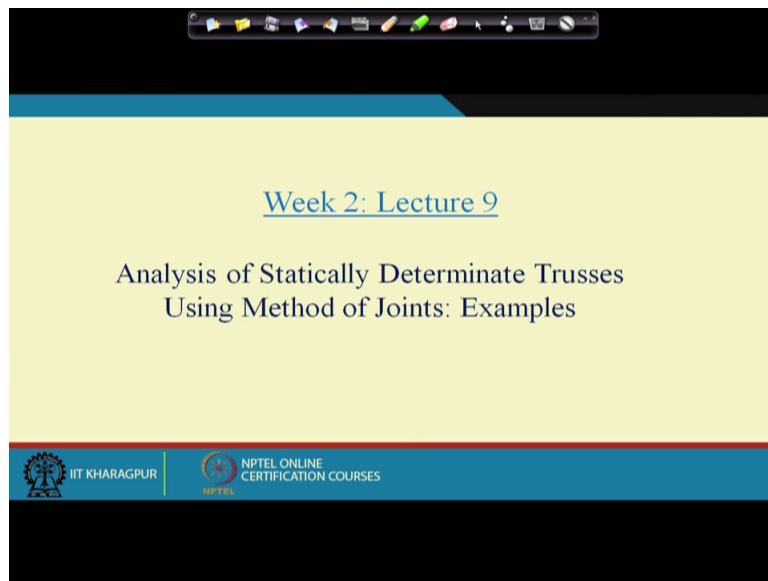


Structural Analysis 1
Professor Amit Shaw
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Lecture 9
Analysis of Truss - Method of Joints (Contd.)

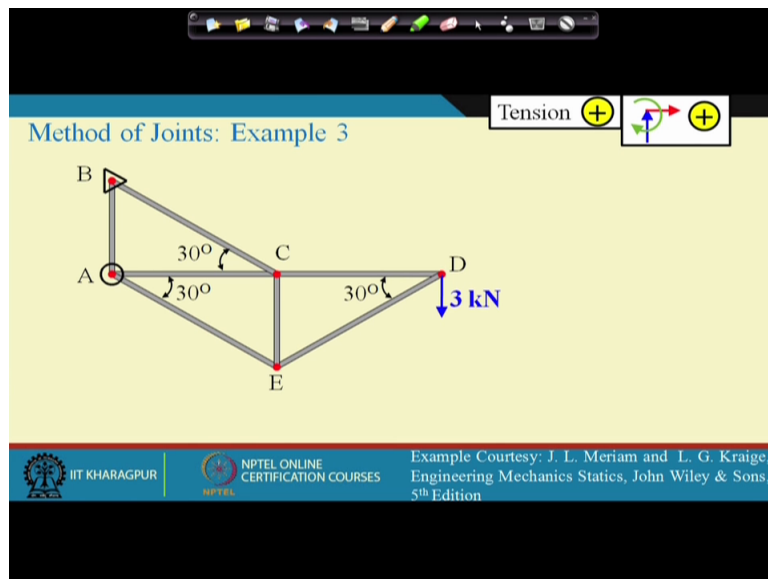
Hello. So what we have been doing since last class is, we are trying to see some examples of analysis of statically determinate truss using method of joints. Let us continue with that.

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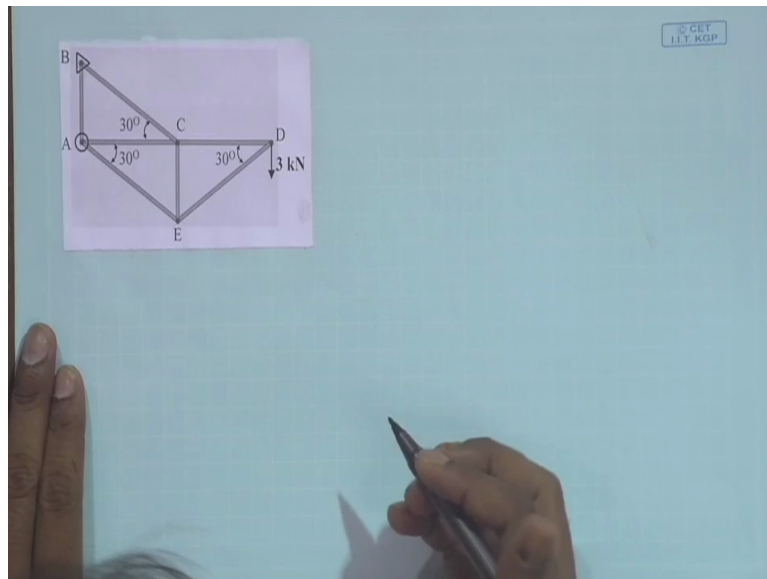
Now next take this example.

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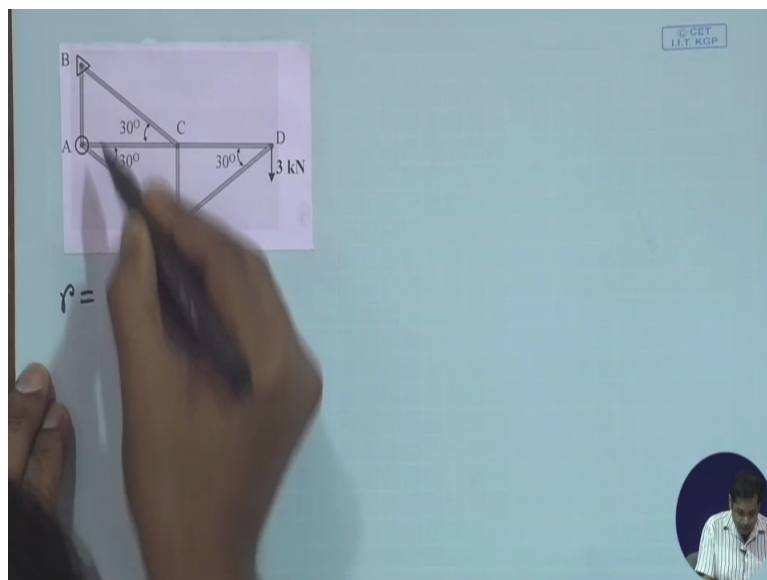
So we are going to solve this truss.

