

**Soil Mechanics/Geotechnical Engineering I**  
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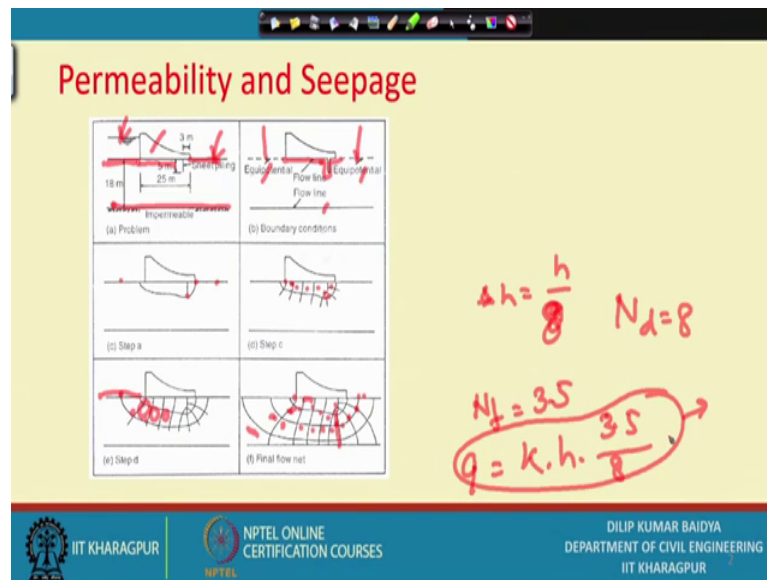
**Lecture -12**  
**Permeability and Seepage (Contd.)**

Once again welcome to this lecture. This is in my last lecture just I have completed how to estimate the quantity of seepage through the body water retaining structure and then just I have shown the application of flow net and if I can draw the flow net and if I can make equal potential lines number of at equi distance and with equal drop and if I can draw few flow lines and between the flow lines I can imagine this is a flow channels. So, each small flow channels I can imagine.

Number of flow channels I can count from the Figure. Number of flow equipotential drop also I can count from the Figure and height different between the up steam and down steam also I can found out and  $k$  for the soil will be known. So, based on that we can find out the quantity of seepage, which will be equal to  $k$  into  $h$  times  $N_f$  by  $N_d$ .  $N_f$  is the number of channels and  $N_d$  is the number of potential drop.

So, that was the things that I have discussed just previous lecture. Now, how to draw the flow net? It is not very easy task and it has to be as I have mentioned that while deriving that equation one thing is from the properties of the flow line and equipotential line. We have mentioned that they generally are meet at right angle to each other and also that.

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We have mentioned that we can try to make while drawing the flow net, we will try to make equally square blocks. The size will not be the equal. All block will not be equal size.

But, while each block looks like a square; that means all sides of the block approximately equal. If I do that and follow that then, one can draw the flow net and impact this is before drawing that one has to identify the initially equipotential line, initial flow line, which are those based on inspection of the. This is suppose dam structure and this the body and here actually suppose something head is 0 and here head this much then; that means, from this to this distance head is constant from this to this head also constant.

So, that mean; this can be considered one equal potential line. This also can be considered as one equal potential line. Now, waters will be at this point, will start to move along this. It is considered as this as a flow line and if there is obstruction then water can generally through this actually it will go and will go like that. That is one part. So, that can considered as a flow line and then also water from here it will move along this also that can be considered also; that means, we have fixed one flow line. This is equipotential line and this is equipotential line. These are the things.

So, now I can this is equipotential line. This is also equipotential line. Now, if I want to draw a flow line, flow line should come out right angles from here. Starting point should

be right angles and when will line will go like this and then there is obstruction, it will not go straight and come down like this. It will smoothly pass over this and will reach here.

So, that is our first trial of. This is one flow line. This may be the second flow line. If I get this second flow line, then I can proceed as I have mentioned I can find out what is head difference. From there I can find out  $\Delta h$  and if I want to draw suppose 10 equipotential line. So, I have to 9 actually there will be drop. There will be 9 drop. Equally I can divide and I can fix it. This way suppose this was 1, 2, 3 like that and there will be number of dams I can fix it.

And then accordingly I can start satisfying these right angles requirement and making square that requirement I can complete those initial and slowly I will extend this one. Suppose, next stage I will extend this one further. I can imagine another flow line from here. This was equipotential line and this is a flow line with come out right angles from here and I will extend slowly this and this. So that they will meet at right angle and same time these become rectangle square. These all become square. Like that I will draw another line like that and then it is suppose was here it is ended here and further I am imagine another flow line and I will extend this side and extend this side in such way that always it will make nearly square and along this there will be another equipotential line.

