Soil Mechanics/Geotechnical Engineering I
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Lecture – 23
Vertical Stress for Distributed Load

Once again, I welcome you to this topic soil mechanics in a topic vertical stress distribution and in the first session of these vertical stress distribution I have shown the Boussinesq point load formula, but aspect of it what are the assumption etcetera and then and the second lecture what I have try to show that though the Boussinesq gave the formula for a point load and our foundation generally is not a point it is always we apply load through definite area.

That means, foundation will have different shape through that actually apply the load, then there is a mismatch with theory and our actual practice and I have tried to show in the last lecture that how the point load formula can be approximately utilized for different shape of footing particular rectangular footing we can divided into number of parts and then apply point load formula to each part and then cumulative effect can be taken to get the effect of the entire loading area.

And while doing that we have shown also that when you are going to make smaller the make size will better will be the results by at the same time also highlighted that at a certain number of mess ah size or certain number of mess; that means, divisions he may not get that improve that much improvement over the result.

So, we can do 4 to 6 8 9 maximum 10 division we can make and finally, we can get the results that will be good enough for most of the civil engineering application with that what I have shown next part is why I will divide into small part and apply point out formula similar separately instead of that if I can integrate the effect it is nothing, but integration I am taking a small part and I am footing another transfer into another point the next point transfer into this point next point transfer into this point. So, instead of doing this discrete one point if I take a infinite number of points now.

I can consider the if the foundation as a infinite number of point and infinite number of load that is like a q and that actually if I imagine and your point load is applied then I can just simply try to integrate that. So, through integration I can do better because I have shown in the previous lecture that when you divide in a in a more number of parts then you are getting better results, but now if I integrate that question will not arise directly I will integrate because a effect of all will be taken through integration.

So, that is the one we are trying to show that integration how we can do the integration for different areas the area of foundation one type of shape is circular foundation circular foundation there, but it is not in a very frequently comes very special area only, but most popularly either rectangle, square or stiff this type of footing will be there sub time a sometime a angle also will be there. So, different types of footing, if it is there how to integrate them to find out of pressure or stress at a particular point using Boussinesq theory this is the intention of this lecture.

So, we are trying to integrate the loading area using Boussinesq point load formula, to do that.

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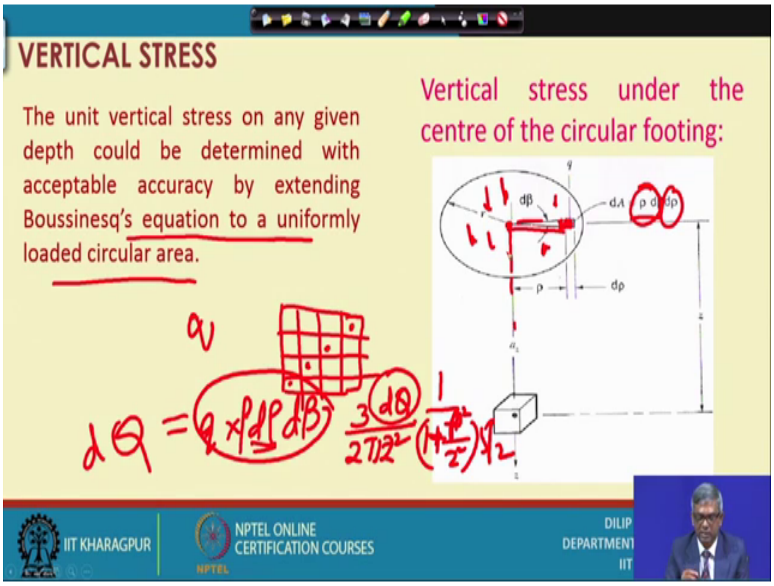

We are taking suppose what I was trying to show here that instead of if there is a area and there is a load is applied through this area and I was dividing into number of parts and then I was taking to take the effect of these 2, here effect of these 2, or effect of these 2 here, effect of these 2 here, like that, instead of doing that I will try to integrate, just continuous not a discrete 1. So, to do this is you see the unit vertical stress on any given depth could be determine with acceptable accuracy by extending Boussinesq equation to uniformly loaded circular area. So, if that suppose this is circular area and it is loaded uniformly with suppose intensity of pressure is q and now what I can do now right now I will apply because if this is the loading area applied with q like this throughout the area and effect of everywhere.