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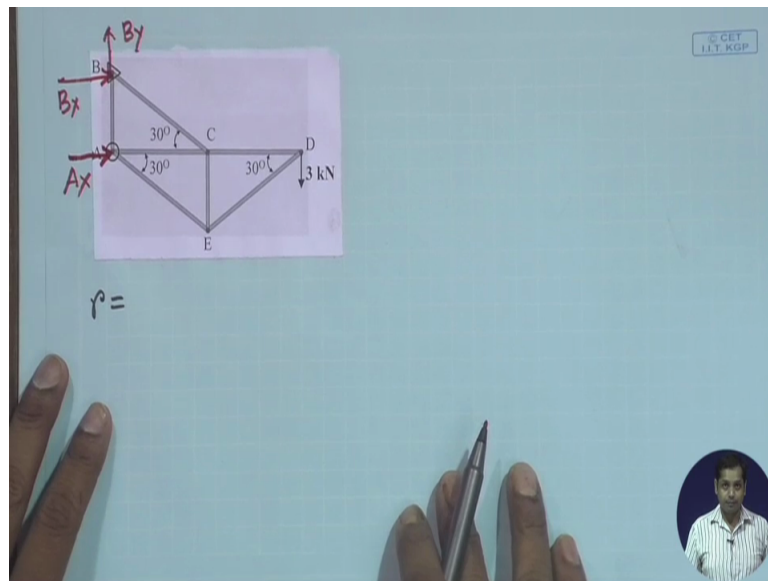
Now, first is, we need to determine the support reactions. So first let us see whether it is statically determinate truss or not. Number of reactions are, we have is 3, here 2.

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Let us draw the free body diagram of the entire structure first. So this is roller support we have. This is A_x and then this is B_x and B_y .

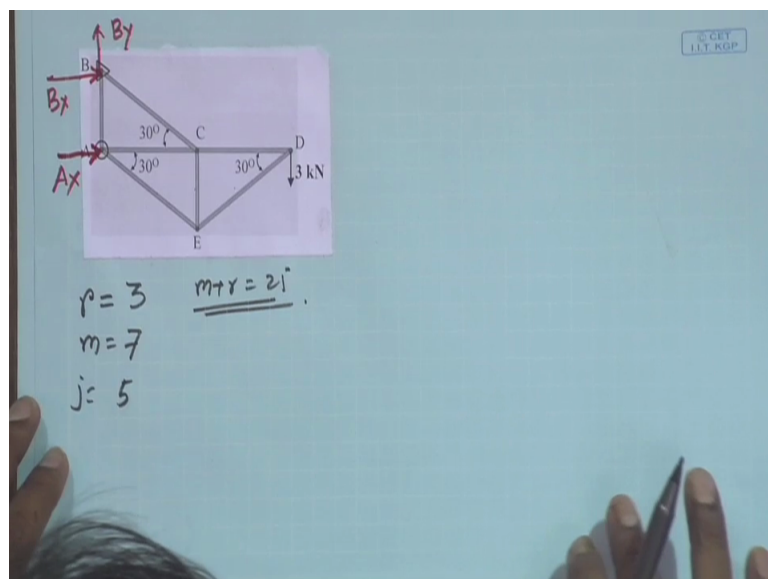
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Again last class also I mentioned. Just to save the time I am drawing the free body diagram of the entire structure and showing this reactions on this diagram itself. But free body means it has to be free from the support, right? So when you are (repl) replacing support by forces, don't show support and the reaction together in the same diagram. Okay, now so number of reactions are 3 and the number of members are 1, 2, 3, 4, 5, 6, 7. Number of members are 7.

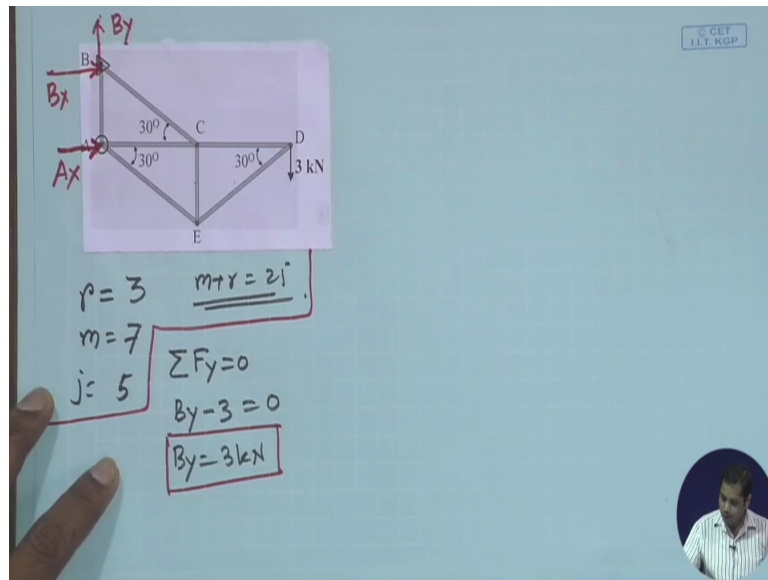
And number of joints J is equal to 1, 2, 3, 4, 5 joints. 5 joints. So $m + r$ is equal to $2j$. So this is statically determinate truss.

