

**Soil Mechanics/Geotechnical Engineering I**  
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**Lecture - 26**  
**Vertical Stress (Contd.)**

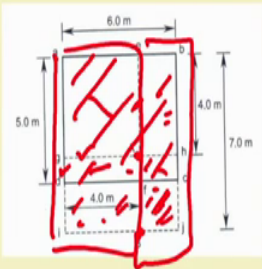
Well let us see some more application I have shown 2 - 3 applications on last lecture and basically I have tried to show the point load application and then some integration part. Now I will try to show all application together and then we can find out which one is better and which are not. So, first problem we will show that and then there will be second problem where actually by corner formula, using corner formula we can find out stress at different point that also I will show in the second problem.

So, first problem we will have everything. So, let me go to the first problem.

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**VERTICAL STRESS: APPLICATION 4**

The uniform contact pressure under a rectangular footing of 6 m by 5 m is 200 kPa. Compute the vertical stress component under point A and B at a depth of 2 m (Aysen).



The diagram shows a rectangular footing with dimensions 6.0 m (width) and 5.0 m (height). A point A is located at the bottom-left corner of the footing. A point B is located at the bottom-right corner of the footing. A vertical line segment of length 7.0 m is shown to the right of the footing, with a horizontal line segment of length 4.0 m extending from the bottom-left corner of the footing to the vertical line. The vertical line segment is labeled with 'h' and '7.0 m'. The horizontal line segment is labeled with '4.0 m'. The diagram is drawn with red lines on a white background.

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So, first problem is this is the first problem; the first problem is the corner formula and no problem. So, this is the uniform contact pressure under a rectangular footing of 6 meter by 5 meter is 200 kPa compute the vertical stress component under point A and B at a depth of 2 meter and this is the problem taken from a particular book by Aysen and this is of course, nothing special, but this all application of corner formula can be shown with this problem and you can see here this is the point. So, this is the area actually loaded. So, this is the area suppose loaded this is the under this form line and point of

interest is A is here and point of another point of interest B is here and at these 2 point actually you have to find out stress and of course, at a depth equal to 2 meter.

So, for that what you have to do as I have shown when giving the procedure that the through the point of interest we can draw line parallel to the sides. So, we have drawn parallel to this side one line I have drawn another line parallel to this side and passing through this point A and then what happened you divide into 4 parts. So, effect of this can be taken here, effect of this can be taken here, effect of this can be taken here.

So; that means, I can apply corner formula 4 times and while applying corner formula 4 times, what I have to do is every time the formula is given in terms of m and n m equal to A by Z and n equal to B by Z and A and B I have to find out from the diagram now. So, if this is the figure if I consider rectangle I can consider this as A. So, A will become 4 meter and this will be B, B will be 2 meter. So, A B if I know then I can find out mn and then I can go back to the equation and apply and calculate what is the pressure at this point.

Similarly, when I consider this area then again I will find out what is A, A is suppose this one 2 and B is suppose here 1. So, again for this I can find out A by Z and B by Z and then I will apply original corner formula by that I can find out because of this area what is the pressure at this point.

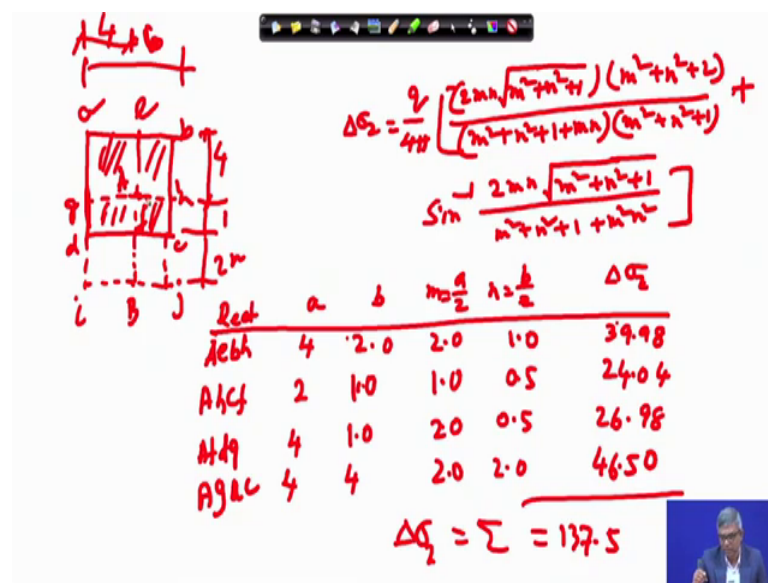
Next suppose if I take these area then I will consider what is the A, this A will be this one suppose 4 meter and B is this one suppose 1 meter again once again A by Z, B by Z that m and n we will find out and then apply once again to get this effect of these 2 this point then when I consider this area once again I will find out what is A, this is A, this is 4 and this is B that is also 4. So, it will be square area 4 by 4. So, there also I will find out A by Z equal to B by Z equal to m equal to n and then apply the formula I get the n another stress. So, 4 values to be added to get this value. So, this is the one, but when it will be at through this point again I can extend the line parallel line passing through this point. So, I extended here suppose and if I extend this point now you can see this is divided 2 rectangle I will cover this, this is A, this is one and this is one these 2 parts are there.