So, you can visualize that the stress intensity will be maximum below the centre of the footing. So, what you are trying to do and most of the time in the design we are interested to know the maximum value. So, because of that we are trying to show the now first integrate with respect to the centre. So, if you want to integrate with respect to center then what I am doing I am taking a small element with d beta angle at a distance rho and thickness is 0 and this is the small area I have shown this is this area will be rho times d beta will be the width and thickness is d rho. So, that become is d a.

So, small area any of the small parts will be I am considering this is a circular arc this is a circular arc then this circular will be what is arc theta since that a distance rho and angle is d beta show rho d beta is the arc length and the arc length outer arc and inner arc the area this will be little different I can considered that that a arc is taken upto the center of the element in that case I can considered that is the average width of the arc. Average length that is the arc length and width is d rho. So, length average length multiplied by width will be your area. So, this is the small area we have got now the small area and intensity is uniformly loaded putting. So, q is there, you if you multiply by q into rho d rho d beta. So, that become actually capital Q this become capital Q.

So, the I have first I have got the small area multiplied by intensity that become your Q; that means, now I want to find out the small area which is supposed instead of Q I can say this is d Q. So, because of this d Q point load I want to find out the effect at the centre. So, how to find out this one the same point out formula will be applied because this is the Q applied here at a distance rho. So, what is the effect, same formula I can apply; that means, 3 Q. So, here instead of Q it will be 3dQ by 2 pi z square and 1 by 1 plus r actually here rho square and r is nothing, but rho square by z square 2 the power 5 by 2.

So; that means, for this small element I can use this equation, but when I want to do suppose for entire area then what I can do I can integrate and you can see d Q is a function of rho and beta d rho and d beta. So, that is variable, I can integrate for beta from 0 to 360 degrees I have taken a small angle d beta and beta that give it I can vary from 0 to 365 I can integrate between 0 to 360 and I can rho I can integrate rho is varying when it is load if I considered this point instead of here if I considered here in a relation distance if I take at the centre itself then become r become 0. So, r can vary from 0 to radius of the 14. So, this thing if I do then I can do simple integration and by that integration I can get the stress intensity that is the thing I let me show you in the next slide in better way.

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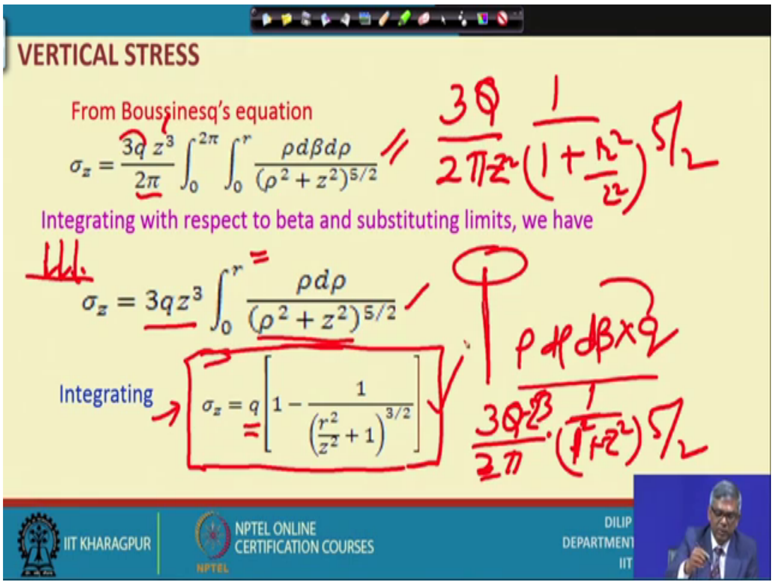

So, you can see this same thing I have done. So, from Boussinesq equation actually it is a double integration is given. So, as I have told you that our expression is expression is 3 Q by 2 pi z square 1 by 1 plus r square by z square 2 to the power 5 by 2.

So, now this q will be changing q will be your rho d rho d beta into q that become q if I have I have a substitute. So, we can see 3 small q 3 small q 2 pi z cube it has gone and 0 to 2 pi 0 to r and this is another form this instead of these form and we can write actually this equation I can write another form that is 3 Q z cube by 2 pi into 1 by 1 plus sorry rho square plus z square to the power 5 by 2 this is also another form 3 Q z q by 2 pi 3 3 q by 2 pi z cube into this is the form. So, I have use accordingly in this, in this here. So, dq is substitute. So, q has gone 3 into q has gone this z q also came here 2 pi is there already and 0 to 2 pi that is d d beta to be integrated and 0 to r rho to be integrated.

