

**Soil Mechanics/ Geotechnical Engineering I**  
**Prof. Dilip Kumar Baidya**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 34**  
**Shear Strength**

Good morning friends. Once again we will continue with the Shear Strength chapter and we are almost towards the end of the chapter, this topic Shear Strength. And last few lectures, we have discussed several aspects and few things critically i need to discuss so that it will be easy to understand and apply.

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**SHEAR STRENGTH**

Behaviour of Soils Under Shear

Before going to this topic familiarity with the following terms are needed:

- Overburden and effective overburden pressure
- Normally consolidated clays
- Over consolidated clays
- Pre-consolidation pressure

$\sigma_v - u = \sigma_v'$

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So, first thing I will discuss about we have Shear Strength we have discussed and to find out Shear Strength parameter, different methods we have discussed. Now the behaviour of soil under shear, the different soil will have different behaviour. So, that part will try to highlight in 2, 3 lecture.

And here, before going to this, I want to discuss about few I introduce few terms that is actually your; you can see that i have written here there is Overburden and overburden and effective overburden, overburden and effective overburden.

So, Overburden is actually, suppose this is a ground and I consider a point here and overburden means above that whatever soil weight, because of that whatever pressure is

giving here, suppose here that pressure is effective overburden pressure and whereas, effective overburden is what it does not water table somewhere here again, then from that overburden actually because of the total weight of the soil, if I get the stress here that is total stress  $\sigma$  and then because of this water pressure if I subtract that one that become effective. So, that is effective overburden.

So, only we are considering, when we are considering the material above the point. So, both soil and water. Soil is giving total stress and then water is giving more water pressure because of this presence of this water table. So, if I subtract this then it is effective overburden pressure. And these are things will be useful because I have mentioned that your shear strain is a function of effective stress and effective overburden pressure. So, this is the one will be applicable everywhere.

Then, normally consolidated clays that means, see I have already pointed out that different soil will behave differently. So here, actually normally consolidated soil and over consolidated soil what it is actually discuss in length in the subsequent topic that is compressibility of soil. But here just I will introduce the term, that you actually normally consolidated means whatever at am particular point or at a particular point if I consider a some pressure and effective overburden pressure and that pressure is the maximum in his history in its history, that means or that point is never been subjected to a higher pressure than the present effective overburden pressure that is called Normally consolidated.

What is normally once again I repeat; that if the soil at this point never been subjected to higher pressure, then the present effective overburden pressure that is normally consolidated. And whereas over consolidated means that suppose is a particular area in the past it is subjected it has been subjected by a higher pressure than the present effective overburden pressure. There may be several example you can give, but i will not coming that point here. We will give this explanation may be later stage while discussing the compressibility of the soil.

So that means, there is a normally consolidated one group over consolidated one and they of course will be behaviour under shear will not be not be same. So, will discuss that and what is Pre-consolidation pressure, that the over consolidation is done by some pressure. So, that pressure is called Pre-consolidation pressure; that means, the pressure at which the soil has been subjected to and that is actually pre-consolidated pressure. So,

these are the term some time will be required to understand this material behaviour. So, just I have introduced those.

Let me continue now. This is the behaviour of sand.

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**SHEAR STRENGTH**

Behavior of sand and other granular materials: Unless drainage is prevented deliberately a shear test on a sand will be a drained one as the high value of permeability makes consolidation and drainage virtually instantaneous.

Dry sand: there will be no pore water pressure and inter-granular pressure will be equal to the applied stress

Saturated Sand: the pore water pressure will be zero due to the quick drainage and the inter-granular pressure will be equal to the applied pressure

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And other is granular materials under shear. So, this is actually you can see that the sand generally will be highly porous. It will have high permeability values and so because of that unless drainage is prevented during the test, shear test on sand will be drained mainly because of the high permeability values of the soil, when we apply the load of if there is a development of pore pressure, it will be immediately dissipated. So, because of that when the sand and granular soil when you conduct test, that will be automatically is a drained until and unless we prevent drainage.

