

Soil Mechanics/Geotechnical Engineering I
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Lecture – 37
Compressibility of Soils

Good mornings, once again you are all welcome to this lecture Soil Mechanics Geotechnical Engineering I and we have completed quite a few topics and just now we have completed the shear stress topic and this is very important topic; obviously,. And now we are going to start new topic that is your compressibility of soils.

And compressibility means what? Compressibility means under loading the volume change and in fact, at that volume change that volume change actually they can be in all direction. But in soil actually what we will see ultimately we want to see the vertical compression though it will be there in all direction.

The compare to other direction vertical compression will be larger. So, that we will try to address and in fact, when a material is loaded; generally it will have change in volume or length or dimension. And in a solid mechanics we have learn that when a rod is applied by axial load or then best one this is compressive or tensile it will be elongated shortened.

So, here also in soil if we apply this type of compressive load generally soil we apply compressive load. So, generally it will be there will be a compression and that will be there because of these some sort of elastic nature of soil and in addition to that in soil mechanics we will experience some other sort of compressibility or shortening or compression that is because of another phenomena called consolidation. And this consolidation cannot it is not a phenomena of all type of soil; this is generally is phenomena of fine grained soil. And particularly when fine grained soil is saturated and we apply load then this consolidation generally take place.

So, exact mechanism of consolidation and we will discuss later part to estimate the rate of consolidation etcetera that time I will elaborately discuss that mechanism of consolidation. Right now we are considering the just the compression because of the reduction in void ratio.

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The slide is titled "COMPESSIBILITY OF SOILS" in red. Below the title, "Consolidation - Introduction" is written in red. A diagram shows a vertical load being applied to a soil layer, with a horizontal arrow indicating the direction of the load. Below this, the text "The effect occurs for saturated fine grained soils" is written in blue. The equation $S = S_e + S_c$ is written in red. Below the equation, the text "A large wheel load rolling along a roadway resting on clay will cause an immediate settlement which is recoverable once the wheel is passed" is written in blue. At the bottom, the text "If the same load is applied permanently there will in addition be consolidation" is written in blue and underlined. The slide footer includes the IIT Kharagpur logo, NPTEL Online Certification Courses logo, and the text "DILIP DEPARTMENT IIT".

COMPESSIBILITY OF SOILS

Consolidation - Introduction

The effect occurs for saturated fine grained soils

$S = S_e + S_c$

A large wheel load rolling along a roadway resting on clay will cause an immediate settlement which is recoverable once the wheel is passed

If the same load is applied permanently there will in addition be consolidation

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And as introduction what I can say the effect occurs of saturated fines; that means, consolidation is for saturated fine grained soil happens. And a large wheel load rolling along a resting on clay will cause and immediate settlement which is recoverable once the wheel is passed; that means, we have actually that elastic whatever I have mentioned that on the road. Suppose if there is a load applied through wheel or anything then we will see initially there will be a depression, but while pass then it will be again coming back to the same position that is actually elastic compression.

But this consolidation settlement is not recoverable and it will do with time so; that means, and. So, that is the if the load same load like it is passing that this is recovering, but if the load is continuously kept here for a longer period permanent there will be in addition there will be a consolidation settlement.

And theses consolidation settlement actually that means, we have if there is a settlement in foundation suppose if there is a foundation here and we have constructed over here, then load is applied through this over the time this settlement foundation will settle by some amount. We will see some you will go downward this amount of settlement; if this is S this is will be combination of three component actually right; now I will let us discuss third component basically I will discuss first two component that is one is elastic another is consolidation.

And so, soil when loaded the elastically it will also settle and in addition to that if there is a fine grained soil saturated. Then we will after loading the within the soil there will be a development of pore pressure and slowly the pore pressure will be dissipated. And slowly that this is because of the dissipation of pore water pressure the load actual load will be slowly transfer to the soil grain and that.

