Soil Mechanics/Geotechnical Engineering I
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Lecture - 30
Shear Strength (Contd.)

Good morning once again. I will; I welcome you to this course soil mechanics and last few classes, I have discussed about many preliminary on shear strength background only I have discussed.

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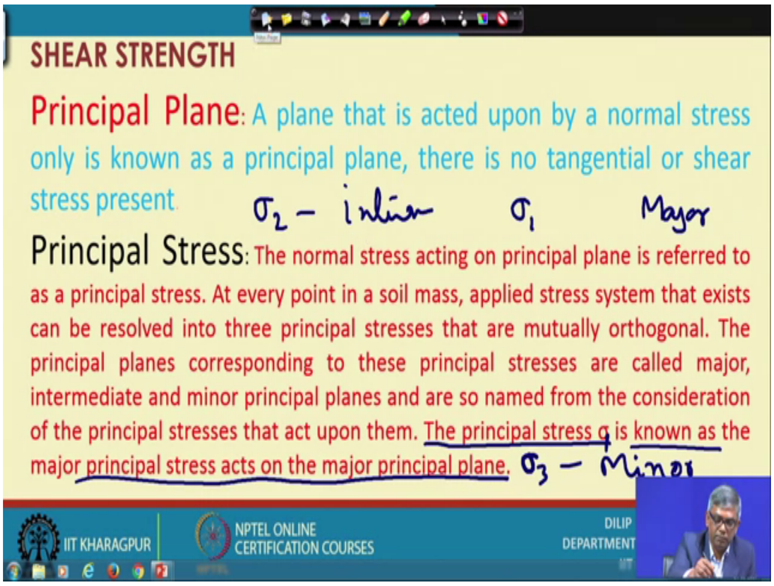

Now I will discuss really on soil shear strength of soil and what I have gone through as background and we have mentioned that when there is a at a particular point inside the mass because of the externally applied load, we can see that at a point state of stress.

And of course, initially you have seen in 3 dimensional form and then we discussed in biaxial form because this is very commonly we use for soil mechanics, though, we can used in the 3 dimensional, but initial understanding with 2 dimensional biaxial problem can be handled first and while doing that once biaxial problem, then we have seen that if you know the state of stress in that respect to the X Y direction whatever the components, then we can find out also any other plane what is the normal and shear stress and also if you find out there is a plane or orientation of a plane or plane oriented in such a way that on that plane principal stress maximum principal stress occur or in some plane maximum stress is occur.

And that also how at which angle or which plane that happens that also we have discussed and finally, in summary we have seen 2 things; one is principal plane another principal stress and what is principal plane that is a plane that is acted upon by a normal stress only is known as a principal plane and in that plane; obviously, tangential shear stress present ok.

And similarly principal stress that is the plane was first of all we have mentioned where the maximum normal stress occur that is the principal plane and principal stress is the normal stress acting on the principal plane is referred to as a principal stress and at every point in a soil mass applied stress system that exist can be resolved in to 3 principal stresses that are mutually orthogonal and the principal planes corresponding to this principal stresses are called major intermediate and minor planes; that means, the plane on which maximum occur that is major principal stress the plane on which the minimum occur that is called minor and plane and plane on which the intermediate stress occur that is called intermediate plane.

And major intermediate minor planes and are so called. So, name from the consideration of the principal stresses that act upon them; that means, larger stress acting on a plane that is major smaller one that is a minor plane and the intermediate one is the intermediate plane the principal stress is sigma is known as the major principal stress is act on the major principal plane ok.

So, the major principal stress sigma is known as the principal stress sigma is sigma 1 actually is known as the measure principal stress acts on the major principal plane. So, there are 3. So, where sigma 1 acting that is major and where sigma 3 is acting that is minor and where sigma 2 acting that is intermediate.

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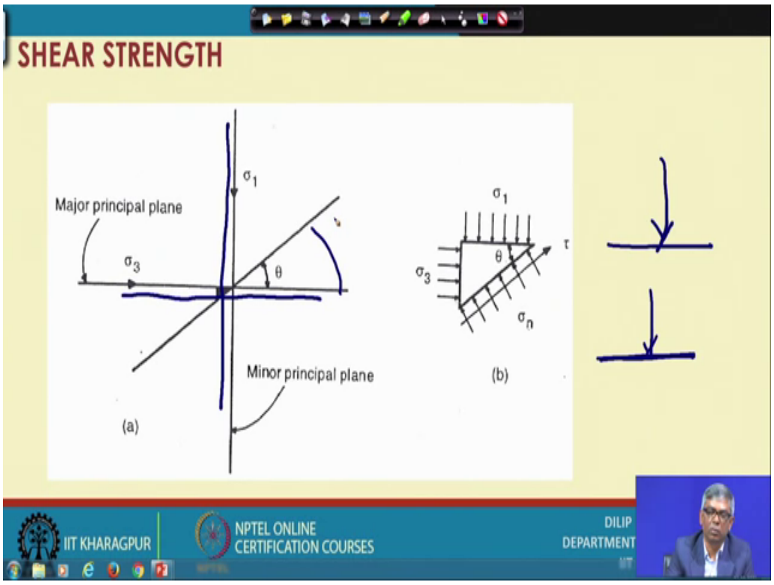

So, this is the way we generally classify next and you can see this one this is the plane this is the sigma 1 is a major principal stress and this is the direction of major principal stress. So, major principal plane must be perpendicular to that. So, this is the major principal stress, then your this one will be the major principal plane.

