Soil Mechanics/Geotechnical Engineering I
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Lecture – 40
Copressibility of Soils (Contd.)

Good morning let me continue over I have stop last class. In the in my last lecture actually I have discussed normally consolidated soil, over consolidated soil and how to determine for a normally consolidated or over consolidated soil total consolidation settlement. And I will take some problem later on to illustrate those whatever we have derived and right now I will just try to continue with the consolidation time consolidation. That means, we know that if the saturated fine grain soil is apply subjected to a pressure then it will undergo consolidation.

And now the question is how long it will take to complete the consolidation. So, that is the one most important and what is the some soil because of its characteristics may quickly consolidate some soil may not it may take longer time. So it depends on soil property and a before going to development of that time consolidation, let me see what is the mechanism of consolidated how it happens. Before that I will just since I am talking about consolidation settlement and I have also mentioned that there is a elastic settlement.

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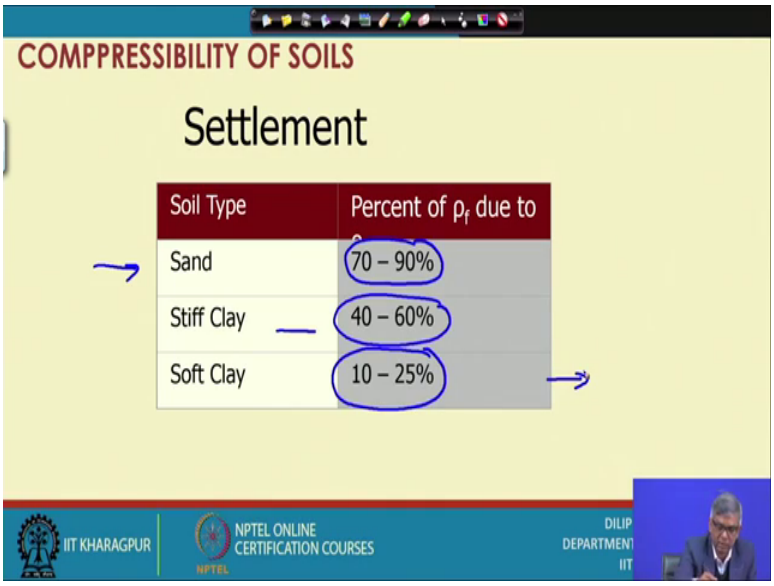

So, in comparison to this actually different soil will have a have different types of soil settlement. And we can classify three different type of soil that is Sand, Stiff Clay and Soft Clay and we know the sand actually majority contribution is from elastic settlement. And we can see that this is due to elastic settlement these 70 to 90 percent is from the consolidation sorry elastic settlement whereas, stiff clay the out of total settlement 40 to 60 percent for from consolidation. Soft clay it is 10 to 25 percent come from elastic settlement and rest will be (Refer Time: 02:54). That means, the when it is when you talk about sand elastic settlement important, while talk about soft clay then it is consolidation settlement in important.

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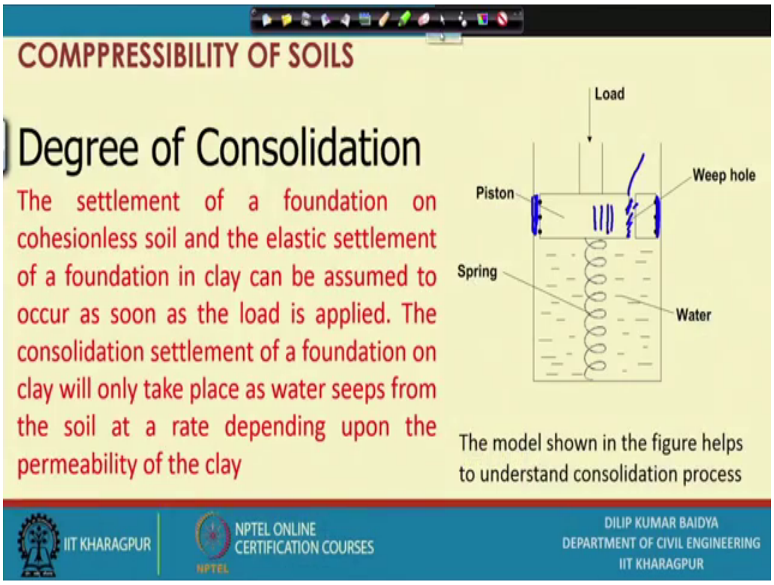

So, now we will see the mechanism of consolidation. That means, you can see this is a mechanism I have shown here. This is little focus sketch little problem is there. So, but anyway I assume that this is a shield actually, they are shield and no friction ok. There is no friction here and initially there is no opening at this point and then if you apply if you apply load though this. This piston suppose assume this is a piston and it is filled up with water and there is a spring. Now, if I apply this load, since the water is incompressible.

