

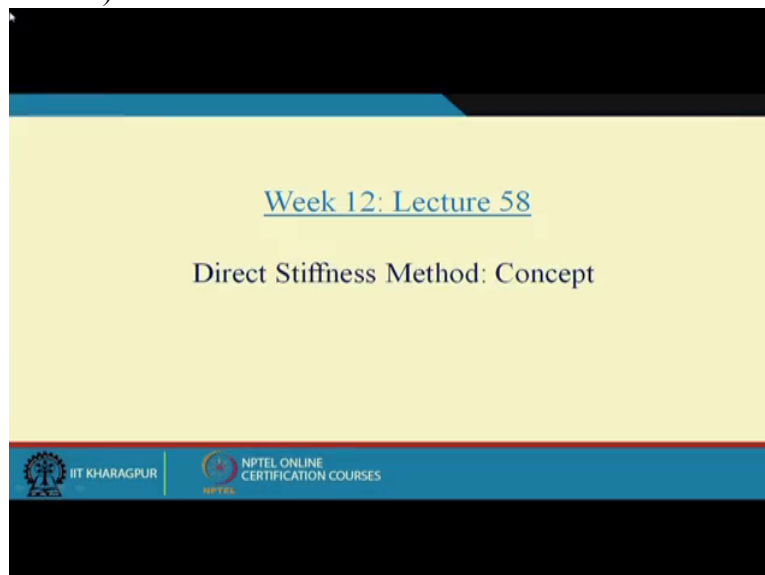
Course on Structural Analysis I
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture No 58
Direct Stiffness Method (continued)

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Hello everyone, welcome to the second lecture of the, of week 12. Today what we are going to do is

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again we will continue direct stiffness method, the concept again. And, in the last class what we learnt is we; we tried to understand the concept of direct stiffness method through analysis

of, analysis of plain truss. We have not done the examples. That we will be doing in the subsequent lecture and today we will be doing the same exercise through analysis of beams. We will see what will be the, what will be the, what would be

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the form of the matrix, stiffness matrix for beams, Ok. Again this is the slide

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Displacement Methods: Summary

$$\begin{aligned}
 F_1 &= -(M_{BA}^F + M_{BC}^F) \\
 F_2 &= -(M_{CB}^F + M_{CD}^F) \\
 F_3 &= -(M_{DC}^F + M_{DE}^F) \\
 F_4 &= -M_{ED}^F
 \end{aligned}
 \quad
 \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}
 =
 \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}
 \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}$$

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which gives you the summary of displacement method, I showed this slide in the last class as well. Ideally what we want to do is we want to get this relation, right. Here are the degrees of freedom. This is the stiffness matrix and these are the nodal force vector.

And if you recall

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what we discussed in the last class, and again the similar exercise we did in the displacement method while talking about slope deflection method and all, we have a beam, then we decomposed the beam, we assumed each segment of the beam as fixed beam and then we determined what are the fixed end moments and the corresponding stiffness coefficients, Ok.

Now suppose, suppose let's consider this beam,

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Displacement Methods: Summary

$$\begin{aligned} F_1 &= -(M_{BA}^F + M_{BC}^F) \\ F_2 &= -(M_{CB}^F + M_{CD}^F) \\ F_3 &= -(M_{DC}^F + M_{DE}^F) \\ F_4 &= -M_{ED}^F \end{aligned} \quad \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}$$

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what is shown on this, on this slide. Now this beam has several spans, A B, B C, C D and D E. And the idea is we decompose this system into small, small sub-systems for that, such that, for each sub-sys/sub-system,

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for each sub-system we have the information, we know the relation between force and displacement through stiffness, through stiffness and once we know this relation then we assemble all the subsystems to get the global system equation for the entire, entire system.

Now

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Displacement Methods: Summary

$$\begin{aligned} F_1 &= -(M_{BA}^F + M_{BC}^F) \\ F_2 &= -(M_{CB}^F + M_{CD}^F) \\ F_3 &= -(M_{DC}^F + M_{DE}^F) \\ F_4 &= -M_{ED}^F \end{aligned}$$
$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}$$

$\theta_B = D_1$ $\theta_C = D_2$ $\theta_D = D_3$ $\theta_E = D_4$

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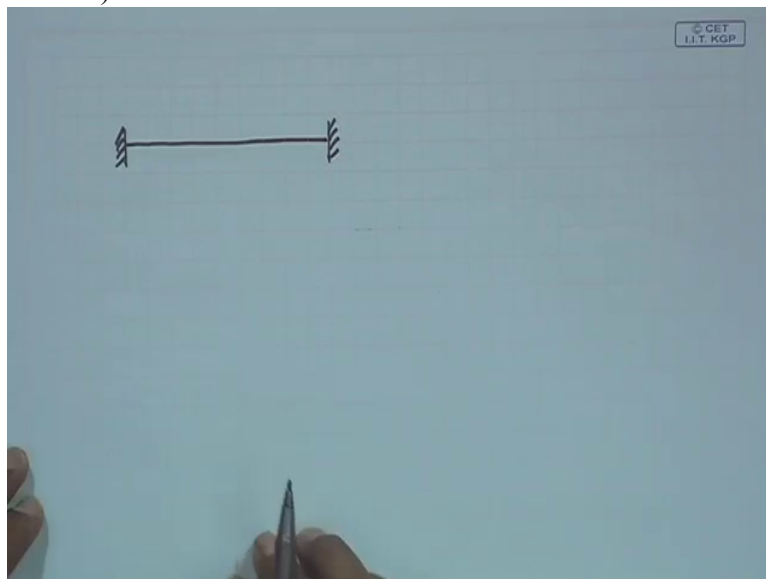
let's, now assume, so let's what we do is let's take one span, any span, any random span of this beam and then see what would be the force displacement relation for that particular span and similar force displacement relation can be obtained for any span, Ok. Now, so let's take one span A B.

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So this end is fixed and this end is fixed, Ok.

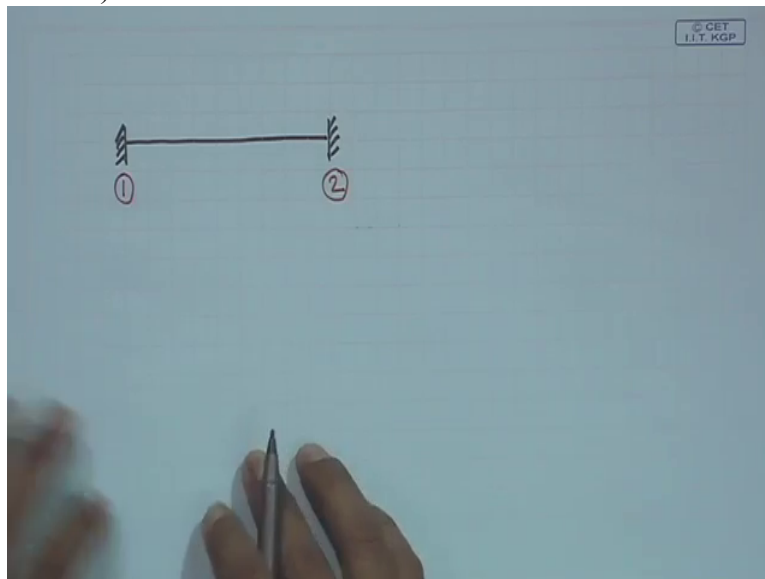
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Now so the entire, the entire beam is now divided into small, small sub-beams and this is one such, one such sub-beam, one such small segment and we are going to derive the force displacement relation for this segment and similar force displacement relation can be obtained for all segments and then we can assemble them, Ok.

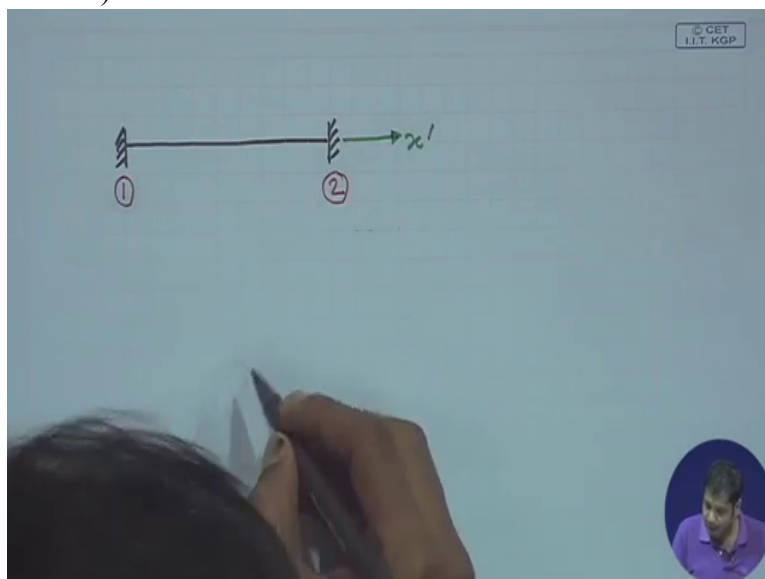
Now first thing is, we need to, suppose this is the point number 1, and this is point number 2, Ok,

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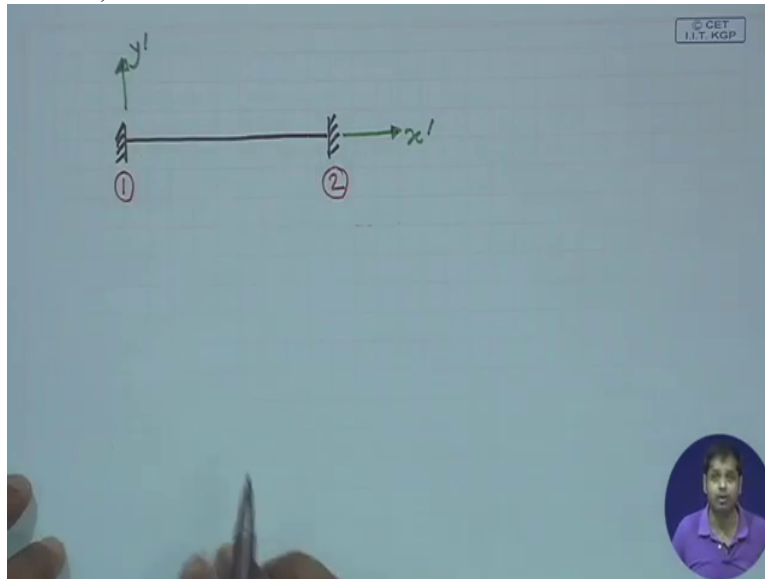
similar in the truss case also we have a point and point number 1 and point number 2. We also have a global, local, local coordinate which is x dash here

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and this is y dash, this is y dash,

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we are talking about, we have a, we have a, we have a, we can also have a z dash which is normal to this plane but since we are talking about plane structure, we have, we have only two coordinate axes, x dash and y dash, this is the local coordinate axis, Ok.

Now, now let's define the degrees of freedom. As we know any, any point, in, in two-dimensional plane has 3 degrees of freedom, two translations and one rotation. In local coordinate system for a truss, only the axial transformation, axial translation is allowed. That is why in local coordinate system for each truss member we had one degrees of freedom at every point. So total two degrees of freedom for any member.

Now in this case, there is no such, no such assumption that the force always has to be axial, there is no rotation, no bending takes place, we don't have such assumptions here. So ideally at every point you should have three degrees of freedom. In this case, translation in this direction, translation in this direction and rotation; translation this, translation this and then rotation, Ok.