And dry sand, there will be no pore water pressure and inter granular pressure will be that equal to the applied pressure. And when we saturated sand that pore water pressure will be 0, due to the quick drainage if do not prevented from draining and automatically inter-granular pressure will be equal to the applied pressure; same as this because of this porous or high permeability of the soil.

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**SHEAR STRENGTH**

Dense sand: dilate during shear and the shear strength of the sand increases because of the development of negative pore pressure if the movement of pore water is restricted from movement

Loose sand: tends to decrease in volume during shear and the shear strength of the soil decreases because of the development of positive pore pressure if the movement of pore water is restricted from movement

Critical Density: The density at which there is no increase or decrease in shear strength when the sand is maintained at constant volume

Handwritten notes:

$$\tau = f(\sigma')$$
$$\sigma' = \sigma - u$$
$$\sigma - (-u) = \sigma + u$$

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Now, this drain sand, when sand is dense generally because of during shear it dilate that is change in volume, increase in volume and because of that the shear strength of the sand increases and so the reason is, that if the volume increases then negative pore pressure develop and that negative pressure.

So, your effective pressure if the tau is sorry, tau is a function of effective stress and tau is function of effective stress and effective stress is  $\sigma' = \sigma - u$  and which will be equal to  $\sigma - u$  because negative pressure. So, it is  $\sigma + u$  it become. So, when in the sand, when you conduct test particular drain sand because of this volume increase, the pore pressure will be negative as a result it will give you increase in Shear Strength.

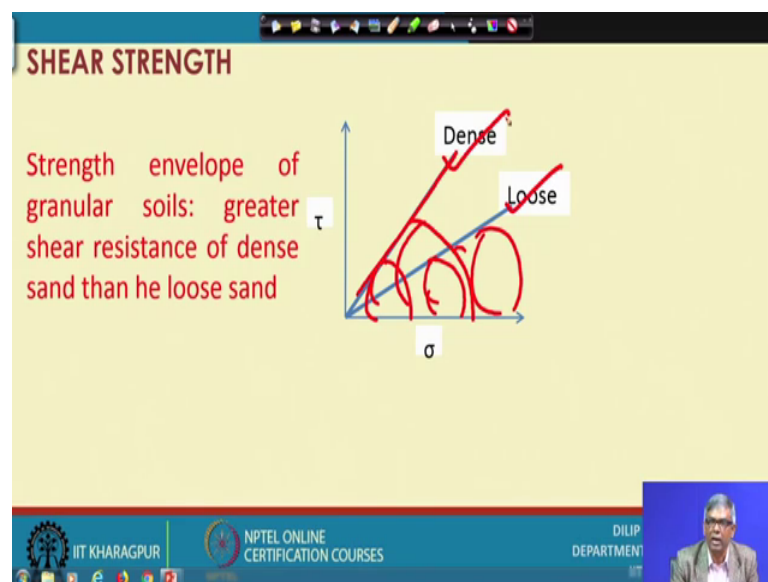
And of course, movement of pore water pressure is restricted, that is another thing. See if it does not of it is not restricted, the water movement is restricted then what it will happen otherwise it will be drained.

And Loose sand, actually when we will shear, it will be volume will be decreased and as a result positive pore pressure will develop and as a results shear strength will be effective shear strength will be reduced and which will result the reduction in Shear Strength.

And this is the critical density that means density at which there is no increase or decrease in shear strength when the sand is in is maintained a constant volume. That means, at a particular density, when you will shear it will neither increase in volume nor decrease in volume and that mean that will give you constant shear strength.

So, that is the thing where you will get no increase or no decrease in shear strength that strength is maintained a constant volume. That is the one; that density is called critical density. That similarly when it is critical densities are, from there we can find out also critical word ratio. So, what is critical word ratio? So, how to find out those thing? Again we will discuss maybe later stage.

