Soil Mechanics/Geotechnical Engineering I
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Lecture – 21
Vertical Stress Distribution

Good morning friend, once again I welcome you to this soil mechanics lecture week 5 and today I am going to start vertical stress distribution over depth; that means, when you apply load on to the foundation. How this below the foundation soil how it will be stressed? Or at any point what will be the intensity or stress? That we are going to find out through this lecture. Will be a several of them will may take to go one by one.

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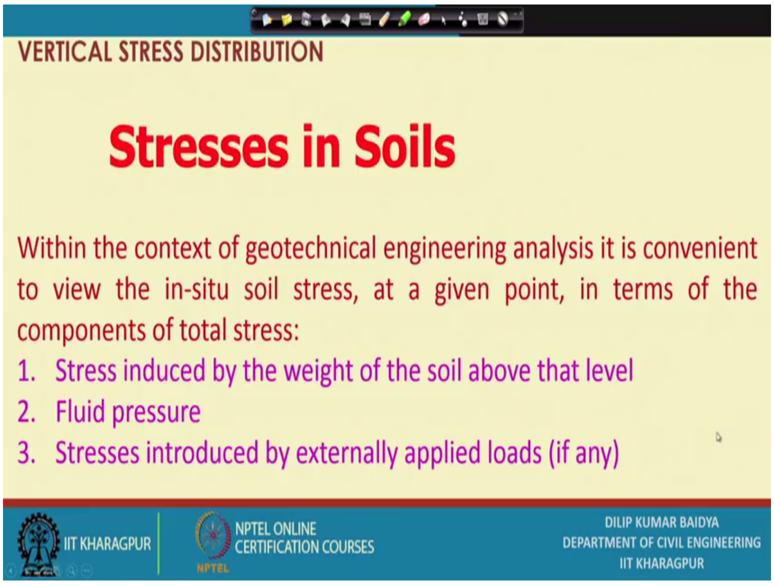

And now this is actually a stress in soils in general with within the soil. In the context of geotechnical engineering analysis we can divide into 3 important parts, one is stress induced by the weight of the soil above that level; that means, soil may be considered at a depth of 5 meter. But, above the above the 5 meter or entire depth of the soil, there will be weight and that because of that weight at 5-meter depth there will be some stress.

That is one that is because of the sulphate weight of the soil and fluid pressure; that means, when there is a soil is also there is a water table and then saturated and because of that level of water table and there will be fluid pressure and then stress induced by the externally applied loads; that means, it is obvious that we try to transferred the foundation structure load through our foundation to the soil.

That structure load how it is transmitted through found foundation to the soil that is another that is third type, stresses induced by the externally applied load; that means, through the atmosphere structure. Whatever load is coming to the soil that is actually stresses induced by the applied stress in a applied loads.

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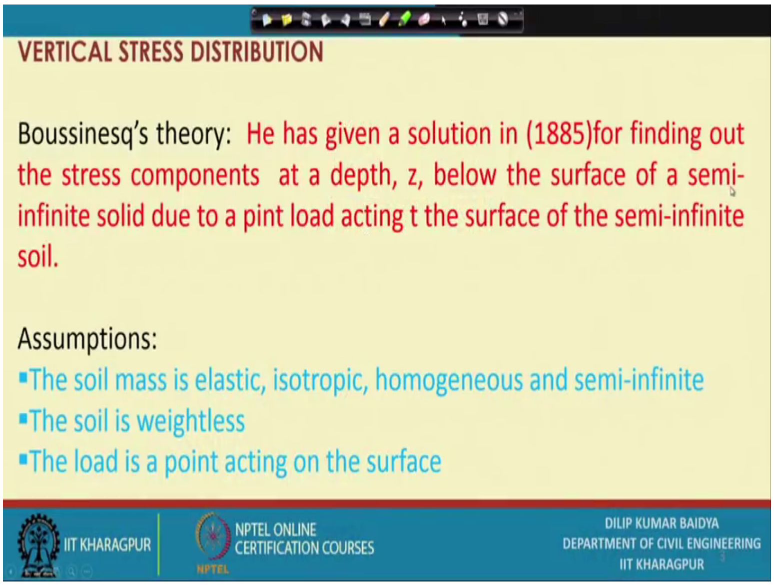

This 3 categories out of this 3 categories will be soil weight actually is easy to do is I will discuss later on. At the beginning, I will try to discuss about that stresses because of the externally applied load and for this we have different theories and that 2 this theories.