So, effect of this I can find out effect of this I can find out while doing this you can see that I have I am taking this is extra, also I am taking this portion is extra so; that means, I will consider this rectangle once effect of these I will find out at this point, I will

consider this rectangle and effect of that I will find out this here while doing this since I have taken these and which is actually not there. So, I consider another rectangle with negative that upward pressure and I will take this is another rectangle with negative. So, subtract these and these we will get the value. So, these for this rectangle, plus this rectangle, minus this small rectangle, minus this rectangle, this is the way I can do the calculation.

So, systematically if I, then it will be like this I can draw the figure.

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Suppose this is point A and this is suppose point g, this is suppose point h, and this is suppose b, this is suppose a, and this is suppose c and this is suppose d and if I extend this and then if I complete this, then suppose this is capital B and this is suppose j and this is suppose i and I hope I have given all and this is suppose e and with these I can do the calculation. So, this is 4 meter up to this it is 4 meter, this is 1 meter and this is 2 meter and this is 6 meter, and up to this is 4 meter this is not looking proportionate just that is only for understanding I am drawing again.

And our equation will be delta sigma z will be equal to q by 4 pi and this will be 2 mn under root m square plus n square plus 1 and then m square plus n square plus 2 and divided by m square plus n square plus 1 plus mn and multiplied by m square plus n square plus 1 these plus sin inverse 2 mn under root m square plus n square plus 1 divided the by m square plus n square plus 1 plus m square n square. So, this is the

equation to be used and this calculation can be done in a tabular form suppose I will do rectangle and then what is value a? What is the value of b? What is the value of m, m equal a by z? What is the value of n, b by z? and what is the value of delta sigma z?

So, like that if I do delta rectangle is Aebh. So, this is the one I am taking. So, this one if I consider this then a will be 4, b equal to 2, then this will 2 and this will be 1 and with this is the mn if I put in this equation and then delta sigma z and with our q whatever given if I put then I will get 39.98, similarly next Ahcf if I Ahc that is the f here that is the f Ahcf then here also is a is 2 and b is 1 and corresponding this is 1 and this is 0.5 and if I put in this equation we will get the value 24.04 and then Afdg, this portion if I take then for here a is 4, b equal to 1 and then it will be 2 and this is 0.5 and then if I put all those thing in this equation we will get 26.98 and finally, Agac, so, Agac; that means, this one in this you will get a equal to 4, b equal to 4. So, this will be n equal to 2 and n also equal to 2 and if I put this value in this equation we will get 46.50.

So; that means, and pressure at this point is pressure because of these pressure, because of these pressure, because of these pressure. So, all 4 I have taken so; that means, delta sigma at 2 meter will be equal to summation of these and that will be equal to 137.5. So, 137.5 and now if I want to this is actually; that means, at point A now if I want to find out at point B.

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Handwritten diagram showing a rectangular area divided into four quadrants (Aebh, Ahcf, Afdg, Agac) with dimensions a, b, m, n, and a table of calculations for delta sigma z.

Diagram labels: Aebh, Ahcf, Afdg, Agac, a, b, m, n, z, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z.