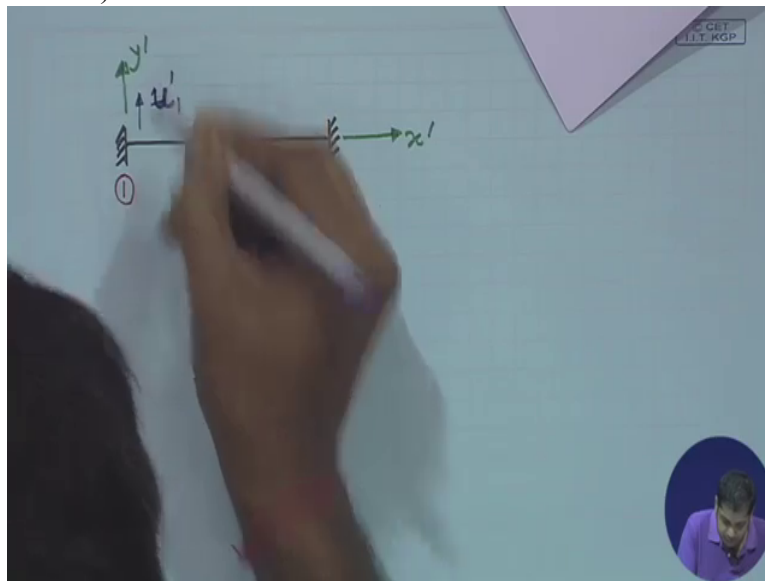
Now to start with, we assume, so here 3 degrees of freedom, two translation and rotation, here three degrees of freedom, total 6 degrees of freedom. In plane frame, in plane truss, for in local coordinate system, it was just 2 degrees of freedom per, per, per member. Now to start with, let us assume there is no axial deformation in this member, Ok. So axial deformation is neglected. Now if we, if we, if we assume there is no axial deformation in the member, then what happens, then the degrees of freedom that we will be interested in only, at

this point, it is the translation in this direction, and the rotation at this point, translation in this direction and rotation at this point, Ok.

So first we will see for a beam, there is no axial translation, axial deformation is allowed. We will derive this stiffness matrix by considering other 2 degrees of freedom per node and then knowing the fact that when, when axial, when axial, axial deformation is allowed, what this stiffness matrix, what is the form of stiffness matrix we have already learned in, in, in truss problem. We can combine them to get when all degrees of freedom present.

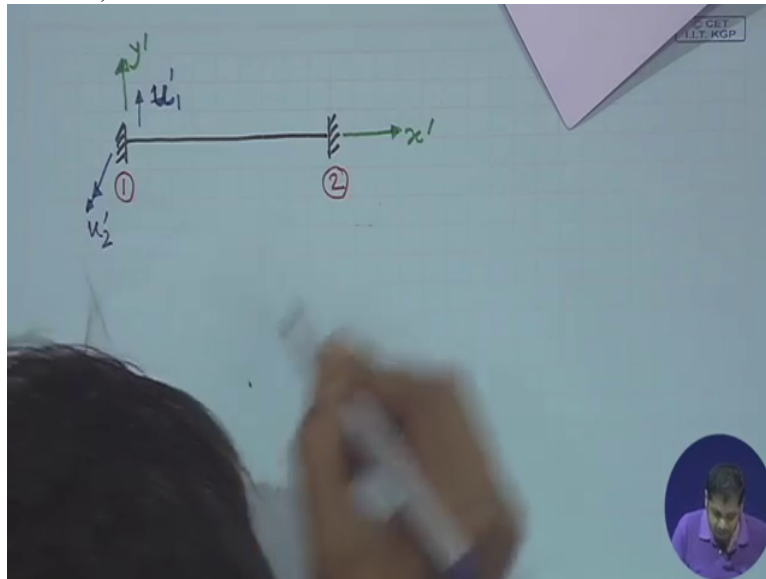
So now the degrees of freedom here are, are, are these. We have, we have degrees of freedom is this which is, say degrees of freedom say 1, Ok. And then we also have, say this is, this is u_1 . This is u_1 , u_1 dash,

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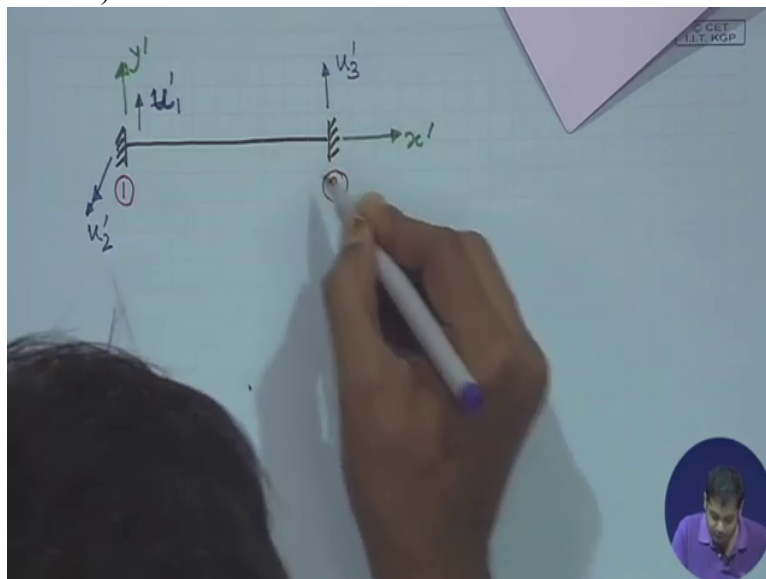
Ok and then we also have rotation at this joint, rotation is shown by this double arrow and this is u_2 dash.

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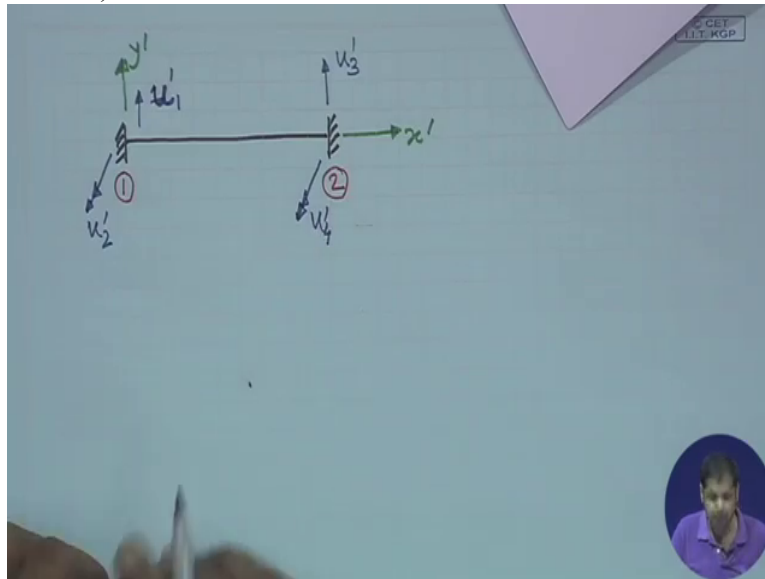
Similarly we have displacement, translation in this direction u_3 dash

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and again rotation at this point which is shown by double arrow u_4 dash,

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Ok.

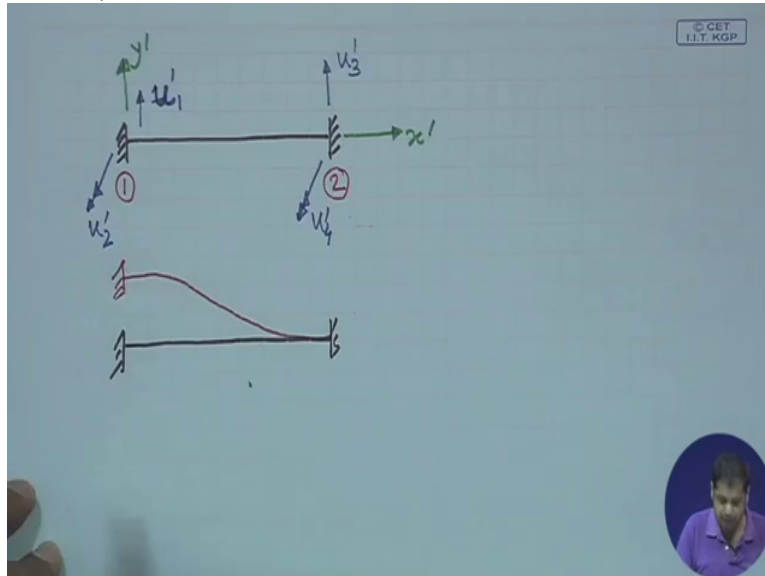
So there is no translation here, no translation here because for the time being we are neglecting the axial deformation. So at every joint we have 2 degrees of freedom, one transverse, translation in transverse direction and one rotation, Ok. Now let us, similar to truss, first we consider that each degrees of freedom and then find out what are the corresponding stiffness coefficient, Ok.

Let us first consider the degrees of freedom, 2 degrees of freedom at this joint considering this end fixed and this end allowed to, allowed to have the, the, the, allowed to have the deformation as per degrees of freedom. And next what we do is we will assume this end fixed and this end is allowed to move, allowed to, allowed to deform as per the degrees of freedom we consider. Ok.

So first is let's take degrees of freedom at u_1 means this point, the point number 1, it translates in this direction but there is no translation, no rotation takes place at this point and there is no rotation takes place at, at 1. So it is only one translation takes place at point 1, so if it is, then, then actually your structure is initially like this, initially like this. This is fixed, this is fixed.

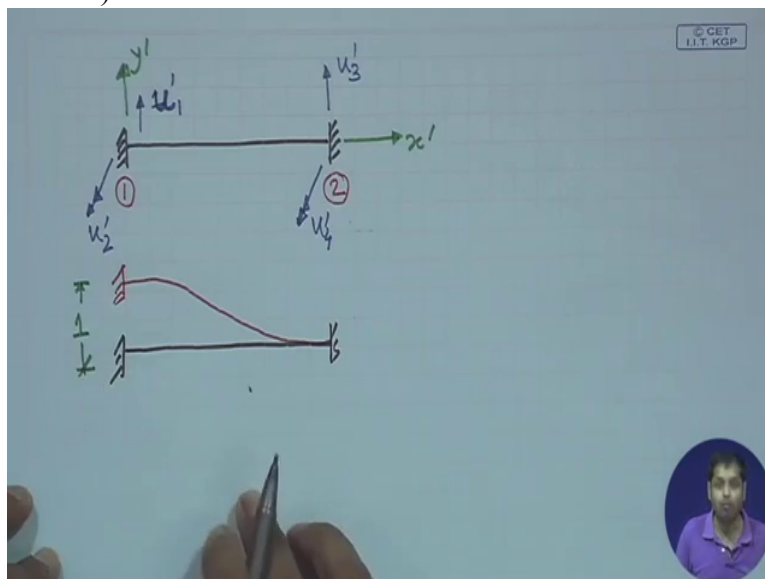
Now what we do is we give a translation, we give a translation in this direction so the deflected shape, the deflected shape becomes something like this. So this is translation

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in this direction, Ok. And suppose this translation; the value of this translation is unit. It is u_1' dash is equal to 1. This is 1, this value is 1 Ok and this value is

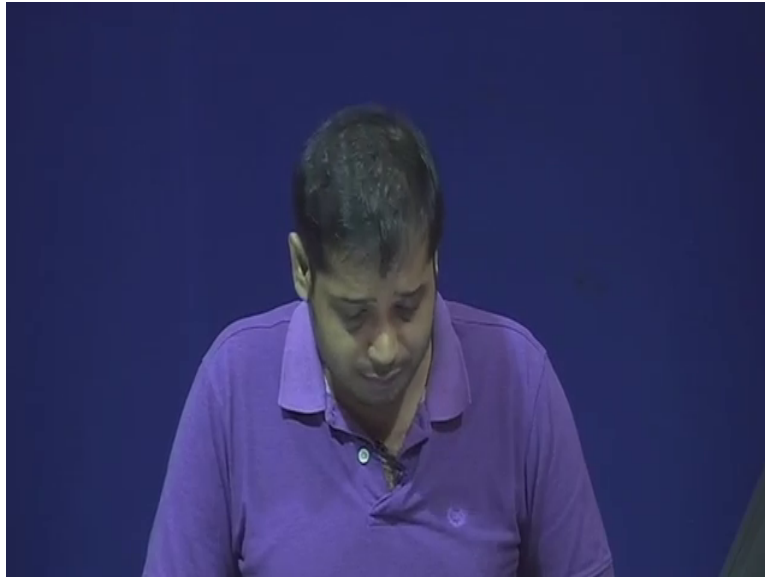
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1.