So, like that once we complete this. Now, I can count how many potential drops. There will be 1 from here. Actually totally head 1, 2, 3, 4, 5, 6, 7, 8. There will be 8 drops. So, your  $\Delta h$  will be 1, 2, 3, 4, 5, 6, 7, 8. So, it will be 8 and how many  $\Delta h$  become this  $h$  by 8. So,  $N_d$  is actually 8 and  $N_f$  will be how many? This will be 1, 2 and 3. Last one you can see it is become narrow.

So, approximately by higher estimation though it is becoming bigger and bigger when you are going away from the body, but this one need not be considered as a full flow channel. So, it can partially 3.4, 3.5 like that.  $N_f$  can be considered partly 3.5 suppose. Then from this Figure I can find out  $q$  equal to  $K$  is known,  $h$  is known there and  $N_f$  suppose 3.5 and  $N_d$  suppose is 8. So, from this equation I can find out the  $q$ . This is the procedure how to draw the flow net and this is the equation. Finally, to use for calculate the seepage. This is of course, is not a very easy job. Everyone has to do large number of trials and practice. Then only one can get this type of flow net.

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**PERMEABILITY AND SEEPAGE**

Most loose deposits are probably isotropic, i.e., the value of permeability in the horizontal is the same in the vertical direction. Practice of constructing earth embankments and dams is spreading of the soil in loose layers and then compacted. This construction technique results in a greater value of permeability in the horizontal direction than that in the vertical direction. The value in the vertical direction is usually 1/5 to 1/10 of the value in horizontal direction

The general differential equation for flow in three dimensional case was derived earlier is:

$$\left( k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} \right) = 0$$

For two-dimensional case:  $\left( k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} \right) = 0$

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So, now most loose deposits are probably isotropic; that means, if the soil is loose condition, then when flow take place, then flow all direction will be almost equal value; that means, value of permeability in horizontal is the same is in the vertical direction, but actually in the practice when there will be this type of structure is constructed, generally it will fill in layer and then it will be compacted.

When you do this then what happens if vertically getting compacted, horizontally it may not be that much. Because of that it is seen that whatever the amount of permeability is in the horizontal direction and whatever permeability is in vertical direction they are not same. They are significantly different. In fact, this is the object. The value in the vertical direction is usually 1 5 to 1 by 10; that means, of the value in horizontal direction. So, horizontal direction whatever value is.1 5th or 1 10th of that value that means, horizontal direction is always higher than the vertical.

So, this is the permeability is there; that means, they are not same,  $k_x$ ,  $k_y$  is not same. The general differential equation flow generally we have derived when  $k_x$ ,  $k_y$ ,  $k_z$  is not same this is the general equation and if it is 2 dimensional if we consider, then this is the generalised equation. So, this equation now if the  $k_x$  and  $k_y$  is not same. We will not be able to draw the flow net. When we have drawn the flow net, there we have shown that  $k_x$  equal to  $k_y$  or  $k_z$ . Because, then only we could draw or apply the law and we could draw the flow net, but if this is different, then we will not be able to draw the flow net,

but there is a solution we will be able to draw the flow net, but you have to do something what is that you have to do that we will see in next subsequently.

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**PERMEABILITY AND SEEPAGE**

Unless  $k_x$  is equal to  $k_z$  the equation is not a true Laplacian and cannot therefore be solved by a flow net described before. To utilise the method it must be modified for an equivalent homogeneous system. Steps:

$$\frac{k_x}{k_z} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \Rightarrow \quad \frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Where  $\frac{1}{x_t^2} = \frac{k_x}{k_z} \frac{1}{x^2}$  or  $x_t = x \sqrt{\frac{k_z}{k_x}}$

The slide includes a diagram showing a coordinate transformation from  $x$  to  $x_t$  and  $z$  to  $z$ , with arrows indicating the scaling of the  $x$ -axis by a factor of  $\sqrt{k_z/k_x}$ .

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Now, unless  $k_x$  as I have mentioned that unless  $k_x$  is equal to  $k_z$ . That the equation is not true Laplacian equation and therefore, be solved by a flow net described before. To utilize the method, it must be modified for an equivalent homogeneous system; that means, when  $k_x$  and  $k_y$  is different and suppose this the soil mass and the  $k_x$  value is this  $k_z$ , this is  $k_x$  and this is  $k_z$  and if they are different then I cannot apply that.