So, until unless this piston goes down load will not transfer to the spring and this spring that the piston to go down what you need to do? That means, water to be has to be compressed, but you know that water is incompressible and because of that what will happen that the water will be pressurized; that means, there will be pressure will develop within the pressure within the water and it will be under pressure actually.

So if I apply some amount of load p, that same magnitude actually equal pressure will be developed within the water. But still there is no transfer of load to the spring. Now in the second stage if I imagine a small opening here and then what will happen it is very. It is not so big. Suppose very fine opening is made. Then since the water is pressurized it will try to get released. So, what will happen then water will try to seep through this small opening and whatever excess pressure is develop that pressure until unless the pressure is becoming 0, it will continue to come out.

And when it will come out this the volume will be reducing and because of this volume reduction this one will be going down. And when it goes down the water no pressure that mean what is the additional pressure where it is going? Going to the spring that way actually that is the way the load finally, transferred to the spring. In the soil also it will be similar thing will happen where the through the grains contact to contact grain the soil mass is formed and when we apply the load through the surface, then until unless this void space is reduced then that that void space is reduced there will no compression will take place because both soil grain and water is incompressible.

So, until unless water is coming out or void is reduced that actually soil will not compress and until unless compressed then whatever additional load you applied to the soil that soil will be will be transferred to the water. That means, water will be pressurized and that pressure is called excess pore pressure. So, within the soil mass when you apply the load immediately after that excess pore pressure will develop and the over the time when through interconnected void spaces the water will try to seep through and release the pressure.

So, when this mechanism will be completed; that means, by seepage of water when completely excess pore pressure is dissipated that time actually you called complete of consolidation. That means, what about load you have applied that load finally, transferred to the soil grain. And then after that because of that the soil grain will come closer and the consolidation will be completed. So, this is the mechanism that similar way where it is a sealed container when you on below that it is a spring, but if I pushed us piston until unless water is coming out the load will not be transferred to the spring.

The similar mechanism where there in the saturated fine grain soil if I apply load onto that until unless the some amount of water is coming out to, I have the volume change that actually this extra load whatever you have applied will not transfer to the soil grain.

So, that is the mechanism of consolidation; that means, immediately after applying the load within the soil whatever pore water is there they will be pressurized in the form of excess development of excess pore pressure at with time this excess pore pressure will be dissipated and the external load will be transferred to the soil grain causing the compression and that compression is called consolidation when the excess pore pressure will be come to the 0; that is called completion of the consolidation settlement under that particular load. So, this is the mechanism.

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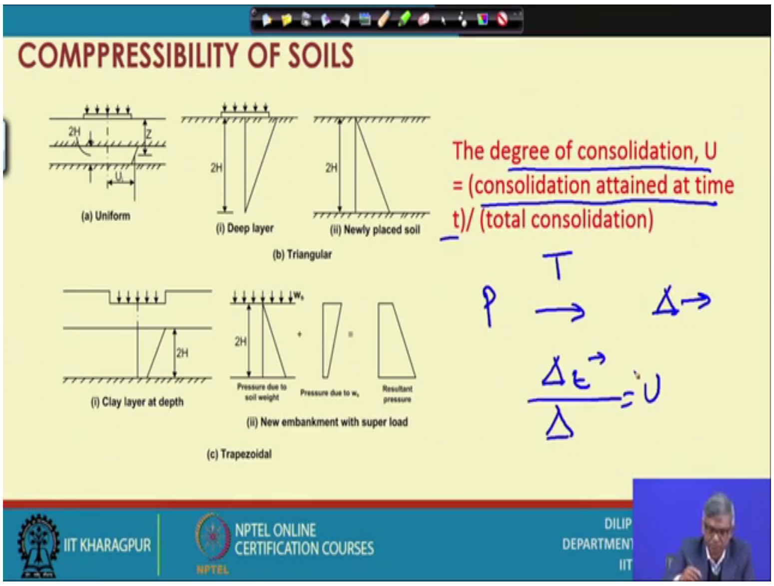

Now, when you apply the load onto the surface generally the pore pressure development will not be uniform generally and it will different places it will be different. When there is a, at a great depth there is a thin layer is there and load is applied here. Bossiness theory and other (Refer Time: 09:07) that it will be with depth the pressure intensity reduced ok. So that means, at this point whatever pressure at this point different pressures will be there and if there is different pressure initially the pressure will be counter countered by the development of excess pore pressure.

So, pore pressure will be here and here then it will be more here in less here. But if the layer is thin we generally we generally consider the uniform and at the middle of the clay layer and with that for all practical purpose we can work. So, whereas if there is a load is applied like this and then it is a over the depth pore pressure from here to here if I want to imagine then pore pressure will be maximum here and pore pressure will be minimum here and at some depth it will become 0. Why it is so because, because of this loading presences pressure is decreasing over depth. So, pore pressure also will be less.