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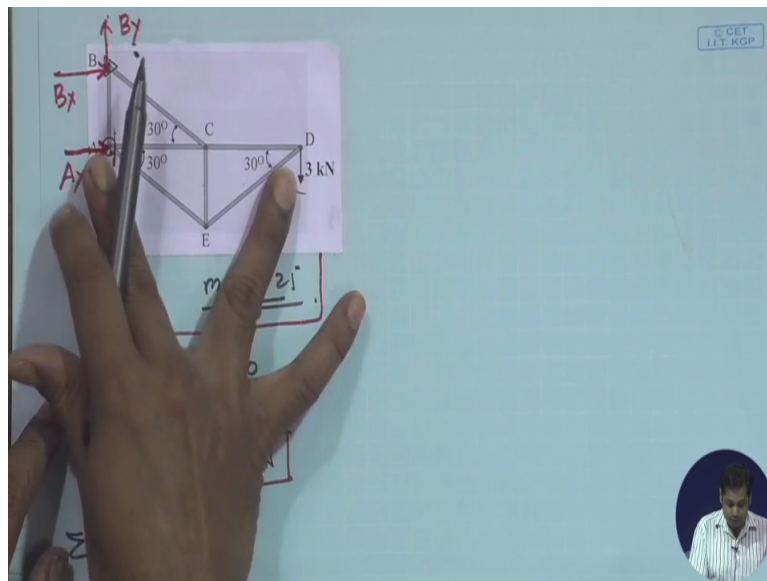
Now first let us find out the support reactions. Now if we take $\sum F_y = 0$. Let's just, this is okay. On this free body diagram, if we apply the equilibrium equation, first is summation of F_y is equal to zero. This directly gives that B_y is equal to what are the forces you have in Y direction? We have B_y and then downward 3 kilo Newton force. So $B_y - 3 = 0$. So B_y is equal to 3 kilo Newton. So this is okay.

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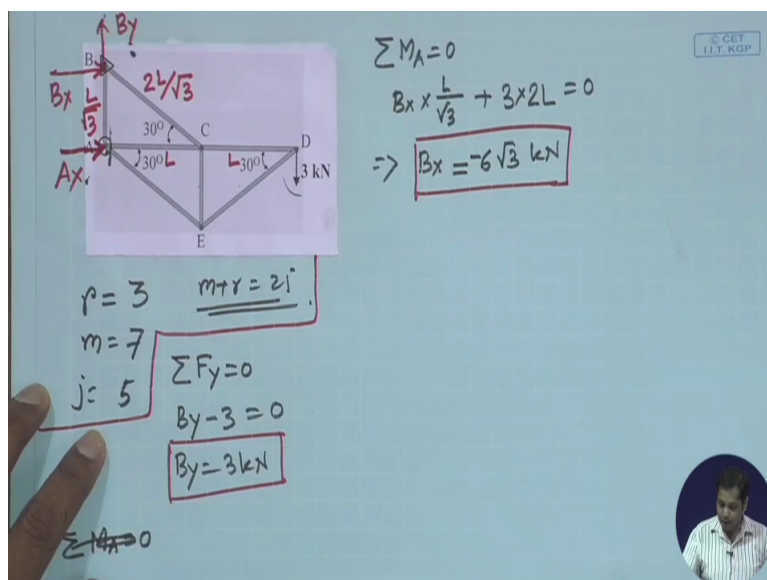
Now if I take moment about A is equal to zero. Say summation of moment about A is equal to zero. Now what are the forces we will contribute to this moment? A_x will not contribute, B_y will not contribute because the point A lies on the line of action of the B_y . B_x will contribute and this 3 kilo Newton load will contribute. Now the forces in B_x will be this distance into B_x . And then clockwise couple by this force is 3 into this distance.

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Okay, let us first write the distances. These distances are L, these are all L. These angles are 30 degree. So this is $2L$ by $\sqrt{3}$ and this is L by $\sqrt{3}$. Now summation of M_A is equal to zero. It means let us write it here. Summation of M_A is equal to zero. What it gives is B_x . B_x into L by $\sqrt{3}$. This is positive because it is clockwise. Plus 3 into $2L$, that is equal to zero and this gives B_x is equal to minus 6 . This equal to minus 6 root 3 kilo Newton. So this is B_x , right?

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Now if I take summation of F_x is equal to zero. Then horizontal component are A_x and B_x . This gives us A_x plus B_x is equal to zero. You could have done like this. You could have showed A_x in this direction and B_x in opposite direction. In that case it will be A_x minus B_x

is equal to zero. Plus try to avoid that because when you draw the free body diagram, whatever sign convention you have fixed for your free body diagram, you please do that. Be consistent with that. This is equal to zero.

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$\sum M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$

$\sum F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\sum F_x = 0$
 $A_x + B_x = 0$

So this directly gives that A_x is equal to 6 root 3 kilo Newton. So this is the support reaction. So we have obtained the support reaction, right?

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$\sum M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$

$\sum F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

Now let us determine the member forces. Now let us identify which joint we shall take. We cannot start with joint E, because joint E has 3 members. So 3 unknowns. Equations will be only 2. We cannot start with joint C because C has 4 unknown, 4 members. So we have only

2 equations. We cannot start with joint A as well because A has 3 unknowns. And equations are only available 2. So either we can start with joint B or we can start with joint D. In both the cases number of unknowns are 2.

(Refer Slide Time 07:06)

$\sum M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$

$\sum F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$p = 3$ $m + r = 2j$
 $m = 7$
 $j = 5$

So let us start with joint D. Now what is the free body diagram of joint D? So this is your support reaction, right? Let us draw the free body diagram of joint D. So this force is F_{CD} and this force is F_{DE} . And then another force 3 kilo Newton. This is D and this is F_{BD} of D .

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$\sum M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$

$\sum F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

$p = 3$ $m + r = 2j$
 $m = 7$
 $j = 5$

$\sum F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

FBD of D:
 F_{CD} (horizontal force to the left)
 F_{DE} (diagonal force down and to the left)
 3 kN (vertical force downwards)

Now take summation of forces in X direction is equal to zero. Or take summation of forces in Y direction is equal to zero first. Then it gives Y direction component as 3 kilo Newton and component of Fde, this angle is 30 degree.

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$\sum M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\sum F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$
 $\sum F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\sum F_x = 0$
 $F_{CD} - F_{DE} \cos 30^\circ = 0$
 $\sum F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

So what we have is Fde which is the vertical component. It is downward that's why I put negative sign here. And then minus 3 is equal to zero. 3 minus because it is acting downward. So this gives me Fde is equal to minus 6 kilo Newton. So Fde is obtained, right?

