Soil Mechanics/Geotechnical Engineering I
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Lecture - 29
Shear Strength (Contd.)

Welcome once again. I welcome you this again to this lecture and I have stopped just how to use Mohr circle for calculation of stresses in different orientation and also before that I have shown by equation method and of course, using graphical or the Mohr circle method is advantageous because once you draw the circle we can get many lot all information from their itself without much calculation of course, though if I draw the Mohr circle with a proper scaling in all directly I can measure the values, but sometime without that also simple sketch Mohr circle and small calculation sometime also it can be done quickly.

So, I will try to show a few examples how to use this before going to shear strength of the soil because these are the things finally, will be used. So, let me come to the first Problem.

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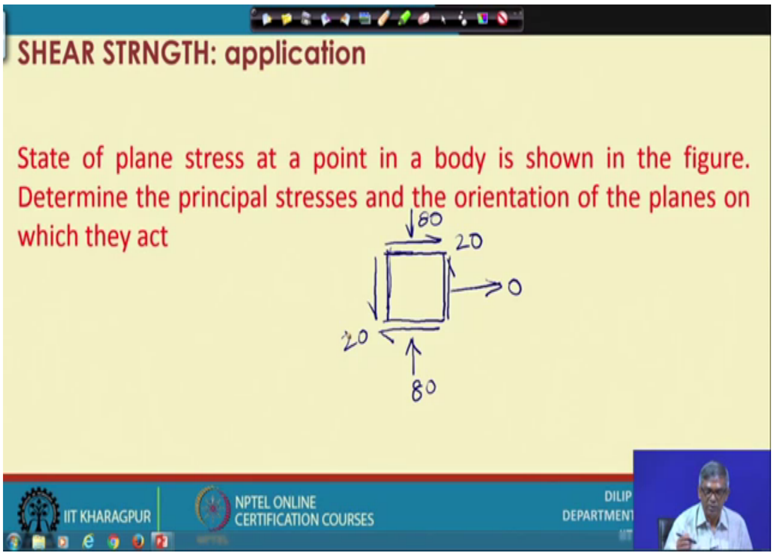

The problem is described here I will just sketch the draw problem here the element is given something like this and you can see and this normal is 80 there is no normal actual

I can draw a figure I can draw narrow, but I can say this 0 and that is a shear that is 20 and; obviously, if this is 80, this is also 80, if this is 20, this is also 20.

So, this is the problem. So, state of plane stress at a point in a body is shown in the figure determine the principal stress and the orientation of the plane on which they act. So, this is the problem we have to do and for this let me this is a very simple problem the sigma y. So, I list the information given and then based on the information how to do by using Mohr circle we will try to see and in combination of the Mohr circle and calculation also I will show you.

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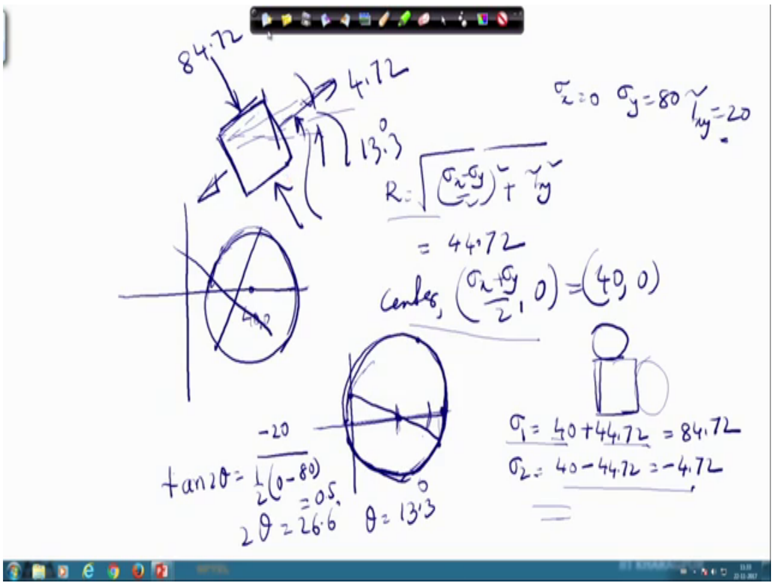

So, this is the one here I can list sigma x equal to 0 and sigma y equal to plus 80 sign compressive I have taken positive and your tau will be xy equal to suppose 20 this is the value given and then if I want to do Mohr circle method the radius of the Mohr circle actually expression is R equal to under root sigma x minus sigma y by 2 whole square plus tau xy square.