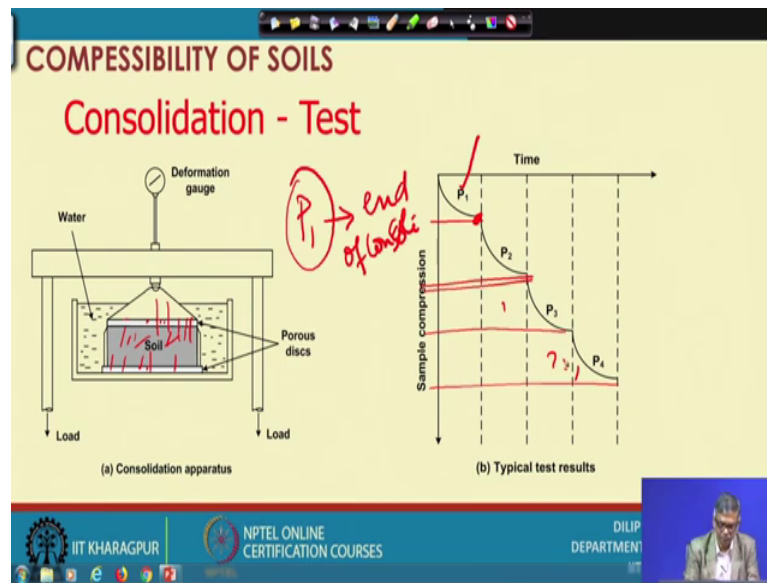
Because of that loading soil grain will try to come closer and that will result the reduction in void ratio and that will be finally, result in the final compression. So, reduction in void ratio basically will happen because of this consolidation process.

And now this one typically the further this mechanism of consolidation etcetera we will discuss later on when we will try to address the issues of rate of consolidation; that means, how long it will take. Suppose I know a particular foundation we will undergo this much amount of consolidation settlement, but my second question will be how long it will take to complete the consolidation settlement? So, that time I will later part of this topic the this lecture I will discuss.

Right now I will just mention just onto highlight that there will be a consolidation settlement. And this consolidation settlement will be this consolidation settlement will be can be estimated or can be based on the observation actual foundation how much is the settlement we can find out. But it is very difficult to differentiate what is the consolidation settlement; what is the how much is the consolidation settlement and how much is the elastic settlement?

So, to asses that we have different theories also we can estimate by calculation how much will be the elastic a settlement and also by consolidation theory I can find out how much will be the consolidation settlement. And these actually in the laboratory also we generally try to understand the compressibility behavior of the soil, we carry out test in the laboratory.

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And once a test is called odometer test and odometer test if I see this is the type of sample we keep here the sample is here and this is the stone at top and bottom. And this is actually odometer actually where we put the sample and this sample entire sample will be immerse in water.

So, that it will be saturated and then the screw this cover we apply the load through here. And if we apply the load then as then under because of this loading the soil that the load will be finally, since transfer to this soil through this and mechanism the when soil; that means, this is the saturated soil and we have applied now the load and when this dissipated happen void spaces are filled up with water.

And then when this load is applied the immediately after this loading application of this loading this load will not will not go to the directly to the soil grain; what will happen, because of this incompressible nature of the incompressible nature of the water this. So, water will be getting pressurized and in the form of excess pore pressure and when excess pore pressure we will develop then that excess for pressure cannot stayed longer. So, it will try to dissipate and how it dissipate when through seepage of water through the porous stones.

So; that means, when you applies a certain amount of load then the excess pore pressure will develop and these excess pore pressure will be dissipated because by the seeping of water through this porous stone. So, like that when because of this application of the load

whatever amount of excess pore pressure develop and if you this load is kept for a longer time, then slowly water will dissipate and finally, the pore water pressure will become 0.

And when this pore water pressure become 0 that time entire load whatever applied externally that will be transfer to the soil grain and that will cause the soil grain to come closer and at way actually it will be volume change will be at that volume change see the other direction restricted, we can assume that it is a 1 dimensional that vertical compression will be there.

So, like that; so, if you laboratory if you do this the soil and if you apply a; so, consolidation here another important thing is if I apply load P_1 under this load P_1 there could be end of consolidation. So, suppose you have applied load P_1 and the sample compression versus time if you plot at some time it will be suppose under pore pressure become 0. So, under P_1 this much is the consolidation settlement.

But it is not that that further compression of the sample is not possible. Now if I apply another load which is magnitude P_2 now. So, if I apply further another load P_2 then it compression will start from here. And under this load this is another amount of compression and which is equal to this. So, like that; so, each under application of each load after apply application the each load there will be comp compression. And so; that means, when you apply particular load with respect to that load we have the consolidation.

