Soil Mechanics/Geotechnical Engineering I
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Lecture - 28
Shear Strength (Contd.)

So once again let us continue to with the topic on shear strength where I have stopped in the last lecture it was basically I am just revising the mechanics part of it before going to shear strength of soil and at a point inside the soil at any depth I can imagine the state of stress that what are the different component of stress are there, that I have shown initially and then I have considered the biaxial condition; that means, only 2 normal stress and 2 shear component and that of course, easy to find out because it a particular known direction generally in vertical direction and perpendicular to the vertical that other direction I can find out by applying whatever knowledge we have.

Now in fact, in the mechanics also when there is a beam and suppose I want to find out normal stress in this direction or normal stress in this direction by applying mechanics I can find out similarly what is the shear at this and what is shear at this plane that also I can find out by applying the mechanics theory whatever we have learned, similar to that suppose in the soil if state of stress is known at a particular point and, but that is not the plane which is critical there may be some plane where it will have maximum stresses.

So, we have to find out those planes where actually a particular stress is occurring. So, sometime we know the maximum, but where it is astringe that we have to find out. So, because of that we have taken a generalised biaxial problem and element and with generalised a state of stress and then I have imagine one plane which is inclined theta with the original plane and with that inclined plane what is the normal stress, what is the shear stress, actually suppose we have to find out and for that what we have done, we have suppose let us take now.

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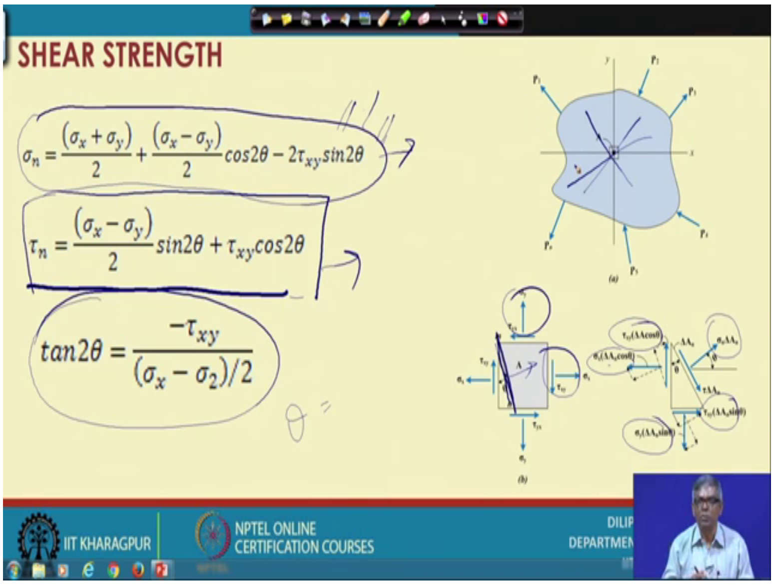

So, this is the plane was there, what I have done previously I have on this plane. So, on this plane actually there is a normal stress. So, normal stress into the area of this is the A. So, this area of this will be A cos theta similarly it will be a sin theta. So, what I have done I have this area multiplied by normal you can see sigma x a n cos theta. So, that is the normal force and that normal force again taken component to this direction, then again another cos theta will come.

Similarly, shear on this plane whatever is there multiplied by area that will be shear force. So, tau x y a cos theta and similarly here on this plane we have normal force suppose sigma y and sigma y n sin theta that become normal force and again I have taken component towards this to get the force equilibrium.

Similarly, on this plane there is a shear stress is the tau x y a sin theta. So, again that become shear force, this is shear force, this is shear force, this is shear force, this is shear force, there are force component and those 4 components I will take along this and what is force acting on this I can I know that this area here is a n and the force acting and stress acting sigma n sigma n in to a n that become the force.

So, this force and this 4 force components that component can be taken towards this and then finally, summation of forces in that direction ultimately at that point will become 0 and based on that I have simplified and I have got this equation that is sigma n equal to this.

Similarly, what is the shear force at this I know area is a n and if I multiplied by shear force acting on this plane tau x y that is shear force and here also number of shear forces I have got those also I have taken component in that direction and then we have equated and then we have got the equilibrium from equilibrium we have got something equation and then we have simplified and then use trigonometrically modification by that we have got a expression tau n equal to this.