So, if I use those values then R become forty 44.72 and your C centre will become that is sigma x plus sigma y by 2 0 and that one; that means, this will be 80 by 2; that means, 40, 0. So, this becomes the Mohr circle and also we have got the centre and we have got radius it is easy to do Mohr circle, but at the same time as I have mentioned that I can read the this face and I can read the this face I will plot first this face one place and this is actually your shear was shear is minus actually here right side half. So, it will be minus.

So, I can draw the Mohr circle suppose this one either I can locate the centre here locate the centre here at 40, 0 and then taking the radius I can draw the circle. So, suppose I will draw the circle like this or I could have done I could have taken then right face is 0 plus 20 it was shown this way. So, upward is taken positive that was convention whatever I have write downwards in mechanics. So, here actually taken positive.

So, if I do this. So, 0, this circle is not drawn properly then. So, better I will draw once again this is not correct. So, this is your. So, 0 and 20 suppose here and circle is somewhere here a centre and this point suppose somewhere here. So, I can join this and draw the circle or taking this as centre and with that radius also you can either way you can do. So, your circle will become something like this circle will become like this.

So, from here actually number of things we can get once we have got this. So, this is C and. So, that is sigma one is this the if I do graph then directly I can read this value what is the value, otherwise I can find out sigma 1 exactly it will be up to this is the C the centre; that means, 40 plus radius is how much I have got 44.72, that become your 84.72.

And similarly I can find out sigma 2 this is 40 and from here actually I can minus R I can to get this side. So, 40 minus R is 44.72, that means, it will be minus 4.72 I can get this and if sigma 1 and sigma 2 we have got either I can do this calculation and get or I can get from this reading this and reading this.

And then now direction you have to find out. So, tan 2 theta will be equal to it will be minus 20 divided by half 0 minus 80. So, that gives you 0.5; that means, it gives you 2 theta will be equal to 26 degrees. So, theta will be equal to 13.3 degrees. So, that is the orientation of the plane on which maximum is occurring.

So, if I want to draw a plane now with that orientation. So, what I have to do, I have to make a horizontal plane I will make a 13 degrees angle and this is the direction. So, our plane will be. So, this is one. So, since it is tensile. So, it will become minus of 4.72 in this direction and this one will be compressive and that will be equal to 84.72. So, at this angle will be 13.3 degrees and; obviously, this side also same tensile this side also same.

So, this is the one so; that means this is the problem. So, we have with the problem it is asked actually to find out major and minor principal stress and their orientation. So, we have got 2 ways we could have done I could have taken the right face and draw the point locate the point read the top face locate the point joining these 2 points I can draw if I join these 2 point then I will get the diameter. So, midpoint of that will be the centre and taking that as a centre and half of that line will be the radius we can draw the circle or based on this calculation radius calculation and centre location these 2 we can do, I can draw the circle after drawing the circle you can see this is the point value direct like entry if I draw in a scale or if I do not draw in scale simple logic actually sigma 1 will be this centre plus this radius. So, 40 plus this we have done and this minor principal stress what I can do logically this centre minus this radius we have got this.

And now the angle you can see original plane was this and this is actually rotated in this direction so, to get maximum, this angle because of that counter clockwise positive. So, this is 13.3 degrees. So, this is the one plane and this is another plane. So, this is another direction. So, these 2 planes will be ninety degrees to each other. So, that is one problem simple problem I have taken.

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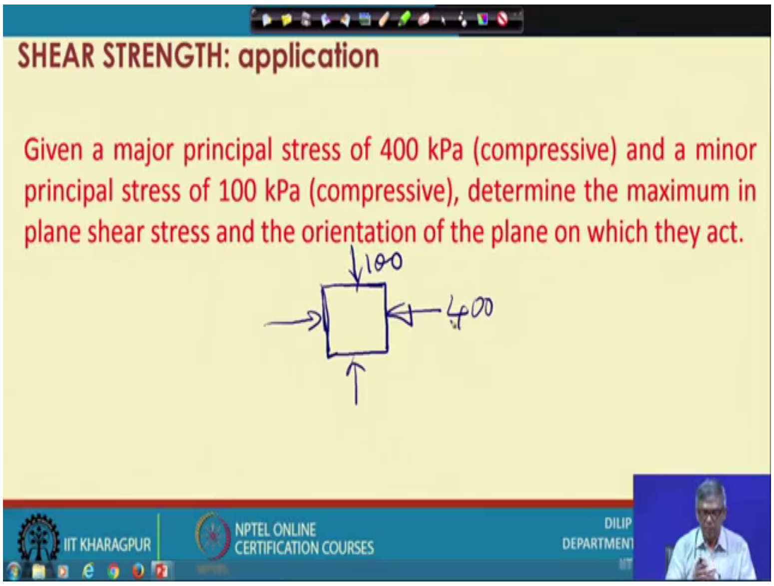

Next problem will be. So, this is another given a major principal stress 400 kPa that is compressive and a minor principal stress of 100 kPa that is also compressive, written the maximum in plane shear stress and the orientation of the plane on which they act. So, this is this problem is reverse actually the principle stresses are given then you have to find out which direction maximum shear stress is occurring so, that to be obtained.