Now if you recall, if you recall the lectures in slope deflection, when we discussed slope deflection with that, we actually, we actually solved these small, small problems. If you have a fixed beam and giving, giving,

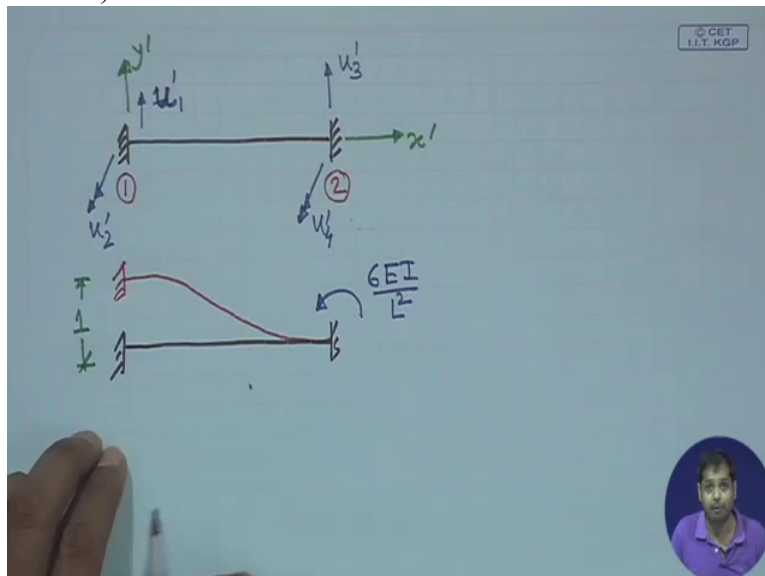
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and given axial transfers, deflection or support settlement in a particular direction, what are the forces and moments generated at the, at the support.

So we have already determined these values and let us just write those values. So we have a moment generated at this point and this, the value of this moment will be $6EI\delta/L^2$ where L is the length of the

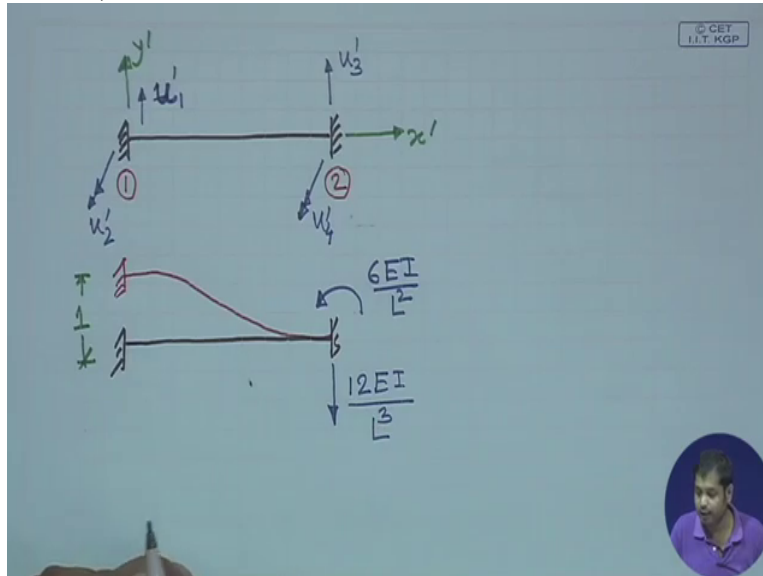
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beam. We have already determined, derived these values if you please refer to the lecture when, when we discussed slope deflection, slope deflection method.

And for this there will be force generated in, reaction force generated in this direction and that value will be $12 E i$, $12 E i$ by L ,

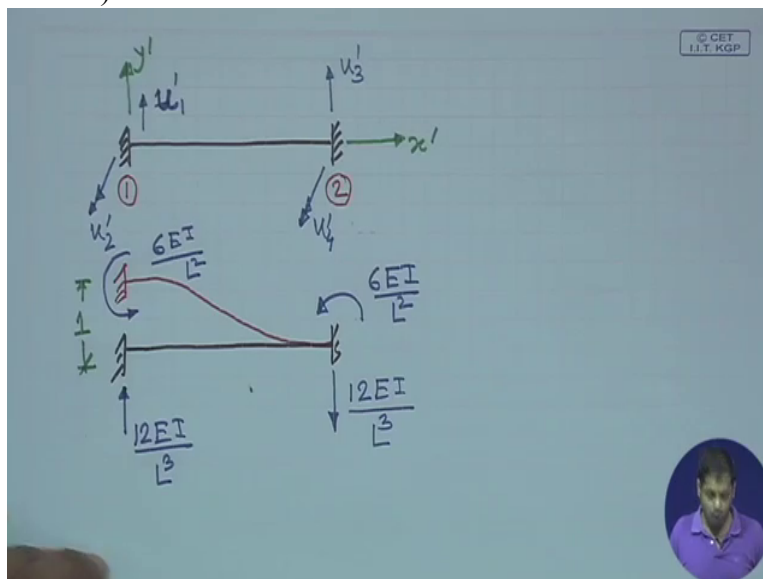
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L cube, Ok. And similarly there will be a reaction generated, generated in this direction and which is value will be $12 E i$ by L cube and then the moment generated will be in this direction and this moment value will be $6 E i$ by L square, Ok.

So

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what you can do is you have a fixed beam which is, where you have a support settlement of unit, unit support settlement at a particular point, at a particular joint and because of that

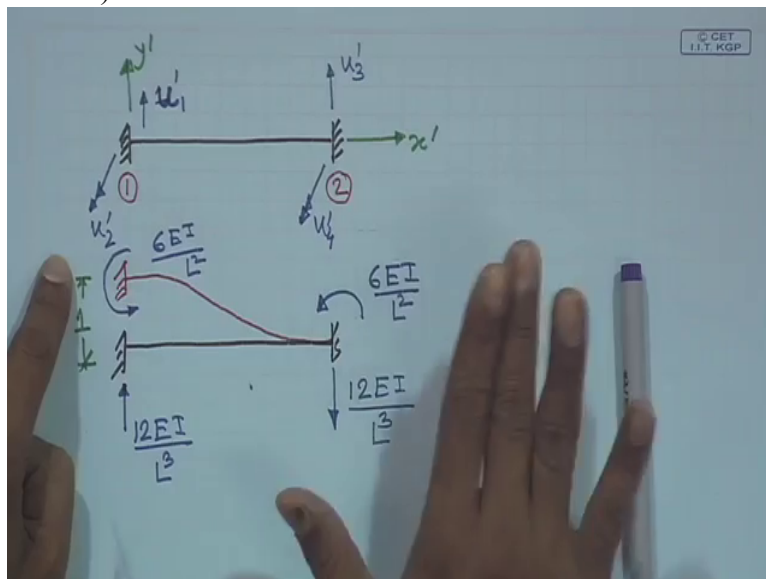
support settlement what would be the support reactions? What would be the reactions at the moment generated at the support? You can use any method to determine

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these values and since you know these methods, different,

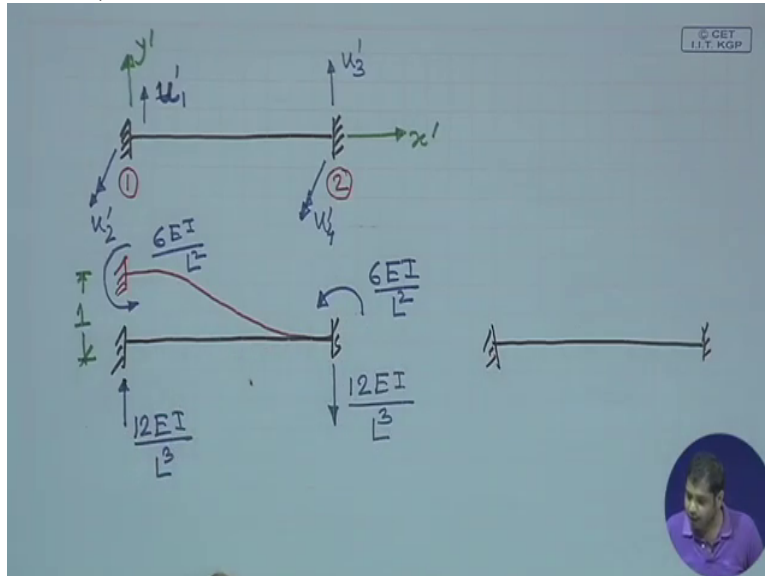
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several methods to determine these values, we are not going to derive these values once again, Ok. So these are the, these are the corresponding reactions you get, Ok. Now similarly let us give unit rotation at this point. Now this is for unit translation. Now let us give unit rotation at this point.

So originally, originally your structure is like this, the member is like this which is fixed here, this is fixed here. Then we

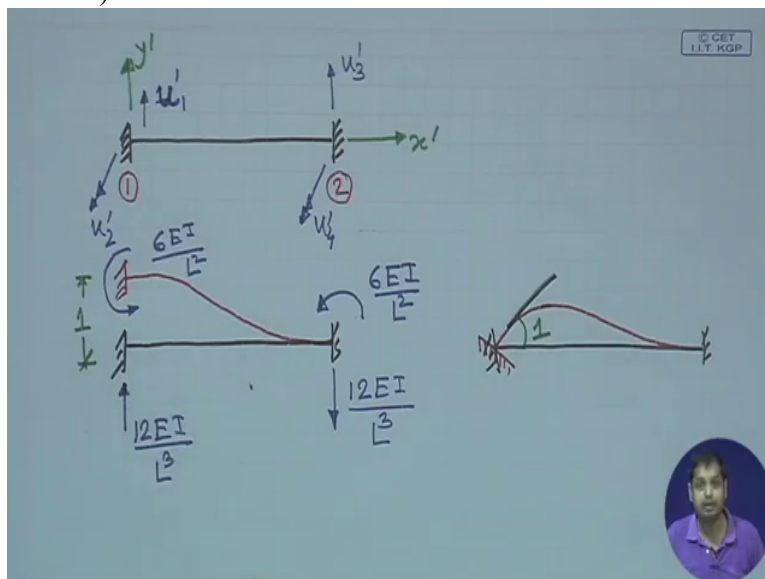
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give the unit rotation at this point and if we give the unit rotation then this is unit rotation and this actually, support becomes like this, Ok. And this value, this value is, this value is unit.

