

Soil Mechanics / Geotechnical Engineering I
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Lecture - 44
Compressibility and Settlement of Soil




Good morning, once again welcome to this course; and perhaps you all remember I have completed compressibility and settlement. And all aspect of the compressibility and time rate of settlement division relevant formula all I have discussed. And I will now try to show the application; that means, how different types of calculation you have to do for finding out the amount of expected settlement particularly here consolidation settlement.

And at the same time, what is the time required to complete the consolidation settlement. And few small, small example perhaps I have taken. And now again and there are different ways actually the question can come and particularly in the competitive exam like GATE and UPSC and many other exam, so I will try to spend about hour about an hour for is to show some different applications on consolidation, competition and time rate of settlement.

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COMPRESSIBILITY OF SOILS

During a pressure increment a consolidation test sample attained 25 per cent primary consolidation in 5 minutes with a mean thickness of 18 mm. How long would it take a 20 m thick layer of the same soil to reach the same degree of consolidation if (i) the layer was drained on both surfaces and (ii) it was drained on the top surface only?

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So, I will start with the first one. This is the problem description. During a pressure increment a consolidation test sample attained 25 percent primary consolidation in 5 minutes with a mean thickness of 18 millimeter. How long would it take a 20-meter thick

layer of the same soil to reach the same degree of consolidation, if the layer was drained on both surfaces and it was drained on the top surface only.

So, this is the problem perhaps I have discussed before also that there are two different ways it can be solved. And if I if you can take one consideration then very quickly you can do, and if you do not even consider that particular consideration or observation, then also you can do it, but it will be little lengthy. So, I will do both ways.

And here actually what is the important point you have to observe here that sample collected from the clay and its consolidation is done and it is it took 25 percent consolidation to attained 25 percent consolidation, it took 5 minutes. And generally consolidation settlement when you do in the lab which is a draining in both sides and so your thickness will be just half of that. And now in the actual field the clay is of 20 meter thick.

And it can have two options either drain only at the top and drained at both sides. So, you have to find out now the same amount of degree of a same degree of consolidation to achieve same degree of consolidation in the field of 20 meter thick clay layer when drained in one side and when drained in both side how long it will take so that is the question.

Now, here what are the observations you have to note that is actually see the soil is same. So, in both case both cases your C_v will be same suppose to be because it is the sample is collected from the clay layer. So, it is it should be representative value as same C_v will be there. And at the same time, both soil is achieving same degrees of consolidation that means, suppose 25 percent. So, time factor t_{25} for field, and t_{25} for the this two time factor will be equal. If the time factors are equal and time factors are equated or correlated or is related to C_v time required to achieve that much of consolidation and thickness of the drainage path.

So, you can express t_1 for t_1 for lab, and t_1 for or t_{four} lab and t for field; both we can express and both the t 's is equal and then if you equate them again we will see C_v will be also equal. So, if you take that observation then ultimately we will have 4 variables the time for lab tests and time and thickness for the drainage path for a lab and time for field to reach that much of consolidation and thickness of the drainage path in the field.

So, this four will be there; out of four, three will be known, one will be unknown. So, this can be done very easily.

If you note this observation then only this is the way very easily you can do; otherwise also it can be done I will show both way that means if I do not consider that based on laboratory information I will find out what is the C_v value. And use C_v value and find out what is the time required for the laboratory field soil. So, both way I will do the so first step one.

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The image shows a handwritten derivation on a whiteboard. At the top, two equations are written: $T = \frac{C_v t_L}{H_L^2}$ and $T = \frac{C_v t_f}{H_f^2}$. These are equated to form $\frac{C_v t_L}{H_L^2} = \frac{C_v t_f}{H_f^2}$, which simplifies to $\frac{t_L}{H_L^2} = \frac{t_f}{H_f^2}$. A boxed version of this simplified equation is shown. Below this, values are substituted: $H_L = \frac{18 \text{ mm}}{2} = 9 \text{ mm}$ and $H_f = 20 \text{ m} = 20000 \text{ mm}$. The final calculation for t_f is shown as $t_f = \frac{t_L \times H_f^2}{H_L^2} = \frac{3 \times 20000^2}{9^2} \text{ min} = \frac{3 \times 2000^2 \times 1 \times 1 \times 1}{9^2 \times 60 \times 24 \times 365} = 4778$. There are also handwritten notes: '① drainage at the surface' and 'one way drainage'.