Equation for delta sigma z:

$$\Delta \sigma_z = \frac{q}{4\pi} \left[ \frac{(2mn\sqrt{m^2+n^2+1})(m^2+n^2+2)}{(m^2+n^2+1+mn)(m^2+n^2+1)} + \sin \frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+n^2m^2} \right]$$

	a	b	m = a/z	n = b/z	Δσ <sub>z</sub>
Aebh	7.0	2.0	3.5	1.0	40.78
Ahcf	2.0	2.0	1.0	1.0	-35.04
Afdg	4.0	2.0	2.0	1.0	-39.98
Agac	7.0	4.0	3.5	2.0	47.74
Δσ <sub>z</sub> = Σ					13.5

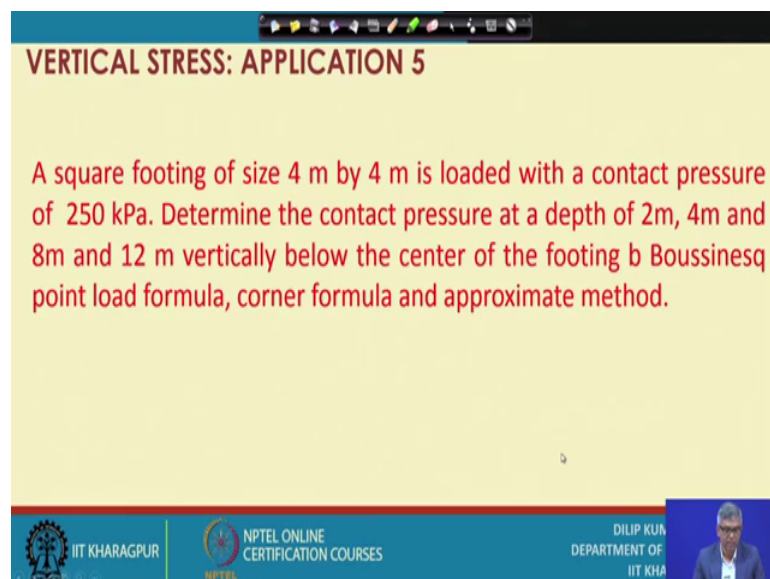
First will be  $B_{ej}$  you can see  $B_{ej}$  so; that means, I will take this portion and for this  $a$  become 7 and your  $b$  become 2 and your  $n$  become 3.5 and your  $n$  become 1 you have put this equation then I will get 40.78.

And next is  $B_{fcj}$  that is  $B_{fcj}$  this portion actually it is not there. So, we have to subtract. So, for this it is 2 and this is 2 also again and this will 1 and this will be 1 if I put in this equation we will get minus 35.04 and next is  $B_{idf}$ . So, this will be not this is not there. So,  $B_{idf}$  this is not there. So, it will be minus and it is 4 and this is 2 and this is 2 and this is 1 and if I put this value in this equation you will get minus 39.98 and finally,  $B_{iae}$  that means,  $B_{iae}$ . So, I am taking now entirely already subtracted. So, this I take then this is 7 and this is 4 and this is 3.5 and this is 2 and if I put all those thing in this equation I will get a value 47.74.

So, algebraically if I sum it plus minus altogether, then I will get a value equal to 13.5 kPa so, you can see; that means, this is the loaded area inside the loaded area; obviously, pressure will be more and we will go away from the loaded area then it will be less that is the thing; obviously, we have got, we have got previously 137 and when I have gone out of this it is only 13.5. So, this is another application how to use corner formula in different ways that is shown in this by same problem considering point A and point B.

Next let me go to your second problem. So, this is of course, as sequence I am considering. So, this is application 4 and next one will be application 5.

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**VERTICAL STRESS: APPLICATION 5**

A square footing of size 4 m by 4 m is loaded with a contact pressure of 250 kPa. Determine the contact pressure at a depth of 2m, 4m and 8m and 12 m vertically below the center of the footing by Boussinesq point load formula, corner formula and approximate method.

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That is the one this is simple problem, but we have to do lot of calculation a square footing of size 4 meter by 4 meter is loaded with a contact pressure of 250 kPa determine the contact pressure depth of 2 meter, 4 meter, 8 meter and 12 meter. So, this and this not required and 12 meter vertically below the centre of the footing by of course, by Boussinesq point formula corner formula and approximate method.

So; that means, we have to do point load formula, corner formula and approximate. So, all 3 methods I have to I will show one by one and finally, I will summarise the values for different methods what are the values we are getting and based on that we can draw some conclusion.

So, if I do that let me take this problem; obviously, I will take a square footing.

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Diagram: A 4x4 square divided into four 2x2 quadrants. The side length is 4m, and the depth is z. The formula used is  $\Delta\sigma_z = \frac{3Q}{2\pi z^2} \cdot \frac{1}{(1 + \frac{z^2}{r^2})^{5/2}}$ .