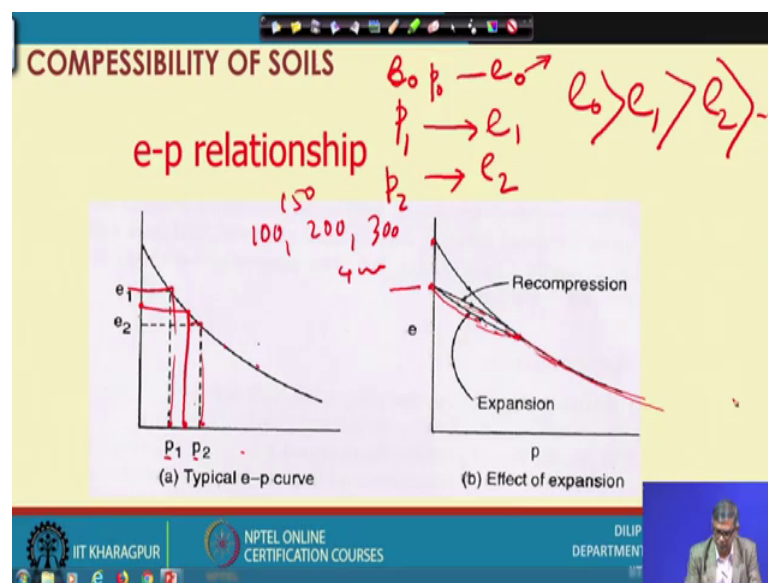
So, when you talk about consolidation you have to mention it is necessary or it is essential to mention under what pressure. So, if I applied pressure P_1 it will have some amount of consolidation if I apply load P_2 there is a consolidation settlement is a something else. So, like that if you continuously change and you will get the different pressure loading, different consolidation.

But here actually you can see when there will be a (Refer Time: 11:31) in soil if you will collect and initially you apply a small amount of P_1 and under that there will be compression, but when under first loading compression takes place if I increase the second equal amount of load, then magnitude of compression in the second stage will be smaller; obviously, the soil already become compact and so, the later stage to get the certain amount of consolidation settlement you need much higher load.

So, though this diagram does not properly in scale, but these magnitude towards end under same loading suppose you P_1 , P_2 , P_3 or all are equal then this magnitude will be reduced. So, this is one sort of; that means, of I apply load and because of this consolidation mechanism there will be a compression.

And that compression actually we need to estimate and to estimate this we have we can introduce some terminology and if I estimate those parameter and then we can find out the settlement.

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So, for that let us go to the next slide and you can see as I have mentioned that under each loading suppose you applied p_1 and then under load p_1 and suppose before loading p_1 the void ratio was before there is no sorry suppose it is load P naught and that time towards e naught.

Now if p_1 is applied then and that is kept for some time that will be compressed and when the you see that suppose after 10 days or after 2 days or after 1 day initially the rate of settlement of compression is quite high and towards end it will be decreased. And over a long period of time if you see that is no decrease or the decrease is very negligible without measuring pore pressure I can declare that the consolidation is completed.

And under that condition if I measure or if you estimate the void ratio suppose this is e_1 . Similarly under load p_2 and like that at the end when there is no change of thickness

then if I stimulate the void ratio suppose e_2 ; obviously, the whatever void ratio was here initially the e naught will be greater than e_1 . And again it will be also greater than e_2 like that; so, if I increase load continuously; that means, your void ratio will be decreasing then only void that decrease of voids will be the resulting in that compression.

So, that is what finally, for different loading some of p_1 p_2 like that the number of loading if I find out void ratio and finally, you plot in a e versus p plot if I do then typically get a curve like this ep curve. And suppose this is the one suppose p pressure p or p corresponding void ratio p_1 p_2 corresponding void ratio is e_2 like that I can draw a curve. Suppose while doing this the developing this curve we apply pressure 100, 200 300 or 400 like that there is a certain ways we have to do.

But now if I want to find out void ratio or some other pressure; that means, suppose 150 then from this curve suppose this is 100 and this is 200, 150 is middle. So, I will just produce here and from there I did the void ratio; so, that is the way actually we can find out void ratio.

And typically during consolidation again you can see that you have applied certain amount of load here. And then under after loading this void ratio that like the similar one this is going up to this point. And sometimes you remove the load then again it will be some sort of expansion will be there and it will go in some it will not go back in the same rope path it will be rebound in this way; it will not go to 0 it will have some permanent compression.

And then if you apply further load from here it will again a at certain path will follow and compression; when reach here and in further increase of pressure. Again this curve and this curve that it will continued further similar it will be going to become continuous. So, up to the here if we removed the load it will go this way and if you further loading, then you will come back at this point and again it continued with the same continuous line a curve.