So, I will take the first one by first method that is your you can see as I have told you the laboratory sample suppose I will say this is T for laboratory and that will be equal to $C_v t$ by H square. And for field also similarly suppose this is also T equal to $C_v t$, so this is about t lab L suppose and this is suppose t field and this is suppose H field square and H lab square.

So, now, this since we have mentioned their same degree of consolidation, how much time is required. Here actual time required is mentioned, and here time is not known. So, these two are equal then it will have $C_v t_l$ divided by H_l square equal to $C_v t_f$ by H_f square. So, again you can see C_v both the same soil the C_v will be equal that gives you t_l by H_l square equal to t_f by H_f square. So, actually t by H square equal to t_l by H_l square equal to this is the when this type of problem, this can be remembered as a formula if you can remember this formula by simple calculation one can do.

And you can see so you have your laboratory average thickness is 18 millimeter. So, H_1 generally in the lab it is not mentioned; otherwise generally it is mentioned; otherwise also you can assume H_1 will be since 18 millimeter is there. So, it will be 18 by 2 millimeter. And our problem is given for two first one is drainage only at surface drainage at the surface. So, H_f will be equal to 20 meter itself. And since it is in millimeter, so it is 9 millimeter. So, here also I may express in 20,000 millimeter ok.

So, then your calculation will be that is t_{field} will be required t_{field} will be equal to t_1 into H_f square divided by H_1 square. So, this will be t_1 is how much it is 5 minutes 5 minutes and that will be equal to drain in when draining in one side it will be same. So, 20,000 square, and this is actually 9 square. So, this actually these units become it, it become minute. So, this again I can divide 5 into 2 20,000 square divided by 9 square into 1 by 60 then become hour then it become 1 by 24, it will become day; then 1 by 365 that become year.

So, if I do this calculation, it comes finally, 47 years. So, if I take drain at this surface only one side one-way drainage. I do not know I will see the problem is given whether reverse that to be noted. If it this is specifically I have mentioned when drainage is one-side, this is the answer.

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drainage in both side

20
 $\frac{20}{2} = 10m$

$t_c = T_f$

$$\frac{t_c}{H_1^2} = \frac{t_f}{H_f^2}$$

$$t_f = \frac{t_c \times H_f^2}{H_1^2} = \frac{5 \times 10000}{9} \text{ min}$$

$$= \frac{5 \times 10000}{9} \times \frac{1}{60} \times \frac{1}{24} \times \frac{1}{365} \text{ yr}$$

$= 11.7 \text{ yrs.}$

Now, I will say drainage in both side in both side that means in the clay layer is here and this is draining layer this is also draining layer and this is 20. So, your drainage path will

be half of this. So, this will be 20 by 2 equal to 10 meter. And your t_{lab} by H_{lab} square equal to t_{field} by H_{field} square. So, t_{field} will be equal to t_{lab} into H_{lab} square by H_{field} square. So, this will be equal to 5 minutes, and this will be 10000 10 meter means 10,000 square divided by H_{field} will be 9 millimeter. So, this will be actually in minute.

Now, again I can do 5 into 10,000 square divided by 9 square into 1 by 60 that become hour, then 1 by 24 become day, 1 by 365 that become year. So, if I calculate this one, you will get the answer as 11.7 years. So, you can see now difference, the 20 meter thick clay layer if we drained in one-side then it takes 47 years; and if the drain is both side, it is taking only 11 years.