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$\sum M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\sum F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$
 $\sum F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\sum F_x = 0$
 $F_{CD} - F_{DE} \cos 30^\circ = 0$
 $\sum F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

Now in the same equation if we take summation of same free body diagram, take summation of F_x is equal to zero. Then what we have? We have F_{cd} minus $F_{de} \cos 30$ is equal to zero, right?. This is again negative.

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$\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\Sigma F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

$\Sigma F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\Sigma F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$

Both are negative because as per our sign convention, both are in this direction and the component of this is also in this direction. Per our sign convention is this direction is positive.

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$\Sigma M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$

$\Sigma F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

$\Sigma F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\Sigma F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$

Now F_{de} is already obtained 6 kilo Newton. And from this equation we can get F_{cd} is equal to uh $3\sqrt{3}$ kilo Newton. So F_{cd} is determined. So this is determined, right?

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$B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\Sigma F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

$m+r = 2i$
 $\Sigma F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$
 $F_{CD} = 3\sqrt{3} \text{ kN}$

$\Sigma F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

Now let's see what joint. We have already obtained the member forces in these 2 member forces. Let us see now which joint to take next. Joint C still has three unknowns. It has originally 4 unknowns. But one is already computed. So still it has three unknowns. So we cannot take next joint C. Joint E, originally it has three unknown. But now this is known to us. Member forces in member ED.

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$\Sigma M_A = 0$
 $B_x \times \frac{L}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\Sigma F_x = 0$
 $A_x + B_x = 0$
 $A_x = 6\sqrt{3} \text{ kN}$

$r = 3$
 $m = 7$
 $j = 5$
 $m+r = 2i$
 $\Sigma F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$
 $F_{CD} = 3\sqrt{3} \text{ kN}$

$\Sigma F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

Now only unknown it has is these 2. Member forces in this and member is this. So let us draw free body diagram of joint E. Now joint E will be the free body diagram Fbd of E, right? Now Fbd of E will be this, this and vertical component like this. All these angles are 60 degrees these angle are 60 degrees angles are 60 degree. This is F_{ED} , F_{AE} and then F_{CE} .

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$B_x \times \frac{1}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\Sigma F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

$r = 3$ $m + r = 2i$
 $m = 7$
 $j = 5$
 $\Sigma F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$
 $F_{CD} = 3\sqrt{3} \text{ kN}$

FBD of E:
 F_{AE} (up-left), F_{FE} (up), F_{ED} (up-right)

Now if we take summation of F_x is equal to zero, it gives us $F_{AE} \sin 60$ minus, this is plus $F_{ED} \sin 60$ is equal to zero. And this gives me F_{AE} is equal to F_{ED} . So forces in this member and forces in this member are same. Now this member already we have determined F_{ED} . F_{ED} is already determined minus 6 kilo Newton. So this we can write F_{AE} is equal to minus 6 kilo Newton.

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$B_x \times \frac{1}{\sqrt{3}} + 3 \times 2L = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\Sigma F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$

$r = 3$ $m + r = 2i$
 $m = 7$
 $j = 5$
 $\Sigma F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$

$\Sigma F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$
 $F_{CD} = 3\sqrt{3} \text{ kN}$

$\Sigma F_x = 0$
 $-F_{AE} \sin 60^\circ + F_{ED} \sin 60^\circ = 0$
 $\Rightarrow F_{AE} = F_{ED} \Rightarrow F_{AE} = -6 \text{ kN}$

FBD of E:
 F_{AE} (up-left), F_{FE} (up), F_{ED} (up-right)

Now next what we have to take? Next we have to take the forces in vertical component. So vertical component will be component of this and this and the force in members C itself.

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$r = 3$
 $m = 7$
 $j = 5$
 $m + r = 2j$
 $\sum F_x = 0$
 $A_x + B_x = 0$
 $\Rightarrow A_x = 6\sqrt{3} \text{ kN}$
 $\sum F_y = 0$
 $B_y - 3 = 0$
 $B_y = 3 \text{ kN}$
 $\sum M_A = 0$
 $\Rightarrow B_x = -6\sqrt{3} \text{ kN}$
 $\sum F_y = 0$
 $-F_{DE} \sin 30^\circ - 3 = 0$
 $F_{DE} = -6 \text{ kN}$
 $\sum F_x = 0$
 $-F_{CD} - F_{DE} \cos 30^\circ = 0$
 $F_{CD} = 3\sqrt{3} \text{ kN}$
 $\sum F_y = 0$
 $-F_{AE} \sin 60^\circ + F_{CE} \sin 60^\circ = 0$
 $\Rightarrow F_{AE} = F_{CE} = 6 \text{ kN}$

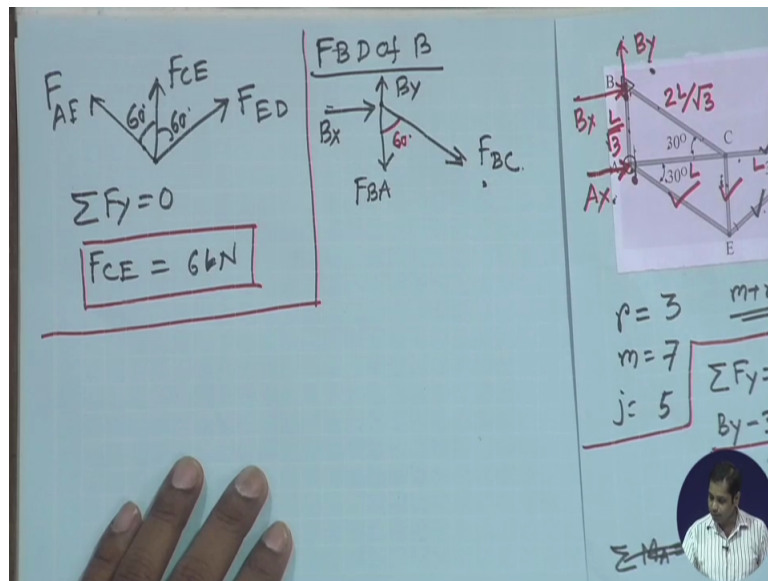
So, let us once again draw the free body diagram of this. It was F_{ed} and it was F_{ae} and the force F_{ce} . This is 60 degree, this is 60 degree. If you take summation of Y is equal to zero, then we get F_{ce} is equal to 6 kilo Newton. I am just writing the final result because it is just some algebraic operation. So F_{ce} is equal to zero.