This value is,

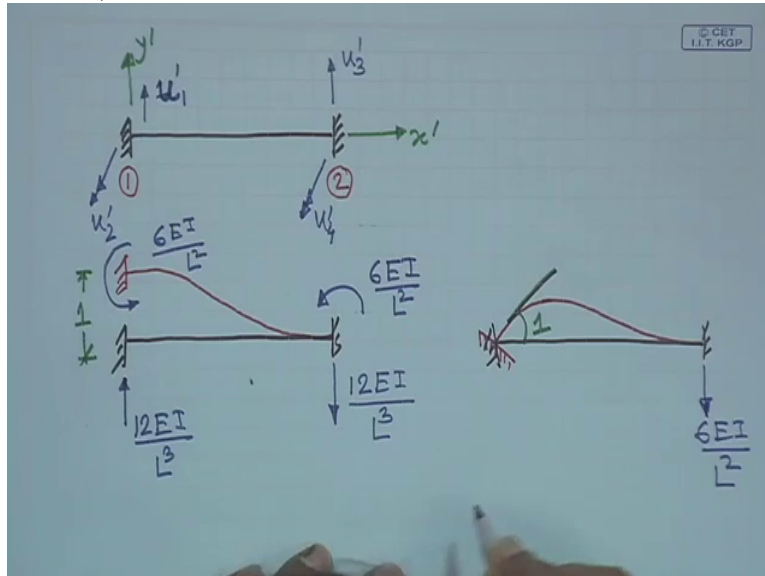
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this value is 1. So again we have a fixed beam, we have a fixed beam and then one end you give an unit rotation or propped cantilever beam, one end, the propped end you give an unit rotation then what would be the, what would be the, what would be the, what would be corresponding moment and corresponding, corresponding reactions at the support, that we can obtain by any method that, any method that we have learnt so far.

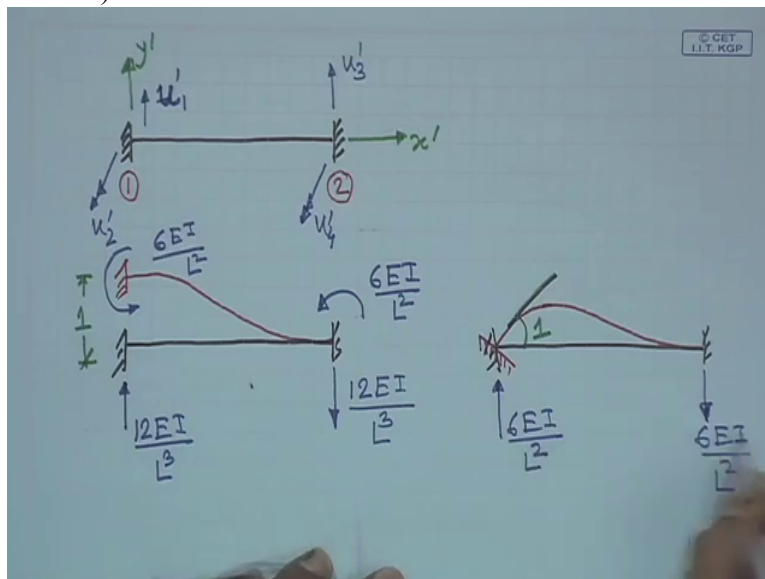
And if we give an unit rotation here, then the corresponding forces, corresponding forces will be, you have a vertical force here, this vertical force will be $6EI/L^2$ and then

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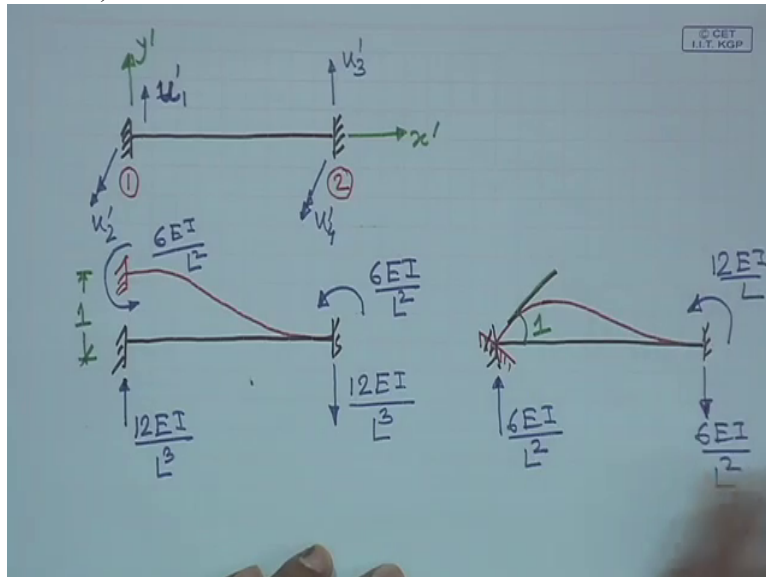
there will be reaction here which is again $6EI/L^2$

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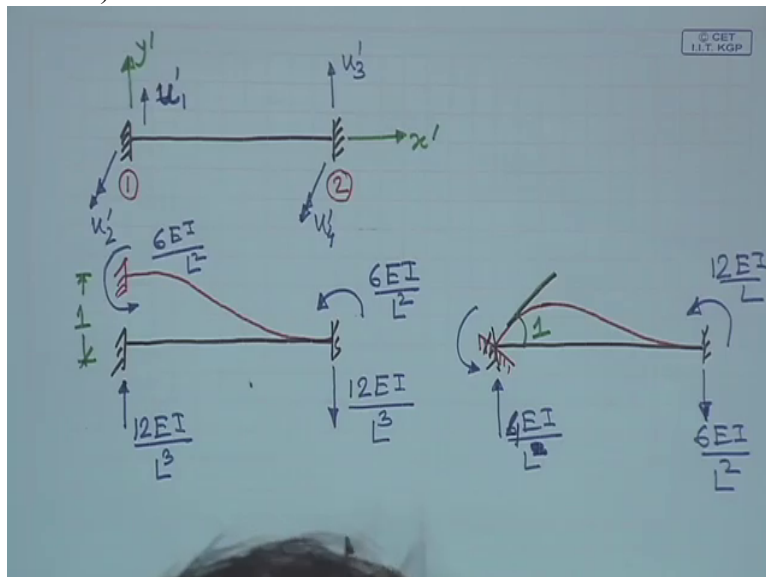
to satisfy the equilibrium, they will be in opposite direction. And the moment here, and this moment will be $12EI/L$

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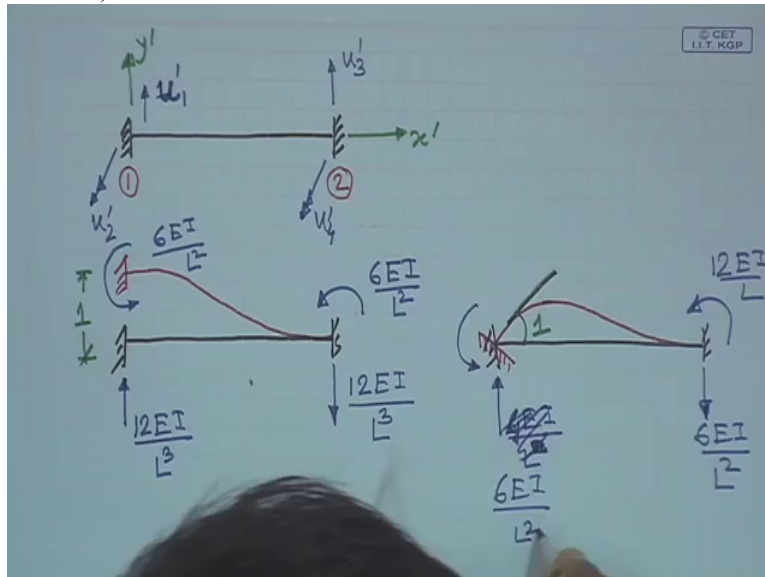
and then corresponding moment will be, this value will be 6, this value will be, this value will be, there is, this value will be 4 E i, this, this, Ok just one minute, this value will be 4 by, this 4 E i by L, 4 E i by L, this

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reaction will be, this reaction, just one minute, this reaction will be 6 E i by L square and

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this value will be $4EI$ by, this value will be $4EI$ by L , Ok.

Now you see what we have done is, we considered, there are 2 degrees of freedom at each joint. Here translation in this and rotation of this joints, translation and rotation at this joint, u_1 , u_2 , u_3 , u_4 . Now we are considering each degrees of freedom separately and checking for if the member is allowed to have that degree, is allowed to have the deformation as per the degrees of freedom, as per the chosen degrees of freedom what would be the corresponding moments and react/reactions, what would be the corresponding reactions generated at the, at the support.

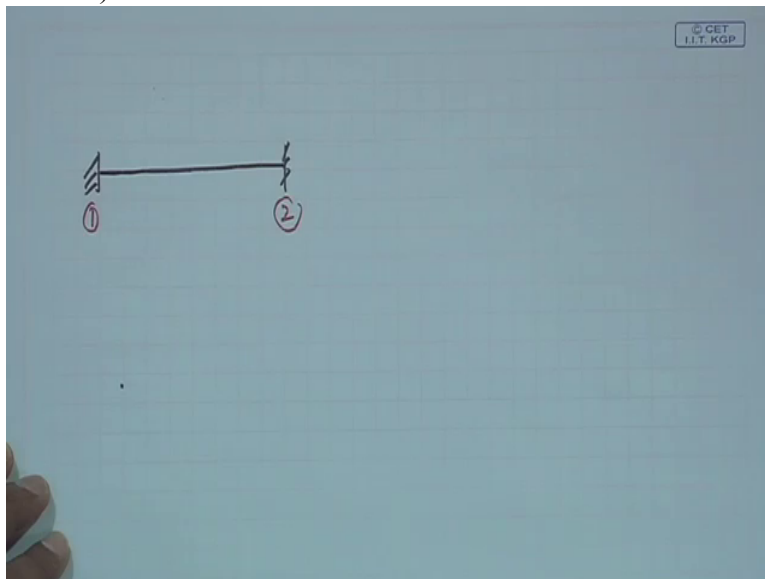
So this is for translation at joint 1, this is for rotation at joint 1. Similar exercise we can do for translation at joint 2 and rotation at joint 2. And if we do that, if we do that

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then what we have is something like this. Now, now before, suppose again this is a structure, this is a structure and now this end was actually fixed. Now this is joint number 1, joint number 2. Now this joint is allowed to have translation in this direction and if,

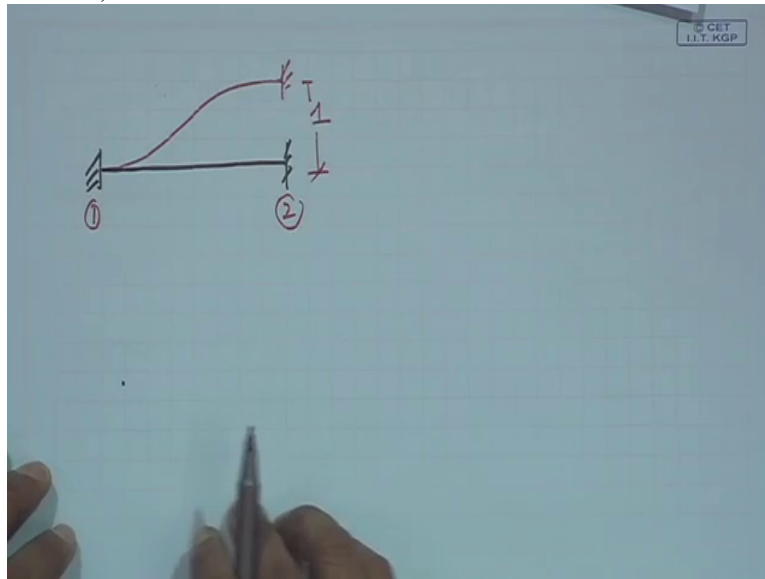
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if it translates, if it translates like this, then the corresponding deflection will be like this.