So, rho by rho square plus z square d rho is there and 5 by 2. So, now, this in this is a integrated double integration. So, for beta actually 0 to 2 pi ultimately it will come 2 pi. So, it will be 3 q by z cube that 2 pi 2 pi get cancel. So, 0 to r rho d rho by rho square z square 5 by 2.

Now by substitution method rho square plus z square I can consider a constant something and that can be against further substituted and integrated and put the value finite integral limit 0 to r and that is where r is the radiation of the footing if I put then finally, I get a expression like this that is your sigma z equal to q into that is q actually is the intensity of the tooting what is applied at the surface this is surface footing intensity of q is given and added at any depth q minus this.

So; that means, 1 minus this; that means, even go deeper and deeper the stress obvious will be less, but through this equation we can get that; that means,. So, far by this integration what I have got when there is a circular loaded applied load is applied to the circular footing along the central line of the footing vertically below the footing what is the stress intensity that gives you this expression, but in not any other point any another point; obviously, intensity will be less and you can also find out, but there is a sum method a complication is there.

So, that I am not discussing right now later on will discuss that. So, right now whatever you have got it is at a an any vertical depth through the centre of the footing and which is critical; that means, when footing is loaded circular footing below the centre of the footing stress will be supposed to be more and we have got the expression for that. So, this is the one that mean circular area to be integrated next one when there is a rectangular area that is the one.

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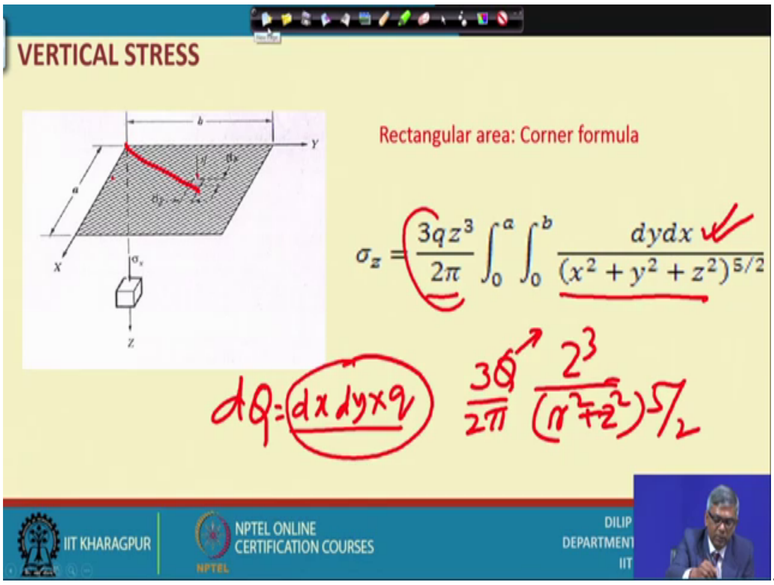

Suppose the rectangle with size a and b and then in that case how to do this integration you can see here this is a x axis from x axis at a distance suppose dy at a distance at a distance y at a distance y I have taken a small width dy and from x y axis at a distance x I have taken dx width.

So, there is a; that means, one small area I am getting of width dx and dy; that means, the area of the small part is dx into dy and intensity of pressure on the footing is q so, that mean this become dQ. So, similarly we can apply that your point load formula that was 3 Q by 2 pi z q divided by r square plus z square to the power 5 by 2 this is one form actually if I simplify it comes other form.

So, this is the form in this Q to be substituted dQ. So, here ah the here actually since I am get calculating. So, d Q to be calculated substituted by this, if you substituted this you can see this q is there 3 q by 2 pi will be there z q also will be there and integral dy dx and here actually instead of r square plus z square if I go back to (Refer Time:16:52) coordinate. So, r is nothing, but under root x square plus y square. So, if I substitute r by x and y. So, it will be under root x square plus y square plus z square and that is the algebraic function we are getting which has to be integrated.

So, this integration is not very simple, but 2 3 time you have to substitute and integrate ones for x and then integrate for y. So, 2 I's you have to integrate when you integrate for x and then x will be varying 0 to a and when you integrate for y will give the limit 0 to b and after doing this double integration we may get a we get a equation something like this I will show you the next slide.

