

**Soil Mechanics/Geotechnical Engineering I**  
**Prof. Dilip Kumar Baidya**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 19**  
**Effective Stress (Contd.)**

Once again this, we have just defined effective stress. Effective concept we have explained and how to calculate effective stress because of the (Refer Time: 00:30) of the soil just we have seen. And now this we have seen that permeability seepage and we have seen effective stress and during permeability.

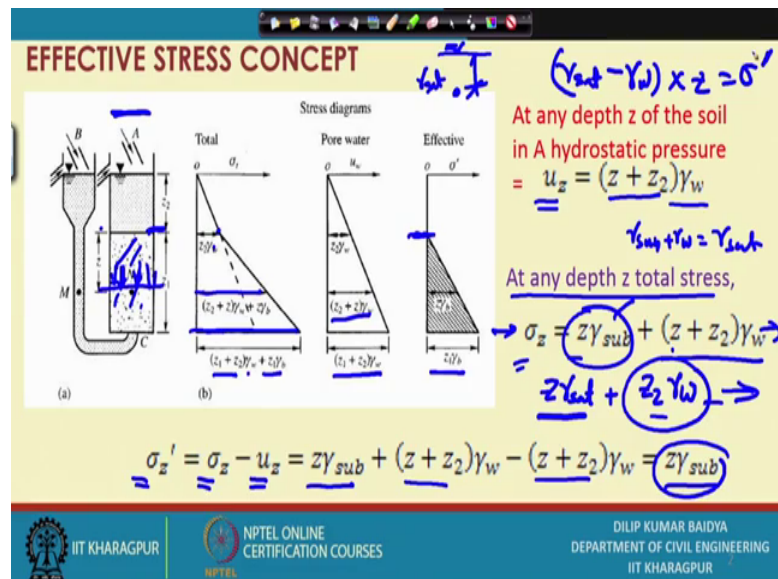
And seepage we have shown that because of this porous in nature of soil and then there if there is a change in head let us say upstream and downstream then water will flow from higher head to lower head. So that means, there can be in the field, there can be a number of situation that soil water in a static condition that is a fixed ground water level and there is a fixed ground surface and for that we can find out what is the effective stress and depth.

Now, the situation may change water table may rise that is one situation and then sometime there is another situation that water may flow through the soil and again flow can be in a different direction flow through the soil, the upward direction. That means, from the bottom to up and also there can be flow of water from the top surface to the bottom another situation and there may be no flow. So, there are different situation can arise in the field and because of that how effective stress will change, that we will examine one by one and we will see the application also simultaneously.

And sometime that in a static condition it is a quite stable, but when there is a flow take place either downward or bottomward, downward or upward and sometime we will see that that sometime flow may cause the danger, different ways, danger. One is actually I have shown already, but once again we will show here that is let critical hydraulic gradient, how critical hydraulic gradient play a role to make the condition bad that we will show one by one and also because of the flow from when it is up from the up to downward side and then sometime we have seen that some seepage pressure will act. So, that actually what is the quantity and that we will see one by one.

So, particularly I will start with a no flow condition.

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So, suppose this is a situation here and you can see that this is a soil, mass this is a soil mass and it is a through this soil and water head is applied here and from the bottom also it is connected to another tank and the water is continuous through this end to this end A to B. But since there is a overflow from both places if there is a water more either here or here it overflow will take place and maintain same level..

So, if this is the level maintained then this will be the very much in equilibrium condition and then there will be no flow of water through the soil. So, this condition already I have explained, but this is a case separately since we are taking, I have taken three cases I will show one by one and this is one of them that is no flow condition and at this state we can say above the soil we have head about  $z_2$  and soil thickness or height is  $z_1$ . And so, under this if I consider since there is a no change in head and then there is no change of head loss there is no head loss. So, there is no flow. So, everywhere if you go it should be same hydrostatic pressure you can get.

So, because of that within the soil mass at a point  $z$  suppose at depth  $z$  from here from here to  $z$  depth we have considered a point. So, if I consider that point then what will be the  $u_z$  at that point? That means, what is the hydrostatic pressure at this point? As we know that hydrostatic pressure is nothing, but unit weight of water multiplied by depth  $z$ . So, what is the total depth at this point? This will be  $z_2$  plus  $z$ . So, this is the total head

at this point if I consider. So, this is the total water head. So, at this point because of this water what is the pressure? So, that is what  $u$   $z$  is  $z$  plus  $z^2$ ,  $z$  plus  $z^2$  into  $\gamma_w$  is the  $u$   $z$ .