One is boussinesq theory, another is westergaards theories and they are almost parallel and can be utilized, but boussinesq theory is most popularly used in soil mechanics and this boussinesq theory based on certain assumptions and there is been developed very long back actually it is in 1885 and it is of course, when is developed for application of some areas, but finally, you have introduced in soil mechanics with certain assumption those assumption may not be truly correct, but 2 some extent it can be utilized to get some values.

The assumptions are the soil mass is elastic, isotropic, homogenous and semi-infinite; that means, when the load is applied on the ground surface, then ground surface with that per with respect to the point assumed horizontal, in infinite both, direct all direction and in also infinite, in the downward direction isotropic homogenous and semi-infinite. Laterally infinite also downward infinite, the soil is also weightless that is another thing considered because we are going to do for stress not because of the externally applied load not because of the sulphate.

Because of that the theory is assumed that the soil is weightless and load is point load is a point acting on the surface the it is a point loading. In fact, there is a typo error this is a point load acting on the surface and with this assumption. In fact, entire derivation is out of the scope of this course because.

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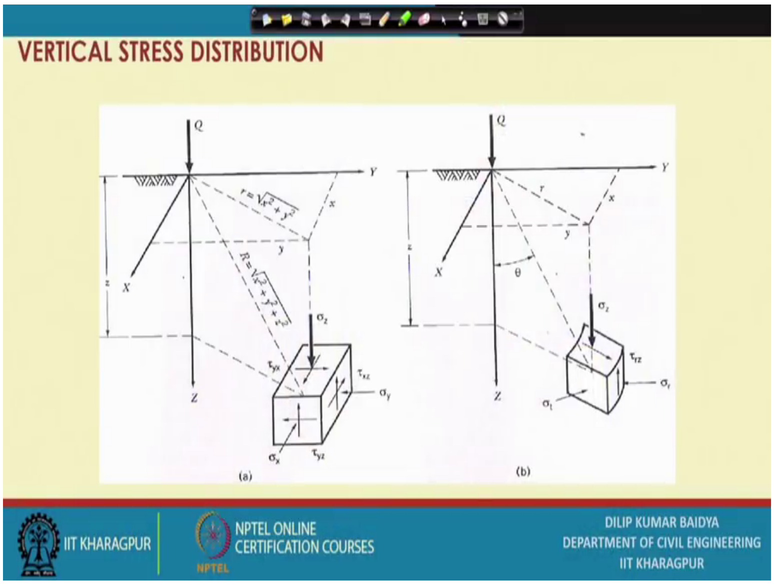

We are as user will be using the boussinesq theory and here actually initially when it is develop the boussinesq theory, it was actually in the rectangular coordinate system XY and we can consider at a point X which is sorry point any point at a depth Z. Which is at a distance X from the Y it is axis, Y from the X axis and at Z from the XY, XY plane.

This is the point and this point is shown which will little enlarged in the exaggerated one and you can see this point. If I considered as a solid of 3 dimensional solid then we will get 6 planes and X plane there will be the number of forces acting normal and shear normal will be one perpendicular to the plane and shear will be 2 parallel to 1 plane parallel to one axis perpendicular to the other. Like that this one and this one both shear tress will be there similarly this plane similarly this plane. All those planes there will be shear and normal stresses and directly will be going to will be we are going to I am going to the expression by which this stresses can be calculated.