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COMPESSIBILITY OF SOILS

Coefficient of Volume compressibility

The volume change per unit of original volume constitutes the volumetric change. If a mass of soil of volume V_1 is compressed to a volume V_2 , the assumption is made that the change in volume is caused by a reduction in the volume of voids.

$$V_1 = V_o (1 + e_1)$$

$$\text{volumetric change} = \frac{V_1 - V_2}{V_1} = \frac{(1 + e_1) - (1 + e_2)}{(1 + e_1)} = \frac{(e_1 - e_2)}{(1 + e_1)}$$

Where e_1 = void ratio at pressure p_1 and e_2 = void ratio at pressure p_2

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Now, if we want to estimate the amount of compression then we use a term that is coefficient of volume compressibility. And this is; what is the definition actually the volume change per unit of original volume constitute the volume volumetric change ok.

So, volume change per unit of original volume is a volumetric change if a mass of soil of volume V_1 is compressed to a volume V_2 ; that means, original volume was V_1 and final volume is V_2 the assumption is made that the change in volume is caused by the issue that is the thing; already I have mentioned that if this is this is the origin initial volume. And then final volume is this and this is the volume change is there these volume change only because of the changing void ratio in this mass soil mass.

So, that if that if that is assumption is made that volumetric change is V_1 minus V_2 by V_1 . And then V_1 actually you can say if I be the V_1 can be written as $V_o (1 + e_1)$ like that V_2 also can be written as or V_1 actually that way can apply $V_o (1 + e_2)$ and V_1 . So, like that if I express in terms of void ratio corresponding volume then even these V_1 will become these and e ; obviously, can assume that V_o is added here it get cancel finally.

So, ultimately $1 + e_1$ minus $1 + e_2$ by $1 + e_1$; so, volumetric change which was ΔV by V_1 what is ΔV the original volume minus final volume by V_1 . And if I express this way then ultimately these become e_1 minus e_2 by $1 + e_1$ sorry e_1 minus e_2 divided by $1 + e_1$.

So, 1 plus e we can consider as the original volume this almost become the change in volume where e_1 is the void ratio at pressure p_1 and e_2 is the void ratio at p_2 and e_1 is obviously higher than e_2 .

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COMPESSIBILITY OF SOILS

m_v = volumetric change/unit of pressure increase, If H_1 is the original thickness and H_2 is the final thickness,

$$\text{volumetric change} = \frac{V_1 - V_2}{V_1} = \frac{H_1 - H_2}{H_1} \quad (\text{AREA IS ASSUMED TO BE CONSTANT})$$

$$= \frac{e_1 - e_2}{1 + e_1} = \frac{a dp}{1 + e_1}$$

Also $\frac{de}{dp} = \frac{e_1 - e_2}{dp}$

$$\text{volumetric change} = \frac{a dp}{(1 + e_1)}$$

Hence, $m_v = \frac{a dp}{(1 + e_1) dp} = \frac{a}{(1 + e_1)} \quad m^2/MN$

And then m_v defined as a volumetric change per unit pressure sorry volumetric change; volumetric change per unit pressure that is in m_v H_1 is the original thickness and H_2 is the final thickness the volumetric change this way you can write also I can write it in this way because area if it is constant.

So, V_1 actually a into H_1 and here also a into H_2 because cross sectional area if you see consider same. And then this one also can be expressed in this way and this is again equal to this previously this one we have shown that this is the one expression.

And again if I e versus p the slope of the e versus p curve that is called a . So, $\frac{de}{dp}$ if I define as term as a and then volumetric change will become $a dp$ by $1 + e$. So, if the combining these; so, your volumetric change become your $a dp$ by because $1 + e$ minus $1 + e$ equal to $a dp$ if I multiply. So, this become e times this become a times dp and divided by $1 + e$. So, that is the thing we have shown here.

Volumetric change $a dp$ by $1 + e_1$ and so, and m_v definition is volumetric change per unit pressure increase. So, $a dp$ by $1 + e_1$ divided by dp . So, dp dp cancelled; so, a by $1 + e_1$. So, a by what is a ? a is the slope of e log P curve how to find out the slope? If

I p_1 here p_2 here then I can find out what is the p_1 p_2 and what is e_1 e_2 . So, that slope will be your a and $1 + e_1$ is the e can nothing, but is the original volume. So, that is m_v be equal to this a by $1 + e_1$.