So, this is the way and this problem I have shown here in a when the sample is collected from a lab representative sample, and tested. And a particular degree of settlement time is known, and then if it is asked same to achieve same degree of consolidation the field for that much think of layer then what observation we are taking that time factor for both case t_{lab} equal to t_{field} this is the one if I do. And then from, that I will get this one and if I take this one or calculation is one-step. But if I do not do this then alternative actually there is a alternative can be is there you can do this way.

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$U = 25\%$
 $T_{1/2} = \frac{\pi}{4} \left(\frac{U}{100} \right)^2 = \frac{\pi}{4} \times .25^2 = 0.049$
 $T = \frac{C_v t}{H^2}$
 $0.49 = \frac{C_v \times 5}{9^2}$
 $H = \frac{18}{2} = 9m$
 $C_v = \frac{0.49 \times 9^2}{5} = 0.7938 \text{ min}$
 $t = \frac{0.049 \times 20000}{0.7938} = 6172839 \text{ min}$
 $= 47 \text{ yrs}$

So, if the laboratory actually 5 minutes for achieving 25 percent degree of consolidation so U here 25 percent ok. Then T_{25} will be equal to π by 4 and that is actually π by 4 U

by 100 square, so that means, π by 4 into 0.25 square and that gives you 0.049. So, if I get I know that T equal to $C_v t$ by H square and H for the lab actually 18 by 2 that is 9 millimeter.

So, if I put here, then you will get time factor is 0.49. So, let me 0.49 will be equal to C_v into t is a 5 minutes, and here actually H equal to 9 millimeter square. So, from here actually we can get C_v equal to 0.49 into 9 square divided by 5, so that gives you zero point not. So, this is 0.7938, it is millimeter square per minute. This is the C_v value, I got. From the information given from the laboratory test results I got the C_v .

And now in the field, in the field, case 1 that is drained one side only suppose drained one side only so suppose the clay layer is here and this is impervious, impervious, and this is a 20 meter. So, H becomes 20 meter. So, I can find out again they are also same thing I will I will apply t equal to 0.049 equal to C_v , and t suppose unknown divided by H equal to 20,000 square. So, from here, I can get t equal to 0.049 into 20,000 square divided by C_v that is 0.7938.

So, if I do these, then I will get answer we will get in minute that is actually 6172839 minutes. And if I convert in year it become 11.7 years. So, this is the way and where actually you mostly do mistake here actually a laboratory sample thickness is given in millimeter, and the actual field soil thickness is given in millimeter, and coefficient of consolidation is also millimeter square millimeter.

So, two main dimensionally correct this meter has to be converted in millimeter, so that is what if you do not do this conversion, if you write 20 here then everything will be spoiled. So, you will not get the correct answer. So, this is the one case that will drain one side.

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(1) drained in both side

Diagram: A rectangle with '00' at the top and bottom corners, and '20' on the left and right sides. A vertical line in the center is labeled '20 = 10m'.

$$0.49 = \frac{0.7938 \times t}{10,000}$$

$$t = \frac{0.049 \times 10,000}{0.7938} \text{ min}$$

$$= \underline{11.7 \text{ yrs.}}$$

But if I want to do next one drained in both side. So, drained in both side that means, the same one draining here, draining here, and this is 20 meter, so your edge become 20 by 2 actually 10 meter. So, here also I can do 0.49, 0.049 equal to C_v actually 0.7938 into t is unknown and H is here 10,000 square. So, it gives you t equal to 0.049 into 10,000 multiplied 10,000 square divided by 0.7938, this will be in minute.

And if you convert into year then it will be 47 years it will be sorry this is I have done drained both sides it, it will be 11.7 years, it will be 11.7 years. And if I the same whatever I have got in drained in both side drained in one side, this is drained both side 11.7 year and drained in one side that will be 47 years I do not know whether I have done mistake there let me see no this is wrongly I have done this is actually drained in one side this calculation I could do it will be it will coming 47 years.