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And strength envelope of granular soil, generally you can see here, that when it is a dense sand the shear strength will be higher and as a result we may have Mohr circle like this and your tangents to the circle will have envelope. This is the one and when it is a loose soil of course, if you have a particular confining pressure and then it will fail at a definite value or deviator stress deviator stress and from there you can find out  $\sigma_1$  and  $\sigma_3$  if you know number of circle, those circle may be falling under this. And so this is enveloped or loose and this is the envelope for dense sand and dense sand will have higher strength.

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**SHEAR STRENGTH**

Saturated Cohesive soil: these soils are defined as saturated clays and silts in either natural and remoulded state

Unsaturated cohesive soil: until the late 1980s it was felt that both the value of shear strength and the volume change characteristics of an unsaturated soil could be considered in as function of single effective stress in a similar manner to that for a saturated soil

This theory has been discarded now. We will consider only behavior of saturated clay only.

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And next is saturated cohesive soil. This is basically this soil are defined saturated clays and silts during particular is direct shear test, we considered saturated condition either whether it is natural or it is remoulded state, we try to make it saturated. If it is not saturated, we will try to saturate before shear.

So, that is one thing. And then unsaturated cohesive soil, that mean unsaturated, we understand saturated soil and we understand if there is water table then how will be the effective shear stress etcetera. But when unsaturated soil, the behaviour is completely different which we understood now, but until 1980's it was felt that, both the value of shear strength and the volume change characteristics can be explained as same as same way as it is done for the saturated soil.

But of course, this understanding has been changed and new understanding developed, new theories have developed and. So, this theory has been now discarded and, but though several things are available now with unsaturated soil, but we will concentrate only on behaviour of saturated clay only, saturated clay. So, this is the part you are restricting to this part that the unsaturated one we are not considering at all because it is my target is undergraduate level. So, i am just keeping away.

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**SHEAR STRENGTH**

**Undrained shear:** The shear strength of soil if expressed in terms of total stress, corresponds to Coulomb's law, i.e.,

$$\tau_f = c_u + \sigma \tan \phi_u$$

Normal - total

For saturated cohesive soils tested in undrained shear it is generally found that  $\tau_f$  has a constant value being independent of the value of the cell pressure  $\sigma_3$ . Hence we can say that  $\phi_u = 0$  when a saturated cohesive soil is subjected to undrained shear.

Hence;

$$\tau = c_u = \frac{1}{2}(\sigma_1 - \sigma_3)$$

$\sigma_3$   $\sigma_1$   $\tau$   $\tau_f$   $\sigma_1 - \sigma_3 = 2 \tau_f$

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Now, when we consider the undrained shear strength of soil expressed in terms of total stress, if we shear strength is expressed on terms of total stress; that means,  $\phi_u$  and  $c_u$ , then your shear strength equation is this  $\tau_f$  will be will be  $c_u$  plus  $\sigma \tan \phi_u$  and this is the normal stress. This is the normal stress and that normal stress also total normal stress.

Suppose if I want to consider a point here and find out shear strength and water table here, then total stress means I have to find out at this point what is the total stress not the effective stress that is  $\sigma$ . And for saturated cohesive soil tested in undrained shear, it is generally found that  $\tau$  has a constant value and being independent of value of the self-pressure; so that means, which as I have mentioned that during triaxial test, we apply different confining pressure and go up to the failure.