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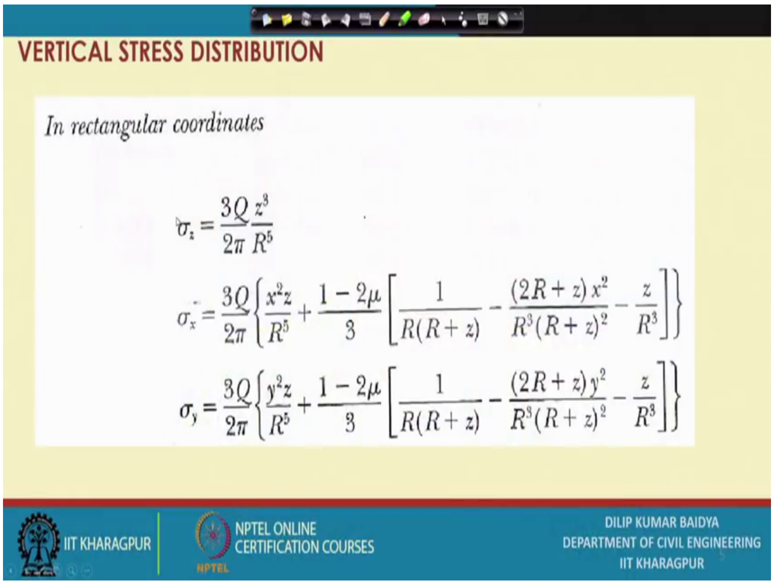

Here this is given sigma Z expression 3Q by 2 pi ZQ by R to the power 5 this R is what actually here it is shown you can see this is the R this is the R. You can find out this small R as under root excess over X square Z square and this is again Z. This R square plus this Z square ultimately will be under root X square plus Y square plus Z square.

This is the one sigma Z is given this sigma X is this, sigma Y is this, these are the expression for different component of stresses similarly these are all normal stress normal stresses.

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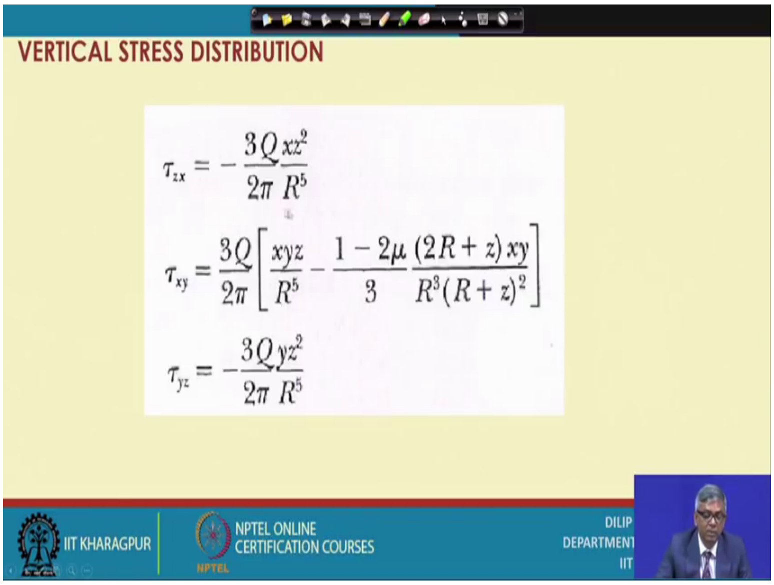

And next with this one is shown all normal shear stresses tau ZX this is the expression tau XY this is the expression and tau YZ this is the expression. This was originally in a state rectangular coordinate.

And now later on it was converted to polar coordinate to reduce the number of variables, but which some assumption that solid should be circular. In that case in this if it is a polar coordinates system the one variable is r another variable is Z and same thing can be converted R is nothing but under root X square plus Y square R and Z with respect to R and Z.

If this is explained this is expressed than those equations are stress components reduces to some other form and they are given like this sigma Z equal to Q by 2 pi Z square 3 cos cube theta and this theta cos theta is somewhere here it is shown. This is the theta. This is this is the one and this theta value if I substitute by this and this and then this one will be reduced to some other form that I will come later on.