And at any time depth  $z$  what is the total stress? So that means, I want to find out at this level what is the total; that means,  $\sigma$  that is total stress I have got  $u$  next I am getting total stress suppose this is like is what is the stress here, what is the stress, here total stress. So, to find out this total stress what you have to do we have to consider whatever there above this line I have to take weight of that and divide by area. And since I can assume that I can assume that I have considering this for a unit cross section area. So, if I can find out the weight, weight itself will be the pressure. So, to find to out this I can first find out the weight of this and then I found out the weight of this soil.

So, this is the way I have done. So,  $z$  plus  $z^2$  water weight is already there and this portion is taken  $z$  into  $\gamma_{sub}$  submerged the additionally up to water height what was there, additionally you have take submerge unit or buoyant unit water soil. And this is the way we have taken I could have taken in other way also, I could have taken this one also like this  $z$  into  $\gamma_{sat}$  saturated if I take this one and in that case I could have taken here  $z^2$  into  $\gamma_w$  I could have taken this way also.

But ultimately, this one actually I can do  $\gamma_{sat}$  and  $\gamma_{sat}$  is nothing, but. So,  $\gamma_{sub}$  minus  $\gamma_w$ . So, this if do and this and this are actually similar and. So, at this point I will calculate either this way if I take to saturated unit where total I take then only  $z$  I will take and when I will take this way then I will take only water height only the  $z^2$  is acting there only this can be taken..

So, either this way or this way,  $\gamma_{sub}$  into  $z$  in that case what are all the  $z^2$  into  $\gamma_w$  and when I take  $z$  into  $\gamma_{sat}$ ; that means,  $\gamma_{sat}$  I am taking means I am taking water weight also in this. So, because of that I will not take separately water weight. So, this will be if you do that if you do that you can see  $\gamma_{sub}$  plus  $\gamma_w$ ,  $\gamma_{sub}$  plus  $\gamma_w$  is the  $\gamma_{sat}$ .

So, because of that you can see if I take  $z$  into  $\gamma_w$   $z$  into  $\gamma_w$ . So, here if you multiply, so  $z \gamma_{sub}$  plus  $\gamma_w$   $z$  plus  $\gamma_w$  is  $\gamma_{sat}$ . So, this is coming. So, that means, if I take  $z$  into  $\gamma_{sat}$  here from this portion saturated depth of the soil; that means, water is already included because of that I am not

considering water separately here only water depth  $z$  into  $\gamma_w$ . So it can be done either this way or it can be done this way. If I do this  $\gamma$  submerged will be used or if I use this  $\gamma$  saturated will be used.

Now, once you do this, this is total  $\sigma$  at  $z$  at depth  $z$  and this is  $u$  at  $z$  then  $\gamma$  effective stress by definition we know that  $\sigma_z$  at any depth minus  $u_z$  and  $\sigma_z$  minus  $u_z$ , if I now put  $\sigma_z$  actually is  $z \cdot \gamma$  into  $\gamma$  submerged this I have taken  $z$  plus  $z^2$  into  $\gamma_w$  this I have taken this minus  $u_z$  plus  $z^2$  into  $\gamma_w$  and you can see  $z$  plus  $z^2 \gamma_w$   $z$  plus  $z^2 \gamma_w$  here this will get cancelled ultimately  $z$  plus  $\gamma$  submerged.

So that means, when there is no flow and then at any depth if I want to find out what is the effective stress, then it is nothing but  $z$  into  $\gamma$  submerged. You can see this that the same thing whatever way I have shown there you can see  $\gamma_w$  is total stress we have calculated. So, this up to  $z^2$  into  $\gamma_w$  has come here and  $z^2$  plus  $z \gamma_w$  plus  $\gamma$  submerged..