Similarly, this is the sigma 3 that minor principal stress and. So, minor principal stress will be acting perpendicular to that plane; that direction. So, minor principal stress is a plane is this one. So, if the direction of stress is this, then direction of the plane will be perpendicular to that or if the plane is defined the normal stress will be perpendicular to that obviously.

So, if I know the major principal stress that is also normal major principal stress or major normal stress major principal stress that is the normal stress so; that means, plane will be perpendicular directions. So, that is the way is defined here, similarly, if you have any plane here like inclined here on that plane what is the if the plane is oriented like this; this plane is rotated and this is new plane suppose, then what will be the stress on this we can calculate whatever wave we have discussed previously.

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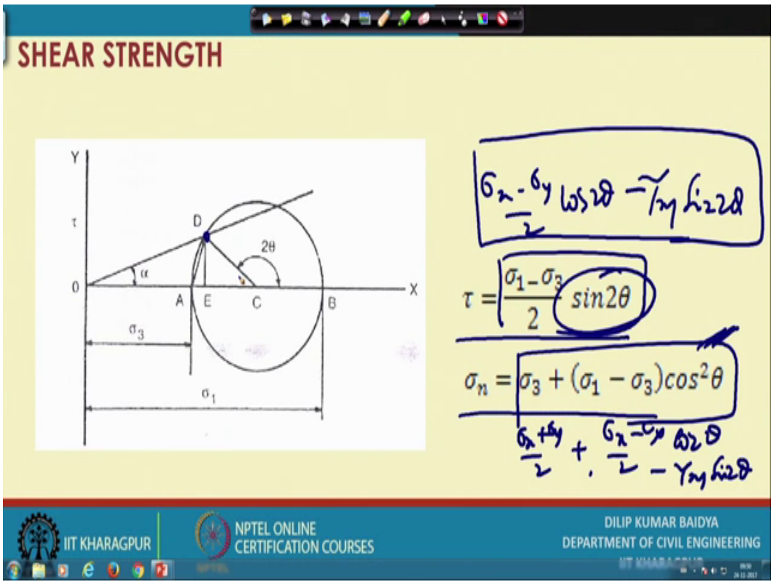

Now this is the one we have shown before also the shear strength that is your at this point; suppose, at this point, what is the shear and what is the sigma that we can find out by this equation and; obviously, this is the initial form we got, but this form also can be trigonometrically simplified and it can be written at as a sigma at X minus sigma Y 2 cos 2 theta minus tau X Y sin 2 theta. So, this thing, this form, this is whatever is the trigonometrically if you simplify you will get this form also.

Similarly, this is the initial form we have got by consider in the equilibrium this is also another form of the sigma X plus sigma Y; Y 2 plus sigma X minus sigma Y by 2 cos 2 theta cos 2 theta minus tau X Y sin 2 theta. So, this is again trigonometrically if you simplify you will get. So, this and this; either this and these are same and these and this they are same only thing is this is expression double angle and we have mentioned that whatever angle we will see the circle the on the plane actually half of that. So, this is the plane suppose if I imagine the 2 theta actual orientation of the plane is theta. So, that is the thing we have already learned. So, same thing I am just will apply to the soil.

So, these are the initial expression and this is in these other important things are shown if I draw a Mohr circle; here on this Mohr circle, this is actually major principal stress sigma 1; this is minor principal stress and if I take a point here suppose point D, then that that is the plane either this is the plane, suppose, if I consider on that plane; what is the normal and what is the shear by using this equation one can find out.

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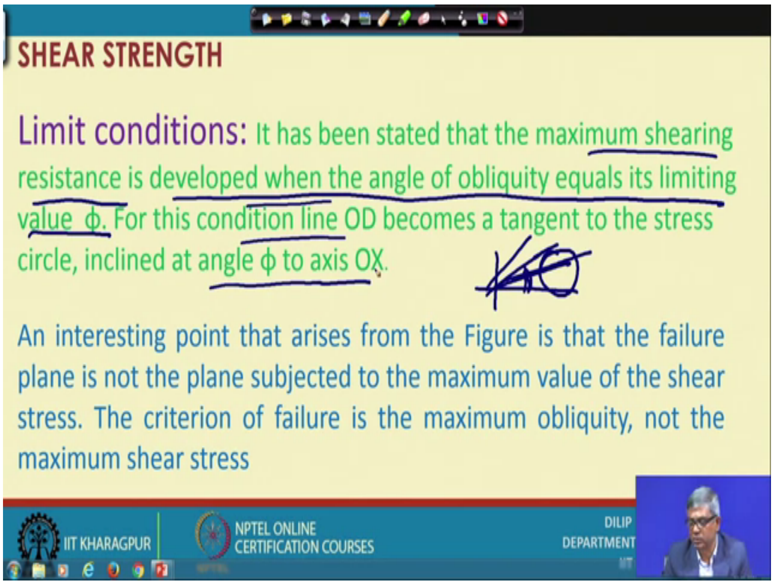

