

Soil Mechanics- Geotechnical Engineering I
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Lecture – 33
Shear Strength (Contd.)

Once again Shear Strength. Just in my last lecture, I have explained different types of tri-axial test and then how to find out different types of soil strength parameter that have taken one application and I may, I am planning to take a few more application and how to, from the different test how to find out the soil strength parameters. So, let me take the first one something like this.

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SHEAR STRENGTH: Application 2

A series of drained tri-axial tests were performed on a soil. Each test was continued until failure and the effective principal stresses for the tests were

Test No.	σ_3 (kPa)	σ_1 (kPa)
1	200	570
2	300	875
3	400	1162

Plot the relevant Mohr stress circles and hence determine the strength envelope of the soil with respect to effective stress

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There was a test, a series of drained tri-axial test, drained tri-axial test means what? Actually this is consolidated drain. So, when it is mentioned drain test, it means it is the consolidated drain because without consolidation draining we do not do. So, that is drain test means it is a consolidated drained, automatically it is understood.

So, series of drain tri-axial test were performed on a soil and each test was continued until failure and the effective principal stress for the test where that is 200, 300, 400 and corresponding σ_1 570, 875, 1165. And in fact, each test was continued until failure. So, this is the initial information is given and since it is a it is mentioned as a drain test; that means, in each in under each increment of $\Delta \sigma_f$, $\Delta \sigma$, the

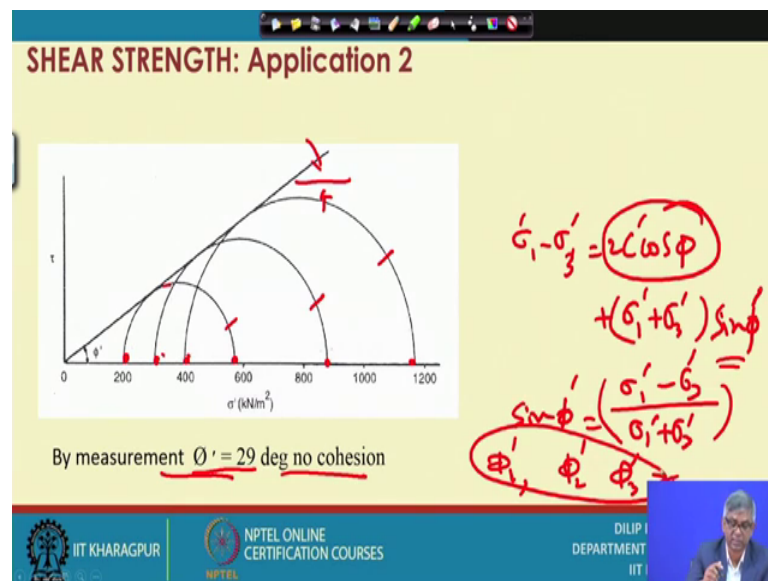
pore pressure also dissipated and until it becomes 0, it we have not we generally do not go, we do not go next increment. So, that is the of thing followed in the test we have to assume because the initial information is given that it is a drain test.

So by the drain test, it is ah 300 corresponding sigma 1 dash 570 and this is 300, your confining pressure and after dissipation of pore pressure, obviously and then corresponding sigma 1 dash is 875 and 400 confining pressure and corresponding 1162. Plot the relevant Mohr stress circle and hence determine the strength envelope of the soil with respect to effective stress.

So, from this test, we are getting this is one set, this is another set and this is another set. So, 3 sets we are getting data and this data can be plotted in the form of Mohr circle and then you can draw the common tangent. So suppose, let us see this is actually your 200, 570; 200, 570 and then 300, 875 and 400, 1162.

So, let me take to next slide and you can see this now.

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The first circle the 570, this is 600 less than that. So, sigma 1 is 570 and sigma 3 is 200. So, we know this is a 2 principal stresses. So, 2 principal stress will be on the x axis and distance between these 2 are actually the radius, diameter of the circle. So, considering this as diameter, that means I can find out midpoint and complete this circle.