So, these 2 are actually most important; that means, I know the stress condition at this direction and this direction this is the this is known and this is known what I want to find out on a any plane which is inclined by theta here suppose on this plane what is the stress. So, this 2 by these 2 equation that if you know the theta and if you know the sigma x and sigma y I can find these 2.

Now, after finding these 2 sometime this not enough find out the maximum value I need to find out at which direction this maximum is occurring to find out that particularly this to find out the maximum I can differentiate this and equate to 0 that is maximize I am doing and based on that for the simplifying I am getting the expression; that means, that theta this is double angle 2 theta equal to this then theta will be half of that; that means, if the theta value is tan inverse half halved half tan inverse this if the value of theta on that plane we will have either maximum or minimum. So, both can be there.

So, from this equation I can get now this equation by using equation again and again sometime it may be tedious. So, in soil mechanics particularly even in the mechanics instead of using this equation sometime we use a method called Mohr's circle, Mohr's circle method and what is the Mohr's circle, actually it is the circle actually now the sigma n and tau n with the variation of theta suppose that at this point I will make a plane along these, I make a plane along these, I make a plane along this, the different orientation I am changing and because of that on that I am getting a sigma and tau. Those points actually locus of those points if you follow then we will see that they are following a circular path so; that means that is actually circle.

So, the using that circle sometime we will be able to find out different things, what is maximum, what is minimum, what is the maximum shear, what is the maximum normal, what is the direction or if we want to find out at a particular orientation, what is the shear, what is the normal, all those thing we will be able to find out.

So, how to find out that Mohr's circle first I will explain that and then use a Mohr's circle in the further detail I will discuss. So, you can see here.

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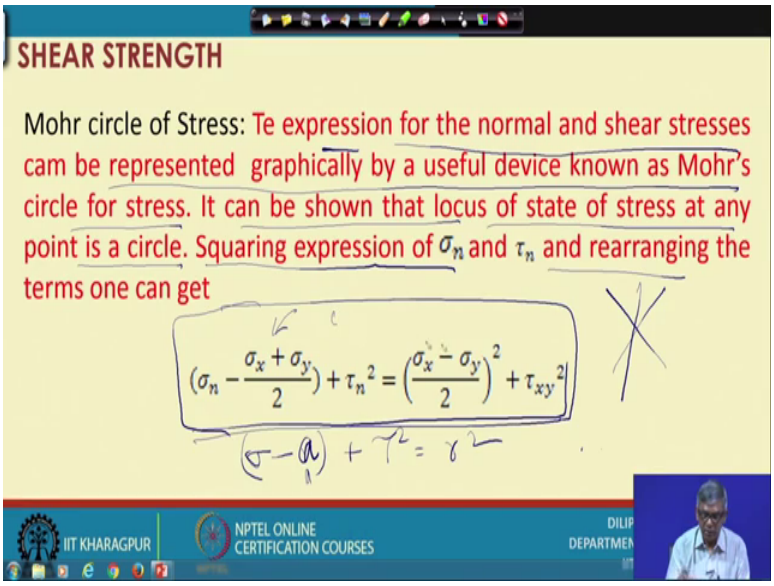

Mohr's circle of stress, the expression for the normal and shear stress can be represented graphically by useful device known as Mohr's circle of stress. It can be shown that locus state of stress at any point is a circle, that what I have shown this point, I take this direction, now I take this direction, now I take this direction, different direction I orienting the plane and then I am finding out the sigma x and tau x then that can be shown that.

So, whatever expression I have shown by previously we can square them and then add and simplify then we can get expression like this and expression like this and this expression if you look it look properly then you can see it will be sigma minus suppose r plus suppose tau square equal to if I say r square or sigma minus a suppose. So, like; that means, this is a equation of circle and this a 0 is the centre of the circle and this under root this will be the radius of the circle.

So, we now by observing the expression of normal and shear we have squared and added and finally, rearrange and we can what we have got, we have got an expression and then that expression is nothing, but a equation of a circle and if you have equation of circle that circle have a definite centre and radius. So, that is what is this radius actually, radius is this and centre is this that is one coordinate is this and another coordinate is 0 and this is the radius, this in the next slide.