So, what we can do? We can consider an equivalent  $k$  or we can convert this geometry in such a way that automatically if I take these 2 are equal. Then also our calculation will not affect or it will not. That way actually; that means, you have to try to make or transfer the actual structure into an equivalent structure so, that considering the variability and permeability in 2 direction to do that what I can do? I can take this original equation and I have taken divide by  $k_z$ . So,  $k_x$  by  $k_z$  and  $k_z$ ,  $k_z$  get cancelled.  $k_x$  by  $k_z$ ; that means,  $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$ . I am getting here.

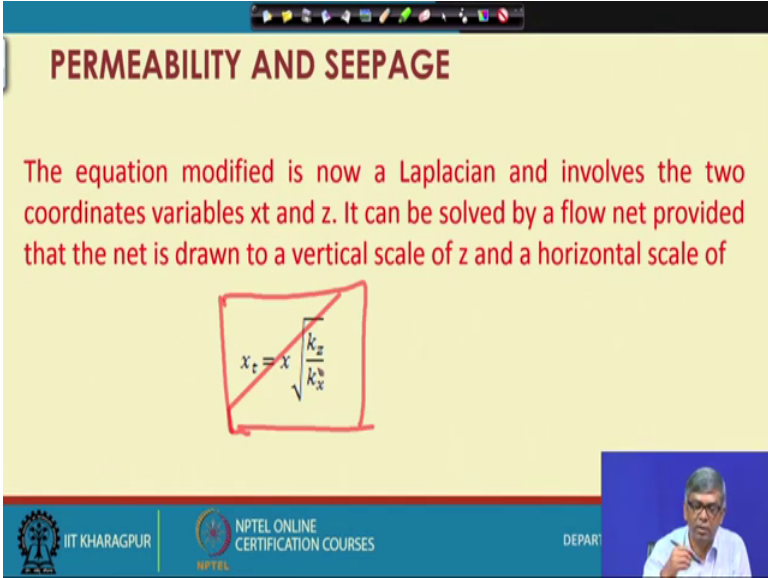
And then if this can be converted, actually  $\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$ . Now, this actually Laplace equation we have converted and your coordinate  $x$  is there actually it was  $x$  and  $z$ . Here your coordinate  $x$  is,  $x_t$  this was before  $x$  and  $z$ . Now, my equation is  $x_t$  verses  $z$ . Now what is  $x_t$ ? What ways it is different from  $x$ ? That we seen. So, that while doing this we have substituted 1 by  $x_t^2$  equal to  $k_x$

by  $k_z$  into  $1$  by  $x$ . Then only we could get that and then  $x_t$ ; that means, what is the  $x_t$  with relation with  $x$   $t$  equal to  $x$  into under root  $k_z$  by  $k_x$ .

So, this is the one; that means, I can keep vertical direction unchanged, but horizontal dimension  $x_t$ , I can convert into  $x$ . How to do that? The value in corresponding using the corresponding value of permeability into direction and that way I can convert see if once I convert then what will happen on that system become homogeneous in terms of  $x_t$  and  $z$ .

And then I can draw that modified or transform section I can draw the flow net as described before; that means, assuming that it is homogeneous and can satisfy those condition that flow line equipotential line meeting at right angles and also while doing free flow net, we will try to make a square blocks; that means, in each block should side and width length and breadth will be equal and if do that, then again going back to original section or I can show that the same equation is valid with little modification. That we will see one by one.

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**PERMEABILITY AND SEEPAGE**

The equation modified is now a Laplacian and involves the two coordinates variables  $x_t$  and  $z$ . It can be solved by a flow net provided that the net is drawn to a vertical scale of  $z$  and a horizontal scale of

$$x_t = x \sqrt{\frac{k_z}{k_x}}$$

The diagram shows a square element of a flow net with a diagonal line representing a flow line or equipotential line. The horizontal axis is labeled  $x_t$  and the vertical axis is labeled  $z$ . The equation  $x_t = x \sqrt{\frac{k_z}{k_x}}$  is written inside the square.

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So, now the equation modified is now Laplacian and involves the 2 coordinates variables. One is  $x_t$  and  $z$ . Originally it was  $x$  and  $z$ . Now it become  $x_t$  and  $z$ . It can be solved by a flow net provided that the net is drawn to a vertical scale of  $z$  and a horizontal scale of  $x_t$  equal to this. So, this is the change actually. The horizontal  $k$  scale

has to be change  $x$  to equal to  $x$  into under root  $k_z$  by  $k_x$ . If I do that, then how that equation can be utilized or used that I have to see now.