Calculations:

- For  $z=2$ ,  $r = \sqrt{2^2 + 2^2} = 2$ ,  $\Delta\sigma_2 = \frac{3 \times 250}{2\pi \times 2^2} \cdot \frac{1}{(1 + \frac{2^2}{4})^{5/2}} = 173.26$
- For  $z=4$ ,  $r = \sqrt{2^2 + 2^2} = 2$ ,  $\Delta\sigma_4 = \frac{3 \times 250}{2\pi \times 4^2} \cdot \frac{1}{(1 + \frac{4^2}{4})^{5/2}} = 88.92$
- For  $z=8$ ,  $r = \sqrt{2^2 + 2^2} = 2$ ,  $\Delta\sigma_8 = \frac{4 \times 250}{2\pi \times 8^2} \cdot \frac{1}{(1 + \frac{8^2}{64})^{5/2}} = 27.63$
- For  $z=12$ ,  $r = \sqrt{2^2 + 2^2} = 2$ ,  $\Delta\sigma_{12} = \frac{4 \times 250}{2\pi \times 12^2} \cdot \frac{1}{(1 + \frac{12^2}{144})^{5/2}} = 12.81$

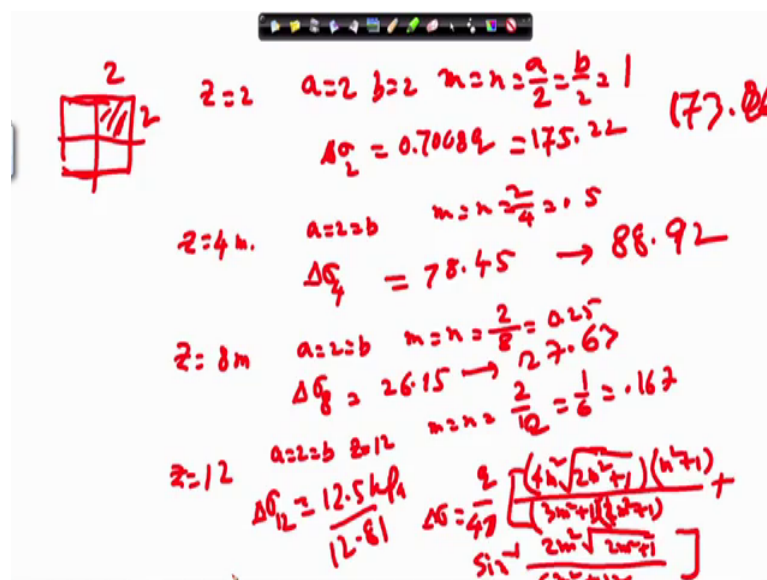
And it is 4 by 4. So, each part will be 2 by 2 and if I consider corner formula when I do for this footing I will divide by 4 parts and this way I can do and at when we have to calculate different depths. So, when I will calculate a different depth. So, for by point formula delta sigma suppose at 2 meter depth will be equal to 3 Q by 2 pi and 2 square into multiplied by 1 by 1 plus, what will be the r here, when I will consider this is the r and this r will be actually root 2 1 under root this r equal to under root 1 square plus 1 square equal to root 2. So, root 2 square it will be 2 and 2 square it will be 4 and this will be 5 by 2.

And this one actually and there are 4 parts. So, I can finally, multiply by 4 I will get the value will be equal to 173. 26, similarly if I take at 4 meter if I take z equal to 4 meter, delta sigma 4 same formula if I put that is 3 Q by 2 pi z square actually it is 4 square and 1 by 1 plus that will be your radius is same 2 divide by 16 to the power 5 by 2 and that to multiply by 4 that gives you 88.92 and when it is z equal to 8. So, delta sigma 8 that will come, I can multiply at the beginning 3 Q by 2 pi multiplied by 8 square and 1 by 1 plus 2 by 64 to the power 5 by 2 and that gives you 27.63.

And now if I z equal to 12 meter. So, delta sigma 12 that gives for multiplied by 3 Q divided by 2 pi and it is actually 12 square and multiplied by 1 by 1 plus 2 by 144 to the power 5 by 2 and that that comes 12.81 and here actually Q will be how much it is 2 by 2 2 multiplied by 2 multiplied by 250. So, pressure is 250 perhaps. So, that if I do then ultimately I am getting this way. So, 173 at 2 meter at 4 meter this and 8 meter this and 12 meter this and you can see nearly less than 10 percent here which reached.

So, now I have to apply corner formula. So, this is the one let me get a new page again I will draw the same figure that is 4.