Similarly, if it is a newly deposited soil and in that because of this soil weight only it will have more compact here and lesser lose here. So, this places immediately pore pressure will be dissipated. So, because of that when go deeper and deeper pore pressure intensity will be more. And suppose if there is a thick layer and load is applied here, obviously there will be different trapezoidal distribution will be there more here and less here. And in the other case suppose a embankment newly constructed embankment with some loading in that case you because of this newly deposited a soil the pore pressure will be like this. Sorry. Because of these load pore pressure is, like this and because of this loading pore pressure will like that is similar to this only.

So, these two together will have a Reverse trapezoidal. For this normal condition, trapezoidal will more will be here less will be here and newly deposited embankment, there actually it will be also trapezoidal. But lesser intensity it will be the surface, more intensity it will be more a pore pressure will be a depth. So, these are actually before going to develop the rate of consolidation you need to know how pore pressure developed at different depth.

So, this is the a visualization. You can visualize the, for different condition how pore pressure distribution will be there water depth. And then another thing is we know that another important term is that degree of consolidation. So, degree of consolidation U that is consolidation attained. Suppose in a particular load suppose if I apply P load and under that at total settlement at the end of consolidation suppose delta and at time t. So, with respect to, now and this one will take suppose a capital T time.

Now, if I want to find out or what is the amount of consolidation you will take place at time t some different time between less than t. So, that is what is defined degree of consolidation is defined as consolidation attained at time t at any time t. So, that is suppose delta t and divided by total consolidation suppose delta. So, total consolidation is delta and delta t at time t. So, that ratio is degree of consolidation. This is the another term will be used for our time consolidation.

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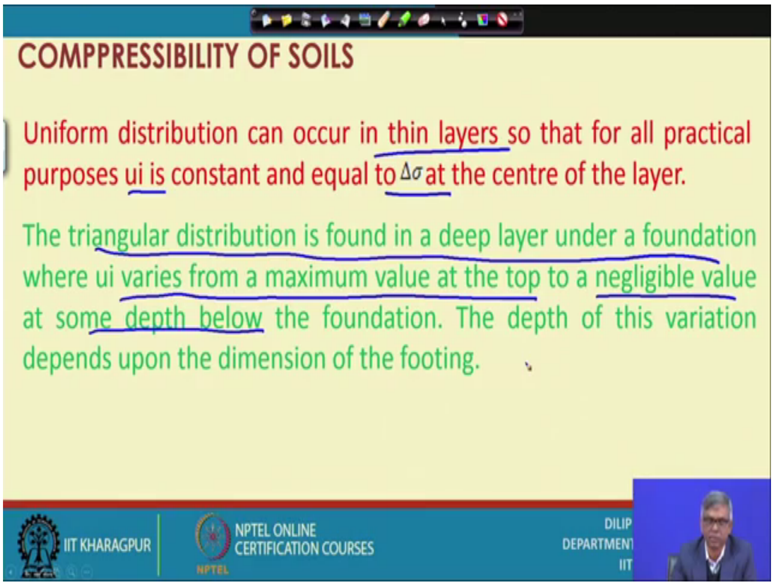

So, whatever I have explained in the last slide in the schematically once again it is listed once again. Uniform distribution can occur in thin layers. That is what I have mentioned thin layers though it is different, but in practical purpose one can use the ui can be as delta sigma then we at that depth whatever delta sigma because of this loading at the surface that can be taken as a ui, that initial excess pore pressure and for triangular distribution is found to deep layer under foundation where, ui varies from maximum value at the top and at a negligible value at some depth.

Because why? Because actually the immediately after loading the pressure will be more on the soil and will go deeper and deeper pressure will be less and this pressure will be countered by the pore pressure only. So, because of that at the near surface pore pressure will be more and at the greater depth pore pressure will be negligible and sometime it will become 0.

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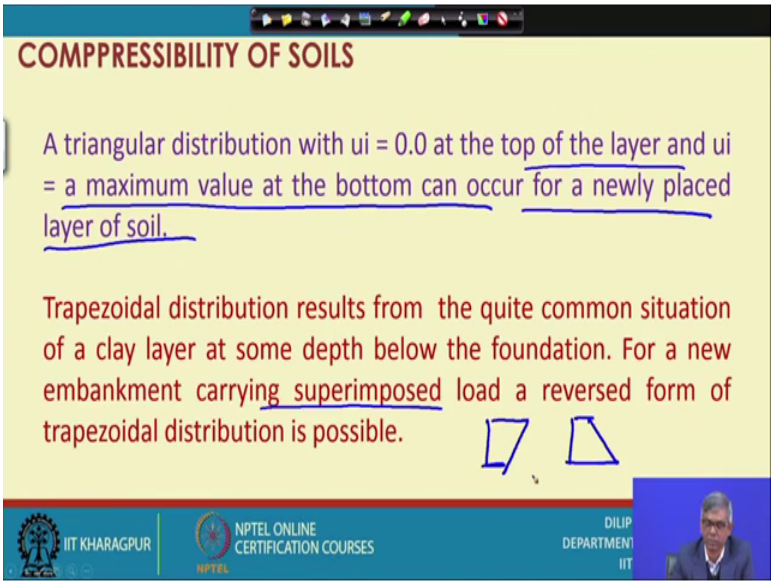

And similarly it a triangular distribution with ui equal to 0 that is newly deposited because they are actually since is a loose and immediately after deposition there if there is a pore pressure development immediately it will dissipated. So, at the top of the layer and ui equal to a maximum value at the bottom can occur for newly newly placed layer of soil. So, because of this reason already I have explained.