So, the limit conditions; that means, you can see I have already explained this that maximum shearing strength resistance is developed when the angle of obliquity equals is limiting value phi. So, maximum shearing resistance developed by the soil when the angle of obliquity is equal to phi.

So, when this the angle it was different angle and circle is there suppose and this circle initially will have this angle then this angle like that when it will just tangential to this that is the limiting condition and that condition actually alpha become phi that angle of obliquity equal to alpha f equal to phi; that means, the maximum shearing resistance developed when the angle of obliquity equals is limiting value phi and for this condition the line O D becomes is tangent to stress circle inclined an angle theta to axis OX.

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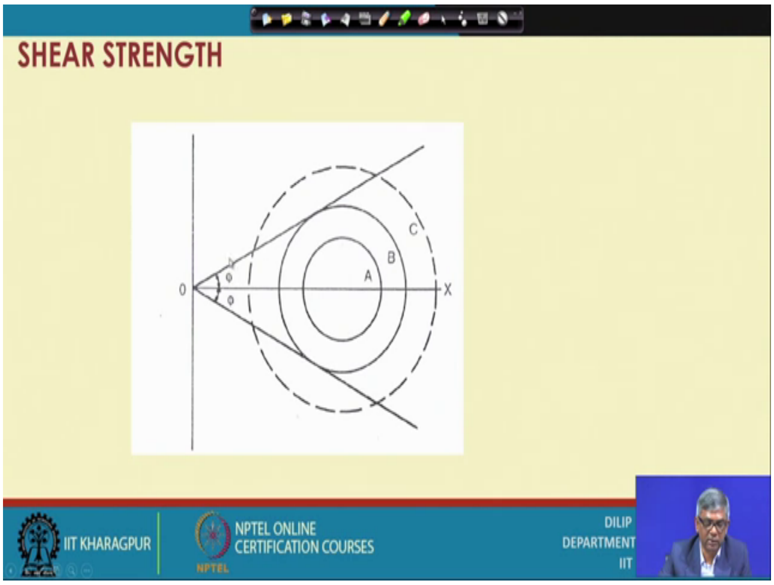

So, that is actually I can show you in the next one you can see in this is the circle and this is one thing. So, this is the circle actually this is circle and on that circle when this alpha will be changing and it will become tangent to this that point only, we will get the limiting value the limiting resistance and so, with this observation another further observation will be done that is you can say oh sorry you can see that is another interesting point that arises from the figure that the failure plane is not the plane subject to the maximum value of the shear stress.

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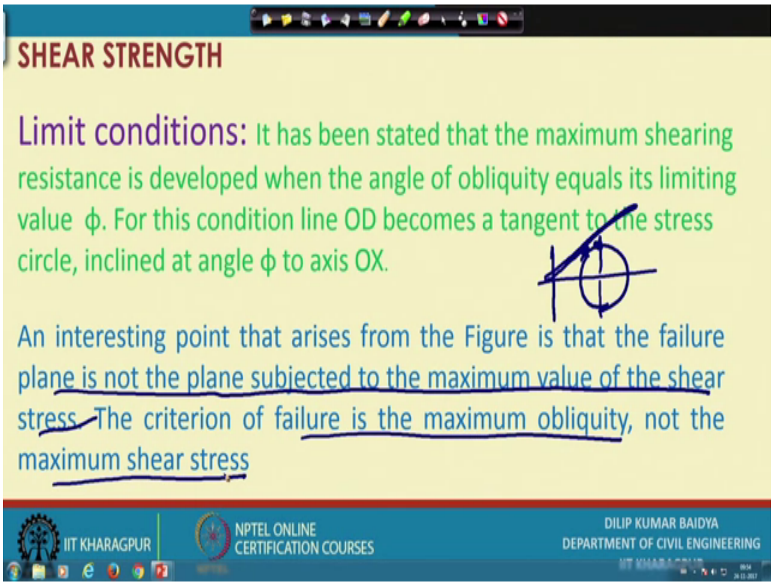

Suppose if I draw the Mohr circle here if I draw the Mohr circle here and suppose this is the origin and maximum shear stress is occurring here, but this is tangents this is tangent somewhere here. So, it is not that is the point interesting point that arises from the figure is that the failure plane is not the plane subjected to the maximum value of the shear stress.