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The image shows handwritten calculations for the corner formula, accompanied by a small diagram of a corner. The diagram is a square with a smaller square inside, and a line from the corner of the inner square to the corner of the outer square, labeled '2'. The calculations are as follows:

- For  $z=2$ :  $a=2, b=2, m=n=\frac{a}{2}=\frac{b}{2}=1$ .  $\Delta\sigma_z = 0.70689 = 175.22$  (177.06)
- For  $z=4m$ :  $a=2, b=2, m=n=\frac{2}{4}=0.5$ .  $\Delta\sigma_z = 78.45 \rightarrow 88.92$
- For  $z=8m$ :  $a=2, b=2, m=n=\frac{2}{8}=0.25$ .  $\Delta\sigma_z = 26.15 \rightarrow 27.67$
- For  $z=12$ :  $a=2, b=2, m=n=\frac{2}{12}=\frac{1}{6}=0.167$ .  $\Delta\sigma_z = 12.54$  (12.81)

The final formula for  $\Delta\sigma_z$  is given as:

$$\Delta\sigma_z = \frac{q}{4\pi} \left[ \frac{(4\pi\sqrt{2a^2+b^2})(n^2+1)}{(3m^2+1)(3n^2+1)} + \frac{2m^2\sqrt{2a^2+b^2}}{(3m^2+1)(3n^2+1)} \right]$$

So, it will be 2 by 2 and here actually z actually suppose 2 meter first and here if I consider this small one then it will be a equal to 2, b equal to 2. So, m equal to n equal to a by 2 equal to b by 2 nothing, but it will be 1.

And same corner formula when m and n equal to here since all square. So, m and n when become equal then your equation will be slightly simplified  $\Delta \sigma$  will be equal to  $q$  by  $4\pi$   $4m^2$  under root  $2m^2 + 1$ , multiplied by  $m^2 + 1$ , divided by  $3m^2 + 1$ , multiplied by  $2m^2 + 1$ ,  $2m^2 + 1$  and plus  $\sin^{-1} 2m^2$  under root  $2m^2 + 1$  divided by  $m^2 + 1$  whole square.

So, this is the formula when m and n it will be equal to. So, I have converted instead of mn only m is used because m and n will be equal for all cases. So, this is the formula to be used and if I use that then  $\Delta \sigma$  at 2 meter will come 0.7008 q; that means it will be 175.22.

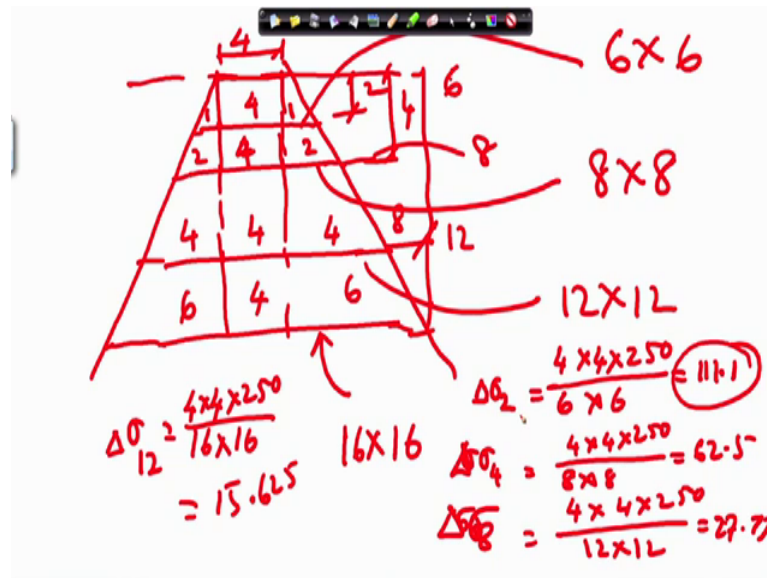
Similarly, if I go z equal to 4 meter, a equal to 2 equal to b. So, your m equal to n equal to it will be 2 by 4 equal to 0.5 if I put in this equation. So,  $\Delta \sigma$  z equal to 4  $\Delta \sigma$  4 I will get the value equal to 78.45, now z equal to 8 meter then your a equal to 2 equal to b. So, m equal to n equal to. So, it will be a by z. So, 2 by 8 it will be 0.25 then I will get from this equal to  $\Delta \sigma$  8 if I put in this equation I will get value 26.15 and one z equal to 12 then your a equal to 2 equal to b and z equal to 12. So, m equal to n equal to 2 by 12 equal to 1 by 6 equal to point 167 I can consider then  $\Delta \sigma$  12 your  $\Delta \sigma$  12 will be equal to coming 12.5 kPa.