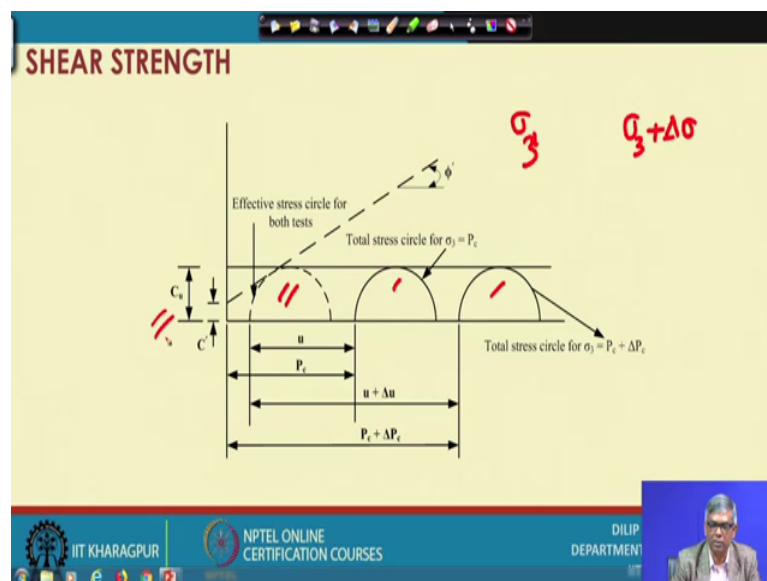
But when saturated soil, we do undrained test and even though change of your confining pressure, this, your stress circle will not be changed or you have  $\tau_f$  will be the constant that is independent of your  $\sigma_3$ . So, if it is  $\tau_f$  become constant; that means, the failure envelop has to be horizontal; that means, it is like this, failure envelope has to be like this that all circle will be below this line. If this is the axis and circle will be below this axis.

So,  $\tau_f$  constant means there will be no  $\phi$ . So, that is what if you consider  $\phi$  zero, then saturated cohesive soil subject to 2 undrained shear and then your  $\tau$  equal to  $c_u$



equal to half sigma 1 minus sigma 3 because this is sigma 1 and this is sigma 3 and then this is sigma, this is sigma 1 and this is sigma 3, then your diameter will be sigma 1 minus sigma 3, then radius here actually tau is nothing but radius here and radius will be half sigma 1 minus sigma 3 whatever I have written here. So that means, when we will do undrained test saturated soil in triaxial operators, then you get the  $c_u$  that is undrained strength which is called undrained stress shear strength and that will be half of sigma 1 minus sigma 3.

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And another important observation here that, if I as I have mentioned that when you do saturated soil undrained test on saturated soil, your shear strength is constant and if we imagine that during test undrained test. If you measure the effective measure the pore pressure and finally you convert the effective stress based on the measurement of pore pressure and we can see and prove that whatever maybe the confining pressure, suppose it is sigma 1, sorry this is sigma 1, sorry suppose 1 is sigma 1 sorry sigma 3.

Next one is sigma 3 plus delta sigma like that different values of sigma 3 we apply, at each stage if you measure the pore pressure develop before failure and that pore pressure if you subtract from sigma 1 and sigma 3, then you will get only same circle. The circle diameter of the circle will not change. So, whether it is these or these finally, based on measurement of effective pore pressure and you find out the effective stress circle, then you will get the this is the circle only circle will get for both the cases. Any number of



test you do with different values of sigma, then you will get the single circle and that circle will not have any phi and then your failure envelop will be horizontal and this will be the c which will be equal to  $\frac{\sigma_1 - \sigma_3}{2}$  or  $\frac{\sigma_1 - \sigma_3}{2}$ .

But most of the time, we get this sometime if I do undrained test and draw the Mohr circle and 3 circle if you draw the tangent, we will see that sometime instead of becoming horizontal. You may get a little bit of phi value and some amount of c value and that is reason actually whatever condition you are assuming that is saturated final soil. So, that condition may not be satisfied there. So, it may be partially saturated or there may be some pressure or other things are present. So, because of that we get some value of phi so, but otherwise in undrained strength, undrained test triaxial test or saturated soil we supposed to get only c value like this which will be equal to half of  $\sigma_1 - \sigma_3$ ,  $\frac{\sigma_1 - \sigma_3}{2}$ .