So, at this plane that point is not maximum shear stress maximum stress is acting here. So, the criteria of failure is the maximum obliquity not the maximum shear stress so; that means, the how much oriented or rotation or orientation of the angle that plane that is important than the maximum value of shear stress there of course, quite close, but not really to the maximum value.

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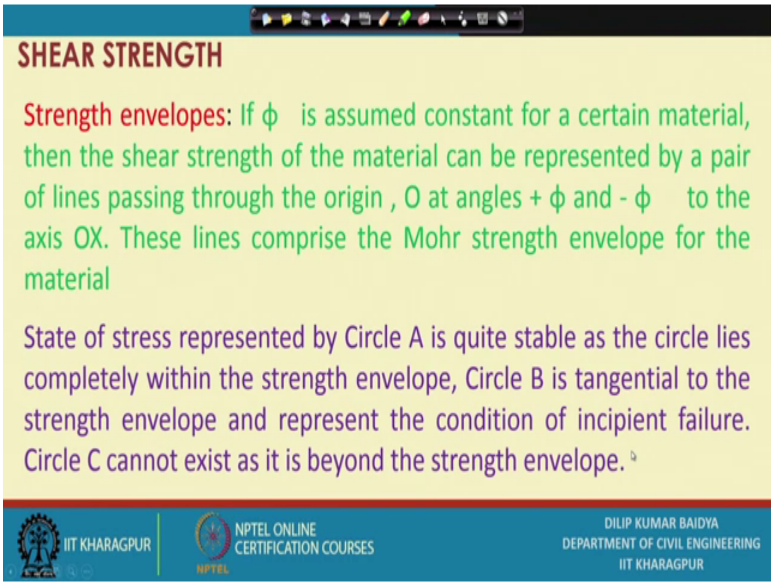

And if phi is assumed constant for a certain materials most of the soil material we will show you later on that your phi value for particular soil different value of phi will be there again if you change the condition and the phi value will change the same soil, but still for the time being suppose all conditions are same and for a for a particular soil phi is constant suppose if I consider.

And then the shear strength of the material can be represented by a pair of lines passing through the origin and O at angles O plus and O minus to the axis OX suppose you can see this is the axis OX and plus phi minus phi this is a phi that line I can draw tangential to this.

And so, if this is the one then to the OX this lines comprised the Mohr strength envelope for the material; that means, what is the meaning of it; that means, the strength envelope; that means, along the failure if I consider sigma and tau if we change then. So, failure will failure will take place whatever failure those points will be within that only

And state of stress represented by circle A is quite stable. So, that is another other 2 observation is you see suppose I have drawn this is the; for a particular soil the phi is constant suppose if I assume, then this is failure envelope this is the orientation actually we know the maximum. So, if I draw a circle tangential to this; that means, it is just at the words of failure.

Whereas if I consider another circle which is also representing the state of stress are located state of stress at some condition ha state of stress and this is the one at a particular point state of stress if I plot and I if get this circle; that means, the soil is much stable and whereas, it is just stable if I change little failure will occur and this is actually much below the stable condition much better than the stable condition.

Similarly, if I stressed a material in such a way the Mohr circle become such a big one. So, it is going out of the envelope; that means, it is totally unstable or it will be failure so; that means, to have the soil a stable condition that is Mohr circle should be within this envelope; that means, if I draw from the origin with plus phi and minus phi 2 lines the circle should be within that 2 envelope. So, that is the thing.

state of stress represented by circle A is quite stable as circle lies completely within the strength envelope that is inner circle; circle B is a tangential to the strength envelope and represent the condition of incipient failure; that means, middle one which is tangential and the circle C cannot exist as it is beyond the strength envelope so; that means, that is failure. So, this one is a failure this is just words of failure and this is a very stable condition.

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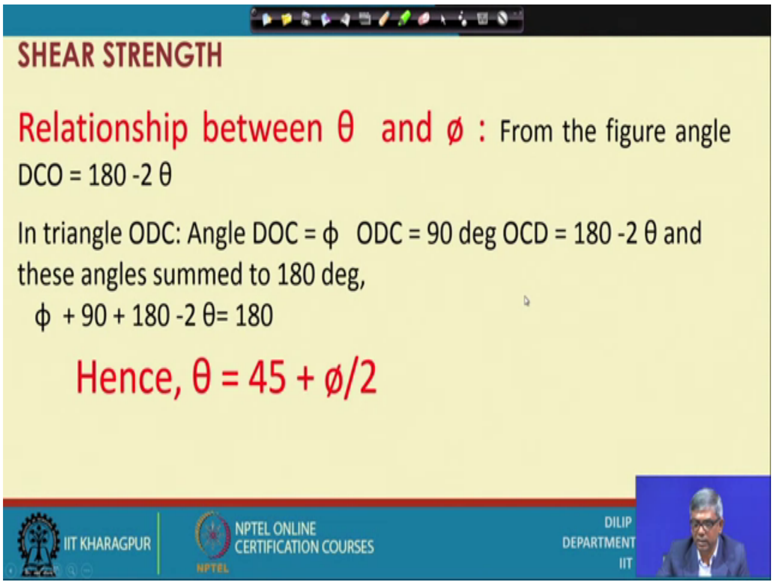

Now, relationship between theta and phi that is orientation angle theta the plane oriented theta and phi is the angle of internal friction from this is the thing actually we have to now find out the relationship and that relation for that relationship we can take the help of the diagram that is in this one.