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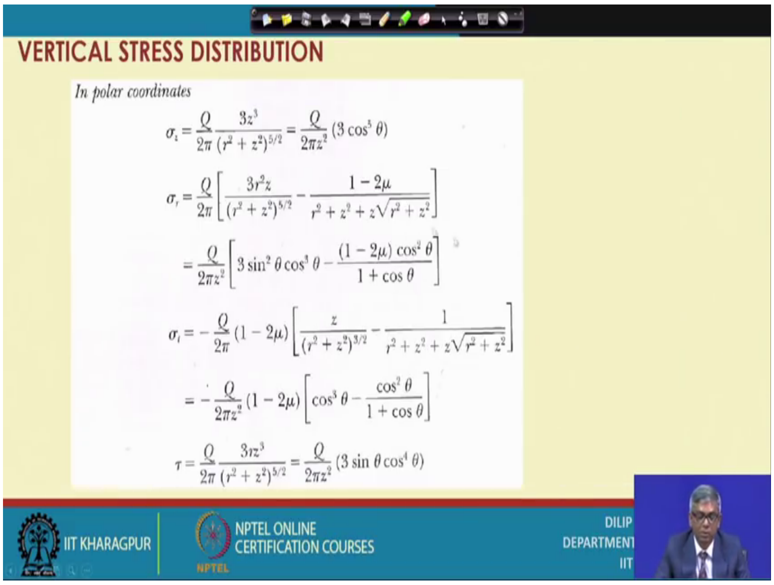

This is actually sigma Z that in terms of theta shown in the previous figure. Similarly, sigma R; that means, radial direction there is a component sigma T there is a tangential that is a component and there is a tau. All those stress components are there.

But out of this generally we apply load vertically and this vertical load always will be created significant large value of stress vertical stress and this verticals because of this vertical stress, there will be soil compression will take place and this actually we generally frequently used for this sigma Z.

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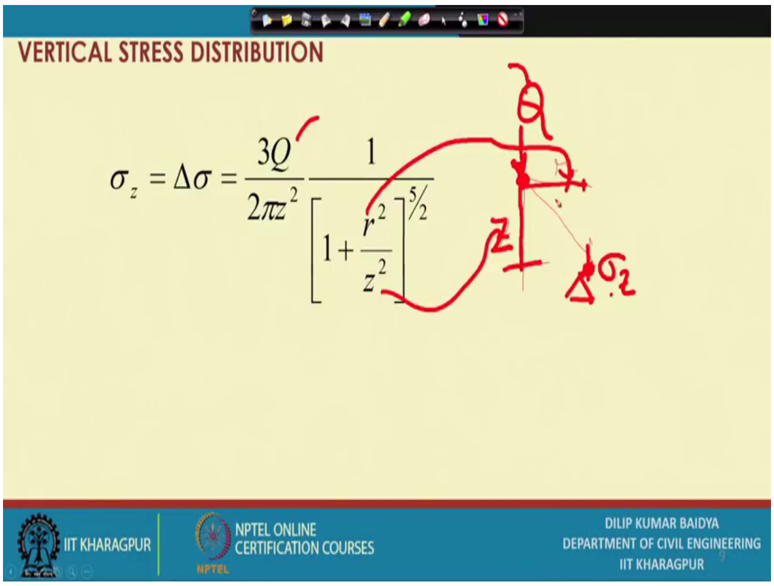

Because of that I will just other stresses is this is not that other stresses are not important, but the sigma Z value or magnitude is large and because of that I am trying to concentrate more on sigma Z. In soil mechanics actually in subsequent we will see many areas the, we have to calculate first sigma Z component sigma, but that force applied at a surface and then at a particular depth what is the value of sigma Z to be obtained? And that Z actually finally, has to be used for some other calculation.

Because of that and that is frequently used will show you subsequently this application. Because of that why I am concentrating more on this sigma Z and as I have shown that sigma Z in the polar coordinate system. It was shown in terms of cos theta and cos theta if you substitute and simplify, then it will equation will be reduces to 3Q by 2pi Z square one by one plus R by Z square to the power 5 by 2.