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**SHEAR STRENGTH**

If we wish to think of the results of an undrained test in terms of effective stress we should consider the nature of test. If for a particular undrained shear test carried out at a cell pressure of  $p_c$ , the pore pressure generated at failure is  $u$  then the effective stress at failure are:  $\sigma_1' = \sigma_1 - u$  and  $\sigma_3' = \sigma_3 - u = p_c - u$   $\sigma_1' - \sigma_3' = \dots$

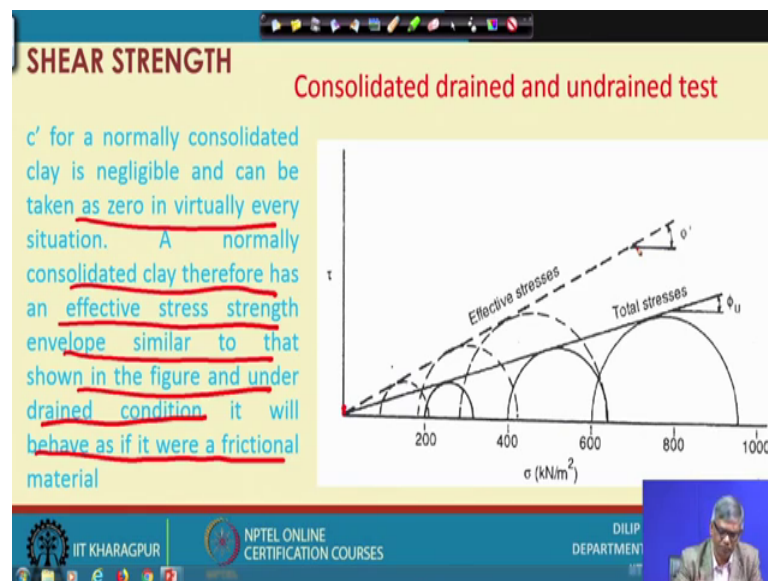
If the test is repeated using a cell pressure of  $p_c + \Delta p_c$  the value of the undrained strength of the soil will be exactly as that obtained from the first test because the increase in the cell pressure  $\Delta p_c$  will induce an increase in pore water pressure,  $\Delta u$ , of the same magnitude  $\Delta u = \Delta p_c$ . The effective stress circle at failure is therefore be the same as for the first case. It is therefore seem that there can be only one effective stress circle at failure, independent of cell pressure value, in an undrained shear test on saturated soil

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This can be proved actually. This is explained, you can see just read those thing. If you wish to think the results of undrained test in terms of effective stress, we should consider the nature of the test. If for a particular undrained shear test carried out at a self-pressure of  $p_c$ , the pore pressure generated at failure is  $u$  the effective stress will be  $\sigma_1 - u$  and  $\sigma_3 - u$  or this value. If the test is repeated using a self-pressure of  $p_c + \Delta p_c$ , the value of the undrained strength of

the soil will be exactly as that obtained from the first test because the increase in the self-pressure will induce an increase in pore water pressure which will be equal to  $\Delta u$  equal to  $\Delta p_c$ . So, that because of that finally, your  $\sigma_1$  dash minus  $\sigma_3$  dash whatever value will be there again another set of test if you do,  $\sigma_1$  dash minus  $\sigma_3$  dash also will be the value same value we will get. So, that is the explanation whatever conclusion I have shown in the previous slide.

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Now this is a consolidated drained or undrained test. As I have mentioned that for  $c$  dash, for a normally consolidated clay is negligible, that is what and can be taken as 0.

So, a normally constituted clay therefore, has an effective stress strength envelope similar to that shown in the figure, undrained condition and it will behave as if it was a fictional material, so that means, if I do whether consolidated it is drained or undrained, both cases actually your pressure envelope will be passing through the origin. So, this is the thing we have to allow as consider. If there is a problem and it is mentioned the drained condition then automatically at one point it is indicated that your failure envelope is passing through the origin. And then relevant calculation can be done.