So, this is the way actually you to do so; that means, two alternative one is if I make the observation and I take t for both the equal then I will get t by a square equal to t by a square. So, t_{lab} by H_{lab} square equal to t_{field} by H_{field} square that relation if I use that is one step calculation. And if I do not consider that I will just from the laboratory information I will get the, I will get this um C_v because degree of consolidation is known. So, T versus U actually there is a chart available you can refer when U is 25 percent what is T , you can collect from the chart otherwise this is the equation available we can always use for calculating the t this equation is valid for T U up to 60 percent. So,

since it is 25 percent, I have used this. And if the you become more than 60 percent, there is another equation that has not come, so I have not used that one.

So, based on that, I get the C_v value. And once I get C_v value then I go I will apply to the field calculation that I know the 25 percent degree of consolidation has to be achieved. So, corresponding to that 25 percent of degree of consolidation time factor is these this is known and then $C_v T$ by thickness square. So, T is the only thing unknown from here I can find out. So, this is the way one can do and. So, this is the whole drained in both side is 11 years.

So, like that I can do the problem in both ways. So, either you do the observation and use the formula otherwise I can do based on laboratory information find out the soil parameter and that soil parameter applied to calculate the field time consolidation. So, this is the way can be done ok.

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COMPRESSIBILITY OF SOILS

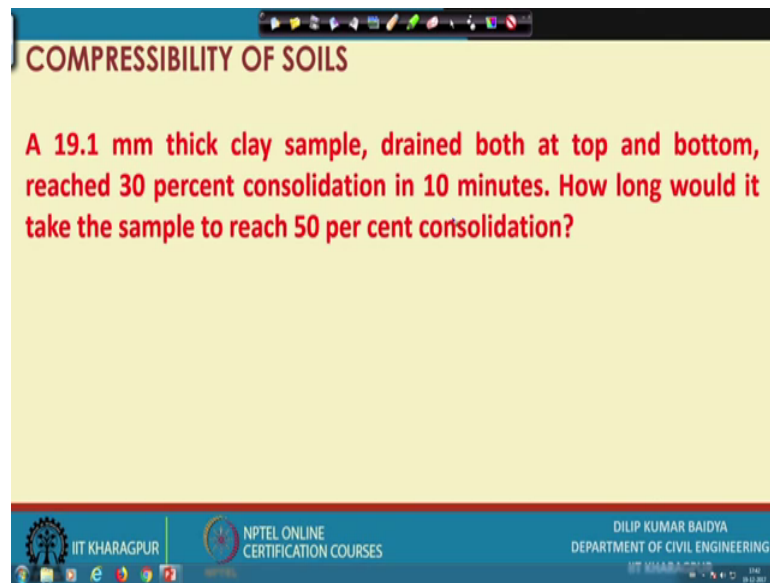
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So, here actually you can see the layer was drained on both surfaces actually while solving I have done reverse. So, please make a note while solving I have taken these are the second case and these has taken five case, so first case.

So, reverse order I have solved otherwise I have written clearly drained one side drained both side whether it is one or two. So, carefully you should see that.

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COMPRESSIBILITY OF SOILS

A 19.1 mm thick clay sample, drained both at top and bottom, reached 30 percent consolidation in 10 minutes. How long would it take the sample to reach 50 per cent consolidation?

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So, next is next problem you can see a 19 milli 19.1 millimeter thick clay sample 19 drain both at top and bottom, reached 30 percent consolidation in 10 minutes. How long would it take the sample to reach 50 percent consolidation ok? So, this is the one question is there. The sample thickness is given 19.1 and drained both side, it is also mentioned and 30 percent consolidation reached in 10 minutes.

So, now suppose I have to estimate I have to go up to 50 or 60 percent, how much time I had spend that to be estimate, so that one actually you have to find out. So, for this the problem can be solved like this.