That means, it is typically it is like this, if it is force is applied here and suppose your point is somewhere here and that is at a radial distance R and at a vertical distance Z. Then you can see this is the vertical tape Z and radial distance R. And so, this these are the 2 things are kept here and load suppose you act. Suppose when load is acting in here at a radial distance R and vertical depth Z what is the value of what is the expression?

If I apply a Q load here I will get sigma Z here delta sigma Z I will get here by this expression and what is this Q is this one applied and what is Z? This Z and what is R? This is the R so; that means, if I know this 3 things then I can find out the stress at the at a particular depth.

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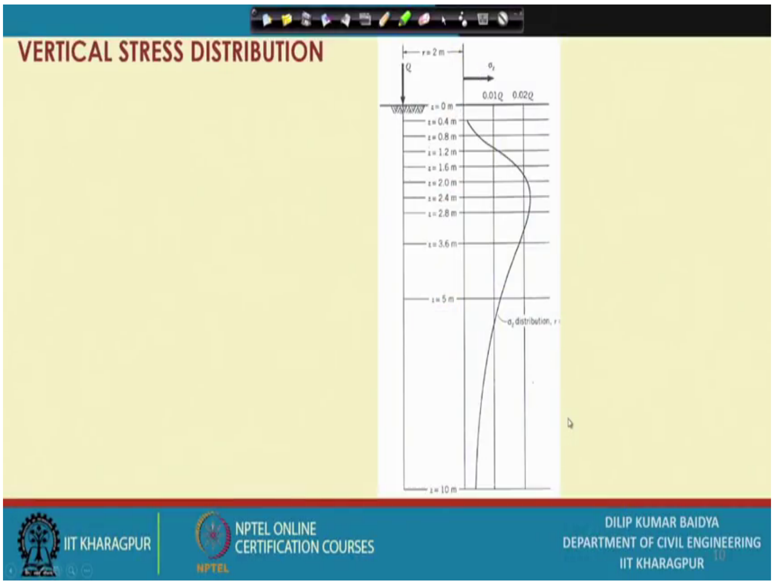

Now, As 2 variables are there one is R another is Z and because of that you can see here if I fix the value of R that at a particular distance of load is applied here and I have taken at a some distance about 2 meter and then I very Z then over the depth how this vertical stress is varying?

Now you can see R is fixed cost constant kept constant and then what the variation of Z the vertical stresses varying like this; that means, close to the surface. It is less and at some depth it will be increasing, it should be the maximum, then it will be decreasing and it theoretically it supposed to go up to infinity, but up to some depth the value when compared to the maximum value become negative we generally ignore the effect.

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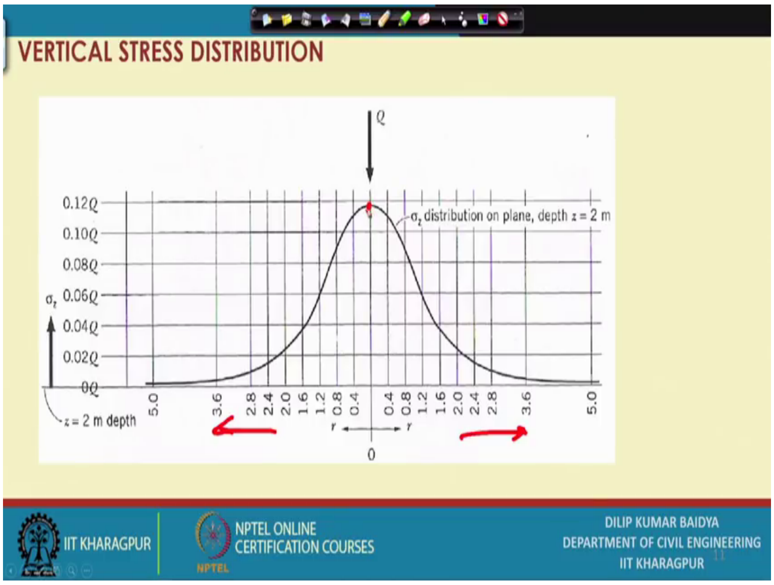

This is one observation second observation is when we are we suppose if Z is fixed; that means, I want to find out at a point load is applied at the surface, but I want to find out a 2-meter depth and from any distance from the point of application of the load. Both side if I go.