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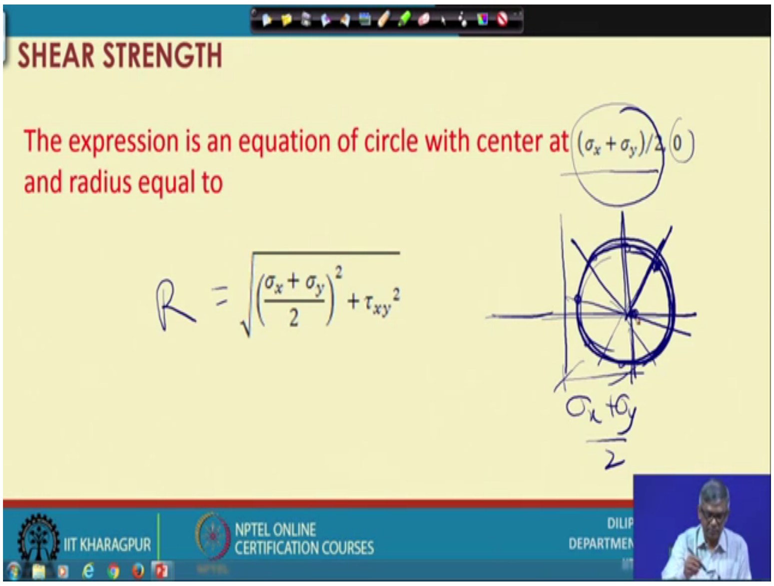

You can see the expression is equation of a circle with centre at sigma x plus sigma y by 2 and 0. So, this is x coordinate and 0 so; that means, if this is one. So, what is the centre sigma x plus sigma y by 2 and 0; that means, on this axis only the centre will be on this axis and what is this distance this is actually sigma x plus sigma y by 2.

And now with this centre and this become the radius. So, I can draw a circle now. So, this is actually Mohr's circle then whatever each point actually giving you tau and sigma represent different plane. So, this is a plane, this is a plane, this is a plane, suppose this is a plane like that. So, all different plane usually it is give you tau and sigma. So, this Mohr's circle can be utilized very effectively to find out the stress at different direction.

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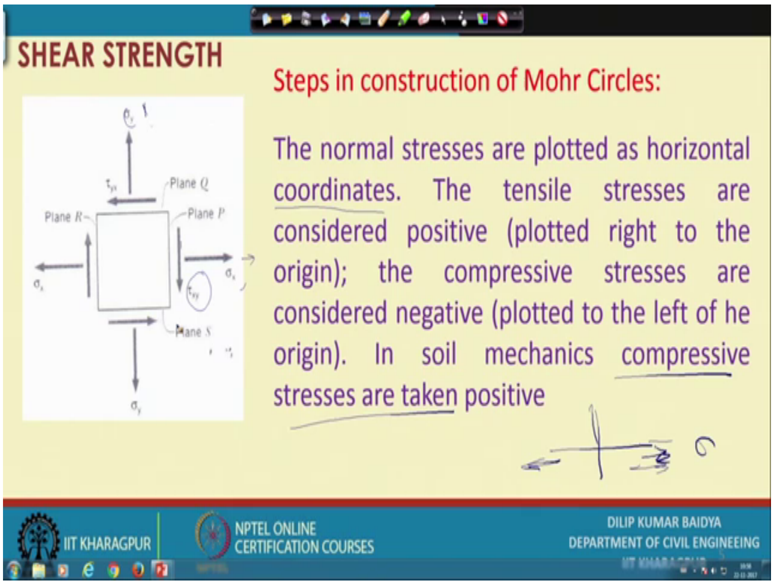

Now, you can see how to draw the Mohr circle steps in construction of Mohr circle. So, if this is the state of stress we can see now this is x direction and this is y direction that this is the sigma x, that is a sigma y and there is a tau x y and this is a if this is tau x y then this will be tau y x and there will be this should be in equilibrium. So, this and this will be oppose this and this will be oppose and biaxial condition we can show that this tau x y and tau x y x is same. So, ultimately you can write without any suffix actually here.

So, now, the normal stresses are plotted as horizontal coordinates, normal stress, this if I consider this. So, this direction will be normal and tensile stresses are consider positive in general and; that means, it will be in this direction and compressive will be negative in this direction, but in soil mechanics we take compressive as positive so; that means, when there is compressive force will take in this direction..

So, now, we will see there is a plane, there is a P plane, there is a Q plane, there is a R plane and there is a S plane, there are 4 planes are there and I can show you that P and R will represent only one point in the circle, similarly Q and S plane will dependent one point in the circle and there in the opposite direction that I will show you in the next slide.