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**PERMEABILITY AND SEEPAGE**

Seepage quantity in an anisotropic soil

Head loss =  $\Delta h$

(a) Transformed

(b) Natural

This is exactly as before:

$$q = kh \frac{N_f}{N_d}$$

The only problem is what value to use for  $k$

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I have taken now the 2 sections. This was the natural section. Suppose  $x$  equal to. This is  $x$  actually. This is the  $x$  and maybe some relation it is there and this is  $y$  direction is  $a$ . And this transformed  $a$  in this form and in this transformed when we were transforming this the same equation is valid; that means, this is exactly as before; that means, I can use this equation  $q$  equal to  $k$  into  $h$  into  $N_f$  by  $N_d$ . Then once you draw the transform section below the structure.

If I draw the flow net, I can count number of flow channel, number of equipotential drop and  $h$  is already know and  $k$  is known. Then I can find out the flow, but doing that how to transform this? The only problem is what value to use for  $k$ ? That  $k$  value, what value of  $k$  to be used? When I will do this? When I am converted transformed the section, the  $k$  will not be the same  $k$  should be different. Because, it  $k$  was originally different in 2 different direction. So, to do that, this case can be used.

You can see this was the original and this is transform original was  $a$  times under root  $k_x$  by  $k_z$  and transform is  $a$  because we have shown previously relation between  $x$  to and  $x$ . This is to prove only. This value can be anything and we can scale to this one in any value, but to prove I have taken this way. So, that if I transform this way, then your equation can be shown properly that how this is valid.

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**PERMEABILITY AND SEEPAGE**

Using the transformed scale a square flow net is drawn and  $N_f$  and  $N_d$  are obtained. If we consider a square in the transformed flow net it will be as shown in Fig a. The same figure drawn to natural scales (i.e., scale  $x$  = scale  $z$ ), will appear as shown in Fig b. Let  $k_e$  be the effective permeability for the anisotropic condition.

From Fig a:  $q = a k_e \frac{\Delta h}{a} = k_e \Delta h$

From Fig b:  $q = a k_x \frac{\Delta h}{a \sqrt{\frac{k_x}{k_z}}} = \sqrt{k_x k_z} \Delta h$

$\Rightarrow k_e = \sqrt{k_x k_z}$

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Now, we can see that in the next slide. We use the transformed scale. A square flow net is drawn. That is a part. That is Figure will have both a and b. And a side is a transform one. You can see this one is the transformed one and this is the original one. And then I am going to this drawn and  $N_f$  and  $N_d$  are obtained. If we consider a square in the transformed flow net, it will be as shown in Figure a, which I have shown already the same Figure drawn to natural scales. That means:  $x$  and  $z$  that will look like Figure b; that means, if you draw, suppose natural scale looks like this and when transformed it looks like this.

Then let  $k_e$  be the effective permeability or equivalent permeability for the anisotropic condition. When there is anisotropic condition what is the value? That is actually suppose  $k_e$ . One value  $k_x$  and  $k_z$  is there, but my intention is to use one value only. Then that is suppose one value, which will be used. Suppose, that is  $k_e$  then from Figure a, I can write this equation. That it is since the transform section. So,  $a$  into  $k_e$  into  $\Delta h$  by  $a$ .  $a k$  and this is  $i$ .  $\Delta h$  by  $i$ . That means, over a distance  $a$ , the head loss is  $\Delta h$ . This is also  $a$ , that is actually area if I take unit height and. This represents a area and this represents actually  $i$  and this represents  $k$ .

So, this one if I do, that is a get cancelled. So, that  $k_e$  into  $\Delta h$  become the  $q$  and if I consider the Figure b, that means original one. Then your  $q$  become a times that is again same because, your flow taking place through height. Actually  $a$  is same and your  $k_x$  is



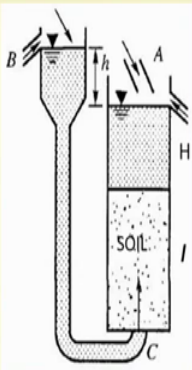
the permeability and your  $\Delta h$  is the head loss and your distance over which it is head loss is taking place that is  $a \times k_x \times b \times k_z$ . In the Figure b, we can see in the Figure a  $a \times k_x \times b \times k_z$ . This is the distance  $x$ . If I do that,  $a \times k_x \times \Delta h$  by this. Now we can see once again. This  $a$  and  $a$  get cancelled.  $\Delta h$  is there and this  $k_z$  will come here and this  $k_x$  and  $k_x$  will become under root  $k_x$ . That means, the under root  $k_x \times k_z \Delta h$ .