And as I have mentioned that consolidated drained test is the best one, but it takes time, but we can do consolidated undrained test and based on measurement of pore water pressure along with the shearing, applying of deviator stress and subtracting those, we can find out the effective stress parameter, effective stress parameter by this. So, initially

total stress circle and then effective stress circle we can do and then we get the envelope for the drained condition also.

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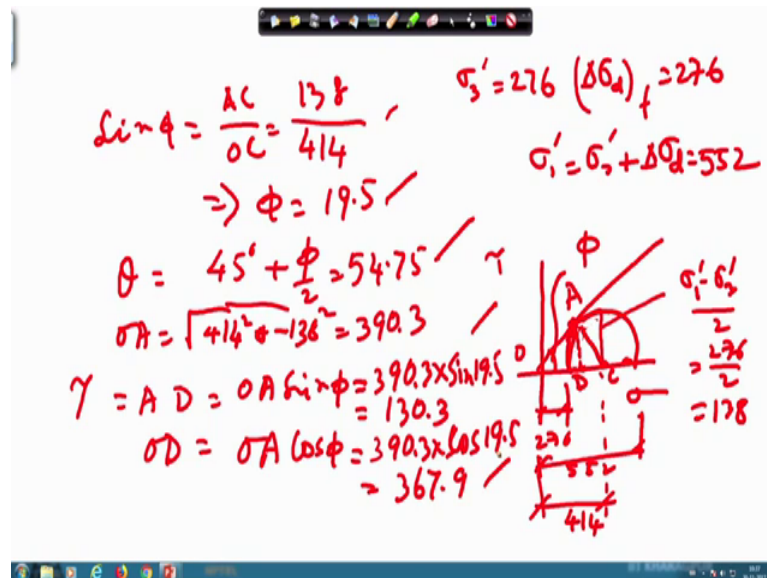
**SHEAR STRENGTH**

A consolidated drained triaxial test was conducted on a normally consolidated clay. The results are as follows:  
 $\sigma_3 = 276 \text{ kN/m}^2$ ,  $(\Delta\sigma_d)_f = 276 \text{ kN/m}^2$   
Determine (i) the angle of internal friction,  $\Phi$  (ii) the angle,  $\theta$  that the failure plane makes with the major principal plane, and (iii) Normal and shear stress on the failure plane.

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And now I will take a one application to this. You can see a consolidated drained triaxial test was conducted on a normally consolidated clay. So, that is the one you can see consolidated drained is mentioned, one thing is mentioned here; consolidated drained normally consolidated clay, so that means,  $c$  dash will be 0 and the results are follow  $\sigma_3$  and  $\Delta\sigma_d$  deviator stress at failure, that is also given this determine the angle of internal friction  $\phi$  the angle  $\theta$  that the failure plane makes with the major principal plane and the normal and shear stress on the failure plane. So, this is the thing I need to do and for that I can do in a different ways and let me take one by one.

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As given by in the problem, it is  $\sigma_3'$  that is confining pressure is given 276 and  $\Delta\sigma_d$  that is also given 266. So,  $\sigma_1'$  will be  $\sigma_3'$  plus  $\Delta\sigma_d$ ; that means, it will be 276 plus 276 it will be 552.

Now, that we have, we know the  $\sigma_1'$  and  $\sigma_3'$ . So, I can draw like this.  $\tau$   $\sigma$  suppose and this is your  $\sigma_3'$  is 276 and this is also 276. So, I can draw a circle. And then I can have the envelope something like this and that means, I have this one. So, we can have this is the circle and this is actually 276 and this is actually 552 and up to this centre, this is actually your 414.