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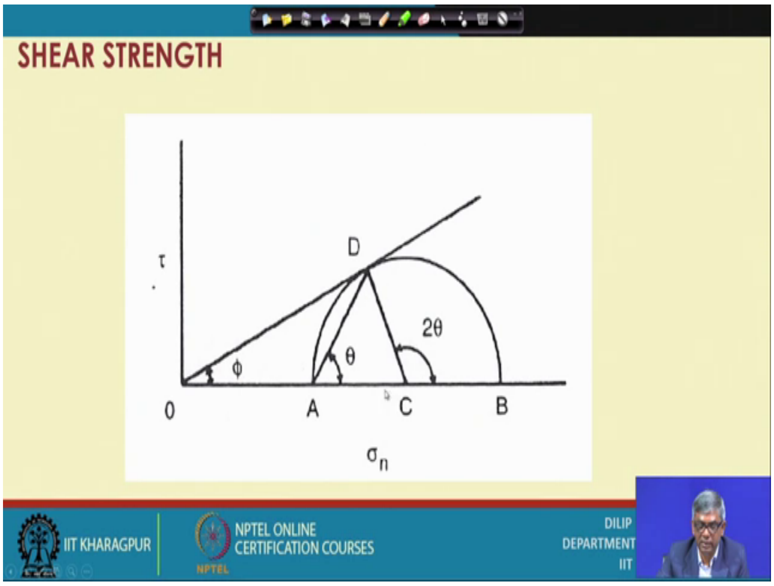

You can see this; I can consider a such triangle; I can consider this triangle and then from this triangle if this is 2 theta that this plus this will be theta and again this and this is same. So, this and this angle will be theta. So, this is theta this is theta.

And so, by based on this observation I can formulate this way you can see D C O is 180 minus 2 theta angle D C O D and D C O this angle 180 minus 2 theta because this total is 180. So, 180 minus 2 theta this angle and your; in triangle O D C angle D O C is phi O D C is 90 degrees D O C is the phi D O C is the phi and sorry and your O D C is 90 degrees and O C D is 190; 180 minus 2; 2 theta O O D C is 90 degrees, you can see O D C; O D C, this is actually since a tangent and this is radius. So, this angle will be 90 degrees. So, this is 90 degree, this is phi and this is 180 minus 2 theta.

So, if that is the one and this angle sum to be summation should be 180 degree. So, phi plus 90 plus 180 minus 2 theta will be equal to 180; 3 angle some circle of triangle if you add, then it become 180 degrees and that 180 degrees we have seen 3 angle one is phi another is 90 and another is 180 minus 2 theta.

So, then this one if you simplify you will see theta equal to forty five degree plus phi by two. So, this is the one. So, this is if this is the plane actually this is the plane and failure plane and this is phi envelope. So, what is the relationship between the failure plane and the angle of internal friction? So, that is theta equal to 45 degrees plus phi by 2.

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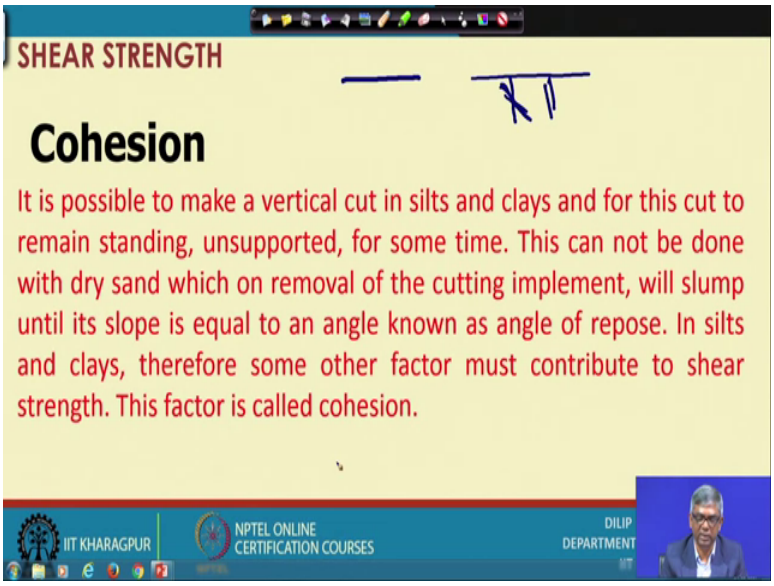

Now, we have discussed about shear strength and we see that one is actually phi angle of internal friction we have that phi we have discussed and now, it is a cohesion it is possible to make a vertical cut in silts and clays and for this cut to remain standing unsupported for some time this cannot be done with dry sand which on removal of cutting implement will slump until its slope is equal to angle known as angle of repose.