And a trapezoidal distribution results from the quite commons a situation of clay layer at some depth below the foundation; that means, if there is a thickness is quite large and because of that distribution of stress will be more at the shallow depth and this will be either greater depth. So, as usual because of that there will be trapezoidal distribution with more at the at the shallow depth and less at the greater depth. And just reverse for a newly new embankment carrying superimpose load a reversed from of trapezoidal can be this can be seen. Then it will be like that ok. So, this distribution are pore pressure distribution will be whatever you could visualize will be useful to analyze in later on.

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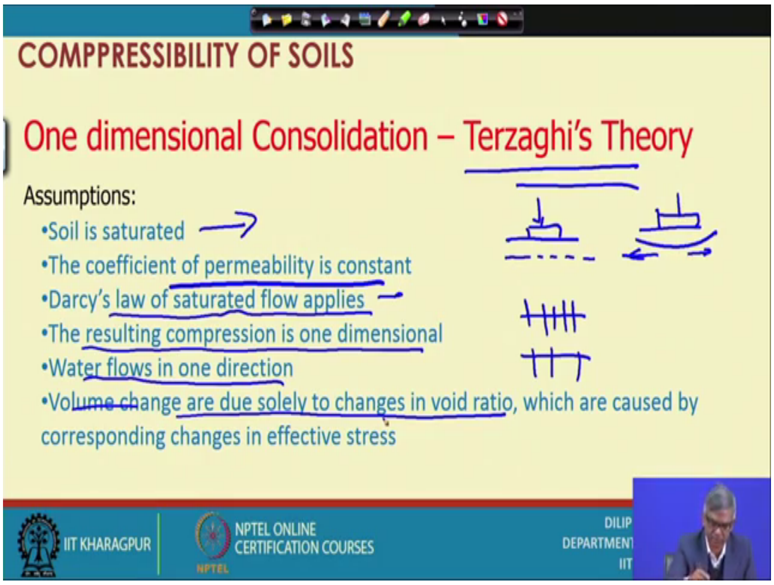

Now, the consolidation settlement that will be theory developed by Terzaghi's firs that is actually one dimensional consolidation theory; later on three dimensional consolidation theory also developed. But we will concentrate right now on one dimensional consolidation theory proposed by Terzaghi. And for any theory you know that we have we need to have certain assumption and if you deviate from the assumption then this will not be applicable.

So, those assumption as given by Terzaghi is, the soil is saturated. We know that is very important because the consolidation is the phenomena for the saturated fine grain soil. So, saturation is the mass, If it is not saturated then consolidation will not be there. The coefficient of permeability is constant. We know the coefficient of permeability can very different direction, but and different depth, but for the development of this theory we assume the permeability of coefficient a coefficient of permeability is constant particularly within a particular layer.

Darcy's law of saturated flow applies k proportional to i that is also to be valid. The resulting compression is one dimensional. That is another important observe assumption. That means if I, there is a clay layer. If I apply load through the soil then your compression will be one dimensional; that means, it will go assume that to go like this. But normally if I apply load like this the compression will be something like this. So, it will be one it will be compressed this will be displaced this side. So, because of that it will it will different, but for the initial development was assumed in the initial development it was assumed that resulting compression is one dimension. That means, only compressing vertically and no other lateral expansion or contraction. Water flows in one direction. Actually see water when there will be water is in the unit and it has to be dissipated. So, water will be flowing in one direction vertical or no not together vertical or horizontal.

Volume change are due solely to change in void ratio. That is another important. You are that means, we are not with soil grain because of the loading if soil grain compressed that is not applicable. Here only as I also mentioned in the mechanism of consolidation that the volume change or compression takes place because of the volume reduction or void ratio is a void reduction. And void reduction how it happens initially voids are filled with water. When water goes out then void become empty and there is no and there is soil grain is loaded. So, that because of that soil grain will come closer completing the void spaces. So, that is volume change also because of only void change of void ratio no other compression of water or compression of solid particles.