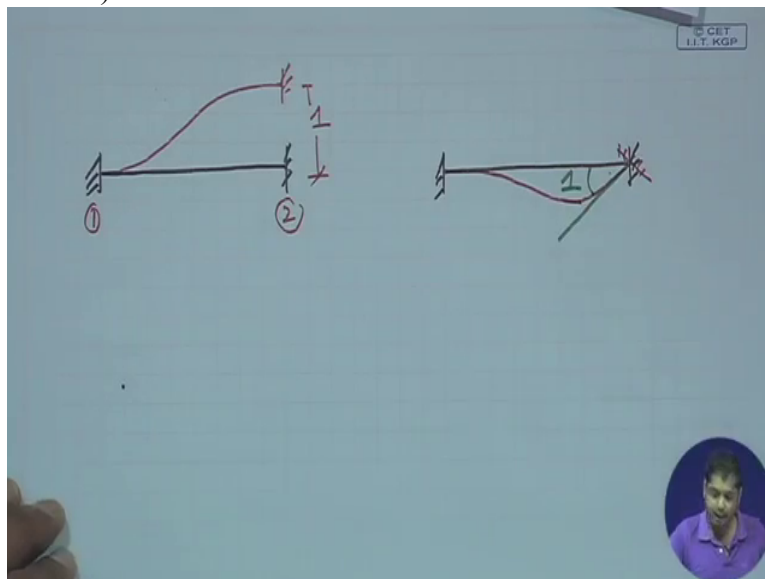
So this will be, this is,

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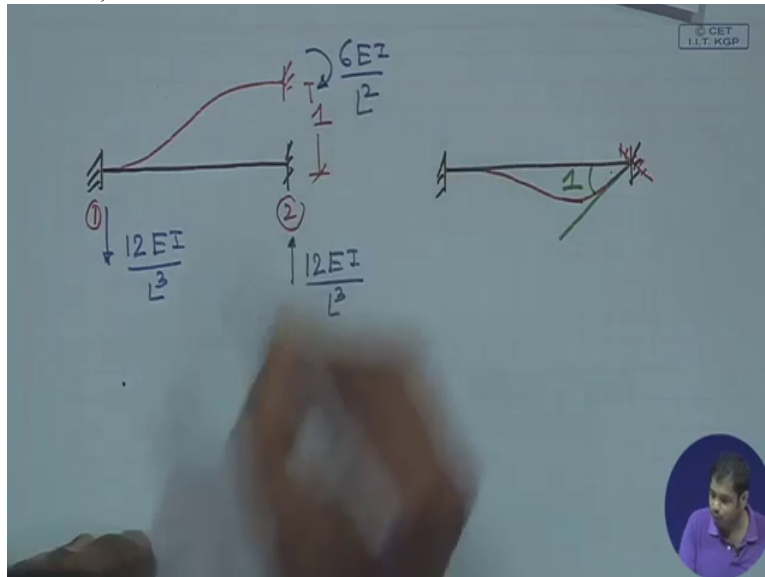
this is unit, unity and then corresponding translation of other joint will be like this. If we have similar structure, this end is fixed, this end is originally fixed then this end you have unit rotation and this unit rotation will be, this unit rotation and corresponding support will be like this. And this is unity, this is 1.

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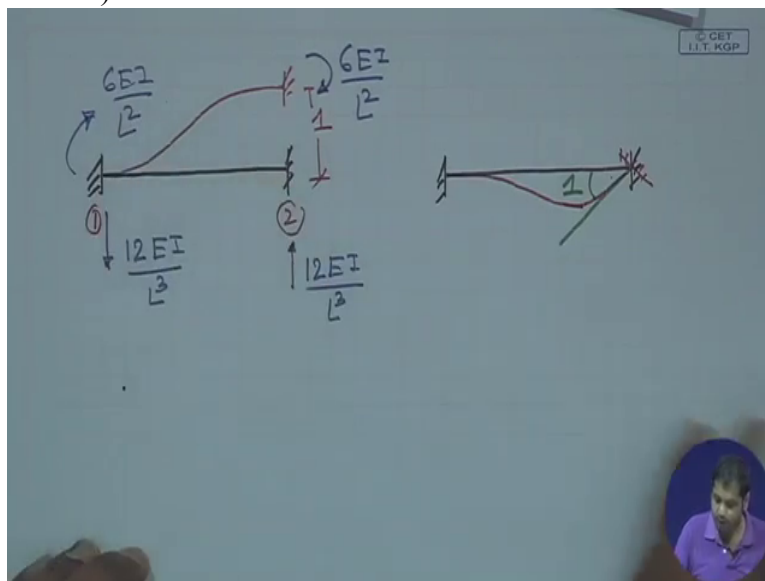
And if we have this, if we have this, then also we have corresponding reactions and moment and those values will be like this and for this we have a reaction force here which is $12 E I$ by L^3 and corresponding reactions here will be $12 E I$ by L^3 and this will generate a moment in this direction and this moment will be $6 E I$ by L^2

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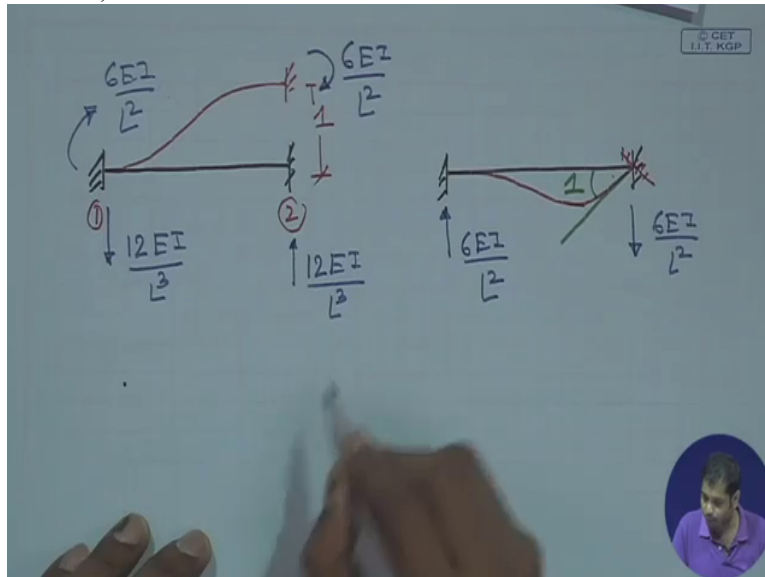
and corresponding moment in this direction it will be, it will be $6 E i$ by, by L square, Ok.

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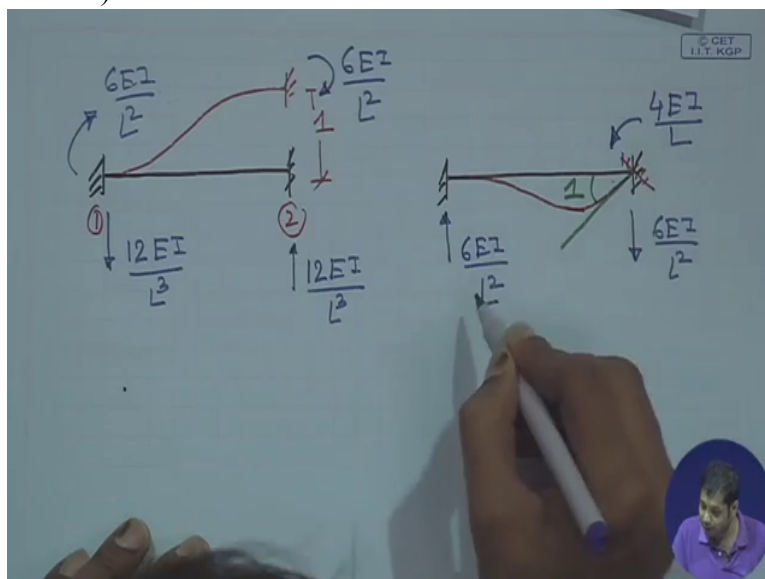
Now similarly we can have reaction force and moments here. Reaction force will be $6 E i$ by L square and then this will be $6 E i$ by L square.

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So you can please verify these values and we are not deriving these values because we have done

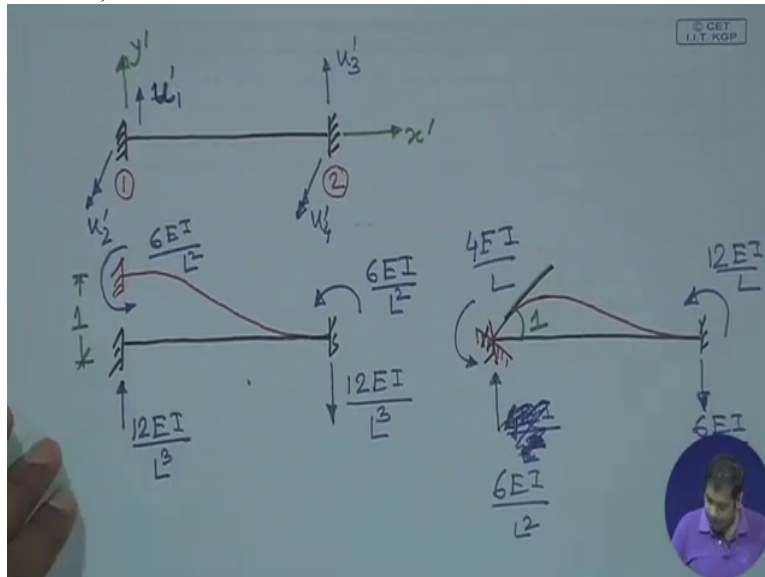
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that and you already know how to, how to do that, how to do that, Ok. So now we have this, right?

Now next what we can do is, you see what,

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if you remember for, for in the case of truss we had, we had 2 degrees of freedom

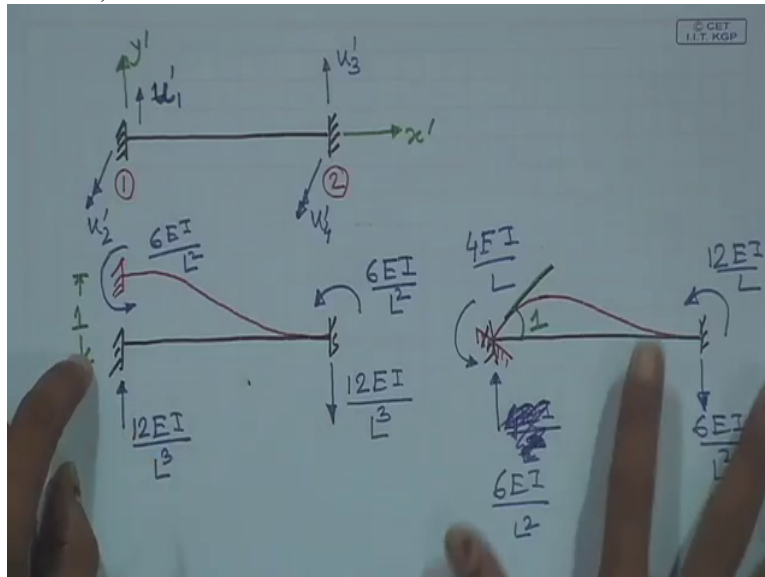
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total at every member, in local coordinate system we have 2 degrees of freedom. Now that's why, for members stiffness matrix, the size of the member, the size of the stiffness matrix in the local coordinate system were 2, were 2 by 2 matrixes, Ok.