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COMPESSIBILITY OF SOILS

For most practical engineering problems m_v values can be calculated for a pressure increment of 100 kN/m² in excess of the present overburden pressure at the same depth

Once the coefficient of volume decrease has been obtained we know the compression/unit thickness/unit pressure increase. It is therefore an easy matter to predict the total consolidation settlement of a clay layer of thickness H :

$$\text{total settlement} = \rho_c = m_v \Delta p H$$

The slide also features a handwritten diagram of a soil layer of thickness H with a vertical arrow indicating a pressure increase Δp of 200 kN/m². The coefficient of volume decrease m_v is also indicated.

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And for most soil most practical engineering problem m_v values can be calculated for pressure increment; 100 kilo Newton per meter square in excess of the present overburden pressure. Suppose; that means, in a soil at point what is the effective overburden pressure at present above more than 100 in excess; that means, if it is 200.

So, I can find out m_v for 300 kpa pressure and that can use for cal once the coefficient of volume decrease has been obtained we know the compression per unit thickness for the unit pressure is the increase per unit pressure increase; that means, compression total compression can be applied the Δc equal to m_v by definition if I multiplied by Δp .

And if I multiplied by thickness then it will be your total settlement of suppose if there is a layer here of thickness H and m value of m_v here to obtained and then and your pressure in the original pressure was something. And because of some loading you have increased the pressure equal to Δp then your settlement will be $m_v \Delta p$ into H .

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COMPESSIBILITY OF SOILS	
Typical values of mv for different soils	
Soil	mv(m ² /MN)
Peat	10.0-2.0
Plastic clay (normally consolidated alluvial clays)	2.0-0.25
Stiff clay	0.25-0.125
Hard Clay (boulder clays)	0.125-0.0625

So, this is one way to find out the compression that is called mv approach; that means, by using coefficient of volume compressibility one can find out the comp compression of the soil or settlement of the soil consolidation settlement of the soil.

Now typical values of mv for different soil different soil; obviously, we will have different mv values you can see peat has the mv very high and plastic clay will have this much compare to less stiff clay will have further less hard clay will be further. So; that means, the organic clay will have maximum compression, plastic clay will have intermediate and these are the soil where we will have lesser compression; that means, mv value based on the value of mv we can assess the which one we will undergo more compression which one we will undergo lesser compression.

So, this is the typical values, but this value not necessarily you have to remember you, you can estimate in the laboratory and for a each soil because these are actually a range is given because 10 to 2 is a very high range. So, for use in calculation you need to find out in the laboratory.

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COMPESSIBILITY OF SOILS

The tabulated results were obtained from a consolidation test on a sample of saturated clay, each pressure increment having been maintained for 24 hrs. ✓

After it had expanded for 24 hrs the samples **was** removed from the apparatus and found to have moisture content of 25%. The particle specific gravity of the soil was 2.65.

Plot void ratio to effective pressure curve and determine the value of the coefficient of volume change for a pressure range of 250-350 kN/m²

Pressure (kN/m ²)	Thickness of sample after consolidation (mm)
0	20
50	19.65
100	19.52
200	19.35
400	19.15
800	18.95
0	19.25

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And now this one I can show with help of a application; that means, whatever you have application of volume compressibility; I can show through this example and you can see the tabulated results were obtained from a consolidation test; that means, whatever tables I have given here that is when 0 pressure the thickness of the sample was 20 millimeter suppose. And then when you apply 50 kpa pressure then the original thickness 20 millimeter become 19.65 millimeter; so, it is compressed.

Then when we applied 100 kpa pressure then it is 19.52 when it made 200 it become 19.35 and you can see when it become 400 it become 19.15 when become 800 it become 18.95. And then at the end when after reaching to this pressure; we can remove all the load and then you allow to expand. And finally, full expansion and in a we have seen the final thing this is 19.25 millimeter.

. So, this is the data actually from the laboratory; so, these thickness is at the end of the consolidation of 50 kpa pressure at the end of consolidation of 100 kpa pressure like that this is given. So, the each pressure increment having been maintained at 24 hours.

And after it had expanded for 24 hours the samples where actually should be where removed from the apparatus and found to have moisture content 25 percent. The particle specific gravity of the soil was 2.65 plot void ratio to effective pressure curve and determine the value of the coefficient of volume change for a pressure range of 250 to 350.

So, coefficient of volume change when you talk about then you have to always mention the oppressive range because with the change of pressure coefficient of volume change will be changing because it is volumetric change per unit pressure change. So, because of that void range pressure you need this that has to be specified.

So, if I see now this problem.