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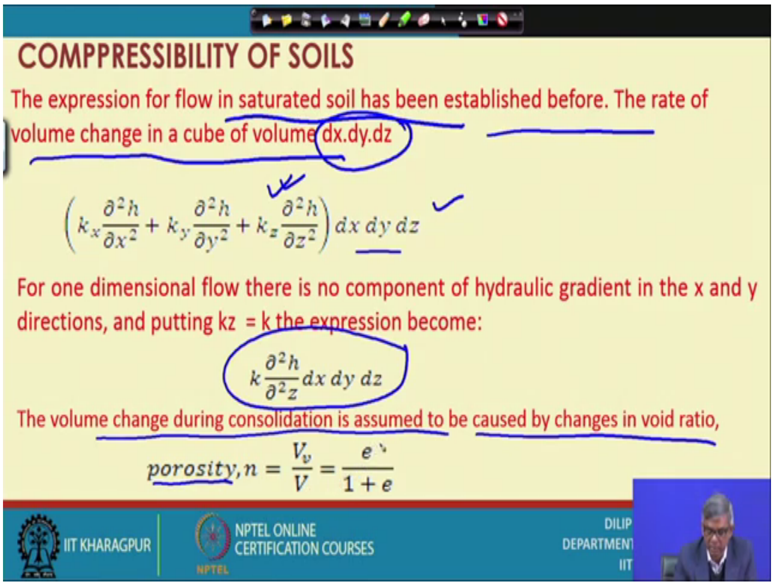

Now to do this, whatever we have done in the while discussing the permeability topic that in a flow in and flow out and that time we have derived the expression you can see. The expression for flow, expression for flow expression for flow in saturated soil already established. The rate of volume change in a cube of volume dx, the volume of cubes about dx, dy, dz and then volume change rate of volume change we have derived this expression before.

So, k x del 2 h del x square plus k y del 2 h del y square plus k z del 2 h del z square dx dy dz. This we have derived before and for one dimensional flow there is no component of hydraulic gradient in the x and y direction and putting kz equal to k the expression become: if I consider one dimensional there is expression which reduces to k into del 2 h del z square del del 2 z del 2 z into dx dy dz. Now volume change during consolidation is assumed to be caused by change in void ratio. So, the porosity I can consider n equal to V v by V and it is again can be explain e by 1 plus e. So, from here I will just see again volume change since it is based on or change of void ratio, I will try to correlate.

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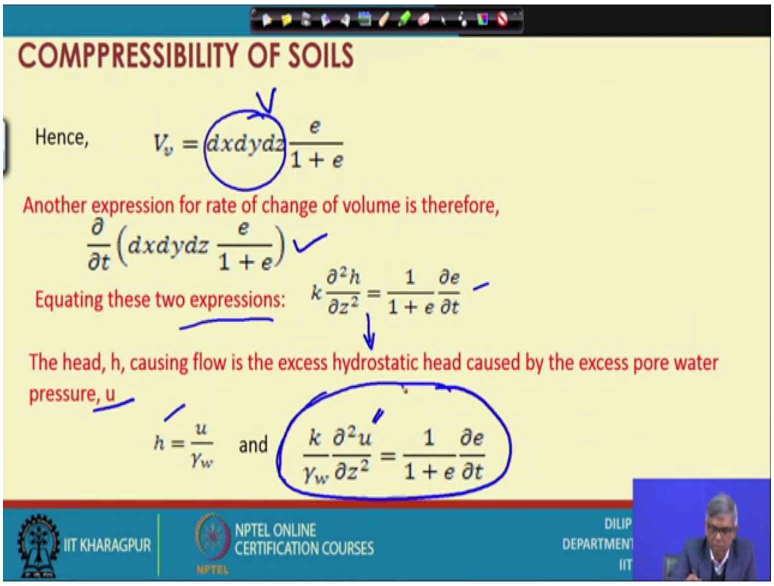

And you can see. So you can see from the previous expression, V v by V equal to e by 1 plus e. So, V v will be now or V is equal to this is nothing, but V. V into e by 1 plus e. So, another expression for rate of change of volume is therefore, so this is the one: del del t with time dx by and this is another expression for volume change. So, equating these two expression we can get k del 2 h del z square 1 by 1 plus e del e by del t ok. The head, h, causing flow is the excess hydrostatic head caused by the excess pore water pressure, u. So that means, there is a h and u there is a relationship. So, h will be called u by gamma w and if you substitute then this become or if I instead of aids if I now express in terms of u then this one this one will be reduced to this form ok.

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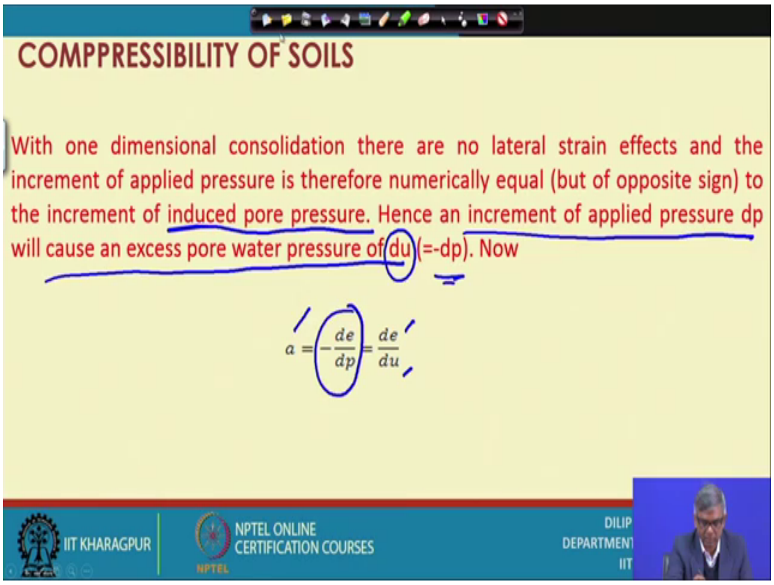