So, this portion up to this is nothing, but this and this portion. So, add  $z$  depth and at this depth it will be  $z^1$  plus  $z^2 \gamma_w$ ; that means, both plus  $z^1 \gamma$  submerged and when this water pressure we are calculating you are getting see, it will be linear and at this depth  $z^2 \gamma_w$  at this  $z^1$  plus  $z^2 \gamma_w$  and this  $z^1$  plus  $z^2 \gamma_w$  and now if I subtract from this to this you can see it get 0 and now this minus this if you do  $z$  into  $\gamma$  submerged that whatever we have shown here and if I take at this point again it will be  $z^1$  into  $\gamma$  submerged.

So, this is the depth so that means,  $z$  into  $\gamma$  submerged. Now, as I have concluded in the previous slide that if the water table change any height above water above the soil, ground surface this is ground surface if water table goes here also suppose it goes here also again similarly up to that diagram will start from here, here also diagram will start from here and it will get cancelled and start effective stress from here..

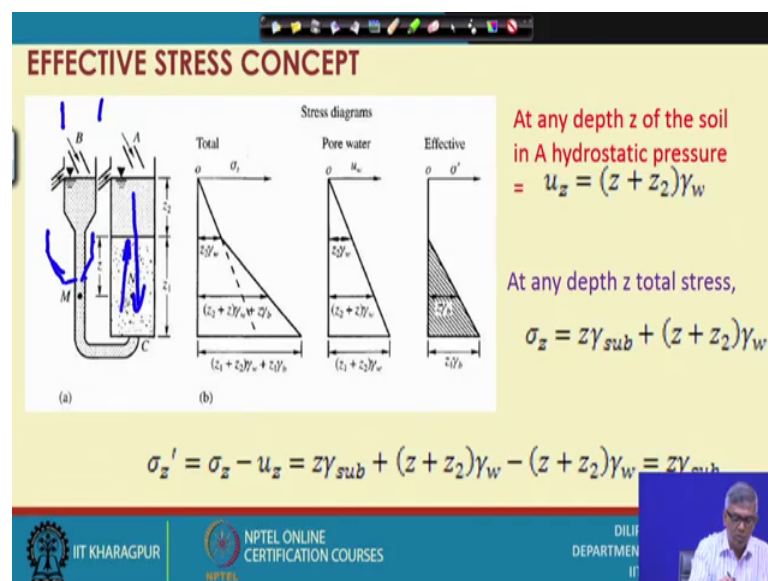
So, there will not be any change. Whereas, if the water table from here if goes down here there will be a change that I will show later on. So, that means so no flow condition if I want to find out effective stress and it is fully saturated suppose there is a ground and it is saturated here, and I want to find out what is the effective stress here. Then the  $\gamma$  if

I know the gamma saturated here then gamma saturated minus gamma w that become gamma sub and into this depth I will consider that z that become sigma dash directly.

So, one can do two ways, one can do first gamma saturated this is the way directly I can do or I can do gamma saturated into depth I can do total and u into gamma w is the pore pressure and difference of these two become effective stress that is that way also one can find out.

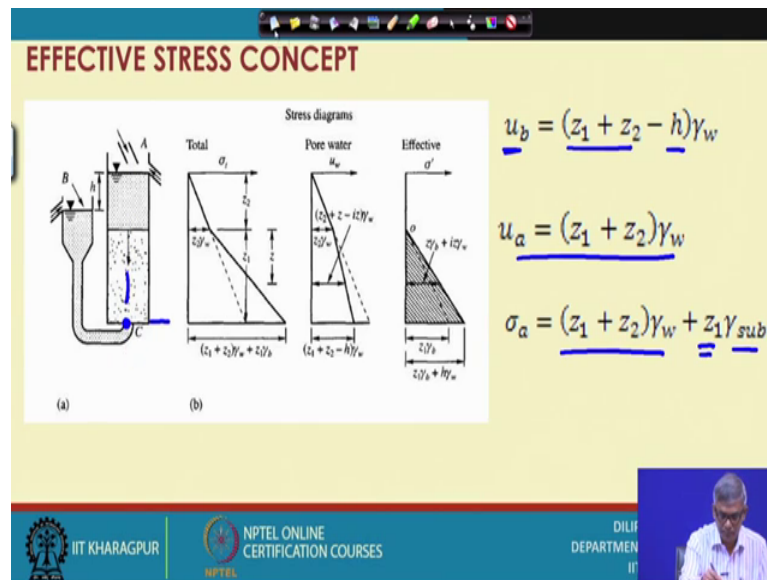
So, this is actually no flow condition that is static is impact whatever I have discussed in the previous lecture they are almost similar only thing I have shown that if I lift this one if this tank is elevated slightly this side and suppose if the tank sorry tank is taken somewhere here, tank height is raised here and this overflow is kept here; that means, head is here at this point hydrostatic pressure is more and at this point hydrostatic pressure will be less.