So, once I am getting  $q$  equal to under root  $k_x \times k_z$  into  $\Delta h$ . Once I am getting original transform section which I have represented by equivalent or effective permeability  $k_e$ . Then that is I am getting  $k_e$  into  $\Delta h$ . So, these 2 must be equal. Because, I am satisfying certain condition I have chosen this  $k_e$ . This 2 flow must be equal and if these 2 flows are equal.


Since,  $\Delta h$  and  $\Delta h$  are both case are same.  $k$  must be equal to under root  $k_x \times k_z$ . So; that means, whatever I have shown previously that equation, when this equation again will be valid. When you have transformed a section, same equation is valid. Only one equation will be where that  $k$  will replaced by  $k_e$  which is equal to under root  $k_x \times k_z$ . So, that is only the difference from that one.

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### PERMEABILITY AND SEEPAGE



Critical Hydraulic Gradient: The total head of water above the base =  $h+H+l$ , and the head of water in the sample above the base =  $H+l$ , therefore the excess hydrostatic pressure acting on the base of the sample =  $\gamma_w h$



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Now, if this is another thing I have to choose here. After actually effective stress I will do more elaborately, but here I just briefly I will try to. Since, Hydraulic Gradient is causing the flow. There is a term called Critical hydraulic. What is Critical hydraulic Gradient? If it is below beyond Critical hydraulic Gradient, there is something happened in the

system and. So, because of that our Critical hydraulic Gradient has to be less than that. What is Critical Hydraulic Gradient? That will examine and how to find out that? What is the expression? We will try to see through this explanation.

So, this is actually soil sample and this is connected to this tank suppose and water is here and this is again this level there is opening and water is collected from here and if water level is here and it is connected, then this water will try to flow through this soil upward direction and since this is the maximum level water can retain. this of the water can go out. So; that means, when this flow is taking place, what is the head under which the flow is taking place? This is the  $h$ . Because, this the maximum head here and maximum head here. So, because of this difference at this and this, the flow will take place.

And here actually above sample the soil, there may be height  $h$  capital  $H$  and soil height is length is  $l$  suppose and this is  $h$ . So, then the total head of water above the base; that means, at this point is  $h$  plus  $h$  plus  $h$  plus  $l$ . So, these 3 together is the total head acting below this.

Now and the head water in the sample above the base and from this side if I consider from side, that is the one and if I consider this one from this side, how much head is acting here from this side? That will be nothing, but  $H$  plus  $l$ .  $H$  and plus  $l$ . Because of some weight acting this direction and because of the water head from at from this bottom, that 2 will be one is upward direction another is bottom downward direction that they will try to balance. How long it will be balanced and when it will be unbalanced that to be obtained.

So, therefore, the excess hydrostatic pressure acting on the base of the sample will be  $\gamma_w$  times  $H$ . If I subtract from here to here, then excess hydrostatic pressure, this minus. So, this  $H$  gets cancelled.  $l$  get cancelled. Only small  $h$  is there and that is small  $h$  into  $\gamma_w$ . This much excess hydrostatic pressure is additionally added from the bottom acting. If it happens, then what is the problem?

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**PERMEABILITY AND SEEPAGE**




If any friction between the soil and the side of the container is ignored, then the soil is on the point of being washed out when the downward forces equals the upward forces:

Downward forces = Buoyant unit weight x volume =  $\gamma_w \frac{G_s - 1}{1 + e} Al$

Upward forces =  $h\gamma_w A$

$h\gamma_w A = \gamma_w \frac{G_s - 1}{1 + e} Al \Rightarrow i = \frac{h}{l} = \frac{G_s - 1}{1 + e}$

$\gamma = \frac{G_s - 1}{1 + e} \gamma_w$

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Problem is here. If any friction between the soil and the side of the container is ignored; that means, what is this? So, there is no friction at this wall. Only if there is a friction, then when are you considering equilibrium flow, I may have to do equilibrium frictional force, but here since is the quantity and force everything is small almost static. We assume that because of this opening through the voids, water is flowing without causing much force in this entire mass. If I consider that, then I can ignore the friction.

