge with height H and angle beta at the base. The weight W acts vertically downwards from the centroid. The active earth pressure Pa acts horizontally to the left from the base.

So, from that you can see for the polygon you can, from the polygon from the polygon p_a actually P_a we can find out in terms of w beta phi. So, that is a thing done you can see p_a equal to $w \tan \beta - \phi$. Again w you know we can calculate from geometry this is half gamma h the soil actually volume of soil and then multiplied by gamma.

So, this was the soil volumes above this is $H^2 \cot \beta$ this is H and then half gamma, half into H into $H \cot \beta$ into gamma that become weight. So, this is the weight and this weight if I substitute here then p_a become this is the expression for P_a which is having beta and phi.

Now, I can differentiate this expression, the p_a with respect to β so; that means, and set to 0; that means that value of β we supposed to get the minimum. So, if you do that $\frac{dP}{d\beta}$ and then differentiate this 1 and set to 0 and then simplify this is not. So, easy of course, we have to do. So, very substitution is that and finally, we will get a result is equal to $2\beta - \phi = 90^\circ$.

If you get this $2\beta - \phi = 90^\circ$ then from there actually I can get that is the; that means, by differentiating and setting to 0 that way we are getting a angle critical angle, be initial you have taken a value β you have taken a β . Now, that β for which it is giving minimum value that is I am defining as β_{cr} . So, that is actually $45^\circ + \frac{\phi}{2}$.

So, what I have shown initially without understanding what will be the plane direction I have just mentioned $45^\circ + \frac{\phi}{2}$ now we have seen call to got mathematically this is $45^\circ + \frac{\phi}{2}$.

Now, this $\beta = 45^\circ + \frac{\phi}{2}$ if you are substitute in this then your p_a will become $\frac{1}{2} \gamma h \tan^2 (45^\circ - \frac{\phi}{2})$ and this is actually can be retained $\frac{1}{2} \gamma h$ square into K . So, actually if there is a wall and if there is a backfill and the soil has been γ then soil pressure diagram with depth will be like this.

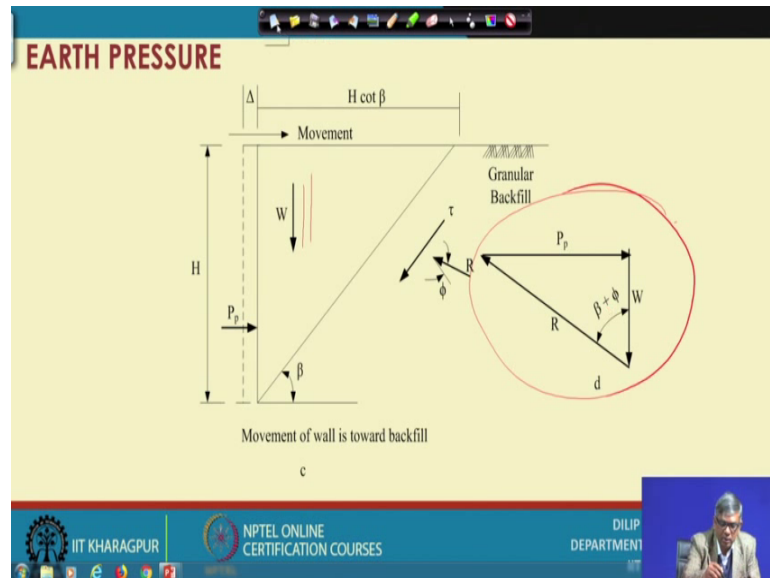
So, area of this diagram will be half into H into γH is it not half H into γH and if you take we need width you need to width. So, that become the your half γH square become the pressure thrust and you can see if I do not do anything then I am getting half γH square. So, half γH square is there and additionally I am getting $\tan^2 (45^\circ - \frac{\phi}{2})$ whatever $\frac{\phi}{2}$ additional things we are getting. So, that we are denoting as a K_a active earth pressure coefficient.

So, K_a will be equal to $\tan^2 (45^\circ - \frac{\phi}{2})$ or it is $1 - \sin \phi$ by $1 + \sin \phi$. So, this is the thing you have to remember, always it will be required. So, active earth pressure coefficient, either \tan from $\tan^2 (45^\circ - \frac{\phi}{2})$ or \sin from that is $1 - \sin \phi$ by $1 + \sin \phi$. So, either way one can remember.