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$\sum F_y = 0$
 $F_{CE} = 6 \text{ kN}$

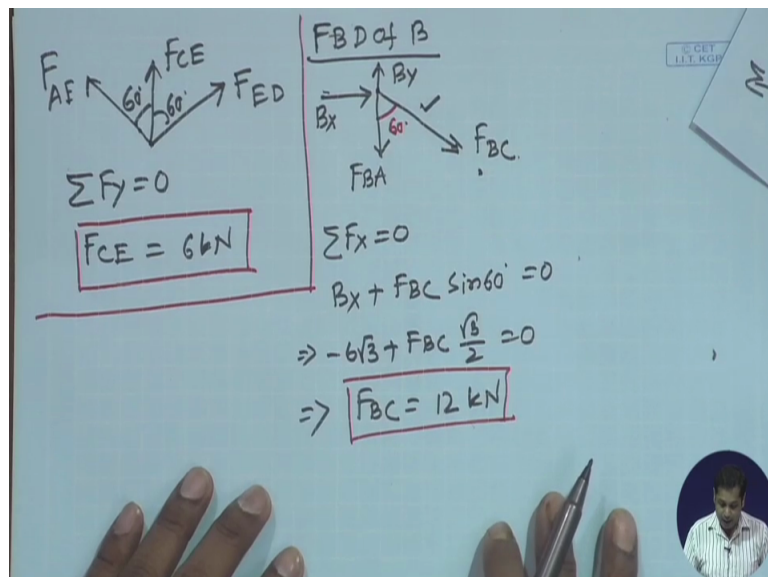
Now in this case we have determined this forces, right? Now next what we can do? Next we can take free body diagram of joint B to determine this force and this force. Free body diagram of joint B if I draw, F_{bd} of B. Then the free body diagram will be, this is and then this, horizontal reaction B_x and vertical reaction B_y . And this will be F_{ba} and F_{bc} and this angle is 60 degree.

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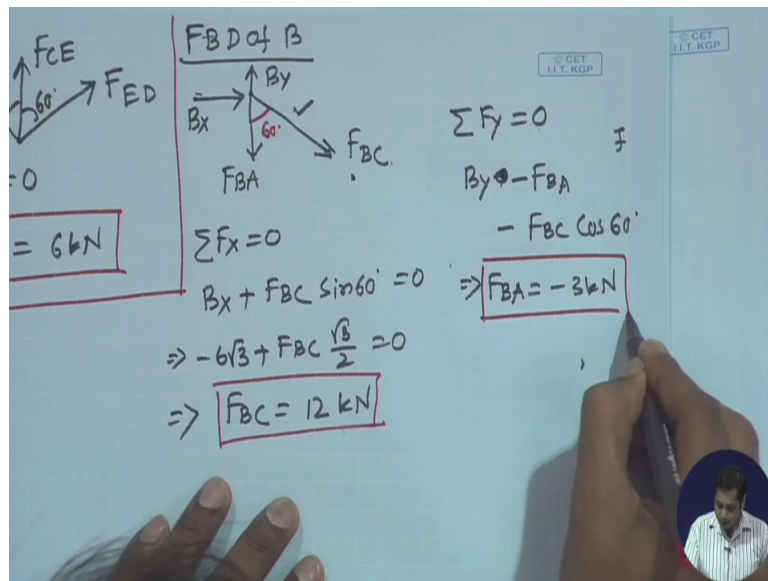
Now if I take summation of F_x is equal to zero, then it gives me, what are the forces we have? B_x is positive. Then plus $F_{bc} \sin 60$ is equal to zero. B_x already we obtained. B_x is equal to minus $6\sqrt{3}$. This is support reaction. And so if I minus $6\sqrt{3}$ plus $F_{bc} \sqrt{3}$ by 2. This gives me F_{bc} is equal to 12 kilo Newton. So F_{bc} is determined.

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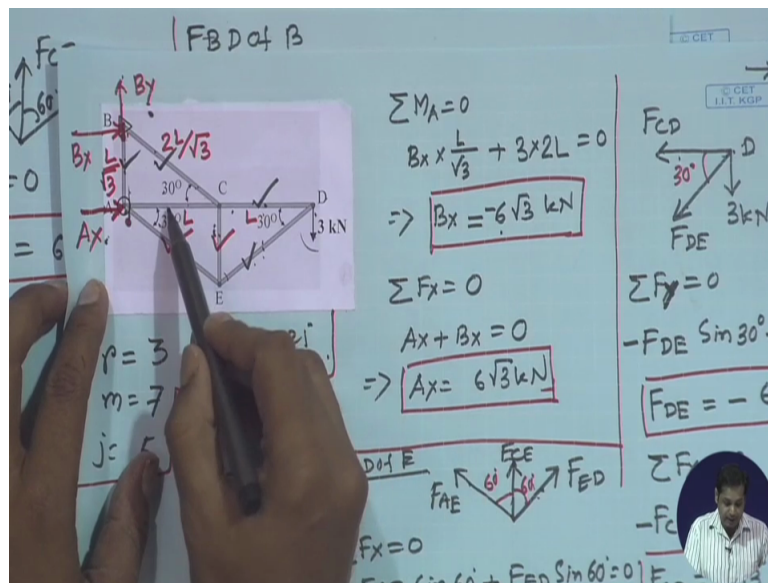
Let's calculate F_{ab} . F_{ab} we can calculate by applying summation of F_y is equal to zero, right? If I take F_y is equal to zero the equations are B_y minus F_{ba} and then minus component of $F_{bc} \cos 60$. And then if I substitute all this. B_c is known, B_y is known. Only unknown is B_a and from this if I substitute all the values, B_a we get is minus 3 kilo Newton.