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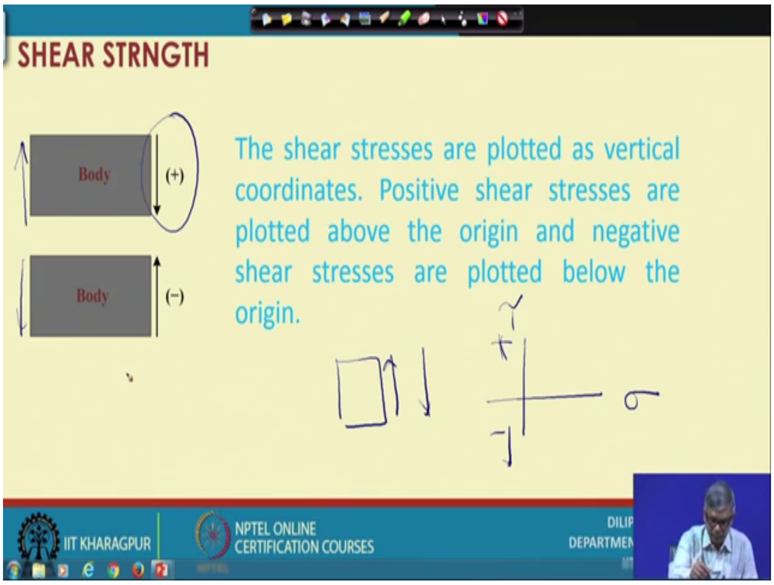

And I have to follow some sin convention actually as I have mentioned that tensile stress is positive and compressive is negative in general in mechanics, but in soil mechanics since compressive force is most important because comes there. So, compressive is taken positive and tensile is taken negative.

Similarly, shear also what shear will be positive, what shear will be sometime directing, some time there is a plane, sometimes shear may be this, sometimes shear may be this which one is positive which one is negative. So, this is the sign convention if this is the body and shear direction right face downward that is positive, the shear stresses are plotted has vertical coordinate; that means, like normal stress I have shown this direction. So, shear stress is this direction.

And positive shear stresses are plotted above the origin and the negative shear stresses are plotted below the origin. So, plus this side minus the side and again in on the when the state of stress shown one element. So, based on this how, what will be taken positive or what will be taken as negative. So, this is the one right face downward or left face upward both are positive similarly right face upward and left face downward that is negative. So, this is the sign convention to use to draw the Mohr circle.

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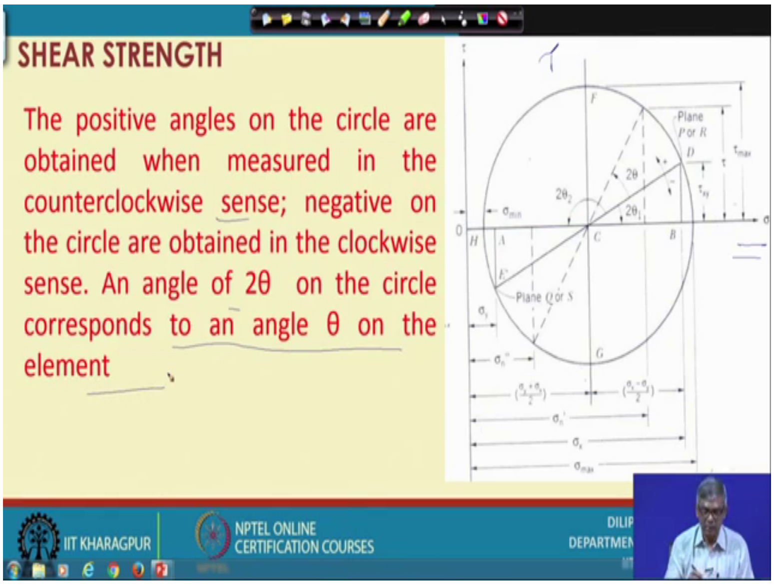

So, now I can show you that this is the Mohr circle, this is sigma direction and this is tau direction and number of things can be shown here you can see the positive angles on the circle are obtained when the measured in the counter clockwise sense that is also another important thing when the plane is rotating counter clockwise that theta is taken as positive and if it is in the clockwise direction that theta will be taken as negative.

An angle of 2 theta on the circle; that means you can see here that is shown 2 theta and actually element it will be theta, when I will draw the Mohr circle on the circle when I will see the it is rotated by 2 theta actually on the element rotation will be theta. So, that is double angle all expression we have shown that that is one important point to be noted an angle of 2 theta, on the circle correspond to an angle theta on the element.