So, if I ignore that, then what happens? Then the soil is on the point of being washed out when the downward forces equals the upward force. Upward force is how much? And downward force is how much? Downward force will be equal to buoyant unit weight into volume. Buoyant unit weight we have derived before. Submerged unit weight we have derived before.  $\gamma_b$  will be equal to  $G_s$  minus 1 by 1 plus  $e$   $\gamma_w$ . This is the thing we have derived before and buoyant unit weight multiplied by volume. That  $A$  is the cross section area of the soil and  $l$  is the length, then this volume that will become the total weight coming from the upward direction additional water and an upward force which will be equal to  $h \gamma_w$  into to  $A$ .

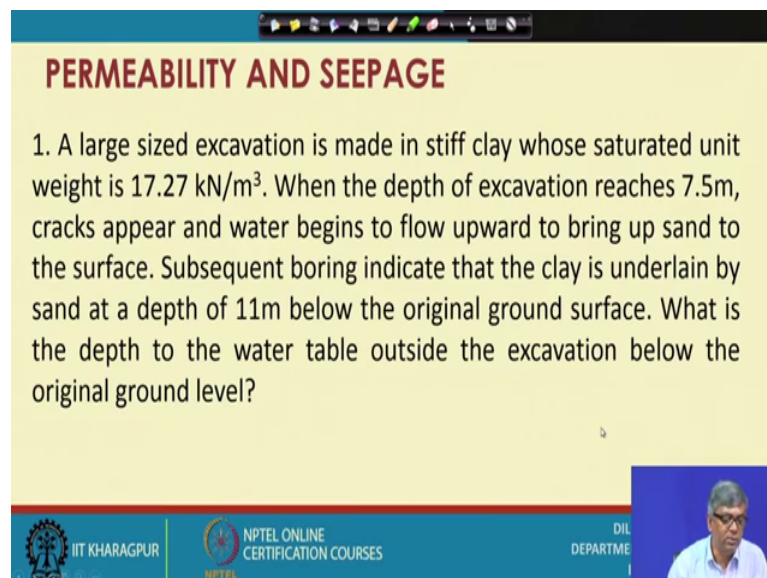
So, that is the excess hydrostatic pressure in the downward direction because of the difference in head is acting. That is additional. So, this and this must be equal. If you want to make these two  $A$ , and  $A$  get cancelled and then ultimately we can see that  $h$  by  $l$  which is nothing, but will be equal to  $G$  minus 1 by 1 plus  $e$ . This is the expression; that

means, Hydraulic Gradient we are getting  $G$  minus 1 by. So, this is the Critical. Why it is Critical? Because, we have just considered the upward forces and downward force, it just equal so, that satisfying that condition. Whatever Critical, the Hydraulic Gradient we have got that is the Critical Hydraulic Gradient and why it is Critical?

Because, if I increase the head this side, then hydrostatic pressure will increase. Suppose  $h$  to  $h_1$ , that will increase the pressure and if that pressure become more and more. There will be sometime because of this excess pressure the soil will be washed out inside. So, that is the void that is the reason we have obtained this Critical Hydraulic Gradient. If the hydraulic Gradient become more than that, then your soil will be washed out; that means, will move upward direction.

So, that is what this is very important relationship Hydraulic Gradient with specific gravity and void ratio. Once you know the possibility of void ratio of soil, then one can find out the Critical Hydraulic Gradient. That is the guidelines which has to be your Hydraulic Gradient has to be less than this. So, if you do that, then that has to be satisfied. Now, what I will do? I will try to take 1 or 2 problem here to apply whatever concept we have got and some application one by one.

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**PERMEABILITY AND SEEPAGE**

1. A large sized excavation is made in stiff clay whose saturated unit weight is  $17.27 \text{ kN/m}^3$ . When the depth of excavation reaches 7.5m, cracks appear and water begins to flow upward to bring up sand to the surface. Subsequent boring indicate that the clay is underlain by sand at a depth of 11m below the original ground surface. What is the depth to the water table outside the excavation below the original ground level?

The slide is a presentation slide from NPTEL. It has a yellow background with a blue header and footer. The title 'PERMEABILITY AND SEEPAGE' is in red. The problem statement is in black text. In the bottom right corner, there is a small video inset showing a man in a blue shirt. The footer contains the IIT Kharagpur logo, the NPTEL logo, and the text 'NPTEL ONLINE CERTIFICATION COURSES' and 'DIL DEPARTME'.

So, this is one application a large sized excavation is made in stiff clay.