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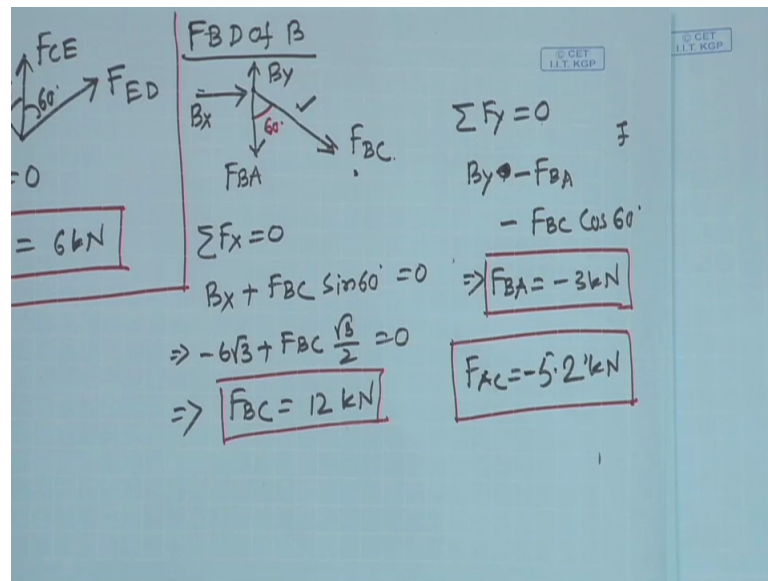
You see now this is also known. This is now known, this is known. Only thing left is this member.

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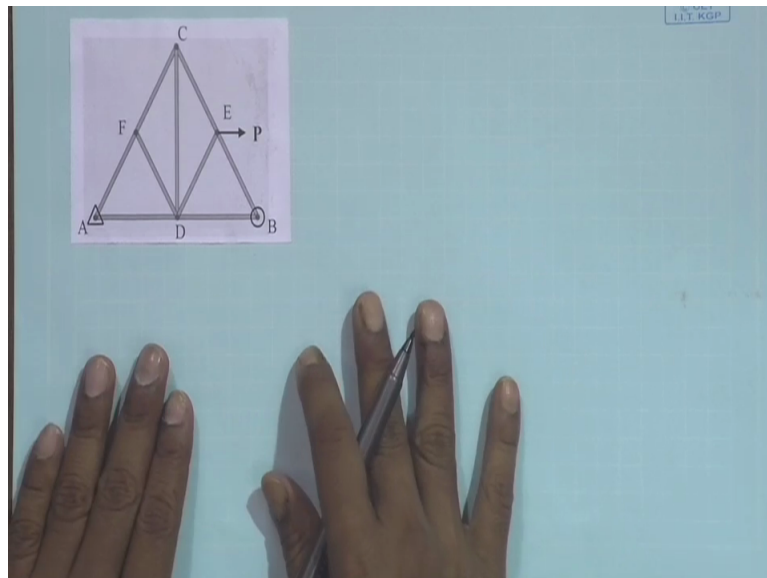
Similarly you can draw the free body diagram of this joint and find out the forces in this member.. The forces in this member F_{ac} will be 5.2 kilo Newton minus. So this is method of joints.

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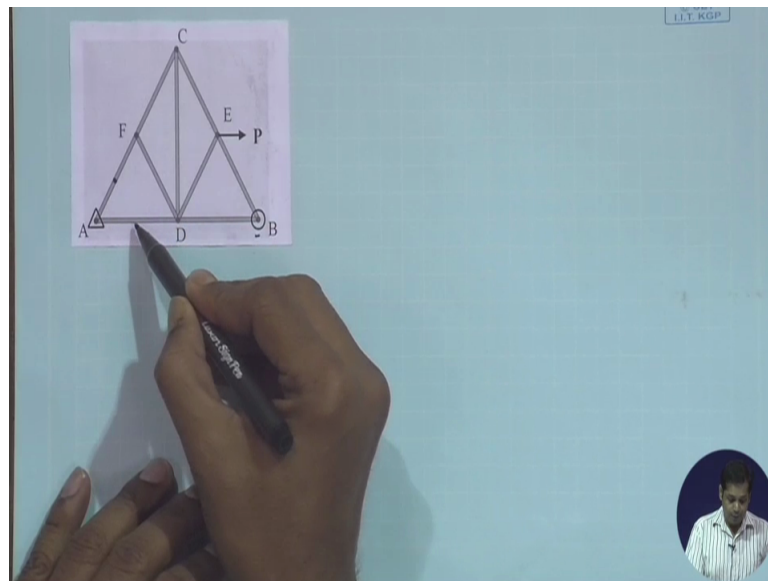
So now similarly once you have all the member forces you can draw the member force diagram of the truss. Just quickly one more example. There is a specific purpose of this example. You see here what we need to determine is, we need to determine the member force in member CD, right? Now this is the problem.

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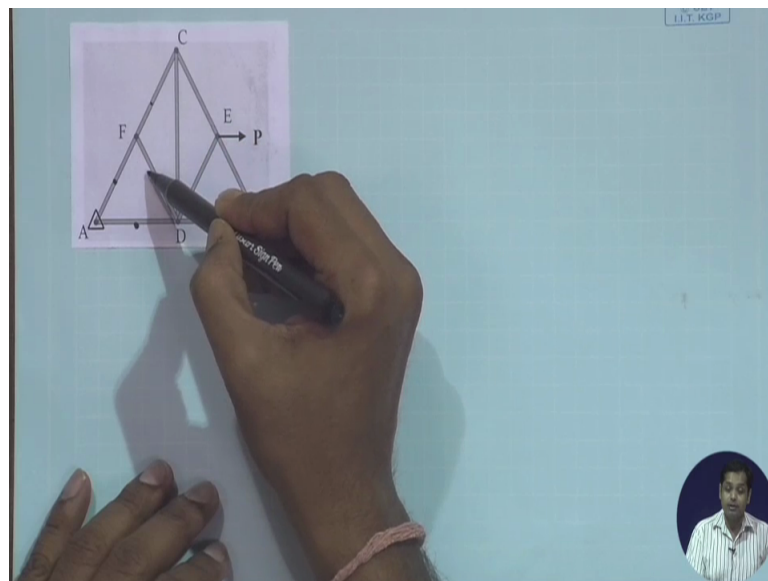
Suppose if I, because you are discussing method of joints today, suppose if I have to find out the force in member CD using method of joints. Now what we have to do is, first we need to find out the support reaction. Once we know this support reaction, we can take this joint and determine the force in this member and force in this member.