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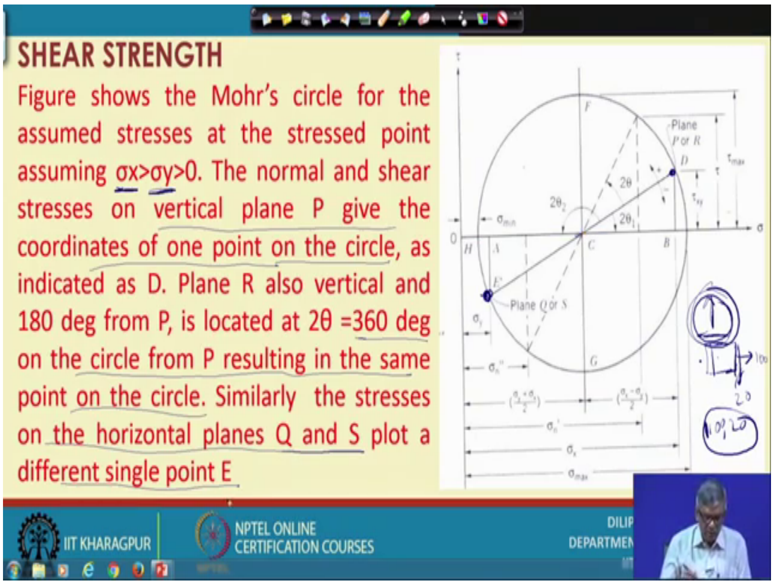

And you can see Mohr circle for the assumed stress at the stress point assuming sigma x was the bigger one than the sigma y. So, based on that we consider major and minor there are big and then 0. So, both sigma x and sigma x value are there in the plane element and they are greater than 0, but sigma is again greater than sigma y.

The normal and shear stress on the vertical plane P the give the coordinate one point of the circle so; that means, this is the P suppose this is the point P this is what actually directly if this element is there and this is normal and this is shear suppose this value this is suppose 100 and this is suppose 20. So, 100, 20 this is the coordinate. So, that suppose point P I have got, right side face of the element represent this point D.

Now, the plane R; that means, this is suppose P and this is suppose Q and this is another plane R. So, this plane R vertical and one eighty degree from P; that means, this plane is 90 degrees and this plane will be 180 degree; that means, it is rotated over by 180 degrees. So, once it is in the circle it has to be rotated twice when element it is rotated by 180 degrees on the circle it will be double of that; that means, 360 degrees so; that means, this point if you rotate this point. So, R phase whatever value is there that will be rotated 360 degrees; that means, it will starting from here the rotated; that means, coming back to this point.

So; that means, the plane R also vertical and 180 degree from P is located at 2 theta; that means, 360 degrees on the circle from P resulting the same point of the circle; that means, P and R will be in the same point that indicating D similarly the stresses on horizontal plane Q and S plot a different single point E.

Suppose now I will read this point. So, suppose there will be shear here and there is a normal here. So, that value again give you another coordinate that coordinate suppose if this is positive this will be negative actually. So, it will come somewhere other side suppose point E is coming here. So, suppose I read this phase then I will get this point and when I will read other phase this one this is again at 180 degrees from this plane.

So that means, in the circle it will be rotated by 360 degree then again we will after rotating we will come back to this point so; that means, this phase and this phase will be representing this point. So, this phase and this phase representing this point and this point. So, this point now we are get it 2 point we are getting on the this is, this 2 are on the circle actually, and there an opposite direction of the circle. So, these 2 points if I join I am intersecting the normal axis that actually it will give you intersecting at a point that will be the centre of the circle.