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Solution: $w = 0.25$, $G = 2.65$

$$e = wG = 0.25 \times 2.65 = 0.662$$
 this is the void ratio corresponding to a sample thickness of 19.25 mm

$$\frac{dv}{v} = \frac{de}{1 + e_1}$$
 or

$$de = \frac{(1 + e_1)}{H_1} dH = \frac{1.662}{19.25} dH = 0.0865 dH$$

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There are two three methods actually can do and one method I will just first explained that is actually from the end actually can start and solution is since at the end of the consolidation where after removal of the load quantity it is fully expanded at that condition also soil will be saturated. And water content was measured 20 point 25 percent and G was 2.65.

And so, we know that saturated condition S into e w into G and as equal to 1 as saturated condition. So, e become w into G ; so, we can find out e equal wG . So, that way is equal to 0.662; that means, at the end finally, after the expansion whatever void ratio as 0.662.

. So, this void ratio corresponding to a sample thickness of 19.25 that one table is and see that 19.25 thickness at the end. So, the corresponding to that thickness we have void ratio is 0.662. Now, we know that dH/H_1 will be equal to $de/(1 + e_1)$ sorry the total volume here.

So, d_e become d_e become 1 plus e_1 by H_1 into dH and if I put original void ratio is 1 plus e_1 become 1.662 and H_1 is 19.35. So, last we are starting from the end actually and it is dH ; so, your d_e you can calculate from here that based on these d_e actually equal to 0.0865 dH ok. So, if this is the one and then I can go back to this one you can see.

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COMPRESSIBILITY OF SOILS

Pressure	H	dH	d_e	e
0	20.0	+0.75	+0.065	0.727
50	19.65	+0.40	+0.035	0.697
100	19.52	+0.27	+0.023	0.685
200	19.35	+0.10	+0.009	0.671
400	19.15	-0.10	-0.009	0.653
800	18.95	-0.30	-0.026	0.636
0	19.25	0	0	0.662

Handwritten notes on the right side of the table:

- $d_e = 0.0865$
- dH
- $d_e =$
- 0.0865
- $\times -0.3$
- $= -0.026$

Handwritten notes on the left side of the table:

- Red bracket connecting the first row (0 pressure) to the last row (0 pressure).
- Red bracket connecting the first row (0 pressure) to the second row (50 pressure).
- Red bracket connecting the first row (0 pressure) to the third row (100 pressure).
- Red bracket connecting the first row (0 pressure) to the fourth row (200 pressure).
- Red bracket connecting the first row (0 pressure) to the fifth row (400 pressure).
- Red bracket connecting the first row (0 pressure) to the sixth row (800 pressure).
- Red bracket connecting the first row (0 pressure) to the seventh row (0 pressure).

Footer information:

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19 point 19.25; 19.25 and thickness and void ratio this one and we have got d equal to 0.0865 d_e equal to 0.0865 dH . So, we know this is the thickness and this is the corresponding void ratio and we have got change in void ratio in terms of thickness we have got this.

So, now for this, what I will do these minus this I will do. So, this thickness minus this thickness if I do you can see this minus 0.3 so; that means, dH is minus 0.3. So, d_e will be equal to 0.0865 multiplied by minus 0.3 and that gives you minus 0.026. So, what is the void ratio will be here? So, this plus this so; that means, if I that if I do plus then it will become ultimately since is a negative; so, it become 636 ok.

And similarly I will do now these minus this then the dH become this and d_e become this and these and these if I add then it void ratio become this. Similarly I will do these minus these then I will change in thickness will be these and if I if I use this equation then d_e become these and then your e_b come with respect to this plus this plus this we can see become 0.671.

Similarly if I do these this and then you are change in thickness become these if I use this equation then your d_e become this and what will be the void final void ratio this plus this. So, this one similarly this minus this if I do I will get the d_h is this use this equation you will get d_e here and what will be the finally, so, this plus this become this.

. So, this is the way one can calculate the void ratio corresponding to each pressure. So, 0 pressure what is the void ratio of 50 pressure what is the void ratio 100 what is the void ratio 400 what is the void ratio now we have got pressure corresponding to corresponding each pressure we have got the void ratio.

Now we can plot the e_p curve and after e_p joint the e_p curve we can find out the slope of the e_p curve a are a by $1 + e$ is the m_v and what range actually you have to find out the slope between 250 to 350.

So, we will do that one in that maybe in my next lecture maybe I stop here I will continue maybe rest of the calculation in my next lecture.

Thank you.