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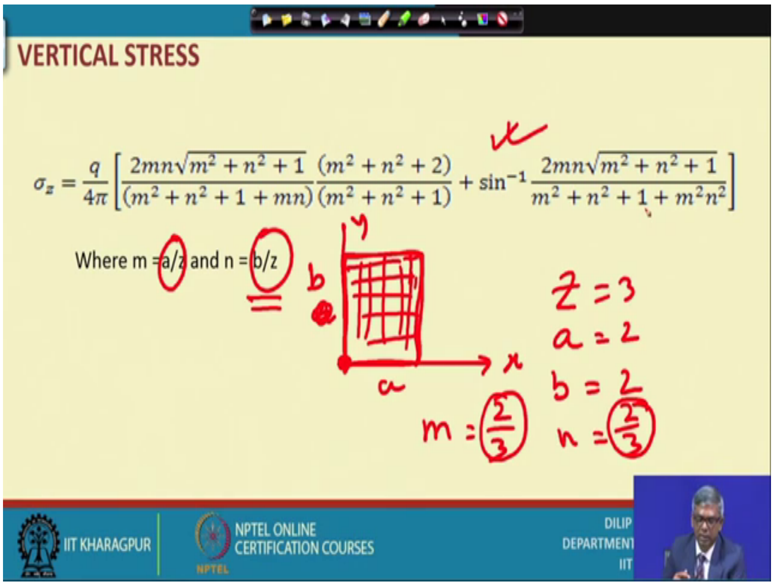

You can see this is the expression you are getting finally, this is looks a very long expression and ah it has a mostly this expression contains 2 non dimensional thing one is a by z and b by z too many times if I write a by z, b by z in this equation looks very clumsy instead of that it is introduce a by z equal to m and b by z is taken as your m and what is suppose; that means, if I have a footing like this and then you have suppose out of this is a and this is suppose b and then suppose I want to find out stress.

So, here actually while integrating what you have done we have taken coordinate axis like this is x and this is y and we have integrated with respect to this part; that means, this formula is applicable for the corner only; that means, throughout the area of the footing when loaded and if I take the effect at this point I am getting that. So, that is typically known as corner formula this is called corner formula; that means, if I get a and b dimension and it is entire area is loaded with intensity q. So, this is the expression. So, while calculating this how what you have to do.

So, our calculation generally you have to find out at 2 meter depth below the corner of the footing support this is a question then what happened I have to I know the a and b and suppose z is given suppose 3 meter and a is given suppose 2 meter b is given suppose 2 meter in that case I have to find out m become a by z. So, 2 by 3 n also equal to b by z b also to. So, 2 by 3 or suppose if it is b is something else then accordingly it will be changing. So, taking this to m and n value you have to calculate this once you get calculate then what will get, I will get the entire area of the footing with dimension a and b when loaded with intensity small q the 2 meter or 3 meter depth I will get the intensity if I use this value in this expression and get the stress.

So, this is a very useful formula though it is applicable for corner, but this formula in several combinations can be used to apply different foundation say or at different foundation location. So, that how this will be done I will show you in the in the next ah in the next slide.

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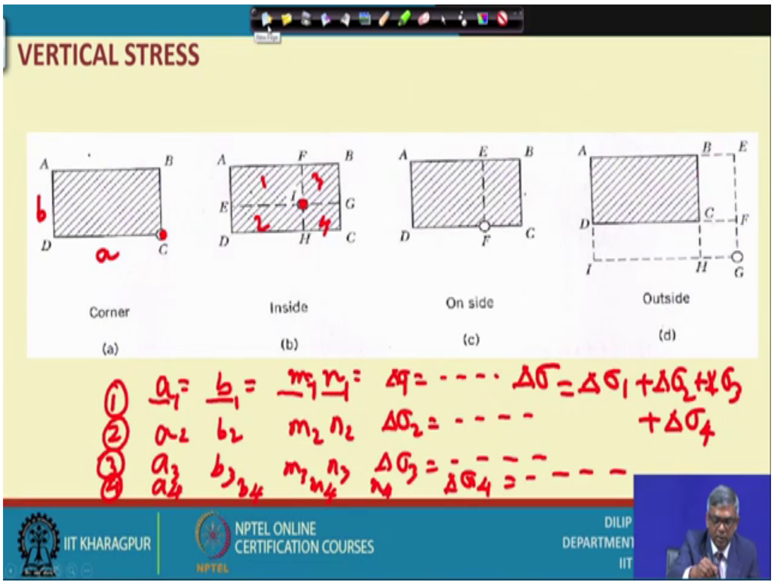

You can see here whatever derivation we have done this is suppose the typical foundation and with dimension is a and b, this is get application of whatever formula we have given. So, we can consider this as a and this as b and at this point suppose we want to find out the stress. So, directly we are applying the corner formula, but if instead of at this corner if I want to find out the stress at some other point suppose at the interior of this point of this foundation.