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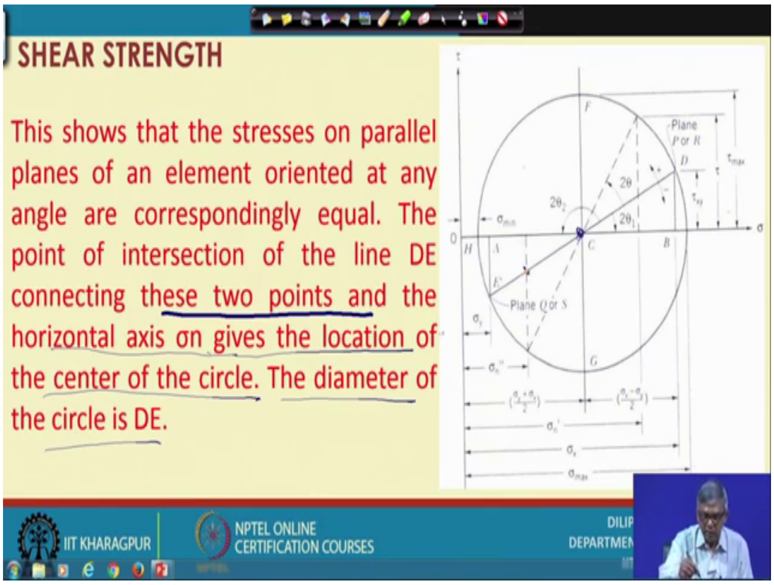

So, next slide I can show you that and this shows that the stresses on parallel planes of an element oriented at an angle are correspondingly equal the point of intersection of the line DE connecting these 2 points that means, D and E connecting these 2 points and the horizontal axis sigma n gives the location of the centre of the circle. So, this, these and this if I join and I get location of the centre of the circle the diameter of the circle is D E these 2 are actually opposite direction of the circle.

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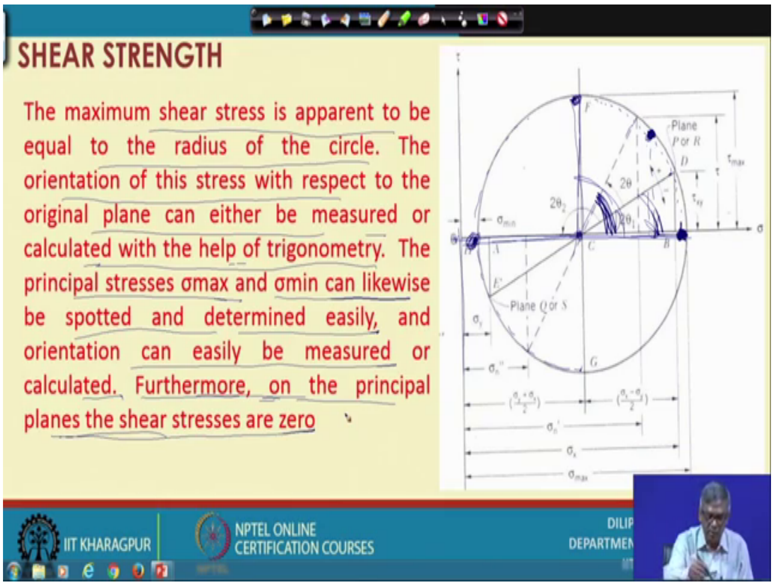

Now, further you can see and now by observing these you can see the maximum shear stress is apparent to be equal to the radius of the circle. So, you can see along these if I take a one point here if I consider a point here this point give you a sigma value equal to this much if I take this as origin this is the sigma value and this is actually tau value so; that means, the tau value is 0 here because on this axis that there is no shear value will be increasing when I will go from this axis to this towards this. So, at this point shear is 0.

Now, if I move by rotating the plane shear value is increasing along this, if I go like this, like this, like this, it is reaching to a maximum value at this point and then if I further rotate again shear value will be decreasing and again at some time it will be reaching to this value. So, this is; that means, the maximum shear stress is possible which will be equal to actually radius of the circle. So, that is the thing first thing is rotated here.

The maximum shear stress is apparent to be equal to the radius of the circle the orientation of this stress with respect to the original plane can be either be measured or calculated with the help of trigonometry; that means, this which plane where actually this is happening. So, between normal direction, maximum normal direction and maximum shear direction is 90 degrees; that means, on the circle you are seeing 90 degrees that on the plane actually what should be it will be half of that 45 degrees; that means, the plane on which the maximum normal stress occurs that is a plane you can see here at this point shear is 0, but normal stress you can see along whatever I have shown that if I move along this normal stress is increasing like this and again decreasing and coming to here.

So, here actually if I move along this circle the shear stress a normal stress becoming maximum here and if I rotate then is coming minimum here and then if I go this way again coming to this point so; that means, the normal stress is here and maximum shear stress is here and when maximum shear stress is occurring you can see normal stress is 0. The plane always maximum sheer stress occur they are actually your normal stress is not 0 similarly the plane on which at this point no normal stress, normal stress is not 0, normal stress is this value is this one whatever value will read here normal stress that is the normal stress value.