Now here we have

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4 degrees of freedom, 2 here and 2 here, so naturally the size of this stiffness matrix will be 4 by 4, Ok. Now, now if you remember, if you, this, this is the stiffness matrix,

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Displacement Methods: Summary

$$\begin{aligned}
 F_1 &= -(M_{BA}^F + M_{BC}^F) \\
 F_2 &= -(M_{CB}^F + M_{CD}^F) \\
 F_3 &= -(M_{DC}^F + M_{DE}^F) \\
 F_4 &= -M_{ED}^F
 \end{aligned}$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}$$

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if you, this is the stiffness matrix. Consider any; this is the stiffness matrix for this member. Now k_{11} is essentially, is associated with, it, it relates the force at, the force corresponding to first degrees of freedom 1 and the degrees of freedom 1.

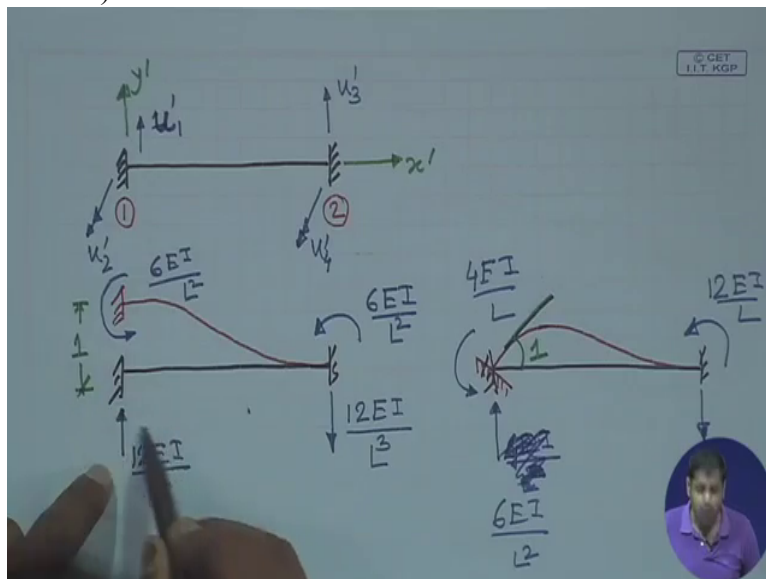
k_{12} is actually the force in the direction of degrees of freedom 1 due to degrees of freedom 2. And then force k_{13} is the force in the degrees of freedom 1 due to degrees of freedom 3. So similarly, now you see here,

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the, this is the force,

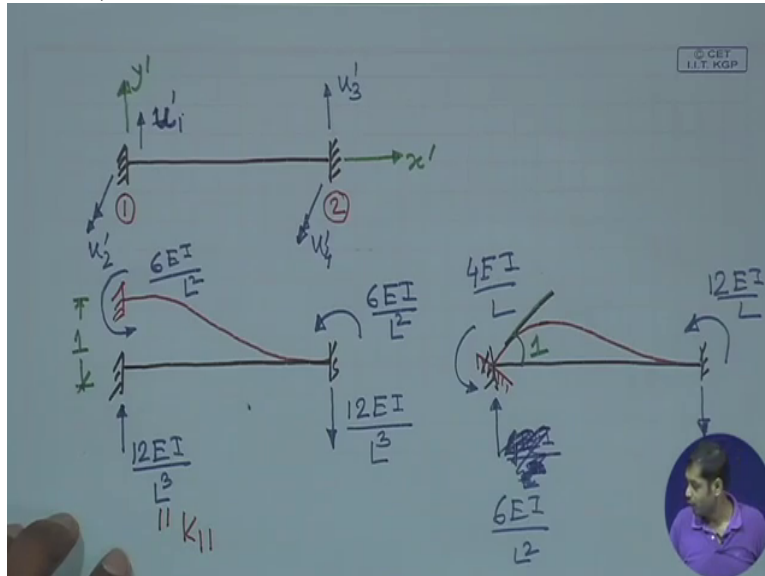
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this is degrees of freedom 1, so this is the force in degrees of freedom 1 and this, this force is generated due to u_1 , u_1 dash. So this is essentially your force in the direction of degrees of freedom 1 due to degrees of freedom 1, Ok.

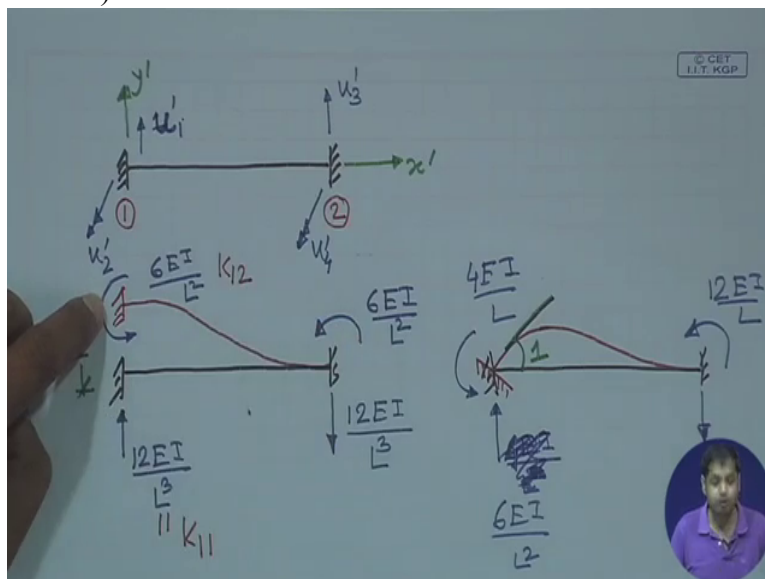
Now, now what is this force? This is essentially; this is in degrees of freedom 2 because that is the rotation, u_2 dash, Ok. This force generated in the direction is corresponding, in the direction of degrees of freedom 2 but this is caused due to degrees of freedom 1. So therefore, this will be, this is in the direction of degrees of freedom 1 due to degrees of freedom 1, so this will be essentially k_{11} , Ok.

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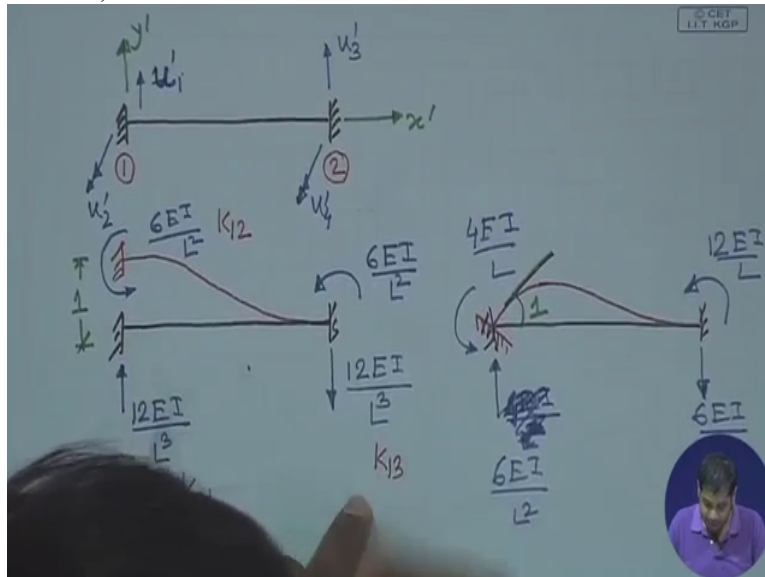
Now this is in the direction of degrees of freedom 2 but due to degrees of freedom 1 so this will be k_{12} .

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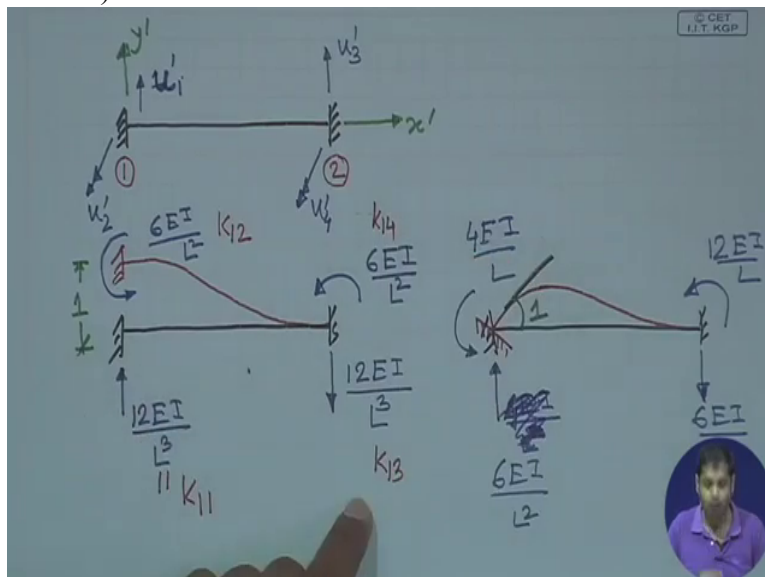
And then similarly this will be, this is direction of, this is in the direction of degrees of freedom 3, but due to, but due to degrees of freedom 1 because this force is caused due to the translation u_1 . So this will be k_{13} . And

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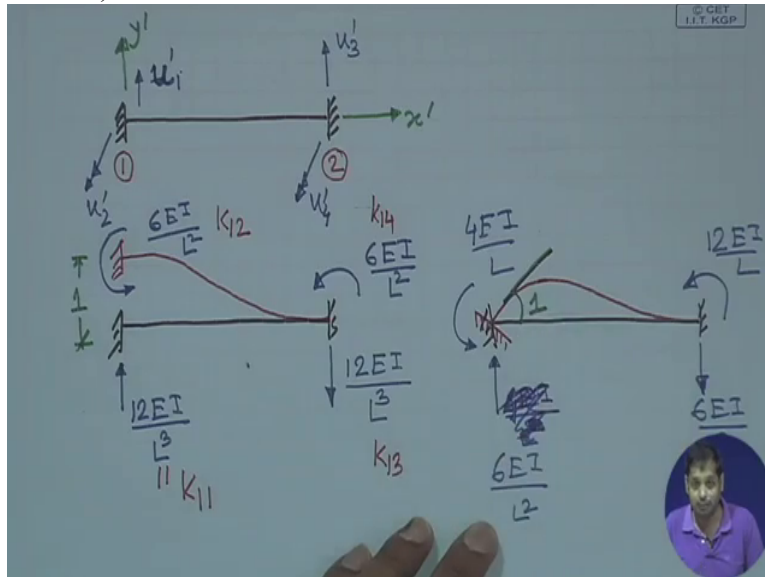
similarly this will be k_{14} ,

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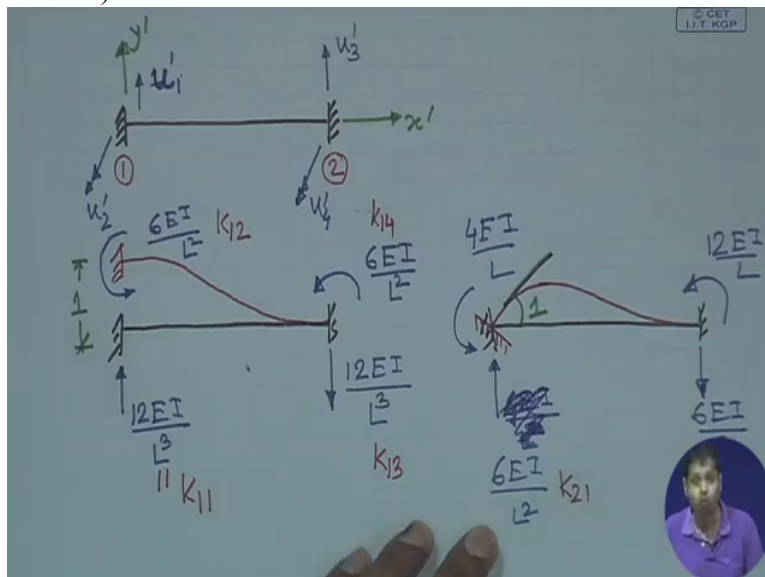
Ok. Now look at this. Now here it will be, this is now degrees of, all the forces here generated due to degrees of freedom 2. Now this is in the direction of degrees of freedom 1 but due to degrees