That is the case suppose if z is fixed suppose a 2 meter or 4 meter and with the variation of R this direction R is increasing, this direction R is increasing, and you can see below the load you have the maximum vertical stress and when will go up every from the from the load than your stress is decreasing. This is another observation.

And now next one is if we now. With respect to 3 different depth suppose you can see now.

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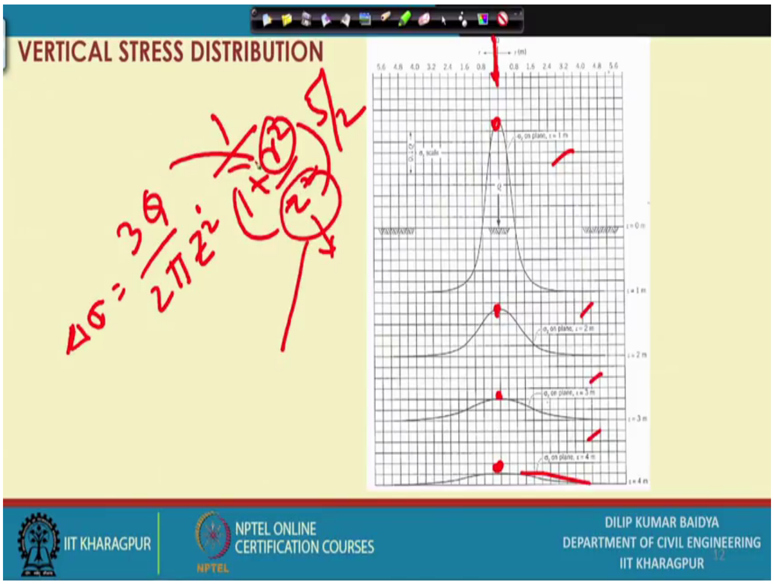

Now, you have considered at 3 different depths or 4 different depths. This is at Z 4 meter suppose 1 meter this is at 2-meter, 3-meter, 4 meters, but you can see that the variation or trend is similar trend is similar; that means, is a maximum at the below the load is applied. Suppose here just vertically below the lord is the maximum stress and where you go away from the point of application of the load the value is reducing and this trend is same as it is in one meter and it is or whatever depth, but finally, what you are go what will another thing is that is one thing, that is maximum values at the below the load and less value when you go away from the point of application.

And with the increase of the depth this peak value also decreasing and also this entire value from the away of the point of application their value become negligible if you go deeper and deeper. These are the trend we can see. This actually by this is actually you are not doing anything simply whatever equation point load equation you have got. By using that point load equation if I change R and Z you wants to keep R is constant where is depth. Then you will get the trend that, how it is good varying the vertical stress what depth.

And then once you can keep Z is constant and then vary R same equation by the same equation that is actually your del sigma equal to 3Q by 2 pi into Z square into 1 for 1 plus R square by Z square to the power 5 by 2.

This is the equation only we are using, but once I am keeping this constant vary this I get the result or I get this constant and vary this then I will get another red. This is the by doing this I have try to show this 2 things. How if I apply the load at this point at 2-meter distance to visit are away from this how much is the stress? And at the same time at 5 meter what will be the stress that can be obtained by this.