Whereas the point plane on which maximum normal stress occurring this is the point suppose this is the maximum value of shear stress or this is the maximum value of normal stress minimum value of normal stress at this point actually we can see since on the x axis only there is no shear value; that means, the plane on, which the maximum normal stress is occurring there actually no shear stress that is actually one condition actually we are getting where show you that is the check that, if you get maximum normal stress on that place you will not have any shear stress.

Whereas, the plane on which we are getting maximum normal shear stress on that plane not necessarily the shear stress that will be some shear stress value that should be some normal stress value, what is that, the normal stress is actually you have to read this from starting from this suppose this is the origin. So, starting from this we can read this value that is the normal stress.

So, now this orientation as we have shown by calculation that sigma n tau n and then tan 2 theta n by that one can calculate what is the value, and what is the orientation. So, by calculation and, but calculation I told that if I want to do many calculation and then it is tedious to do So many calculation, but you once you draw the circle I can find out everything graphically. So, that is the thing. So, either it can be done trigonometrically or by measuring the angle. So, if I this plane how much angle suppose I can just measure the angle and this angle is in the circle, but actual element it will be half of that.

The principal stresses m sigma max and sigma min can likewise be spotted and determine easily; that means, this and this and orientation can easily be measured or calculated, further so, orientation. So, this is the originally state of stress and the maximum happened here; that means, it is rotated this direction 0 clockwise direction; that means, negative angle. So, this is negative angle and how much you measure the angle here, that is 2 theta, what is the value of theta half. So furthermore on the principal planes the shear stresses are 0 you can see the principal planes. So, principal planes this is one principal plane, this is another principal plane. So, on that principal plane maximum principal stress is occurring, maximum principal stress is occurring where actually shear stress is0. So, these are the important observation from the Mohr circle which can be noted.

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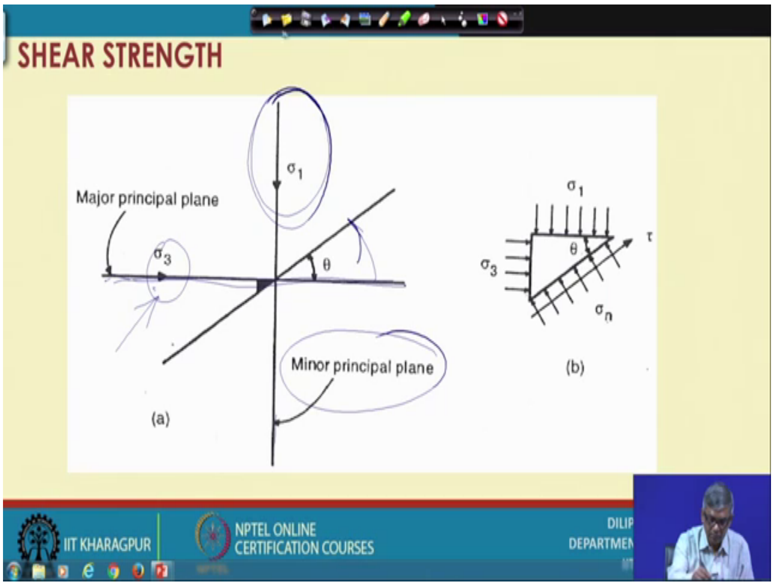

And you can see now I have shown with this sketch actually you can see this is actually suppose sigma one that is major principal stress this is the direction of major principal stress and then what is the major principal plane that must be the major principal plane. So, this is the direction of major principal plane stress, but what is the plane is this because normal stress is the perpendicular to a plane. So, if this is the normal stress, maximum normal stress then major principal plane is this similarly if this is sigma 3; that means minor principal stress then what is the minor principal plane. So, this will be on this plane on which minor principal plane is acting. So, this is the minor principal plane and suppose on any angle theta suppose this is the theta shown. So, I can rotate by these and I can find out other planes actually to find out the stresses..