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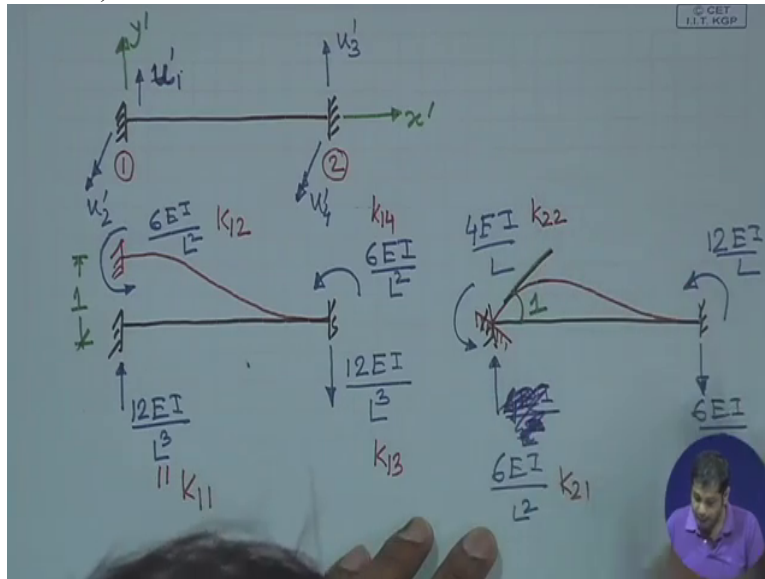
of freedom 1. So this will be k_{21} .

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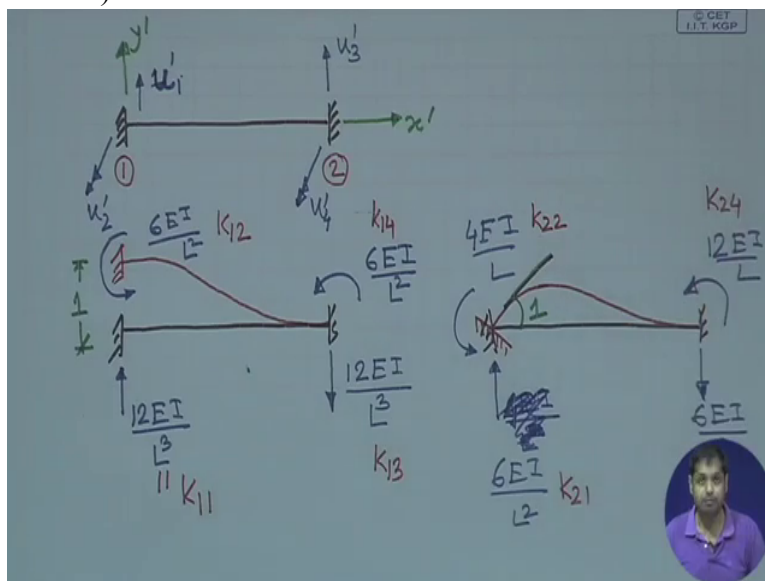
And this is in the direction of degrees of freedom 2 due to degrees of freedom 2, this will be k_{22} .

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And this is in the direction of degrees of freedom 3 but due to degrees of freedom 2, this will be k_{23} and this will be k_{24} ,

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Ok.

Now our sign convention is this direction is positive, this direction is positive. So k_{11} will be $12EI/L^3$, k_{12} will be $6EI/L^2$, k_{13} will be $-12EI/L^3$ because it is in opposite direction and k_{14} will be $6EI/L^2$. Similarly this will be k_{21} will be $6EI/L^2$, it will be $4EI/L$, and this will be $-6EI/L^2$ and this will be $12EI/L^3$, $12EI/L^3$.

Similar thing you can have for, for member, for degrees of freedom 2 as well. And similar expression you will get, Ok. Now if we put that in a, in a matrix form then this matrix will look like this,

(Refer Slide Time 20:58)

$$[K^m] = \begin{bmatrix} \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix}$$

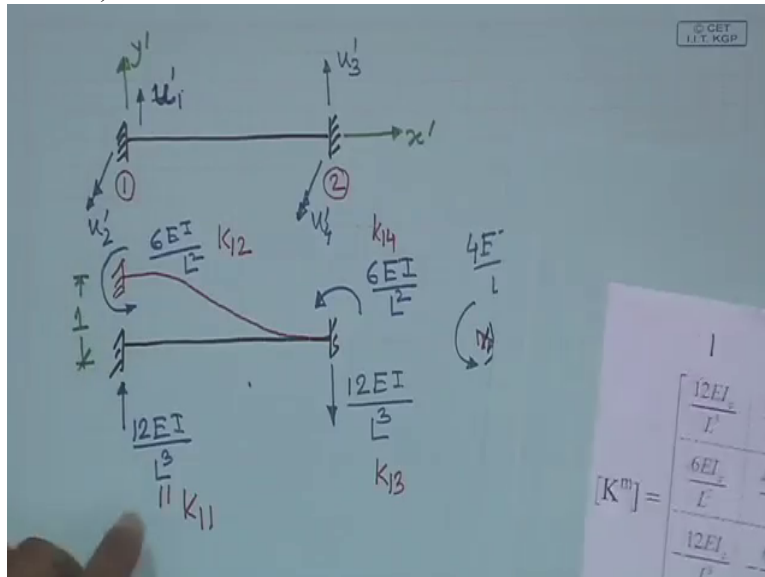
Ok. So this is degrees of freedom 1, this is degrees of freedom 2, this is degrees of freedom 3, this is degrees of freedom 4. This is 1, this is 2, this is 3 and this is 4.

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$$[K^m] = \begin{bmatrix} \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

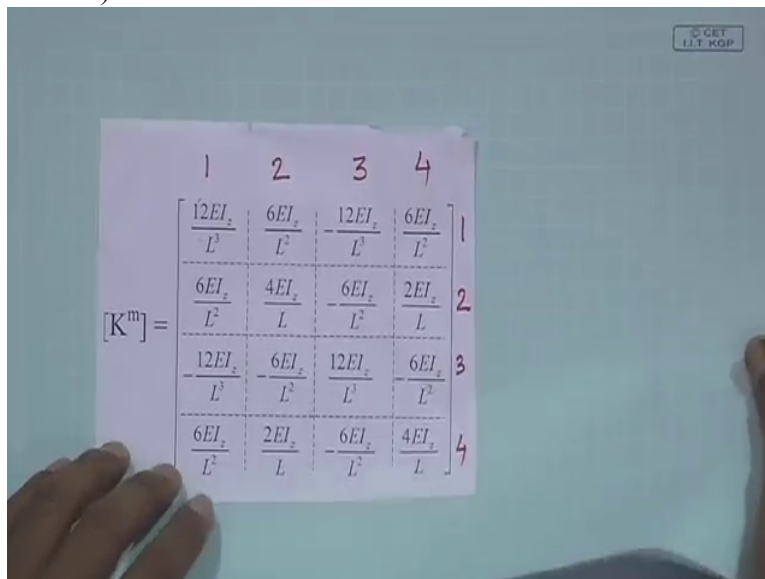
You see, now this is, this gives you, this is the stiffness matrix, the stiffness is essentially force for unit displacement. So this gives you the force due to degrees of, unit degrees of, due to degrees of freedom 1. So $12 E i$ by L cube

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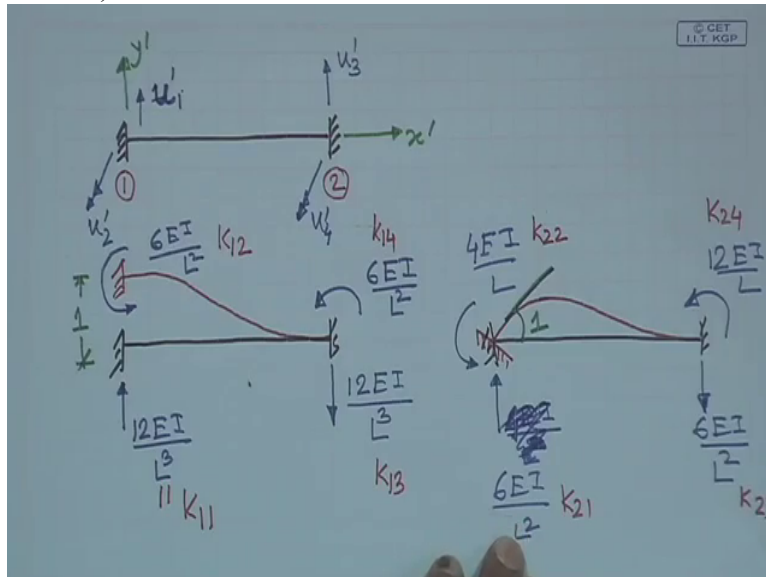
which we have already obtained, $12 E i$ by L cube. This gives you

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force in direction in degrees of, degrees, force F 1 due to degrees of freedom 2 which is $6 E i$, $6 E i$ by L square which we already obtained, $6 E i$ by L square.

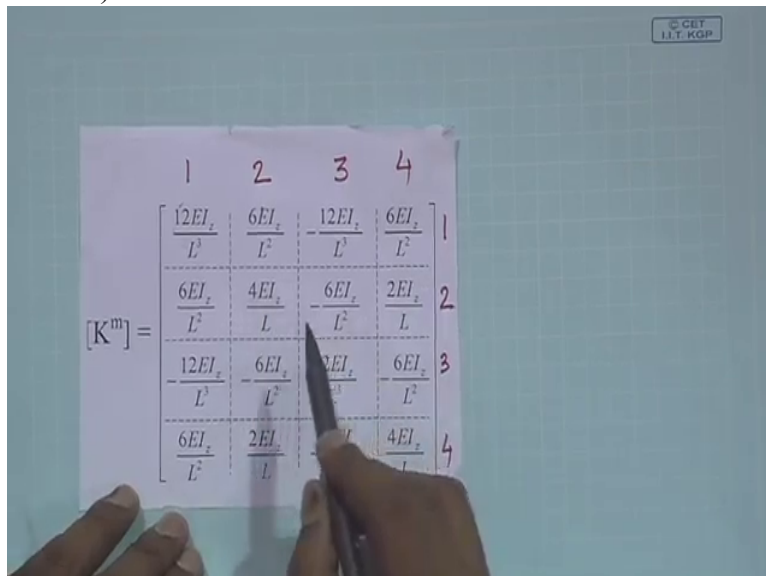
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And similarly this.

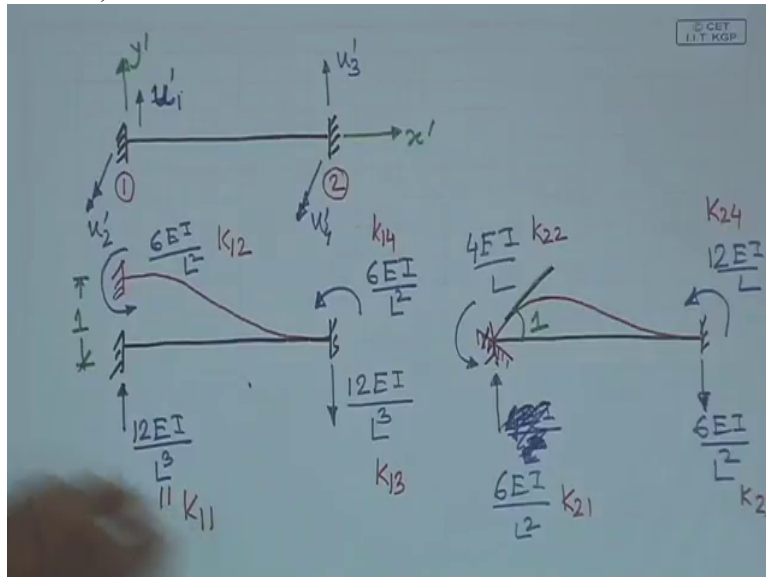
Now,

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so if you substitute that in a matrix form you get like this. And another important, and another important thing, please look at this, this matrix is symmetric matrix. Means the force generated, generated in direction 1 due to degrees of freedom 2 is equal to force generated in direction 2 due to due to degrees of freedom 1. That we can, we can check here also.