Similarly, second set was 300, 875. So, this is 300 and this is 875. This is again the diameter of the circle. So, you have to find out midpoint and then complete the circle going through this 2 point. So, we are getting another circle. So, that is second circle and third one is 400 and 400 is a sigma 3 dash and 1162 is the, this is 1200. So, let the 1162 is the your sigma 1 dash, principal stress 1, major principal stress.

So, again this and this distance is the diameter of the circle. I can find out the midpoint and taking this as half of this as radius, I can complete another circle. So, this is circular 1, 2 and 3. And you can see, through all 3 circle if you can draw a now common tangent and we can see that it is passing through the origin and we are getting a inclination on that angle we can directly read from the graph. And that if you read, you get the whatever, since is the drain test is mentioned. So, from this test whatever soil parameter is called we get that is called drain test parameter.

So, that is phi dash, we have to use phi dash. Phi dash is 29 degrees and cohesion is since it is no intercept, it is 0. And this problem also can be done in another way actually, we have done actually $\sigma_1 - \sigma_3 = 2 C \cos \phi + \sigma_1 \sin \phi + \sigma_3 \sin \phi$. We have got this equation also, if I consider this as 0 and then I can get $\sin \phi$, $\sin \phi$ dash in fact equal to $\sigma_1 \text{ dash} - \sigma_3 \text{ dash}$ by $\sigma_1 \text{ dash} + \sigma_3 \text{ dash}$.

So, from here also directly I can find out the phi dash and here if you find out phi dash. So, there are 3 circle, so corresponding to first circle, we get 1 phi dash 1. suppose then say for second circle phi 2 dash and then we will get phi 3 dash though the same soil is used, identical soil sample is used all 3 supposed to get the same value, but because of the problem in the experimental setup or many other reason, we may get little different, but finally, average of this 3 can be reported as the actual angle or (Refer Time: 07.16) angle effective angle shear angle of this sample.

So, this is one application. So, drain test when you do.

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SHEAR STRENGTH: Application 3

A series of undisturbed samples from a normally consolidated clay was subjected to consolidated undrained test. The results were: ✓

Cell pressure (kPa)	Deviator stress at failure (kPa)	Pore water pressure at failure (kPa)
200 ✓	118 ✓	110 ✓
400 ✓	240 ✓	220 ✓
600 ✓	352 ✓	320 ✓

Plot the strength envelope of the soil (a) with respect to total stresses and (b) with respect to effective stresses

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Next one is something like this; you can see this problem is a series of undisturbed samples from a normally consolidated clay was subjected to consolidated undrained test. This is you have to just note it. Consolidated undrained test, that means, it is consolidation is the first stage is done, but second stage during is not allowed and, but instead making the draining to draining and pore pressure to 0 instead of that draining is path is closed and pore pressure develop to because of the each increment of load is measured.

So, this is the thing is additional information is given 200 self-pressure is applied and then deviator stress at failure was 118kpa and pore water pressure at failure was 110 and then 400, this is 240 and sorry this will be one, 2 extra, 220. It has to be 220 and then 600, then this is 352 is the deviator stress at failure and then it is 320 this is u.

So, plot the strength envelope of the soil with respect to total stress and with respect to effective stress. This is the thing I just explained in my last lecture, how this undrained can be utilised as utilise to find out drain parameter of the soil. So, this one we can show here, we can see now this can be done this way. So, this is actually you can see sigma 1.

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Handwritten calculations for three tests:

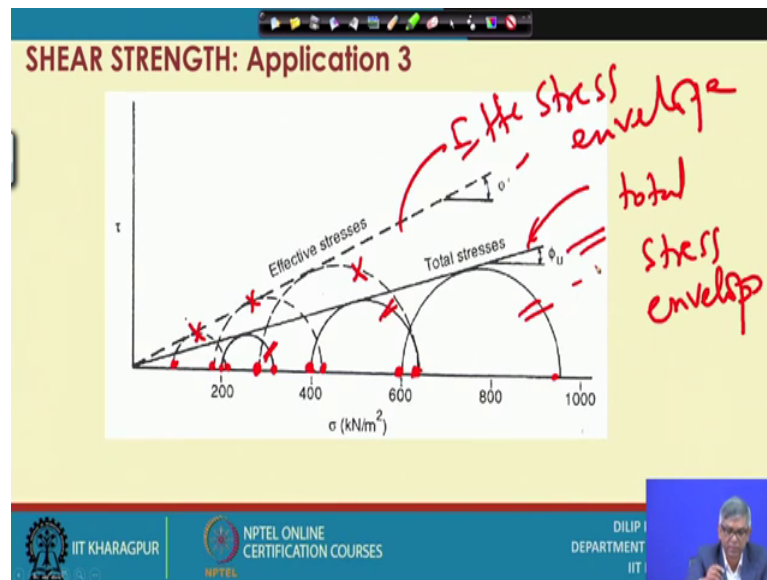
- Test 1: $\sigma_1 = 200 + 118 = 318$, $\sigma_3 = 200$, $\sigma'_3 = 200 - 110 = 90$, $\sigma'_1 = 318 - 110 = 208$
- Test 2: $\sigma_3 = 400$, $\sigma_1 = 400 + 240 = 640$, $\sigma'_3 = 400 - 220 = 180$, $\sigma'_1 = 640 - 220 = 420$
- Test 3: $\sigma_3 = 600$, $\sigma_1 = 952$, $\sigma'_3 = 600 - 320 = 280$, $\sigma'_1 = 952 - 320 = 632$

Sigma 1, test 1 actually sigma 1 is your 200 plus 118, 200 plus 118, that will be equal to 318 and your sigma 3 equal to 200, then your sigma 3 dash, sigma 3 dash will be equal to 200 minus u that is 110. So, that is equal to 90 and sigma 1 dash will be equal to 318 minus 110, 110 that is actually 208. So, we have got 1 data point.

For test 2, similarly sigma 3 is 400 and sigma 1 will be equal to 400 plus 240 that is actually 640, then sigma 3 dash will be equal to 400 minus 220, that is equal to 180 and sigma 1 dash will be equal to 640 minus 220 that is 420. So, this is another data set we are getting. And for sigma third test, you can see sigma 3, 600 then sigma 1 will be equal to 952 because we have to add principal stress difference and then sigma 3 dash will be equal to 600 minus 320 that will be equal to 280 and sigma 1 dash will be 952 minus 320 that is your equal to 632.

So, we are getting sigma 1, sigma 3, that is in terms of total stress, sigma 1 and sigma 3 in terms of total stress, sigma 1 and sigma 3 in terms of total stress. So, this is 1, this is 1 so that means, I get 3 circle, 1 circle for this, 1 circle for this, 1 circle for this. If I draw 3 circles, this represents actually total stress, stress circle and if I use this sigma 3 and sigma 1, this sigma if I draw another circle, by using this I can draw another circle, by using this I again another circle. So, if I draw circle based on these, this will be the Mohr circle representing effective stress.

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So now, you can we can do that and you can see when I have done for total stress circle. So, σ_1 and σ_3 was it was 200 and 300 200 this one 200 318 and the second one is 400 and 640 this was 400 and 640 and this was 600 and 952 600 and 952. So, I have got circle 1, circle 2 and circle 3. So, you can see this is this 3 circle and I have drawn a tangent and whatever parameter I am getting now that is ϕ_u and similarly now when I have done effective stress.

So, that was actually 90 and 208 this is 90 and this is 208 and similarly here 180 and 420 this is 180 and this is 420 and then sorry 180 and 420 and last one was 280 that is 280 and 632. So, this circle 1, circle 2 and circle 3 and this 3 circle if I drop tangent then we will get another angle of these, this is another envelop. So, this is actually total stress envelop and this is effective stress envelop.

And directly by measuring, I can find out what is the ϕ_{dash} , what is the ϕ_u . Instead of doing that, as I have shown in the previous problem it can be done little different way; that means, I can mathematically also or whatever relationship between principal stresses c and ϕ that relationship can be utilised, for example if I do this one, if I do this one, that is will get $\sigma_1 - \sigma_3 = 2c \cos \phi + \sigma_1 + \sigma_3 \sin \phi$.