So; that means, in a silt and clay silt and clay suppose this is the one, but you can make some cut. So, if you make an excavation here up to some depth, it will stand without any support, but if it is a sandy soil suppose this is sandy soil and then, if you try to excavate then it will actually this slope will fail and it will have some final inclination. So, this cannot be done with dry sand which on removal of the cutting implement will slump until its slope is equal to an angle of angle known as angle of repose; that means, the soil will form a angle which is equal to the angle of repose which is nothing, but equal to phi.

In silts and clays therefore, some other factor was contribute to shear strength; that means, silts and clays where sand is not able to stand whereas, silt and clay able to stand what is the reason the reason is that silt and clay also polishing something that is some other factor must contribute to shear strength that is what and this factor is called cohesion; that means, when the dry sand the shear strength contributes from the angle of internal friction that is phi whereas, in slit and clay there may be phi any addition to that there is a some other parameter the which actually can help to stand unsupported that is because of the appearance of cohesion that; that means, cohesion also contribute to the shear strength.

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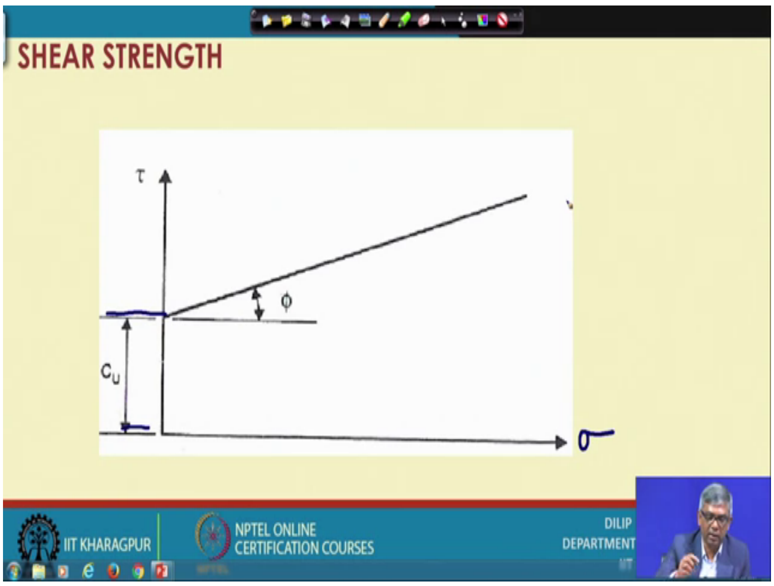

Now, you can see that shear strength; this is actually if I plot this is as a sigma we generally plot one axis sigma another axis is tau and then your failure envelope typically will have with the this form there will be intercept here that is equal to the cohesion and this is this inclination will be phi and; obviously, this C and phi if you have then it will have a definite equation and shear strength equation and that is shear strength equation can be written.

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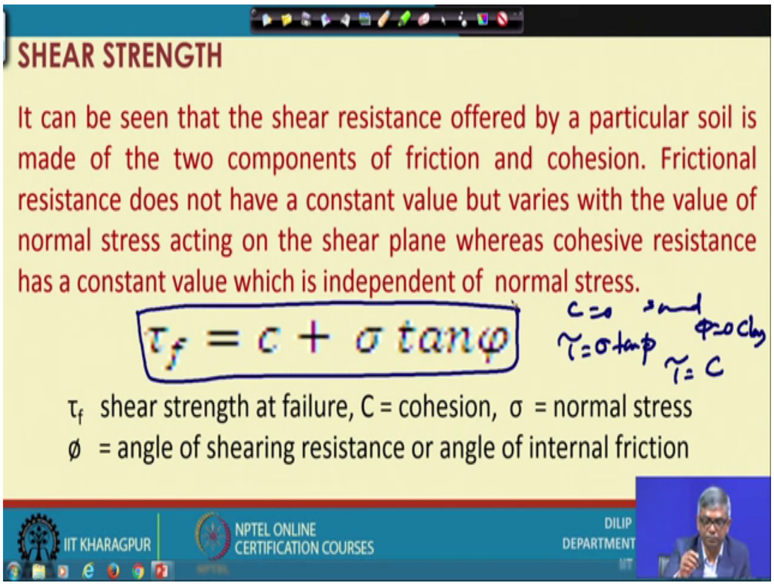

In this form, it can be seen that the shear resistance offered by a particular soil is made of the 2 components of friction and cohesion; that means, as I have mentioned that some soil may contain or may possess only the fiction some soil also may possess only cohesion and more soil actually may most of possess actually both the component actually cohesion and friction.