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So, because of this head change the work flow will take place in this direction. And again if I take the tank somewhere here tank I lower and bring it somewhere here and overflow takes place from here in that case water if I apply here because of this change in head water will move in this direction. When water will move in this direction water moves in this direction there is two different situation and lot of changes will be there within the soil mass particularly the effective stress that we will see in the subsequent slides.

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So, you can see that this is the one as I have mentioned there that I have considered a tank is a lower height and we are supplying flow here at a particular height and the difference is  $h$ . So, because of these changes, change in head at this side and at this side then; obviously, flow will take place in this direction.

And if this flow take place this way then what is the changes, what are the changes in effective stress that we will see. And under this condition I can draw the effective, total stress; I can draw the pore water pressure and then total minus pore water pressure that will gives you effective stress and from there you can draw some conclusion.

You can see here at this point if I consider from because of this head we are getting some head, but if I consider from this side then I will get different water head. So, there is a change. So, if I consider  $u_b$  here, so  $z_1$  plus  $z_2$  minus  $h$ . So,  $z_1$  plus  $z_2$  is the total height, minus  $h$  is the head at this and whereas,  $u_a$  that is if I consider at from here at a then you can get  $z_1$  plus  $z_2$  into  $\gamma_w$ .

So, this it cannot have different at equilibrium from both sides it has to be equal. How it will become equal? Because of during the flow there will be loss of head and final it will become from both sides it should be equal; that means, you have to consider equal here only. So, that will be equal to  $z_1$  plus  $z_2$   $\gamma_w$ .

So, if I want to find out sigma a at this point from considering this side then you can see z 1 plus z 2 gamma w is there z 1 of what the entire depth z 1 plus z 2 gamma w. So, z 1 plus z 2 gamma w is there water head up to this acting. So, generally that will be there at total stress at this point I am calculating and what is the additional because of the soil since soil is under water it is submerged condition, z 1 into gamma submerge. So, that I can consider otherwise I can take any depth I can vary it. So, at this point I get this total stress equal to z 1 plus z 2 gamma w and z 1 gamma submerge and so I am getting stress considering from this side.

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**EFFECTIVE STRESS CONCEPT**

$$\sigma_a' = \sigma_a - u_a = (z_1 + z_2)\gamma_w + z_1\gamma_{sub} - (z_1 + z_2 - h)\gamma_w = z_1\gamma_{sub} + h\gamma_w$$

$$\sigma_z' = \sigma_a' \frac{z}{z_1} = \frac{(z_1\gamma_{sub} + h\gamma_w)z}{z_1} = z\gamma_{sub} + \frac{hz\gamma_w}{z_1} = z\gamma_{sub} + i z\gamma_w$$

Diagram illustrating the effective stress concept. It shows a soil element of height  $z_1$  and width  $z_2$ . The total stress  $\sigma_a$  is the sum of the weight of the soil and water above it,  $(z_1 + z_2)\gamma_w$ , and the submerged weight of the soil,  $z_1\gamma_{sub}$ . The pore water pressure  $u_a$  is the weight of the water column above the point,  $(z_1 + z_2 - h)\gamma_w$ , where  $h$  is the height of the water table. The effective stress  $\sigma_a'$  is the difference between the total stress and the pore water pressure, resulting in  $z_1\gamma_{sub} + h\gamma_w$ . The diagram also shows the vertical stress  $\sigma_z'$  at a depth  $z$  from the water table, which is  $z\gamma_{sub} + i z\gamma_w$ , where  $i = \frac{h}{z_1}$  is the pore pressure ratio. A cross-section of a soil element is shown with dimensions  $z_1$  and  $z_2$ , and a water table at height  $h$  from the bottom of the element. The total stress  $\sigma_a$  is shown as the sum of the weight of the soil and water above it, and the pore water pressure  $u_a$  is shown as the weight of the water column above the point.