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①

$$\sin \phi_u = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = 0.228 \Rightarrow \phi_u = 13.2^\circ$$

$$\sin \phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = 0.396 \Rightarrow \phi' = 23.3^\circ$$

②

③ —

$$\sigma_1 - \sigma_3 = 2 \cos \phi (\sigma_1 + \sigma_3) \sin \phi$$

$$\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

$$\phi_u = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

Here c component is 0, this normally consolidated drained test is mentioned. So, c is 0. So, then you have this one; so sin phi will be equal to sigma 1 minus sigma 3 by sigma 1 plus sigma 3.

So, this is the one when I use when I will be using dash here, then all will be dash here that be effective stress and when I will be using u, then it will be sigma 1 sigma u sigma 1, total stress minus sigma 3 by sigma 1 plus sigma 3. So, when it is total stress it be this expression; when is effective stress then this is the expression. So, if I use sin phi u, sin phi u will be sigma 1 minus sigma 3 divided by sigma 1 plus sigma 3, if I do then we are getting that is 0.228 whatever sigma 1 sigma 3 I have got if I put, we are getting this that gives u phi u equal to 13.2 degrees.

And if I do sin phi prime, that would be sigma 1 dash minus sigma 3 dash divided by sigma 1 dash plus sigma 3 dash and if I put all the value effective stresses, I can find out I am doing this one with respect to the first test. So, and if I do , then we will get 0.396, 0.396 and this keeps u phi dash equal to 23.3 degrees. So, like that I have used test 1, for both similarly I can do test , I can use test 3. So, from one test I get this, from second test I get this, third test I get this. So, little variation may be there. So, final results will be your average of this 3. So, this is the way finally, though it is undrained test, but you can get the effective stress parameter, how to do it that I have just explain here.

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SHEAR STRENGTH: Application 4

The following results were obtained from consolidated undrained tests on specimens of a saturated clay. Determine the shear strength parameters (effective and total)

σ_3 (kPa)	Deviator stress (kPa) at peak	u (kPa)
100	137	28
200	210	86
300	283	147

Handwritten notes on the slide:

- $(\Delta\sigma)_f$ (above the table)
- c, ϕ, c_u, ϕ_u (circled and written below the table)
- c', ϕ' (written below the table)

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Now, another application, this is the application, the last application and you can see here that the following results were obtained from consolidated undrained test on specimens of saturated clay. Determine the shear strength parameters effective and total. The problem is same almost, but I will just explain without using suppose your Mohr circle. I do not want to do, I do not want to use graph paper I want to do mathematically. So, how to do that, I have already explained last few lectures that this one. So, σ_3 is given, deviator stress being that mean $\Delta\sigma$ is given at failure is given and u is given. So, based on that, I have to find out shear strength parameter; that means, c and ϕ and c_u and ϕ_u .

So, c dash ϕ dash and c_u ϕ_u . So, this thing I have to find out mathematically; that means, by solving equation, we have some equation in terms of relating c , ϕ and σ_1 and σ_3 . We have some equation relating all of them and by that actually I can, those equation I can be utilised to solve or finding out this unknown c and ϕ based on the value a known value of σ_1 and σ_3 .

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Handwritten notes showing calculations for three tests (Test 1, Test 2, Test 3) to find principal stresses σ_1' and σ_3' using Mohr's circle equations. The equations involve normal stress (σ_n), shear stress (τ), and cohesion (c).

Test 1: $\sigma_1' = 100 + 137 - 28 = 209$
 $\sigma_3' = 100 - 28 = 72$

Test 2: $\sigma_1' = 200 + 210 - 86 = 324$
 $\sigma_3' = 200 - 86 = 114$

Test 3: $\sigma_1' = 300 + 280 - 147 = 436$
 $\sigma_3' = 300 - 147 = 153$

General equations for principal stresses:

$$\sigma_1' = \sigma_3' + c \tan^2(45^\circ + \frac{\phi}{2}) + 2c \tan(45^\circ + \frac{\phi}{2})$$

$$\sigma_1' - \sigma_3' = c \cos \phi + (\sigma_1' + \sigma_3') \sin \phi$$

Suppose test 1, σ_1' equal to 100 plus 137, 100 plus 137, it will be equal to this is minus 18, minus 228 this gives you 209 and σ_3' will be equal to 100 minus 28 that is equal to 72.