So; that means, total shear strength will be coming from the contribution of both shear friction and cohesion. So, that is the one; the 2 components a friction and cohesion and friction resistance does not have a constant value, but varies with the value of normal stress acting on the shear plane whereas, cohesive resistance has a constant value which independent on normal stress; that means, because of these shearing resistance from friction it depends on the normal stress if the normal stress is more the shearing resistance will be more normal stress is less shearing resistance less.

Whereas shear resistance contribution from cohesion is constant independent of any normal stress; so, because of that we can finally, write the expression tau shear strength of soil at any point will be equal to C plus sigma tan phi where tau f is the shear strength at failure and C is the cohesion and phi is the normal stress and phi sorry sigma is the normal stress and phi is the phi is the angle of angle of shearing resistance or angle of internal friction.

So, now there are 3 typical thing can be there when particular soil C equal to zero that is sand in that case your tau will be equal to sigma tan phi and another soil can have phi equal to zero; that means, clay for that tau will be equal to C only and if the soil possess both; that means, phi and C that this is the general equation for shear strength.

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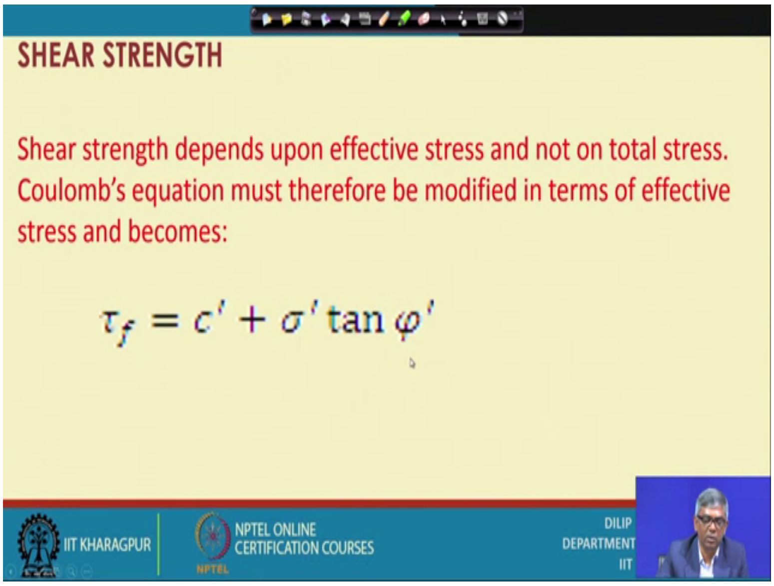

And shear strength depends upon effective stress and not on the total stress. So, I have discussed again the total stress effective stress and that time I have mentioned that the effective stress is very important particularly because the shear strength depends on very much on effective stress only and coulombs actually more coulombs equation.

Therefore, modified that whatever equation we have given before that is coulomb more coulomb equation of shear strength tau equal to c plus sigma tan phi that can be modified in terms of effective stress parameter that tau equal to C prime plus sigma prime tan phi prime. So, these are all call c dash is called the effective c dash and phi dash is called effective stress parameter and phi dash is the sigma dash is the effective stress acting at the point where we are calculating the shear strength.

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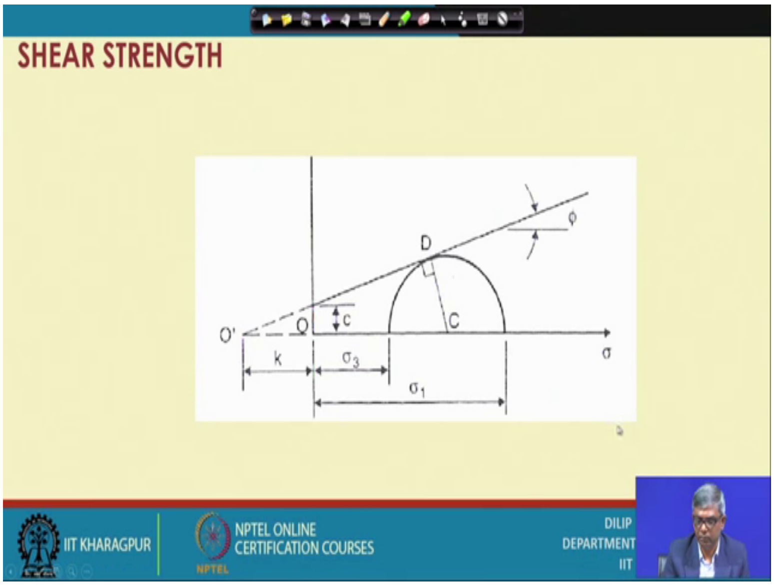

Now, we can see we can we have taken we have we can see a general form of shear strength Mohr circle I can take and in this Mohr circle you can see that the suppose failure envelope is this one is the failure envelope which is having a phi and there is a value of c and based on this and this will be sigma 3 and this will be sigma 1 now we can relate this sigma 1 sigma 3 phi and C where geometrically.