So, already at point load we have got some values corresponding this one I had got 173 you can compare here 173.26 corresponding to these from point formula we have got 88.92 and here instead of 26 we have got 27.63 and last one the 12.5 there we have got 12.81. So, comparison between point load formula and the corner formula when you divide by 4 parts you can; obviously, see here there quite close.

Now, there is one more application I need to show that is your approximate method that is how it is distributed what depth then I can show you. So, this is the footing suppose 4 meter by 4 meter.



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This is the 4 meter footing and suppose different depths 2 meter, that is suppose 4 meter and this is suppose 8 meter and this is suppose 12 meter.

And now dispersion suppose taking place like, this dispersion is taking place like this. So, this is the square footing. So, this is the ground mark, this is suppose 4 meter since this square footing similar other direction also it will happen. So, here this is 4 and since it is 2 meter. So, this will be 1 this will be 1. So, ultimately at this depth width equal to 6 and what will be here at this actually since it is up to this it is 4 meter then it will be this portion will be 2, this will be 2 and this will be 2. So, it will be sorry this is 4. So, for this will be 8 and at this depth what is the width. So, since up to these your is 8. So, this will be 8 by 2 4 this is 4 and this is 4. So, that will be 12 and if I go up to these, this is up to this is 12 meter. So, this will be then this portion will be 6, 4 and 6, this will 16.

So, at this level footing size is 16 by 16,, at this level footing size is 12 by 12, at this depth footing size is 8 by 8 and at this level footing size is 6 by 6. So, this is the simple way I am what we do widening the footing; that means, I have applied pressure at the surface and I am assuming a dispersion that we are loading disperse over a larger area. So, at different depth what is the footing size, I now we have got and now I can find out delta sigma z 2, 2 it will be 4 meter by 4 meter footing multiplied by 250 is the pressure that was load divided by 6 by 6 that will be 111.1 and delta sigma 4 that will be again 4 multiplied by 4 multiplied by 250 divide by 8 multiplied by 8 that is equal to 62.5 and

then delta sigma 8 that will be 4 multiplied by 4 multiplied by 250 divided by 12 multiplied by 12 this value gives you 27.77 and delta sigma 12 that is equal to 4 multiplied by 4 multiplied by 250 divided by this is the footing size last footing size 16 by 16 that will be equal to 15.625.

So, now, we have got a different depth different pressure except pressure at the shallow depth at 2 meter all pressure actually quite close to other method I will at the end summarise by different method what we are getting suppose by point load formula what we have got.

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	Point load	Corner	Approx
$\Delta\sigma_2$	173.26	175.22	111.1
$\Delta\sigma_4$	88.92	78.45	62.5
$\Delta\sigma_8$	27.63	26.15	27.77
$\Delta\sigma_{12}$	12.81	12.5	15.625

$\frac{4}{4} = 1$

Point load delta sigma z 2, delta sigma z 4, delta sigma z 8, delta sigma z 12 by point load we have got 173.26, then we have got 88.92, then we have got 27.63, then we have got 12.81.

And now corner formula we have got corner formula we have got 175.22, then we have got 78.45, then we have got 26.15 and then we have got 12.5 and then approximate method we have got. So, now, this one approximate method we got 111.1 and then 62.5 and then 27.77 and then 15.625, you can see these 2 are throughout the depth quite comparable this one little error up to this actually up to 4 meter, 4 meter actually nothing but it is if I footing dimensional if I consider 4 and depth is 4. So, is B up to B depth

actually approximation is not very good, but beyond B; that means, that means beyond greater depth than with B it is approximate value also quite closer with this rectangular corner formula and point load formula.

So, because of that many application is not very important then quickly we can calculate by using this method otherwise point load method can be used and corner formula; obviously, you have to do lot of calculation and there is a chance of doing mistake some time people use chart actually the chart I have not shown; obviously, if you see any book we will see number of charts given with m versus n and all with I q influence factor that also can be used, otherwise in exam calculation most of the time we do not expect corner formula we generally either approximate or point load formula otherwise anything can be used if you want to get accurate value; obviously, corner formula is preferred, but we have to do lot of calculation. So, with this I just complete my today's application how to calculate vertical stress using Boussinesq and other methods thank you. So, I will continue sometime with new topics.

Thank you.