Similarly, test 2 we get σ_1' will be equal to 200 was σ_3' plus $\Delta \sigma$ is 210 minus u is 86. So, that gives you 324 and σ_3' will be called to 200 minus 86 that will be equal to 114.

Similarly test 3, σ_1' will be equal to 300 is the confining pressure, 280 is the $\Delta \sigma$ minus u is 147. So, that gives you 436 and σ_3' will be equal to your 300 minus; that means, confining pressure minus u 147. So, that is 153.

So, now we have got the 3 sets σ_1' σ_3' , σ_1' σ_3' , σ_1' σ_3' . Now what we can do from test 1, another thing is we have the expression $\sigma_1' - \sigma_3' = c \cos \phi + (\sigma_1' + \sigma_3') \sin \phi$. This is one form and this form if I simplify this will be equal to in terms of, σ_1' directly if I want to find out σ_1' will be $\sigma_3' + \tan^2(45^\circ + \frac{\phi}{2}) + 2c \tan(45^\circ + \frac{\phi}{2})$.

So, this is the another form of equation; that means, better I will clean this one; this is $45^\circ + \frac{\phi}{2}$.

So, this is the form I can use, also this is the form I can use. So, if I use this form; from test 1 I get one equation, from test 1 I get from test 1, I get equation that is 324, 324 equal to no 209, let it be there this is about test 2. So, test 1, 209 it will be equal to $72 \tan^2 45^\circ + \phi$ by 2 and plus 2 c dash and this is ϕ dash $\tan 45^\circ + \phi$ by 2.

Similarly, from second test I can do, 114 $\tan^2 45^\circ + \phi$ by 2 plus 2 c dash $\tan 45^\circ + \phi$ by 2. This $45^\circ + 5$ by 2 or $\tan 45^\circ \phi$ by 2, I can give a name and this then it will be easy to handle the equation. And from test 3, I can write another equation 436 is the sigma 1 here. You can see your 436 was the sigma 3 and sigma 1 $\tan \phi$, sigma 3 $\tan \phi$ sigma 1 is 153, 153. 0. So 153, 153 $\tan^2 45^\circ + \phi$ dash by 2, all are ϕ dash, these are c dash plus 2 c dash $\tan 45^\circ + \phi$ dash by 2.

So, now, I have got 3 equations. I have got 3 equations in terms of ϕ dash and c dash. So, I have 2 unknowns. So, I have to; that means, I have 2 unknown but I have got 3 equations. So, what I can do I can do in combination; that means, I can solve equation 1 and equation 2. So, I get c dash and ϕ dash, one set then similarly I can do 1 and 3, I will get another c dash and ϕ dash and then I can get 2 and 3, then from there also I will get another set of c dash and ϕ dash.

So, like that if I do, then I can get 3 different values and then I can take average, I can take the average. Suppose it is 10, 11, 12 suppose or 10, 12, 11, 10. So, average of this 3 will be the c dash and this average of this ϕ will be ϕ dash. So that means I have got 3 equations, 1, 2, 3. How I have got this equation? I have already shown that sigma 1, sigma 3 from the test 1; sigma 1 and sigma 3; what is the relationship between sigma 1 and sigma 3? This is the 1. So, using that appropriate value of sigma 1 is given 209. You can see and sigma 3 sigma 372. So, sigma 3 $\tan^2 45^\circ + 2 c \tan$ this.

Similarly for the second test, we can see 324 is a sigma 1 and sigma is the 114. So, 114 $\tan^2 45^\circ + 2 c$ this. And third test is the 436 is the sigma 1 and 153 is the sigma 3. So, I have got 3 equations. So, by using 3 equations, I have to find out 2 unknowns. So, I have x s, why it is; because from the test results we may not get the accurate results. So, we have to get the average value. So, because of that combination of 1 and 2, 1 and 3, 2 and 3 like that I can solve and get value of c and ϕ from each set and then finally

average value I can find out. So, this way when we get, then that value that will be your effective stress parameter c dash and ϕ dash.