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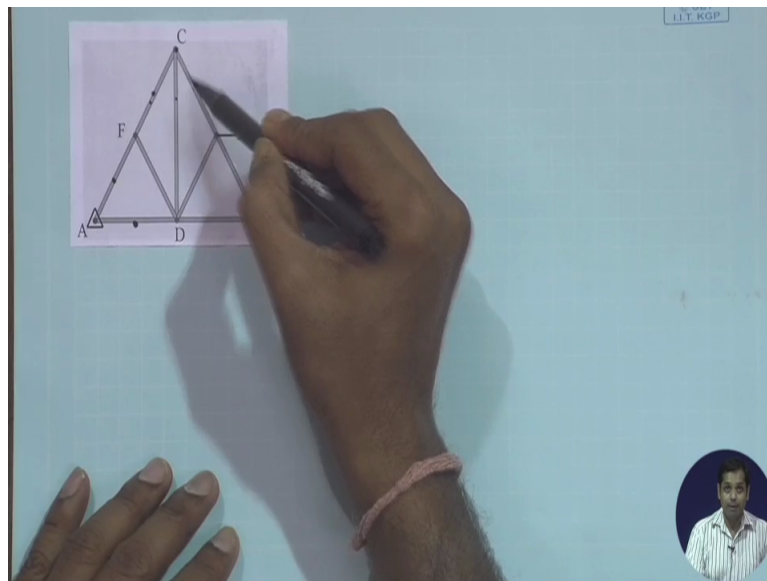
Once we know the force in this member, we need to take this joint to determine force in this member and force in this member.

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Once we know the force in this member, we can take this joint to determine force in this member and force in this member.

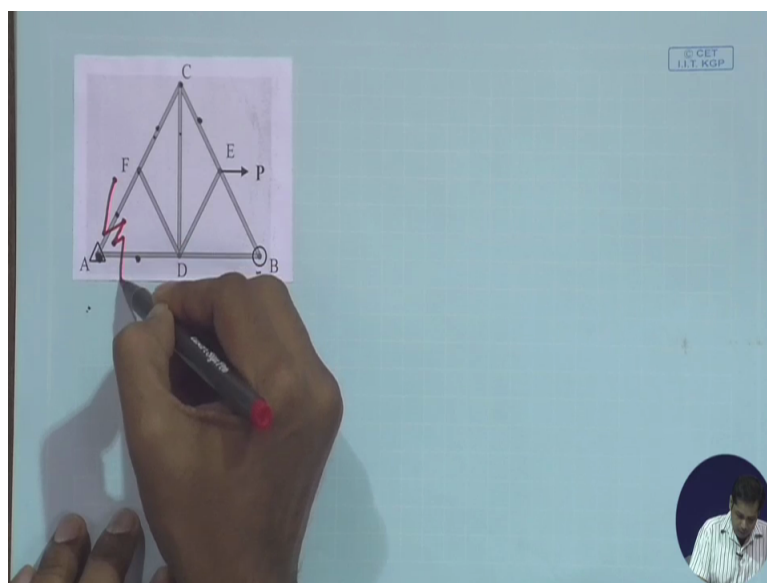
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If you go this way, this way is also similar thing we need to do. So in order to get the member force in CD, what are the operations we need to do? We need to determine the support reactions, then free body diagram, one joint, apply equilibrium equation, second joint, apply equilibrium equation and third joint, apply equilibrium equation. Similarly here also we need to take 2 or 3 free body diagram to find out member force in CD.

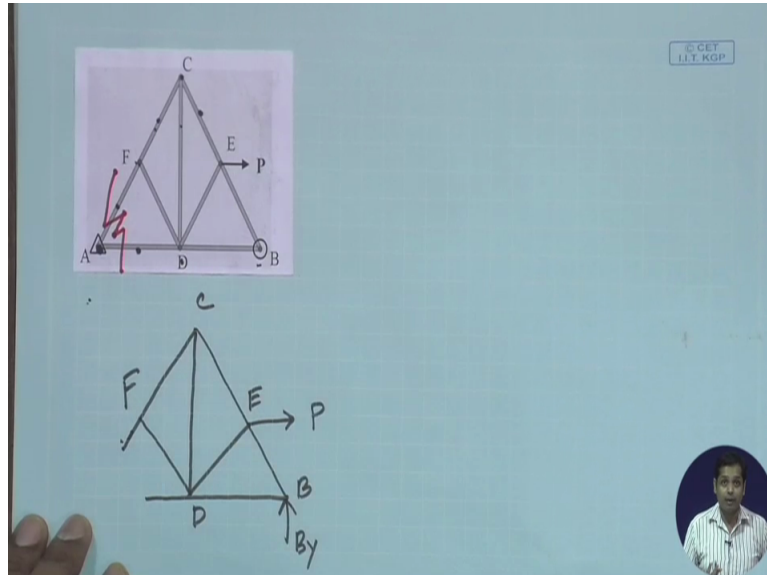
Now is there any other way we can determine the forces in this members. Let us see how we can avoid using so many free body diagram of joints to determine the force in member CD. First now if I break the structure, suppose for instance, suppose if I break this structure here.