So, we got at this level what is the K_a and what is the p_a so; that means, if there is a wall what is the thrust because of the soil pressure and amount and how to find out K_a from

this if you know the ϕ of soil then you can find out k and from there you can find out the active thrust.

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Similarly, you can see if the wall is moving towards the backfills if the wall was here sorry, the wall was it is moving this direction it was here and then moving the direction. So, pressure will be p_p and vertical pressure is acting here p_p is acting here and then in this plane there is shear acting, then resultant r is acting and then what is the angle everything is known from the mechanics. We can find out this the we can find out this post polygon what are the force acting here taking that one you can draw this post polygon.

Now, from this post polygon you can find out also p_p in terms of w and β plus ϕ . So, if you see that similarly as we have done last one you can see you can see that p_p equal to p_p equal to $w \tan \beta + \phi$ and w again we know half γ is a $\cot \beta$.

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EARTH PRESSURE

$$P_p = W \tan(\beta + \phi)$$

$$W = \frac{1}{2} \gamma H (H \cot \beta) = \frac{1}{2} \gamma H^2 \cot \beta$$

$$P_p = \frac{1}{2} \gamma H^2 \cot \beta \tan(\beta + \phi)$$

$$\frac{\partial P_p}{\partial \beta} = 0 \parallel \boxed{\phi + 2\beta = 90}$$

$$\beta_{cr} = 45 - \frac{\phi}{2}$$

$$P_p = \frac{1}{2} \gamma H^2 \tan^2 \left(45 + \frac{\phi}{2} \right) = \frac{1}{2} \gamma H^2 K_p$$

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Diagram: A right-angled triangle representing a soil mass with height H, base H cot beta, and slope angle beta. The resultant force P_p acts at a distance of H/3 from the base.

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So, that is also geometry show this is you want $H \cot \beta$ and this is H .

So; obviously, this angle is taken β like active case we know that finally, β critical is 45 degree plus ϕ by 2, but initially what we have to taken we have taken as β is and for a particular value of β we consider weight and then I concerned the force equilibrium then will polygon we have done then I have find out the expression for a passive pressure and then W I have I can calculate and you substitute then you will get the p_p .

This is the expression for p_p which is having half $\gamma H \cot \beta \tan \phi$; so, β and ϕ . So, β general is not known is it not ϕ is known for soil, but β is unknown. So, that is why we need to find out the value of β . So, similar to that whatever we have done in the previous case $\frac{\partial p_p}{\partial \beta}$ by $\frac{\partial}{\partial \beta}$, that mean if you differentiate this one with respect to β and set to 0; that means, we have optimizing it and then after these days and several substitution, modification, simplification then something cannot be 0 etcetera all those things eliminate.

And finally, you will get a relationship between ϕ β like this. So, like we have got ϕ β they are also here also you have got the relationship between ϕ and β like this. So, this from this I can get whatever β I get that can let me consider as a β critical that β critical will be will be 45 degrees minus ϕ by 2 ok.

So, as I have shown that at the beginning that when wall moves towards the backfill we have shown that that failure plane will make an angle with equal to 45° minus ϕ by 2 without knowing just we have mentioned as a fact, but now the best of this calculation we have got that this angle is 45° minus ϕ by 2.

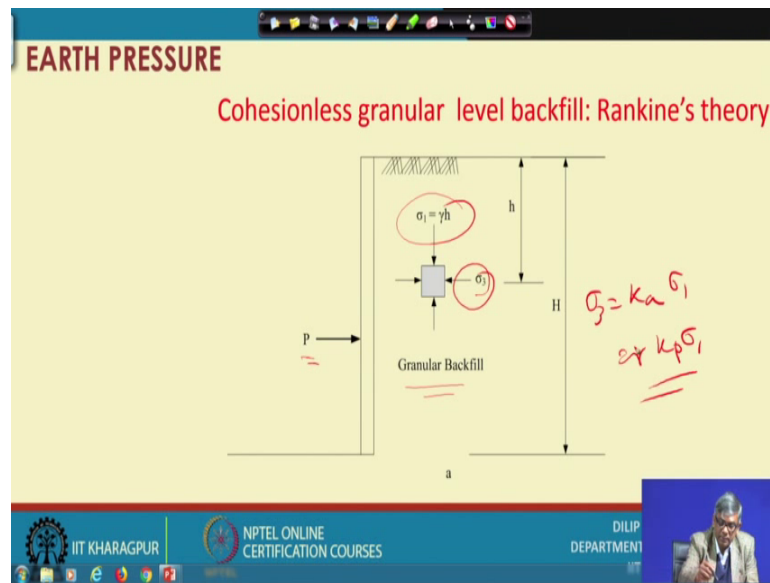
Now, this beta value if I substitute in this then and simplified then we will get p_p equal to this and. So, you can see that p_p equal to the half gamma. So, this is and then similarly again if I draw the pressure diagram and this is h and. So, this is gamma h . So, half gamma h half gamma h into h this is the if I do not do anything this is the thrust, but this is multiplied with a factor that is called K_p . So, this K_p is now K_p equal to $\tan^2(45^\circ + \phi/2)$ or it is $1 + \sin \phi$ by $1 - \sin \phi$.