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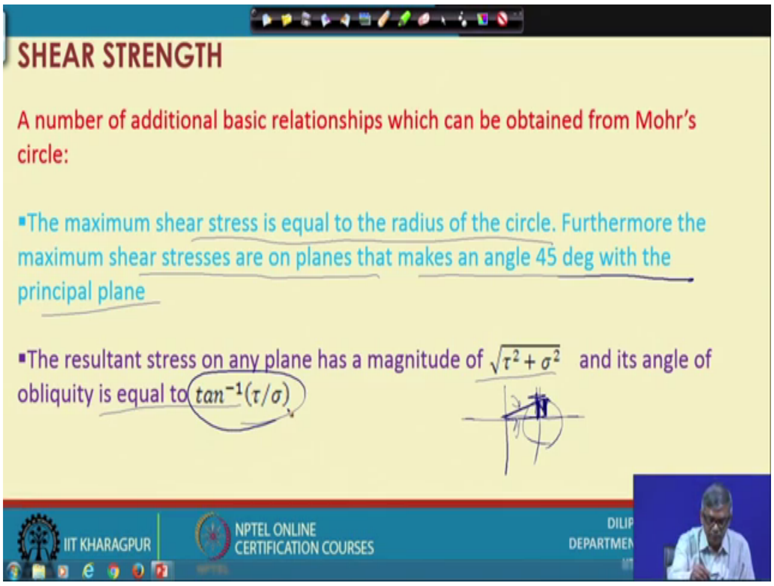

A number of additional basic relationship which can be obtained from Mohr's circle the that is actually you can see the maximum shear stress is equal to the radius of the circle as I have shown here this is the versus maximum shear stress this is nothing, but radius of the circle furthermore the maximum shear stress are on plane that makes an angle 45 degree with the principal planes. So, if this is the principal plane and this is a plane on maximum shear this is angle 90 degrees. So, in the actual element it will be 45 degrees.

The resultant stress on any plane has a magnitude of this; that means, and it is angle of obliquity is equal to 10 inverse suppose if I take like this through this at this point resultant, this is the resultant, this is the value of normal and shear and this is the value of normal. So, resultant will be under root tau square by sigma square and their angle this angle will be tan inverse tau by sigma..

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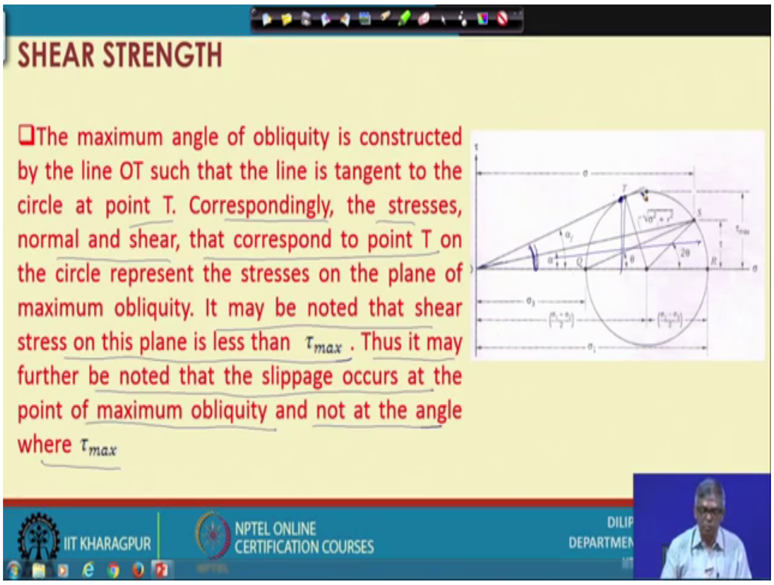

The maximum angle of obliquity is constructed by the line OT that means, you can see similar thing I have drawn here you can see here this line from point O, I can imagine number of line a one line I can dig here this line actually I can change and then finally, it will become tangent at this point suppose at point t it becoming tangent.

The tangent to the circle at point T correspondingly the stresses normal and shear that correspond to the points; that means, at this point whatever normal we can get the normal stress and shear stress at this point I can find out the stresses on the plane of maximum obliquity, that is the maximum obliquity the angle of maximum obliquity and at that point what is the shear stress what is normal stress I can find out by drawing a tangent on the circle.

It may be noted that shear stress on this plane is less than tau max. So, tau max is here, but this is tau max here is less than this; that means, it may be further noted that slippage occurs at the point of maximum obliquity and not at the angle of at the angle where tau max occurs.

So, if I join a line with tau max which will be angle will be smaller than the angle of obliquity so; that means, slippage will be occurring at maximum angle of obliquity not on the plane where the maximum shear stress occurring, that is the observation. So, using these I can utilize the Mohr circle to do so many several calculations and that I will showing some application 1 or 2 application in the next module with this I will stop here.

Thank you.