Then what I will do, I will pass draw a line parallel to the size of the footing and passing through this point along length wise I will also draw another line passing through point parallel to the side of the footing and all through then we can see by this drawing 2 lines

I have got 4 parts 1 2 3 4 now this for this 4 parts what I have to do I can consider now it is apparently.

Suppose one I am considering footing 1 what is a what is b I will write a equal to how much b equal to how much; that means, a is b is not constant. So, initially foundation dimension was a b now I am using another small part what is a for 1, what is a, what is b, what is your m, what is n, I have to find out and then apply the corner formula then what I will get, I will get the effect of this foundation at this point so; that means, by doing this I will find out I will take foundation 1 I will see a what is I had see b then calculate m calculator n and then I calculate del sigma by using corner formula. Then; that means, at a particular depth because of this footing area I will get at this point. So, that will be that is one thing similarly I will take 2 footing 2 what is a or suppose a 2 b 2 I will denote a 1 b 1e m 1 n 1 something like that delta sigma 1 I will say.

So, a 2 b 2 similarly m 2 n 2 similarly delta sigma 2 by using the formula similarly I will take footing 3 suppose this one what is a 3, what is b 3, what is m 3, what is n 3, similarly what is delta sigma 3 from the equation by corner formula similarly if fourth one what is f 4 what is b 4 what is m 4 what is n 4 and then after doing this what is delta sigma 4 by using corner formula; that means, independently I have now got 4 rectangles and 4 times we have applied corner formula and then I have got 4 values delta sigma 1, delta sigma 2, delta sigma 3 and delta sigma 4 then what will be my delta sigma, delta sigma will be effect of all, delta sigma 1 plus delta sigma 2 plus delta sigma 3 plus delta sigma four. So, this I will get. So, this is the way; that means, in various ways I can apply various combination I can use the corner formula to find out the stress at different points.

Now, suppose I want to find out at this point I will just clean this one and then I will once again I will see supposed this is another footing having the footing and I want to find out pressure at this point then what I will do I will do a line I will draw parallel to the edges and passing through this point like this.

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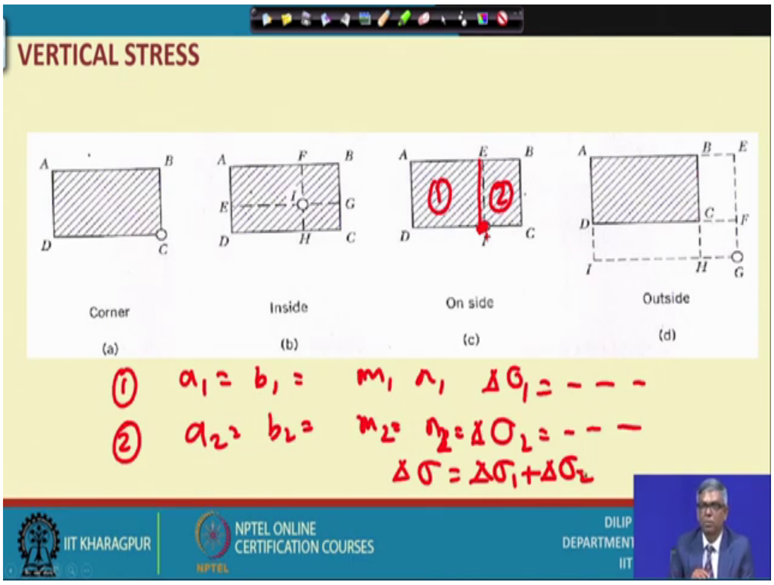

Then it divided into 2 parts 1 and 2 then similarly for 1 I will find out what is a 1, what is b 1, similarly what is m 1, what is n 1, what is delta sigma 1 from the equation and similarly for 2 I will find out what is a 2 equal to, what is b 2 equal to, what is m 2 equal to, what is n 2 equal to, and then what is delta sigma 2 I will find out. Then what after finding out this, then what will be what will happen because at this point effect of this rectangle effect of to both the rectangle will be there. So, because of that what I have to do, I have to find out delta sigma equal to delta sigma 1 plus delta sigma 2, this is the way one can do.