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Handwritten calculations on a whiteboard:

$$U = 30\% \quad T_{30} = \frac{\pi}{4} \left(\frac{U_{30}}{1.0} \right)^2 = \frac{\pi}{4} \times 0.3^2 = 0.07$$

$$H = \frac{19.1}{2} = 9.55 \quad 0.07 = \frac{C_v t}{H^2} = \frac{C_v \times 10}{9.55^2}$$

$$\Rightarrow C_v = 0.6384 \text{ mm}^2/\text{min} \rightarrow$$

$$U = 50 \quad T_{50} = \frac{\pi}{4} \times 0.5^2 = 0.197$$

$$0.197 = \frac{0.6384 \times t}{9.55^2}$$

$$\Rightarrow t = 28.1 \text{ min}$$

You can see for the laboratory 30 percent. So, U is 30 percent either you can refer T versus U curve or T versus U chart or you can take T_{30} equal to π by 4 U_{30} divided by your 100 square. This is the equation up to 60 percent, so it is π by 4 multiplied by this is 0.3 square and this gives you a value equal to 0.07. And so 0.07 will be equal to $C_v t$ by H square, and H here H will be equal to 19.1 by 2. So, this will be equal to 9.55 because the sample [laughter] thickness was 19.1 millimeter, and it is it has drained both top and bottom. So, this is the thickness.

So, if I put this so this will be equal to C_v into t is actually 10 minutes so that will be 10 divided by H is actually 9.55 square. And from this you will get C_v equal to 0.6384 millimeter square per minute. And now I have to find out how much time is required to reach 50 percent. So, U is 50 percent, so T_{50} will be how much again same thing π by 4 into 0.5 square that gives you 0.197.

So, I can find out 0.197 equal to C_v zero point same soil 6384 into t is unknown, and H is again same that is actually 9.55 square ok. So, this information initial information from the initial information, I got the C_v and then to find out the second condition I use that C_v . So, if I do this from there actually we will get t equal to twenty one point 28.1 minute ok; So, this problem almost similar to that which we have done in the in the in the say alternative method ok. So, the first you calculate the C_v then apply that one to find out the time. So, now this is the second problem is done.

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COMPRESSIBILITY OF SOILS

Results obtained from a consolidation test on a clay sample for a pressure increment of 100 – 200 kN/m² were:

Determine the coefficient of Consolidation of the soil

Thickness of the sample(mm)	Time (min)
12.200	0
12.141	0.25
12.108	1.0
12.075	2.25
12.046	4
11.985	9
11.922	16
11.865	25
11.827	36
11.809	49
11.800	64

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Now, let me see one more problem. So, this is a problem given you can see the results obtained from a consolidation test on a clay sample for a pressure increment of 100 to 200 kilo Newton per meter square where this is the table actually is given. You can see when thickness is this much, the time is this; thick is this, time is this that means, different time thickness change is shown determine the coefficient of consolidation of the soil.

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COMPRESSIBILITY OF SOILS

How long would a layer of this clay, 10 m thick and drained on its top surface only, take to reach 75 per cent primary consolidation?

If the void ratios at the beginning and end of the increment were 0.94 and 0.82 respectively, determine the value of the coefficient of permeability

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And then after that you can see how long would a layer of this clay, 10 meter thick and drained on its top surface only, take two reach 75 percent primary consolidation? And then; if the void ratios at the beginning and end of the increment were 0.94 and 0.82 respectively, determine the value of the coefficient of permeability.

So, time versus thickness data is given and that means, you have to first find out coefficient of consolidation. What can I do I can do route time method I can use. So, if I use the route time method from there actually I can get t_{50} ok. And then t_{50} from there I can equating with time factor 50 percent time factor so I can get the value of C_v so that way actually first calculate the C_v and then applying that C_v I can solve the other part.