And you can see; that means, if you know the ϕ value then we can find out the K_p and after knowing the K_p and I am knowing this half gamma h square you can find out the active passive thrust.

Now, you can see that what is the relationship between k_a and K_p you can see k_a will be equal to $1/K_p$. So, if you soil have a ϕ equal to 30° then you have k_a equal to $1/3$ and K_p will be 3 if the ϕ of the soil is 30° then if I put k_a equal to $1/3$ minus $\sin \phi$ by $1 + \sin \phi$ if I do then it gives you it gives you k_a equal to $1/3$ or it is point 3, 3, 3 and if I find out K_p value then that will become 3 term that we inverse of that.

So; that means, from the coefficient of earth pressure active and passive this is k_p ; obviously, you can see that is a very large value. So, earth pressure at passive curve desired only pressure is very high ok so, this.

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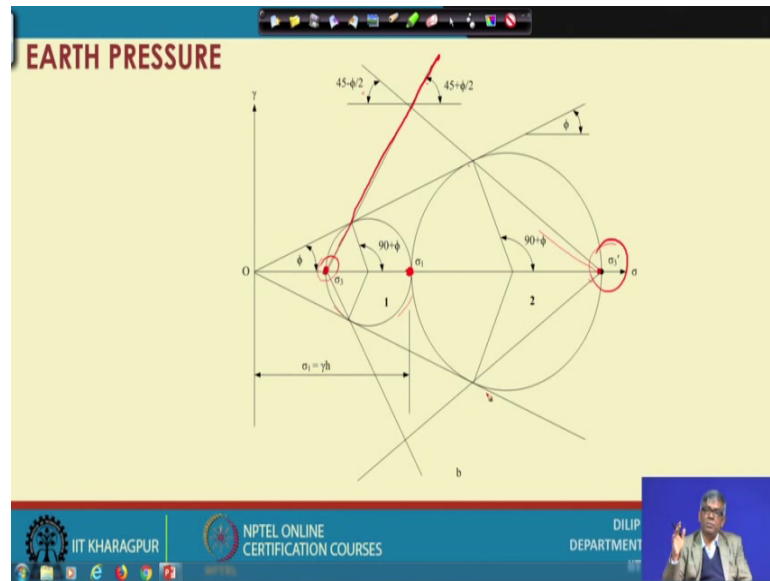


So, now you can see this is the one way we have tried to geometrically force polygon.

We have done and then finally, introducing K_a K_p a name we have got, but based on Mohr circle also applying an Rankine's theory we can also get this one. So, suppose we are we are concerning a wall here p active here granular backfill at a H actually σ_1 is γh and σ_3 will be it is depending upon the whether it is active or passive. So, σ_3 will be either $K_a \sigma_1$ or $K_p \sigma_1$ either this or that ok.

So, the I will show you the same diagram both are happening. So, it go to next slide you can see this.

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You can see this initially was sigma one here initially was sigma one here and then it become sigma three become here and this become the Mohr circle and this become the failure plane and this is 45 degree plus phi by 2. So, geometrically you can find out and this was sigma 1 and for passive case the this becomes sigma 3. So, this is failure. So, from here if you join these were and if you get this will be 45 degree minus phi by 2.

So, this is the 1 now I separately I have shown. So, if I draw a envelop phi both sides of both the circle will be between this and so this will be actually active and this is passive. So, now, the this is some observation because you can see the sigma 1 whatever value sigma 3 is larger. So, these become major actually in passive case and whereas, in active case this is by the for passive active case.

So, this observation now taking this so, I will now next discuss about Rankine's theory based on the Rankine's theory how to find out the earth pressure both for granular backfill and then level ground surface with granular backfill with inclined surface, then if there is a c phi soil also how to find out. So, almost thing 1 by 1 we will proceed. So, right now I will just stop here maybe I will continue in the next lecture.

Thank you.