Now, I will just consider another point where suppose I will show to the next one you can see this is a case where I have to find out pressure at this point, which is outside the power loading area, in that case what I have to do again similar to that through this point I will draw number of lines parallel to the sides. So, I have drawn one line here, another line here and then from this I will extend to intersect this from here also I will extend this to intersect this one then I will get a bigger rectangle. So, instead of A B C D now I am getting A I G E and now again I will draw this line I will extend up to this and this line I will extend up to this.

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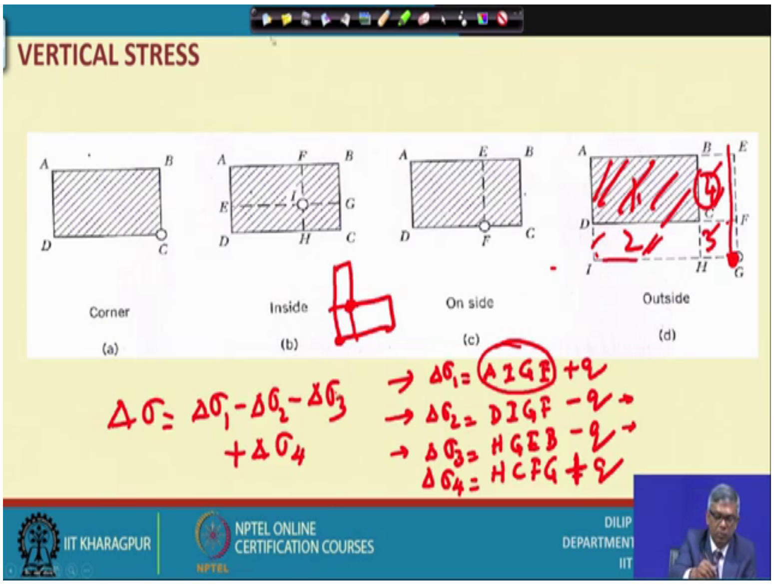

So, I will get now 1 part, 2 part, 3 part, 4 part, now what I can do if I want to find out pressure at this point what I have to do I have to consider actually now I will consider now this is a corner of this big rectangle. So, I can apply corner formula for this big rectangle and then you can see that, but this portions it is not loaded. So, what I can do, I can imagine that this portion is negatively loaded. So, I will again apply corner formula for this and with negative loading.

So, it will be subtracted and again I will apply corner formula again with negative loading then again it will be subtracted and by doing these what you can see that I am subtracting twice this area. So, that is again reducing. So, finally, I can again apply corner formula for this and add so; that means, I can find out delta sigma 1 for area A I G E with loading q plus q I can find out delta sigma 2 with for D I G F, DIGF this is rectangle with minus q, I can find out delta sigma 3 for H G E B with minus q and you can see I have calculated entire area with q and this area I have twice I have done. So, I can find out delta sigma 4 will be equal to this area H C F G with plus loading with plus q.

So; that means, what is the meaning of this area when I will consider this area I have to find out what is a what is b and then apply corner formula find out delta sigma 1 similarly when I will find out delta sigma 2 what I will do I will find out this rectangle what is a what is b what is m what is n then apply corner formula then find out what is delta sigma 2 with negative loading then again I will considered this rectangle I will find out for this rectangle what is a, what is b, what is m, what is n, apply corner formula that I will again with find negative loading. I will calculate delta sigma 3 and I will consider at last this area what is a, what is b, what is m, what is n, and then delta sigma with positive loading and then ultimately final loading will sigma will be delta sigma will be delta sigma 1, minus delta sigma 2, minus delta sigma 3, plus delta sigma 4. So, like that I can get.

So, now if I do; that means, I have subtracted this, I have subtracted this, initially I have considered entire area, now I have subtracted this, now I have subtracted this, but I have subtracted twice because of that I have added and then I am getting delta 4 this is the way I can find out stress at this point because of this loading.

So, like that any shape suppose if there is a foundation shape is like this and I want to find out pressure at this point or pressure at this point accordingly I can divide into like or divide into like this and several points can be consider and method of superposition can be applied to find out the stresses at different points like this is the application of corner formula again some more approximate work I will do some time some other methods I will discuss in the subsequent lecture.

Thank you.