So, after that you can see again. With one dimensional consolidation there are no lateral strain effects and the increment of applied pressure is therefore numerically equal to the increment of induced pore pressure. Hence an increment of applied pressure dp will cause an excess pore water pressure du. So, always it will be reverse. So, du will be minus dp. So, if I do that then we know de by dp equal to a and that will be equal to de by du. So, we have got actually initially versus pressure relationship. We have got de dp de by dp is a. And here actually we can write actually de by du this one in this form. That means, I want to since the consolidation is a result of change of void ratio I want to express in terms of void ratio. That is what we are planning.

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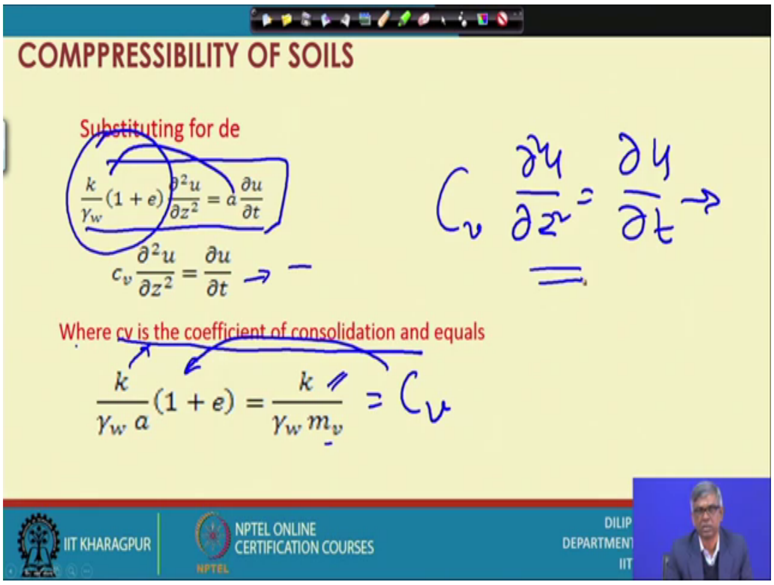

Now you substitute those. Then it become in this form. The equation will come to this form k gamma w 1 plus e del 2 u del z square a del u del t or C v into del u del 2 u del z square equal to del u del t. And now we are taking this one cv is defined, a coefficient of consolidation which will be k gamma w a into 1 plus e equal to k by gamma w. So, C v that is C v. C v equal to k by gamma w m v or C v equal to k by gamma w a k into 1 plus e by gamma w into a.

So, either way, so that means whatever term we have got these if you take this side. Then whatever things we are getting those all together we are defining it or you are introducing by a term that is called C v which is denoted as denoted as coefficient of consolidation Cv. And so, this is also time to time we require some time. By using consolidation settlements you can sometimes find out the permeability if you if remember this formula. So, C v equal to k by gamma w m v you can be utilize in some cases or by using this equation also you can find out k. So, these are also sometime consolidation test sometime used for determining indirectly these are being permeability also.

So, ultimate equation become our ultimate equation become C v into del 2 u del t del z square equal to del u by del t. This is a particular form of equation and mathematical equation, partial differential equation and this will have a standard solution. So, we can try to solve it for you.

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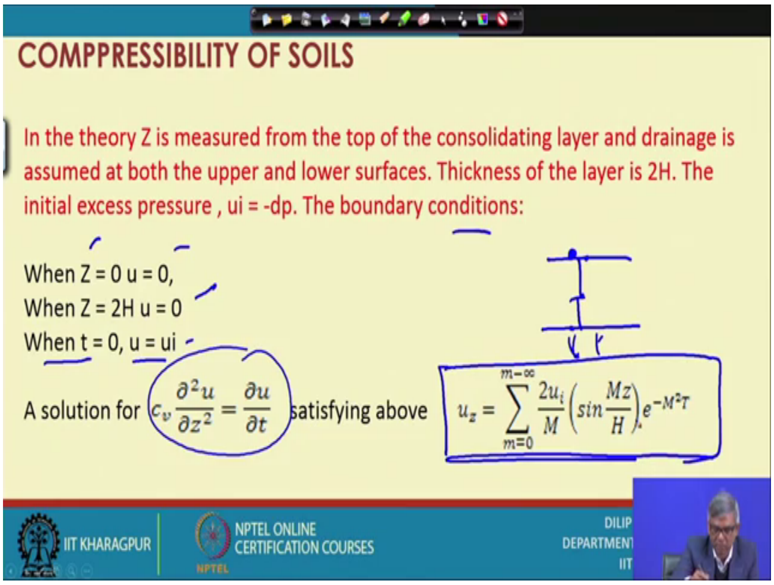

So, in the theory Z is measured from the top of the consolidating layer and drainage is assumed at both at that upper and lower surface. Thickness of the layer is 2H; that means, so upper and downward if this thickness 2H and the initial excess u i equal to minus dp and the boundary conditions are at z equal to 0, u equal to 0 at the surface when if there is a load is applied. So, at this point immediately the pore pressure will be dissipated because it is a surface. So, it will become 0 and at z equal to 2H, that mean at this point if there is a drainage provision there also become 0. At t equal to 0, u equal u i at and t equal to 0, u equal to u i.