And similarly if we get another; suppose if you want to find out in terms of total stress, the total stress was σ_1 was test 1.

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① $\sigma_1 = 237$ $\sigma_3 = 100$
 ② $\sigma_1 = 410$ $\sigma_3 = 200$
 ③ $\sigma_1 = 583$ $\sigma_3 = 300$

① $237 = 100 + c_u \tan^2(45 + \frac{\phi_u}{2}) + 2c_u \tan(45 + \frac{\phi_u}{2})$
 ② $410 = 200 + c_u \tan^2(45 + \frac{\phi_u}{2}) + 2c_u \tan(45 + \frac{\phi_u}{2})$
 ③ $583 = 300 + c_u \tan^2(45 + \frac{\phi_u}{2}) + 2c_u \tan(45 + \frac{\phi_u}{2})$

①	c_u	ϕ_u
②	c_u	ϕ_u
③	c_u	ϕ_u

c_u, ϕ_u

Sigma 1 was 237 and sigma 3 was 100. So, suppose from 2, sigma 1 was 410 and sigma 3 was 200 and from test 3 it was 583, sigma 1 was 583 and sigma 3 was 300. So, I can get one equation; that means sigma 3, 237 equal to sigma 1, 100 tan square 45 degrees plus phi by 2 plus 2 c tan 45 degrees plus phi by 2, phi by 2. Then from test 2, I can get 410 equal to 200 tan square 45 degrees plus phi by 2 plus 2 c tan 45 degrees plus phi by 2.

And from test 3, I can get 583 equal to this is 300 of course 300 tan square 45 degrees plus phi by 2 plus 2 c tan 45 degrees plus phi by 2. So, again we have got 3 simultaneous equations for 2 unknowns. So, I can do 1 and 2, then 1 and 3 and 2 and 3 from each c and ϕ_u , c_u and ϕ_u , c_u and ϕ_u , c_u and ϕ_u . 3 sets of c_u ϕ_u you will get finally average of this will be final c_u and ϕ_u .

So, this is the way actually sometime many people do not like to draw in graph paper wanted to do analytically, then this is the equation can be of same thing only instead of drawing Mohr circle, I can write the equation. And in each equation, will have 2

unknowns from each of single equation will not be able to solve but you need 2 equation at least or 2 test minimum required. But we in the standard procedure, in the laboratory is generally we conduct 3 or more test.

So that, if there is any error because of sample preparation equipment etcetera. So, we can get the average values. So, we have plenty data, 3 or 4. So, we require only 2, but we do not know which is erroneous. So, because of that we can take the combination of 1, 2; 2, 3; 3, 1 like that and 3, 4 sets of simultaneous equation can be formed and by solving them we can get, c_u and ϕ_u from which and finally, 3 sets of c_u 3 sets of ϕ_u or 4 sets of c_u 4 sets of ϕ_u get and then we can get the average of them to get the final value of c_u and ϕ_u .

So, this is the way one can avoid use of graph sheet. Actually graphs it is also not required, when we use Mohr circle; generally conceptual you can draw the Mohr circle, you can just draw the line like this and I can give value something here give value something there and then I can conceptualized I can draw the Mohr circle, not necessarily we have to draw using Mohr circle. If we use, sorry not necessarily you have to use the graph sheet. If use graph sheet, what is the advantage?

Advantage is, the directly you can read the value from the graph whereas if you use plain sheet and conceptually you do the Mohr circle, then you have to do small calculation to find out the stresses and other things. So, this is the way one can do or if they completely avoid this type of sketch and diagram etcetera, they can use this equation setting to a different equation solve it get the value. So, this is sometime many people feel convenient. So, it is up to them to use it.

So, like that I have shown 4 applications already in shear strength. I will take some more topic also in the shear strain and if I feel required, some more application I will show with different types of test condition, how to utilise the test results to find out the value. So, with this I will just stop here.