Now, if I take sigma dash sigma a dash the sigma a minus u a and sigma a will be z 1 plus z 2 gamma w plus z 1 gamma submerge already just in the previous slide we have seen and u, effective u is what? Whatever am seeing from the other one. So, z 1 plus z 2 minus h; that means, that is it is here and tank is here. So, this is h. So, at this point head is nothing, but whatever z 1 plus z 2 was this is the z 1 plus z 2, but actual head acting here, actual head acting here z 1 plus z 2 minus this h; that means, because of this flow the head h is lost. So, you have to find out this value. So, actual at sigma a, sigma a is calculated because z 1 and z 2 are there with gamma w and soil was that into z 1. So, z 1 into submerge and this is total stress and at this point because of this what is the hydrostatic pressure is giving.

So,  $z_1$  plus  $z_2$  minus  $h$  and this into  $\gamma_w$ . So, these two actually difference of these two will give you at that point at bottom at this point effective stress. So, this will be if you see this  $z_1$  plus  $z_2$   $\gamma_w$  minus  $z_1$  plus  $z_2$   $\gamma_w$ . So, this is actually minus  $z_1$  plus  $z_2$   $\gamma_w$  plus  $h$   $\gamma_w$ . So, this and this will get cancelled. So, you will have  $z_1$  submerge plus  $z_1$   $\gamma_{\text{submerge}}$   $z_1$   $\gamma_{\text{submerge}}$  plus  $h$   $\gamma_w$ .

So, this is the, so that means, whatever I have seen we have seen in the previous one that the only when there is no change in head when the two sides then only we have got  $z_1$   $\gamma_{\text{submerged}}$ . Now, when the flow is taking place from the soil downward direction we are getting a fifth stage additional term that is  $h$   $\gamma_w$  the  $h$  is the head loss which causing the flow because of this head loss flow is taking place. So, additionally; that means, this much  $h$  into  $\gamma_w$  is coming.

And now this can be very much generalized that if I consider  $\sigma_z$  at any depth  $\sigma_a$  dash. So, that  $z$  is actually I can go back to the previous slide I will go, I will go up to this after this. So, it is linear. So, that is what it is proportionate  $\sigma_z$  dash is it will be  $z$  by  $z_1$   $\sigma_a$  dash  $z$  by  $z_1$ . So, that was actually it was here like this diagram was like this. So, this depth is  $z_1$  and this depth is  $z$  and at this  $z$  depth actually  $\sigma_a$  dash this one is  $\sigma_z$  dash.

So, if I proportionally  $\sigma_z$  dash if I find out the  $\sigma_a$  dash  $z$  by  $z_1$  it will be the this from the equal symmetric into a tubular symmetry of triangle. So, you can write  $\sigma_z$   $\sigma_z$   $\sigma_a$  we have got and this is the diagram and  $\sigma_z$  dash we can get  $\sigma_a$  dash  $z$  by  $z_1$ . And if I do that, so  $\sigma_a$  dash is this and multiplied by  $z$  by  $z_1$  and you can see  $z_1$  and  $z_1$  get cancelled. So, it will be  $\gamma_z$  into  $z$  into sorry. So,  $z_1$   $z_1$  get cancelled  $z$  into  $\gamma_{\text{submerged}}$ ,  $z$  into  $\gamma_{\text{submerged}}$ . So,  $z$  into  $\gamma_{\text{submerged}}$  is here and you can see here it will be  $h$   $z$ , it will be  $h$   $z$   $\gamma_w$  by  $z_1$ . So, this is coming  $h$  into  $z$  by  $z$   $h$  into  $z$  by  $z_1$   $\gamma_w$ . So, this term is coming.

So, at any depth; that means, this is the effective stress now this can be further simplified this is  $z$  into  $\gamma_{\text{submerge}}$  already we know and  $h$  by  $z_1$ ,  $h$  by  $z_1$ ; that means, head loss because of the flow through how much depth. Your sample was this much this is  $z_1$  and through this flow is taking place and because of this flow through this head loss is  $h$ .