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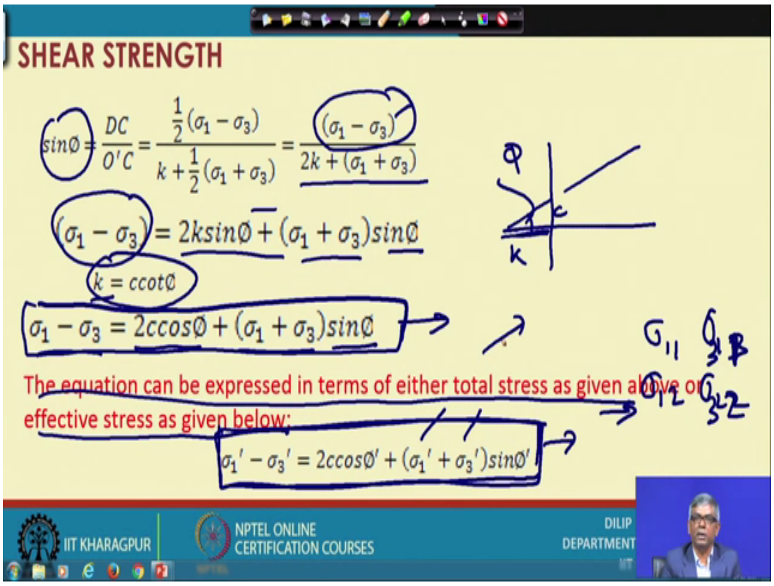

So, let us consider this one and I will see that in the next slide this is you can see; now that sin phi; that means, we can sin phi in the in the previous one sin phi if I consider this is actually phi this angle is a phi this angle is a phi. So, if I consider sin phi; that means, I can this triangle I can consider and based on this triangle sin phi equal to actually this is 90 degrees. So, this by this; this by this. So, this C D by C O dash.

So, if that is if that is the case, then sin phi I can write in those in terms of some known parameters. So, this is this is how much up to this; how much up to this; how much all those and this is what all the in terms of that I can express. So, sin phi equal to D C by O dash C as I have mentioned and D C will be you can see half sigma 1 minus sigma 3, this is D C is the D C is the radius D C is the radius.

So, it will be half sigma minus sigma 1 minus sigma 3 and O dash C will be k plus half sigma 1 plus sigma 3. So, k is what actually we have shown here k is taken as this much. So, these plus this and this one actually half sigma 1 plus sigma 3 so; that means, I can sorry. So, half sigma 1 minus sigma 3 I have done half sigma 1 minus sigma 3 and k plus half sigma 1 plus sigma 3 this is the sin phi if I express in this form it comes. So, if I simplify then it becomes sigma 1 minus sigma 3 by 2 k plus sigma 1 plus sigma 3.

Then if I take in terms of sigma 1 minus sigma 3 which is known suppose sigma 1 minus sigma 3 if I express sigma 1 minus sigma 3 that will come 2 k sin phi plus sigma 1 plus sigma 3 sin phi and if I go back to the previous figure then we will see that k which we have noted that is actually C cot phi suppose this is the diagram and so, this is the k actually and this is actually phi and this is actually C and so, from here I can write k equal to C cot phi k equal to C cot phi.

So, this is this is the k. So, this k actually C cot phi if I substitute C cot phi in this equation then sigma 1 minus sigma 3 become 2 C cos phi plus sigma 1 plus sigma 3 sin phi. So, this is the very generalised expression of in terms of relating sigma 1 sigma 3 and phi of the soil.

So, and this when I have written this equation this equation in terms of actually total stress similarly the equation can be expressed in terms of either normal stress as given above or in terms of the effective stress as given below; that means, we can express the in either total stress form that is the one which without prime. So, this is equation and if I express in terms of effective stress form, then I will get this is the equation.

So, now; so, this these 2 important expression sometime will be useful as I have mentioned beginning that there are number of field and laboratory test available and from the test from the we apply certain amount of normal stress and then we apply we shear the soil and then at the time of shearing what is the stress we calculate find out and then if you have those values and based on those values we have to find out the shear strength parameter C and phi.

So, we measure generally this and sigma 1 and sigma 3 during test suppose and then if I do at least 2 test, then I can express this equation suppose, one is sigma 1 1 and another is sigma 1 3 that is one set and based on that I will get this equation in terms of C and phi, but since C and phi will not change the same soil we are testing identical sample again I will suppose another test I will do sigma 1 2 sigma 2 2; sorry, sigma 1 2 and sigma this is 3 1 suppose 1 1, 3 1 and this is 3 2. So, this is another set the same equation I in terms of this I can express since C and phi will unchanged.

So, from the 2 equations by solving I can find out C an phi. So, this is the way actually this; this is what actually as I have mentioned before that we need some theoretical background along with the experimental data to understand the shear strength. So, that is the one application that whatever we have find out now relationship between C and phi and C phi and normal stresses the major and minor normal stresses whatever relation we have got and experimental if we get some data correlating them we can find out the value of C and phi of the soil with this I will stop here and I will take next.

Thank you.