So, here actually all this is 2 principal stresses are there since principal stress there is no shear stress I need if I want to draw the element now the element if I draw it would be like this. So, 400 compressive and 100 again compressive this is 100 and this is 400 and there is no shear on this.

So, I have to find out that there will be some orientation where shear stress will be maximum and also we know the plane on which shear stress is occurring, shear stress occur there actually normal stress also will be there. So, we have to find out that plane where shear stress is maximum and on that plane what is the normal stress also those thing those 2 parts we have to calculate.

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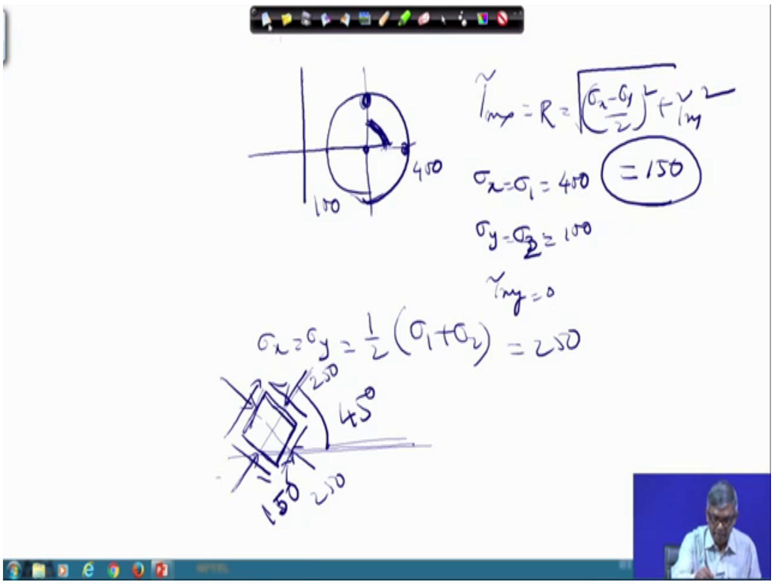

So, let us do that for doing this you can see that tau max the shear stress we know the tau max will be equal to under root equal to is the radius equal to under root sigma x minus sigma y by 2 whole square plus tau xy square and now I can assume sigma x equal to sigma 1 equal to 400 and sigma y equal to sigma 3 or sigma 2 equal to suppose 100 and tau xy equal to 0 and if I do that than your R will become 150 and so; that means, shear stress is maximum shear stress is 150 we can get and now we have to find out the orientation and maximum in plane shear stress is oriented 2 theta equal to 90 degrees from the major principal stress because we know that if I do this if your circle is here and this is major principal stress this is shear stress.

So, angle is 2 theta, your actual orientation will be 45 degrees and then principal stress direction then the magnitude of the normal stress acting on the x and y plane associated with this will be how much at this point, will be how much this is 400 by graph and this is 100 and then it will be you can find out the centre and sigma xy divided by 2 and then that will be your sigma x equal to sigma y equal to half sigma 1 plus sigma 2 graphically also you can find out that become your 250.

So; that means, now the orientation of the plane as I have told you 45 degrees. So, this is the x direction suppose I can imagine a 45 degree direction and now I can draw the plane. So, this is the plane and your both normal stress will be both sigma x and sigma y will be equal to that is 250, this is also 250, and shear stress will be maximum this way and that is actually your 150 which we have calculated and this angle is 45 degree. So, this is the orientation of plane where actually you are getting maximum shear stress.

So, second problem is actually maximum normal stress is actually principal major principal stress and major minor principal stress 100 and this is given then I can find out the plane on which actually your we know that major principal stress direction is this. So, in element since it is in the circle it is 90 degrees on the element will be 45 degrees. So, I can draw line horizontal then I can draw a line 45 degrees on that direction I can draw an element and then I can show the shear stress and normal stress whatever calculated from here.

Maybe third problem is something like this you have one general element is like this.