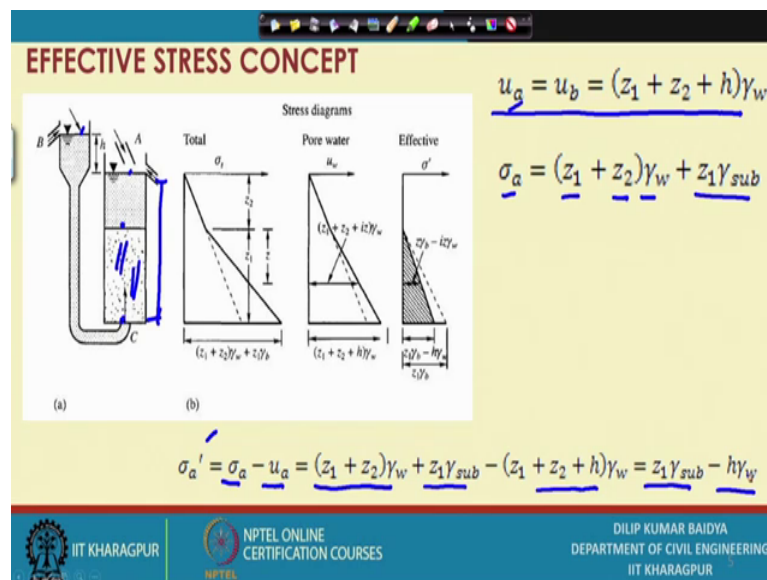


So, as per our definition of hydraulic gradient actually head loss over how much depth  $z$  I so; that means,  $h$  by  $z$  I can define as  $i$ ,  $h$  by  $z$  I can define as  $i$ .

So, it will become, so ultimately  $z \gamma_w$  submerge  $i z \gamma_w$ . So that means, what is the additional stress coming when flow is taking place from downward direction? That means, whatever pressure already there in the static condition plus hydraulic gradient time  $z$  times  $\gamma_w$ .

So, that is  $i z \gamma_w$  additional you have to  $i z \gamma_w$  to be added to the actual static effects stress, effect stress in the static condition this much to be added to get the effective stress in the flowing condition when the flow is taking place from upward direction to downward direction. So, this is one case; that means, additionally first case no condition no flow; that means, it was  $z$  into  $\gamma_w$  submerge and when there is a flow is taking place it will be additionally  $i z \gamma_w$  to be added to get the effective; that means, effective stress is increasing because of the flow taking place from upward direction to downward direction.

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Now, there is a third case we will consider that is actually as I have shown before that the third condition whatever I have shown that when this tank is elevated or lifted some by some height and you can see here and this is the level here and this is the level here. Now, if I continuously supply water here and then because of this the water is continuous throughout this then this water will try to flow through this soil in the upward direction.

Previous case whatever we have taken flow was taken place downward direction now flow is taking place in the upward direction. If it goes upward direction then what will happen that to be seen.

Again here actually if I consider from this side you will get head actually equal to  $z_1$  plus  $z_2$  plus  $h$ , but if I consider from this side actually your it is  $z_1$  plus  $z_2$  only  $z_1$  plus  $z_2$  gamma  $w$ . So, this way, but at this point because of this extra head it flow will take place, but in equilibrium condition you will consider this is the head. So, this is the head we are putting and finally, at this point head will become equal, because of the loss  $h$ .

So, now at this point will be head will be  $z_1$  plus  $z_2$  plus  $h$  that is the head which cause the flow actually and  $\sigma_a$  at this point; that means, what is the total stress it will be  $z_1$  plus  $z_2$  gamma  $w$  and; that means, the gamma  $w$  is there from here to here already. So, that is there plus because of the soil height this is  $z_1$  gamma submerge this is done for all cases. So, it is done. So,  $\sigma_a$  is this and  $u_a$  is this, now  $\sigma_a$  dash again  $\sigma_a$  minus  $u_a$ .

So, if I do this then you can see  $z_1$  plus  $z_2$  gamma  $w$  plus  $z_1$  gamma submerge minus  $z_1$  plus  $z_2$   $h$  gamma  $w$ . So, this is becoming  $z_1$  gamma submerge minus  $h$  gamma  $w$ . So, you can see now when flow was taking place from upward to downward direction we have got one extra term compared to the static one that was  $h$  gamma  $w$  which was plus. Now, when flow is taking place from bottom to upward then it is also giving an additional term which is equal to  $h$  gamma  $w$ ; that means, when flow takes place from bottom to upward direction then that means, effective stress is decreasing and this decrease again how much that can be generalized with respect to hydraulic gradient.