Ah So, so, a solution for this equation this particular equation generally a mathematics you people are handle, you handle mathematics and you know if this is the type of equation what will be the form of solution. This is the solution can be assumed and then this boundary condition can be applied. And if we apply this boundary condition then one will get.

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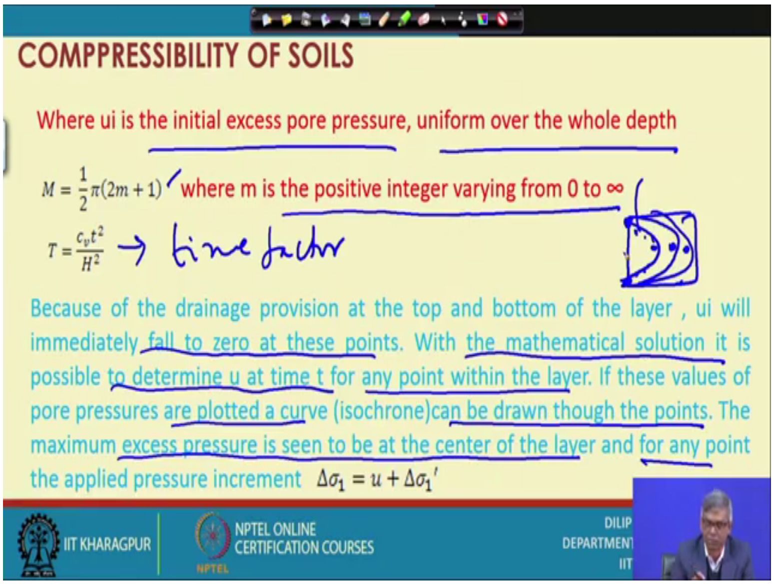

You can see where ui is the initial excess pore pressure uniform over the whole depth and sorry. So, that whatever expression we have assumed or sorry a solution they are ui. There ui is the initial excess pore pressure uniform over the whole depth. Though I have shown at the beginning the pore pressure distribution can be different. More there a top one less at top bottom, less at top more at bottom but for the initial development we are assuming ui is uniform throughout the depth. And capital M is this one, where are small m is the positive integer varying from 0 to infinity.

And T T is a time factor. This is a time factor and this is the expression very useful expression you have to remember always for finding out the time to complete a particular degree of consolidation. So, C v t square by H. Because of the drainage provision at the top and bottom of the layer ui will immediately a fall to 0 at those points. With a mathematical solution it is possible to determine u at time t for any point within the layer.

If these values of pore pressure are plotted a curve can be drawn through the points. The maximum excess pressure is seen to be at the center of the layer and for point. So that means, you have this is the initial excess pore pressure and immediately it will become 0 it will be 0. But pore pressure will be at any time at any depth. So, it will be not 0. So, it will be something else. So, that so different point actually who find and join them. If you got a curve like this, that is called isochrone. So, for different time interval different time isochrone can be obtained.

So, that I will show you next slide the maximum excess pressure is seen to be at the center. That means, if you see at time T1 this is maximum or T2 this is also maximum, T3 also maximum here. So, like that whatever may be the time that isochrone giving you middle value at the midpoint or midpoint of the layer. The isochrone are shown here in the next place. So I will show of course, in the next one.

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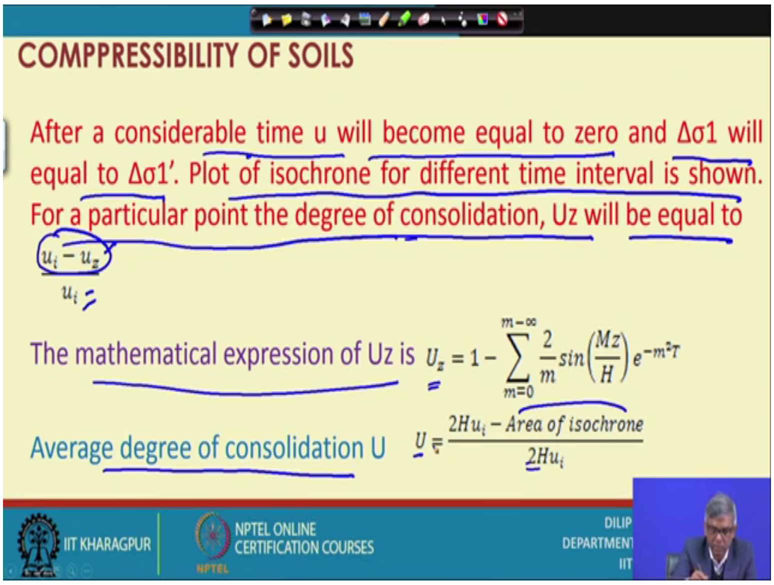