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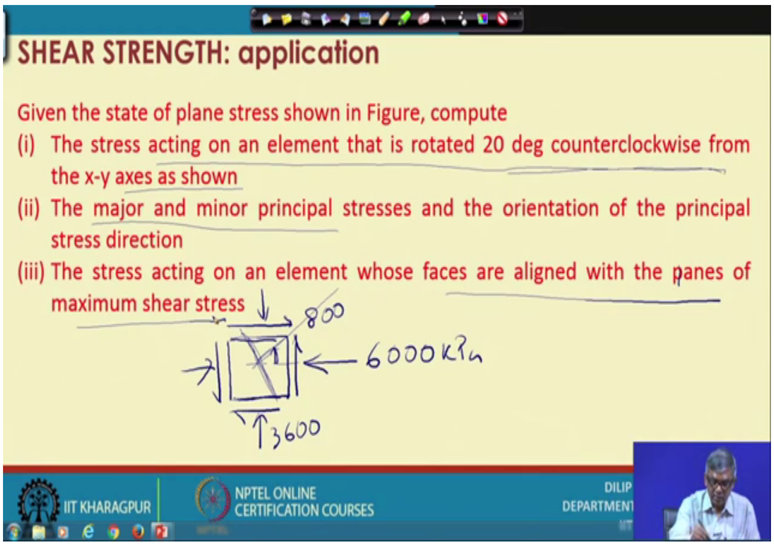

You have the element something like this is the very general state of stress is given suppose your shear is this, shear is this, shear is this, shear is this and then you have this is 6000 and this is your 3600, and this is 800 they have some same unit actually kPa. So, and of course, this is the state of stress.

So, we have given the state of plane stress shown in figure and compute the stress acting one an element that is rotated 20 degrees counter clockwise; that means, I can imagine this one something like this on this plane so; that means, this is horizontal direction. So, I can consider 20 degree the normal. So, this is counter clockwise 20 degrees. So, around this I have to find out normal stress rotated 20 degree counter clockwise from the x y axis as shown so; that means, I have to find out there the major and minor so; that means, I have to find out on this plane what is the normal and shear first part.

Second part is the major and minor principal stresses and the orientation of; that means, because of this condition what is the major and minor principal stress, I have to find out and then stress acting on an element whose faces are aligned with the planes this is missing planes of the maximum shear stress. So, 3 parts I have to do. So, let me see one by one.

So, I can list the parameter that is sigma x is given 6000.

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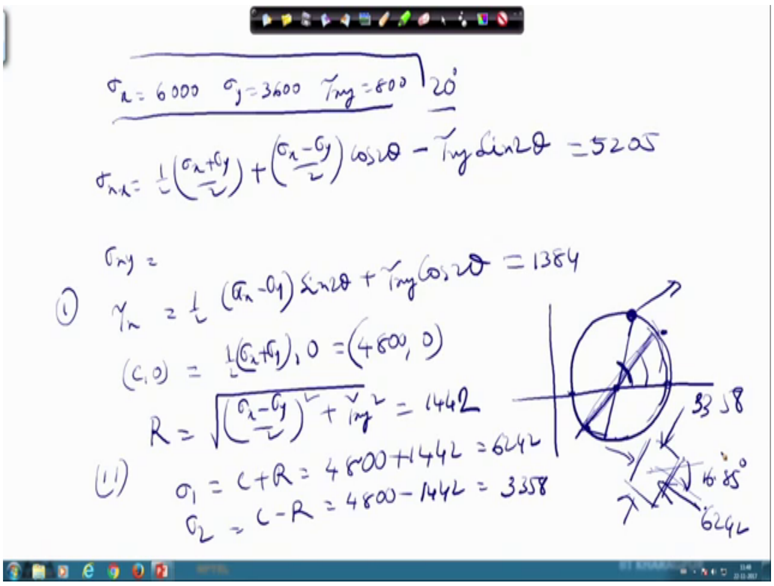

And your sigma y given 3600 and your tau xy your given 800, you can find out by equation method sigma n will be equal to half sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos 2 theta minus tau xy sin 2 theta.

So, now I have listed the values quantities now you put all those then I will get a value here which will be equal to 5205, similarly I can find out this is suppose x component and sigma n y. So, theta value 180 plus 2 theta if I do I will get this value also that let me I am not discuss much on this now I will say tau n which I have given expression that is half sigma x minus sigma y into sin 2 theta plus tau xy cos 2 theta.

Now, again if I put all those values this value will come here 1384 and then I can find out the orientation also by using the equation alternately I can find out centre C 0 suppose which will be nothing, but half sigma x plus sigma y into 0 this is the one; that means, this will give you 4800 and 0 and your R become under root sigma x minus sigma y by 2 whole square plus tau xy square and that gives you 1442.