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**EFFECTIVE STRESS CONCEPT**

$$\sigma_z' = z\gamma_{sub} - i z\gamma_w = 0$$

$$i_c = i = \frac{\gamma_{sub}}{\gamma} = \frac{G_s - 1}{1 + e}$$

Quick Sand Condition

The slide includes a diagram of a soil element with vertical dimensions  $z_1$  and  $z_2$ . It shows downward and upward arrows representing forces. Handwritten notes include  $2\gamma_{sub} z_1$  and  $2\gamma_{sub} - i z\gamma_w$ . The bottom of the slide features logos for IIT Kharagpur and NPTEL, along with the name Dilip Kumar Baidya, Department of Civil Engineering, IIT Kharagpur.

So, if I take next slide you can see that sigma z dash again same way if I do your, this was this was z 1 and this was sigma a dash and this is z and this is sigma z dash. So, proportionally if I do then it will be again if I similar way if I do and simplify you will get z gamma submerged minus i z gamma w. That means, when flow is in downward direction then it will be z into gamma submerge minus i z gamma w and when flow is in upward direction it is z gamma submerge minus i into z. So, that was plus and now i z into gamma w. So, it is everything similar only when you have to remember that when flow is in the downward direction it will be adding and when flow is in the upward direction it is decreasing, so minus.

So, now this decrease can actually to be can be sometime. So, much that effective stress become 0, it can happen so. So in that case when this effective stress become 0 that time we can say i equal to i c and that actually if I come from that definition critical hydraulic gradient equal to i and then you can see this if i equal to 0 then i c become gamma submerge by gamma w and gamma submerge by gamma w you know the G s minus 1 by 1 plus e.

Already defined derived before once again we are getting from this that is gamma submerge by; that time actually what we have done? We have considered equilibrium; that means, force from downward direction force in downward direction force in upward direction and equated and from there we have got the critical hydraulic gradient as per

definition and again once again here considering the flow condition whether it is upward direction and downward direction.

And as I have mentioned when flow is in downward direction it is adding some extra stress or effective stress when flow in upward direction it is decreasing the effective stress and this decrease can be so much that effective stress will become 0 and that condition the condition called critical condition and that is called the corresponding to that whatever hydraulic gradient we get that is called critical hydraulic gradient.

So, the critical hydraulic gradient if you know the soil parameter like  $G$  and void ratio then I can find out the amount of critical hydraulic gradient. And if your hydraulic gradient is more than that then actually then the soil will collapse that is the thing. So, what is the significant? So, whatever we have got here now these are theoretical. Now, what is the significance of this?

Significance is this that when there is a flow take place from downward up to downward direction then that extra stress to be calculated the extra pressure will come to the foundation that to be considered and when there is a flow is taking place in the upward direction that time your stress will be reduced and it can be reduced. So, much that soil may not have an effective stress and soil do not have effective stress it means that is it is like a fluid.

So, because of that there is a when the hydraulic gradient become critical at that point generally that called the situation is called quick sand condition quick sand condition. That means, many times in some places people where standing and suddenly they sink inside the mud or the sand that is what actually there may be a sand layer here and because of some reason there may be some source of water coming from here and it is causing the floor in their upward direction like this.

And if it happens if the flow happens like this and if the head is so much that here effective stress is 0 then if you stand here then will need not be able to stand there you will just because the soil does not have any strength. So, you will sink inside. And that is a very dangerous situation many time it happens in the seashore and in many places people sink.

And to get escape from this immediately when you feel that your feet is going downward immediately you can flat your body flattening the body it what it helps because it will help the way because you are distributing the stress your entire body weight is coming through the feet whatever stress giving at this point and if your entire body will flatten over the soil then intensity of stress is reducing.

That way actually sometime quickly you reduce the stress and come to, you can escape from the danger and. So, this is a quick sand condition the significance of this critical hydraulic gradient is that if your head is so much or which because of that its hydraulic gradient exceeds the critical hydraulic gradient and if the hydraulic gradient exceeds the critical hydraulic gradient then your soil strength almost come to 0, and when there is a 0 strength then anything we put on it, it will sink, simply.

So, I think with this I will stop this one, maybe I will go next slide and do something next.

Thank you.