After a consider after a considerable time u become equal to zero and delta sigma 1 will equal to delta sigma dash. Plot of the isochrone for different time interval is shown. I will show the next one. For a particular point the average degree point degree of consolidation from the definition is ui minus uz by ui. At a at a particular point, ui minus uz by uz ui. So, uz is the total and ui minus uz that will be whatever happened actually at particular time divided by the in initial and what happened at a particular time, ui minus uz that what is the present consolidation and ui is this.

This two ratio this is the ratio that is called degree of consolidation we have defined before the mathematical expression for these if I do this then from there, uz become these. And average degree of consolidation expression I will show the isochrone from there I will show you that what is average degree of consolidation U; 2H u i the area I mean whole area initial minus area of isochrone; that means, whatever consolidation took place divided by these that is u.

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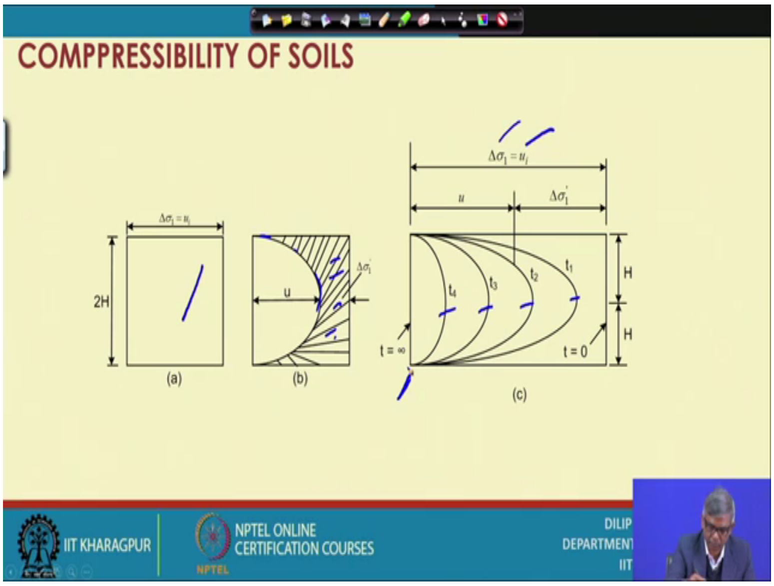

So, let me go to that one. You can see these are the isochrone. So, initial pore pressure is like this. Initial pore pressure is this one and and at any time suppose this must dissipated. So, this is actually transfer up and this is yet to be dissipated. So like that, at different in time interval at t infinity when full pressure is dissipated, so your u become 0; u become 0. And at an initial time t equal to 0 delta sigma 1 equal to ui. That mean whatever stress you have applied that itself u equal to ui and slowly ui will be reduced at at a infinite time which is a sufficiently long time. After that it t at infinity your u become 0. So, when u become 0; that means, is full consolidation.

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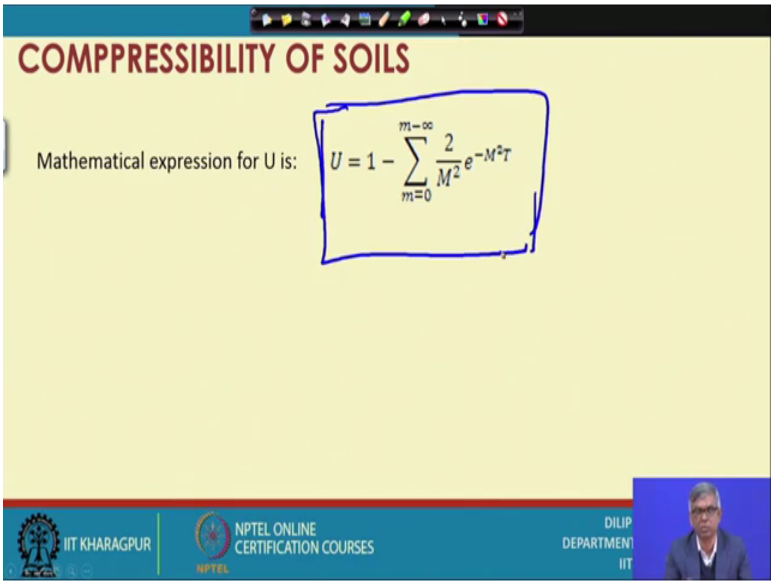

So, now based on this a final expression for mathematical expression for u based on that, this is the final expression. And I will see in the in my next lecture how to utilize this relationship between u and t to find out the time consolidation.

Thank you.