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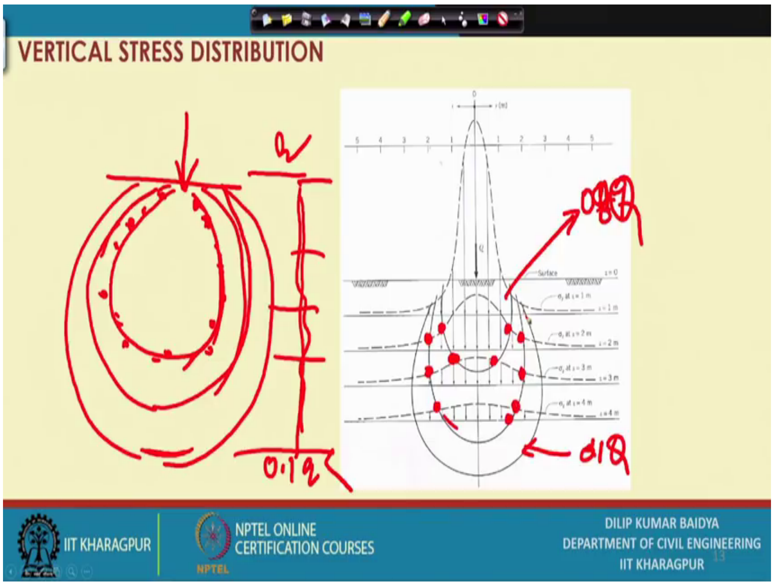

Next one since you can see now I can go back to this previous 1 and a, these are the stresses stress distribution. Actually, this is your stress distribution at some depth, this is your stress distribution at some depth, this is the stress distribution at some depth, like that and now you can see that. Since with the increase of depth stress is decreasing, and with the increase of R stress also decreasing. We may get a several combination of R and Z where we can get the intensity of vertical stress is same.

That is the thing. Maybe intensity of stress here a intensity of stress here is equal. Because you can see here actually R is less and here is Z is less, but R is more here actually Z is more, but R is less. Because of this and, but a particular combination this point whatever stress, this point also stress is same similarly, this point also stress may be same this may be this point also stress also will be same.

Similarly, I may find that at this point the, whatever R and Z this point the R and Z different and if we apply to this equation you may get this value this value are same. Similarly, this value this value and this value also same, this value also same, this value also same. Like that in when I apply the load on a one a ground surface and then I will get a several point like this, like this, like this, like this, like this, like this maybe this is actually this point and this points R and Z same, but these are 2 and this point R is R and Z same, but they are different from here.

As a result, we may get the stress value equal. It will be like this. Suppose similarly, I get another state I will get something like this equal stress I get another I may get equal stress like this. Those actually; that means, we know the lord point we will get several points. Those points which several combination of R and Z where we will get the equal vertical stress and if this equal vertical stress then that is called actually your pressure bulb actually the soil mechanics this is the we use.

This is the this term as pressure bulb; that means, what is this pressure bulb this is actually the line joining line joining the pressure of equal value magnitude. That is called actually your pressure bulb and this pressure bulb finally, I will show you later on that this when the pressure bulb which is close to the load. The intensity is more and we will go away from the load the intensity will be smaller.

Suppose here actually you are getting 0.1q or 0.8Q suppose q and then you will get suppose this is 0.1Q suppose, and if you go further you may get still smaller and, but theoretically speaking up to infinite depth you will get this type of special bulb.

Now, in soil mechanics we suppose if I apply a load at the surface up to what depth it will we will consider; that means, soil under loading it will behave differently and because of that soil may go undergo compression subsidence and because of that you have to consider upto certain depth of soil. And so, up to what depth you have to go. That is what I will show you later on that if I apply a stress magnitude of suppose Q and at a particular depth when this pressure, intensity, become less than point 0.1Q less than 0.1Q.

Suppose I have applies Q at the surface and at a particular depth it become point less than pointer and Q. And so, that is what we considered in soil mechanics or geotechnical engineering as a depth of significance or significance depth. What is the meaning of it? That means, up to this depth we have you should know all details about the soil if there is any layering, if there is a variation in soil strength, soil properties, etcetera you have to in which; that means, when you do soil investigation then we have to do at least up to the significant depth. You may go deeper and deeper to get more information, but at least you have to do up to this.

This is the calculation by which we can get up what depth we have do the investigation. Like this, this is actually some application of boussinesqs theory by applying when applied a load at the surface at different depth what is the intensity of vertical station can calculate? Also, we can find out up to what depth we need to investigate the soil that we can design the structure considering the variability in the soil. This is the one and with this I will close this module.

Thank you.