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Plot thickness vs square root of time

$$\sqrt{t_{90}} = 6.54 \Rightarrow t_{90} = 42.7 \text{ min}$$

$$\text{Mean thickness} = \frac{12.168 + 11.8}{2} = 11.984$$

$$H = \frac{11.984}{2} = 5.992$$

$$\textcircled{1} C_v = \frac{T_{90} \times H^2}{t_{90}} = \frac{0.85 \times 5.992^2}{42.7} = 0.715 \text{ m}^2/\text{min}$$

$T_{90} = 0.85$

So, let me see how to do this you can see we have to plot first we have to plot thickness versus thickness versus square root square root of time that will get. And from there will from there corresponding to (Refer Time: 28:01) root t_{90} will get at t_{50} we will sorry not t_{50} we will get t_{90} here. We will get root t_{90} , root t_{90} we will get from here. And from there we will get t_{90} actually.

So, if I plot this root time versus thickness, I will get root t_{90} actually it is equal to 6.54. So, from there it will t_{39} , t will be equal to 42.7 minute. And mean thickness mean thickness equal to 12.168 plus 11.8 divided by 2 that would top at the end at the beginning it is 11.984 and so 11.984. So, H will become 11.984 divided by 2 that is 5.992.

And then I can find out C_v equal to T_{90} into H square divided by t_{90} . So, this is the formula actually t_{90} equals to C_v by H square. So, since I need C_v , I express in terms of C_v . So, T_{90} actually from the graph I can find out T versus U plot I get U corresponding to 90 percent T_{90} will be equal to 0.85, 0.85. So, I can put 0.85 into 5.992 whole square divided by t_{90} , t_{90} actually 42.7. So, if I put those then I will get this one will be 0.715 millimeter square per minute.

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Handwritten calculations on a whiteboard:

$$U = 75\% \quad T_{75} = 1.781 - 0.933(100 - U) = 0.48$$

field drainage path = 10 m = 10,000 mm

$$t_{75} = \frac{T_{75} \times 10,000^2}{C_v} \Rightarrow t = 128 \text{ years}$$

(iii) $a = \frac{de}{dp} = \frac{0.94 - 0.84}{100} = 0.0012$

$$\bar{e} = \frac{0.94 + 0.82}{2} = 0.88 \quad m_v = \frac{a}{1 + e} = \frac{0.0012}{1 + 0.88} = 0.000638$$

$$k = C_v \times m_v = 4.48 \times 10^{-6} \text{ mm/min}$$

And second part actually for a same soil suppose for same soil for 75 percent consolidation. So, U is 75 percent, then t will be equal to t_{75} will be 1.781 minus 0.933 100 minus U because this is above 60 percent, this is the equation to be used. And if I use this equation you will get T equal to 0.48. And drainage path 10 meter because one side drainage. So, in the field drainage path will be equal to 10 meter or equal to 10,000 millimeter. And then t_{75} will be equal to t_{75} into H square that means, divided by C_v that gives you t_{75} equal to if I this will be in minute, then if you convert t equal to 128 years.

And there is a third part that is d by a equal to de by dp which will be equal to 0.94 is given thickness and 0.82 into pressure divided by 100. What this much pressure, what is the change in void ratio it is 0.0012, and then e equal to the average e will be equal to 0.94 plus 0.82 divided by 2 that is equal 0.88. And so m_v equal to your a by 1 plus e . So, a is already known; e is known. So, if I put those values that is 0.0012 divided by 1 plus

0.88 that gives you $m \cdot v$ equal to 0.000638 meter square per kilo Newton. I have to convert the unit accordingly.

And then k equal to $C \cdot v \cdot \gamma \cdot w \cdot m \cdot v$, and all three values are known $\gamma \cdot w$ take 9.81. So, if you put those all those values then we will get the this calculation, we will get 4.48 into 10 to the power minus 6 millimeter per minute. So, appropriately when you will use those values appropriately you have to use the your unit, so that I will get you get in this unit.

So, with this I will just say stop here because three problems I have taken. So, I will see some more problems maybe in the next module.

Thank you.