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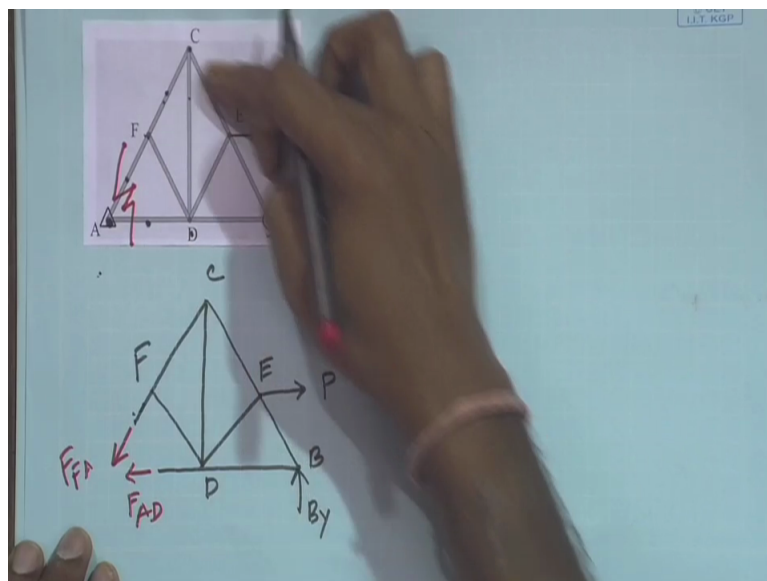
So now I have two part in this structure. One part is this and another part is this. What will be the free body diagram of this part? If I draw a free body diagram of this part and free body diagram of this part. Free body diagram of this part will be, you seenow this line breaks only member AF and member AD. All other members are remain unaffected, right?

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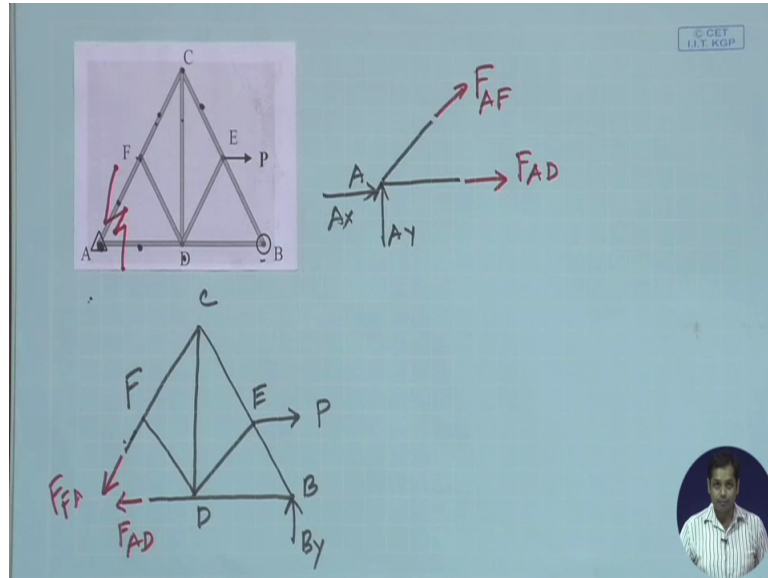
So this is the, this part of the structure. Now since this member AF and AD is broken now, so that has to be represented. It was originally continuous, that has to be represented by forces. So this force will be F_{FA} and this force will be F_{AD} . So this is the free body diagram of this part of this structure.

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Similarly I can draw the free body diagram of this part of the structure. What was this part of the structure? This part of the structure was only this AF and AD is broken. So this is A and this is the support reaction A_x , A_y and then this is F_{AF} and F_{AD} .

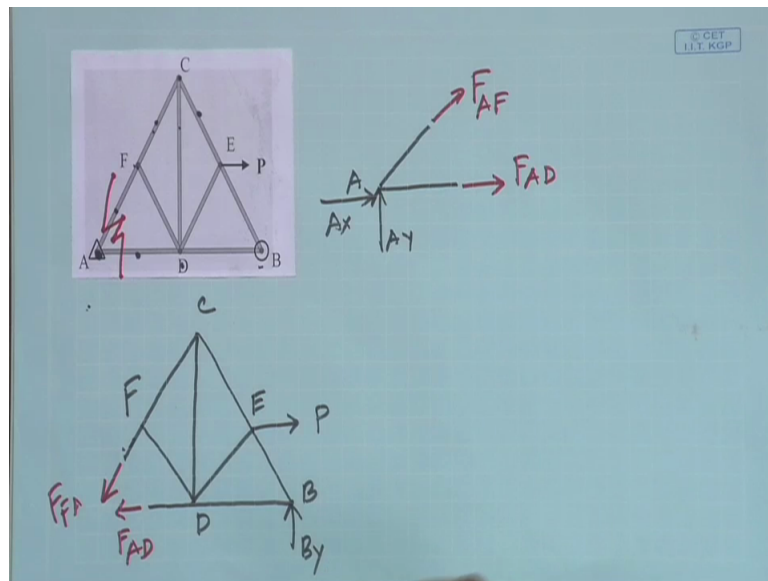
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Now what is the difference between what we have just now done and the method of joints we have discussed so far is, in this case what we have done is we take a section from the structure and this is the free body diagram of one section, this is a free body diagram of the other part of the section.

Now instead of drawing the free body diagram of the joints and apply the equilibrium equation on those free body diagram, we can also take several sections and draws a free body diagram of the sections and apply the equilibrium equations on this sections.

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Now this method is called method of sections. Now what we will do is, we will stop here. Next class we will start with the same problem, this problem and then see using method of section, just now which is demonstrated here, using method of section finding member force in CD becomes much easier. So next class we will do is, we will discuss analysis of statically determinate truss using method of sections.

We will start with this example. We will start with demonstration of this method of section. We will start that with this examples.

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Method of Joints: Example 4

Determine force in member CD. ABC is an equilateral triangle. D, E and F are mid points of the respective side.

Example Courtesy: J. L. Meriam and L. G. Kraige, Engineering Mechanics Statics, John Wiley & Sons, 5th Edition

We will stop here. Thank you.