And I can draw the circle with this; that means, I can take the axis here and from the right phase I can draw a point here locate a point from here and from the left phase I can locate a point here I can join them to get the radius and then and then this as centre I can draw the circle.

So, once you get this circle and from this circle I can find out of. So, I have to find out first that this corresponding to theta equal to 20 degrees. So, I can take theta equal to 40 degrees from here from here I will rotate 40 degrees I will draw a plane and then it will be intersecting here I will read this value that gives you ultimately based on calculation whatever we have got we may get same thing from the graphical method also.

Now, second part that is actually you have to find out the major principal stress. So, from this circle from this circle directly we can read or sigma 1 will be equal to second part this is the first part the second sigma 1 will be equal to C plus R; that means, you have 4800 plus R is actually is 1442. So, that which become 6242 and sigma 2 will be equal to C minus R that is actually your 4800 minus 1442; that means, it is becoming 3358 and it is orientation also can be drawn you can see this was the plane through we have drawn state of stress and maximum occurred here it has gone this direction.

So, I can imagine one point something like this horizontal direction is this and plane inclined. So, on this I can draw the element like this and I can have this, I can have this, I can have this, I can have this. So, this one is your 6242 and this one is 3358 and this angle is negative angle that is actually 16.85 degrees. So, this is second part.

Similarly, third part also we can do. So, third part we can do that is maximum in planes shear stress that is actually your tau max.

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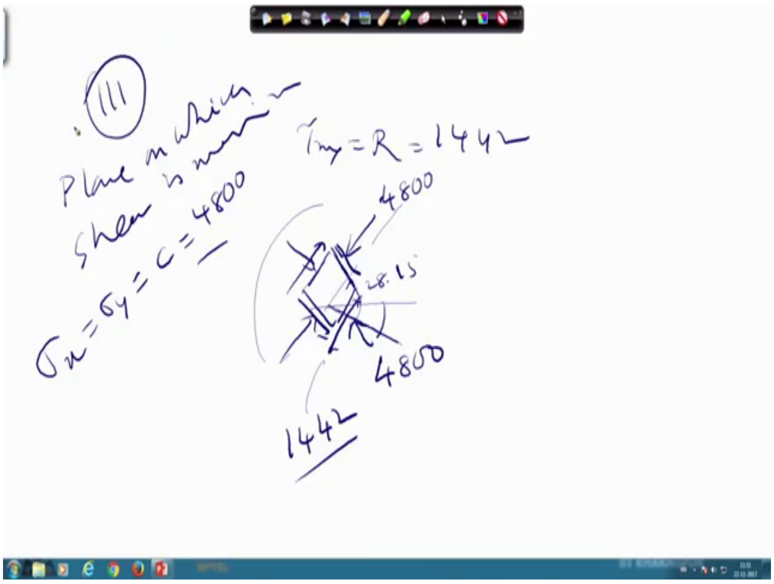

Again we will be equal to R which equal to 1442 and your associated normal stress actually already the plane was 16.85 degrees if I draw horizontal the normal was 16.4 degrees; that means, we have to and normal shear stress is 45 degrees with this.

So, I can go further to make 45 that is 28.0 from here to here I can go 28.15 and draw line. So, this is the direction. So, on that direction I can complete the element suppose this and then on that element I can show the stresses. So, this stress actually magnitude was 4800 that your normal stress will be on that plane on which shear is maximum the plane on which shear is maximum the normal stress will be sigma x will be sigma y will be equal to C will be equal to 4800. So, that is what 4800 this is also 4800 and the shear will be here, here, here, and here and this will be 1442. So, this is 4800 for all will be 4800.

So, this is the orientation; that means maximum principal stress was here in this direction and; that means, with this shear stress will be there 45 degrees. So, how much; that means, we have to rotate from here I will take what 28.15 degrees, 28.15 plus 13.0 something that become 45. So, this orientation I complete the element on that element now I have shown the maximum shear stress and the normal stress on that plane also. So, this is the third part.

So, like that either by using the equation or by using the graphical method one can find out the stresses maximum shear stress, maximum normal stress or any plane what is the normal and shear stress. So, these are all some of application we have to do frequently, we have to use frequently while dealing with the shear strength. So, because of that though it is a part of mechanics I have taken 3 parts actually 3 modules to explain or revise this topic.

So, that I do not have to speak again how we are doing actually I will now when I will discuss with shear strength chapter shear strength of the soil I directly taken this will be taken this one as if you understand all those things I will not repeat again this part. So, because of that I just revise this mechanics part and a next class I will really start the shear strength of the soil and with this I will just stop here.

Thank you.