So, this is actually if is  $\phi$  then your  $\sin \phi$ , if I say this is O and this is A and this is C, then your  $\sin \phi$   $\sin \phi$  will be equal to AC by OC, AC by OC and then, if I put those then it will be your radius will be the radius of the circle will be  $\sigma_1' - \sigma_3'$  by 2 and that will be it will be 276 by 2; that means, 138. So, it will be AC is nothing but radius. So, 138 divided by OC is nothing but this 414 and this gives you  $\phi$  equal to 19.5. And if you know this  $\phi$  and then I can find out  $\theta$  equal to or you can measure the  $\theta$  here actually directly, otherwise  $\theta$  equal to 45 degrees plus  $\phi$  by 2 and that will be equal to your 54.75, 54.75.

And now I can geometrically, I can find out, if I drop here and this is suppose a point d if I take and then your value of way will be equal to under root 414 square plus sorry minus 138 square and that will be equal to 393.3 and then I if I take now OAD circle and from

there if I write from there, I can write AD equal to, AD equal to OA sin phi and which is nothing but AD is nothing but the normal stress at this point sorry shear stress at this point. So, tau and it will be OA, OA you have already got, 393.3 multiplied by sin 19.5.

So, geometrically (Refer Time: 25:53), you are using geometry and Mohr circle all together, then gives you the value equal to 130.3. Similarly I can find out this is actually AD. Now at this point what is the normal stress which is OD. So, AD similarly I can write OD equal to OA cos phi, OA cos phi which is nothing but again 390.3 multiplied by cos 19.5 and that gives you a value equal to 367 point, 367.9. So that means, by the help of the Mohr circle, I could get these are the phi value, I got the plane inclination of the plane, I got the normal and shear stress on that plane. So, this is analytical way and this is actually graphical, but I can get also the same thing analytically whatever I have given.

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$$\sigma_1 = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$

$$\sigma_1' - \sigma_3' = \left( \frac{\sigma_1' + \sigma_3'}{2} \right) \cos \phi + \left( \frac{\sigma_1' - \sigma_3'}{2} \right) \sin \phi$$

$$\sin \phi = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{276}{828}$$

$$\Rightarrow \phi = 19.5$$

$$\theta = 45^\circ + \frac{\phi}{2} = 54.75$$

$$\sigma_n = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta = 367.9 \rightarrow$$

$$\tau_n = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\theta = 130.08$$

Suppose we have given 1, sigma 1 minus sigma 3 dash equal to 2 C dash cos phi plus sigma 1 minus sigma 3 dash sin phi. This is the equation also i have given and Since this part is C is 0, then sin phi will be equal to, sin phi will be equal to sigma 1 dash minus sigma 3 dash by sigma 1 dash plus sigma 3 dash and that if you do, then you will get ultimately 276 divided by 828. And this gives you the phi value equal to 19.5. And once you get this and then you get theta equal to 45 degrees plus phi by 2, then again we will get something 54.75. And then we have the, if you know the plane and on that plane

what is the normal and shear that expression we know  $\sigma_n$  equal to  $\sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$ . So, if I put now all values and then finally, you will get 367.9 that value same value also we have got in the previous method.

And similarly our expression for  $\tau_n$  we have given  $\sigma_1 \sin \theta \cos \theta - \sigma_3 \sin \theta \cos \theta + \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$  and that if I put the value now, then you will get here 130.08.

So, it was the value it was before by the graphical method, semi graphical method, graphical and geometry together whatever we have got identical values we are getting. So, either way we can solve the problem because it is easier to use graphical Mohr circle method along with the analytical method because you can simply geometry you can apply and get the value. Otherwise we have to remember this type of equation and in any time if you forget or if you wrongly, instead of plus here you write minus, instead of minus here if you write plus or instead of plus here write minus here everything will go wrong.

So, because of that best actually to understand the Mohr circle properly and apply along with the Mohr circle, if you draw in a proper graph sheet, directly you can read those values otherwise as I have shown that, you can apply your simple geometric and find out the appropriate terms from there. And with this, I will just stop here for this one ok.

Thank you.