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Force generated in degrees of freedom 2 due to degrees of freedom 1 is $6EI/L^2$ and force generated in degrees of freedom 1 due to degrees of freedom 2 is $6EI/L^2$. This and this are same, Ok.

So this we already discussed

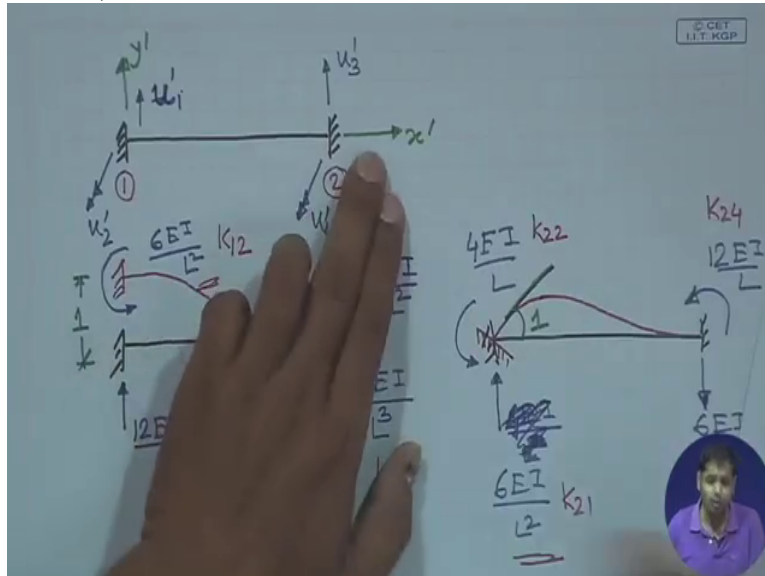
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	1	2	3	4	
[K ^m] =	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	1
	$\frac{6EI_z}{L^2}$	$\frac{4EI_z}{L}$	$\frac{6EI_z}{L^2}$	$\frac{2EI_z}{L}$	2
	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	3
	$\frac{6EI_z}{L^2}$	$\frac{2EI_z}{L}$	$\frac{6EI_z}{L^2}$	$\frac{4EI_z}{L}$	4

during, in a reciprocal we actually discussed reciprocal theorem. It is, it is, it is one demonstration of that, Ok. Now this gives you the stiffness matrix of a beam segment when, when we neglect the axial deformation. Now similar stiffness matrix we can obtain for all the, all the beam segments and then we can assemble them, we can assemble them.

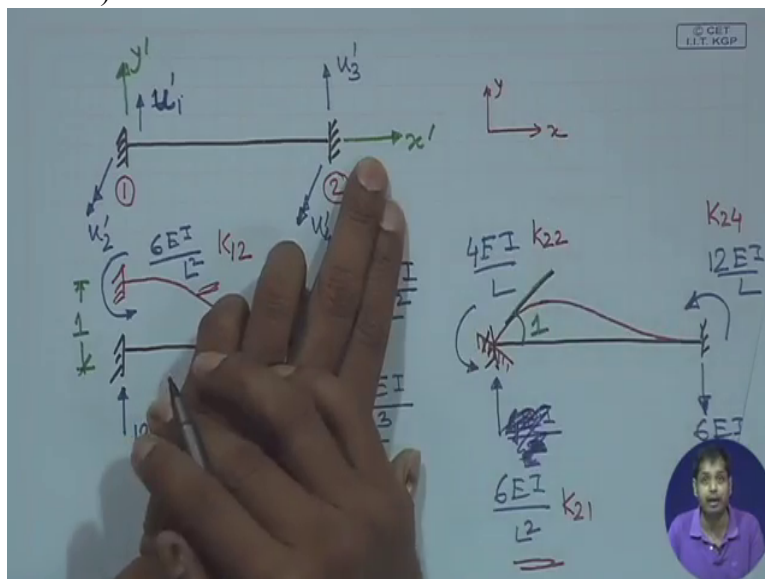
Now as far as beam is concerned, you see this is our local,

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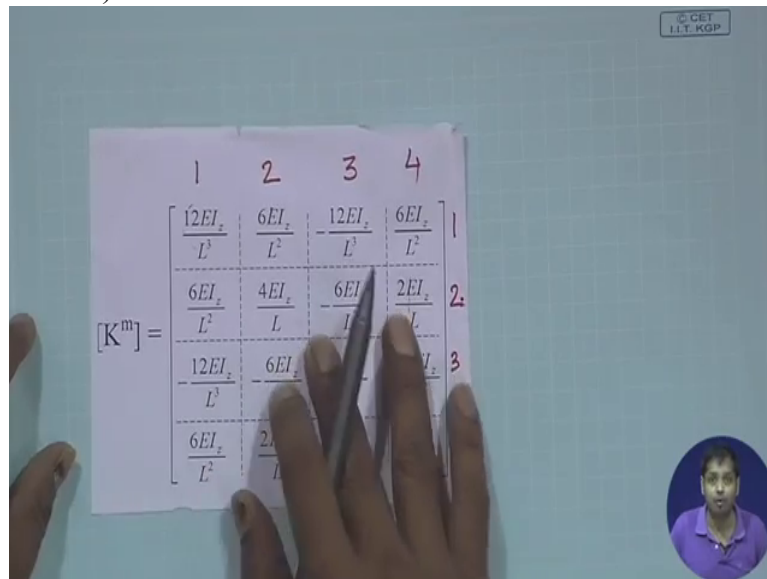
local, this is our local coordinate system and our global coordinate system is also, if you global coordinate system also

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if you take like this, then local coordinate system and global coordinate system, they are same. If they are same, this stiffness matrix that you obtain for local

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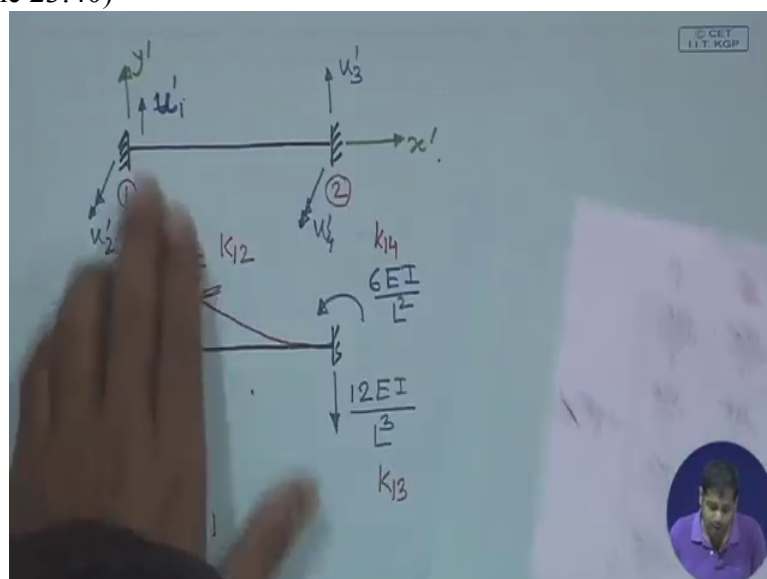


coordinate system, the same stiffness matrix you can obtain for that beam segment for global coordinate system as well.

So this will give you the global coordinate, stiffness matrix for a given member for global coordinate system. Similarly you can have the stiffness matrix in global coordinate system for all the members and then assemble them. And how to do that assembling and all, when you do actually some numerical example we can see that. Ok.

Now what happens if we, so this we obtain when there is no axial deformation considered.

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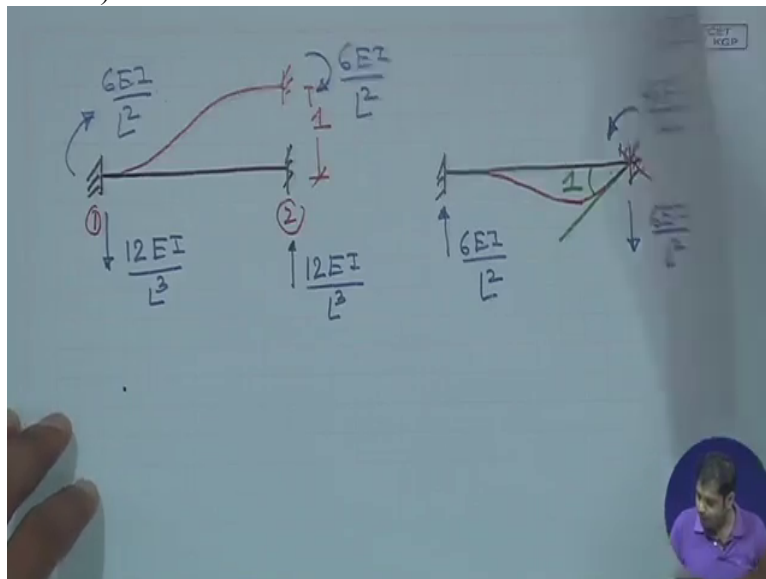
Now if we, if we, if we allow some, allow some axial

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	1	2	3	4
1	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$
2	$\frac{6EI_z}{L^2}$	$\frac{4EI_z}{L}$	$\frac{6EI_z}{L^2}$	$\frac{2EI_z}{L}$
3	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$
4	$\frac{6EI_z}{L^2}$	$\frac{2EI_z}{L}$	$\frac{6EI_z}{L^2}$	$\frac{4EI_z}{L}$

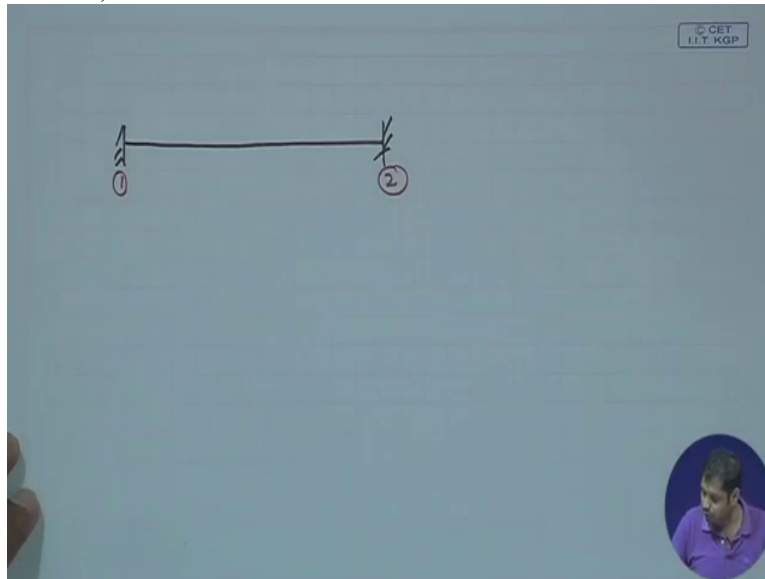
deformation then what happens? Now suppose

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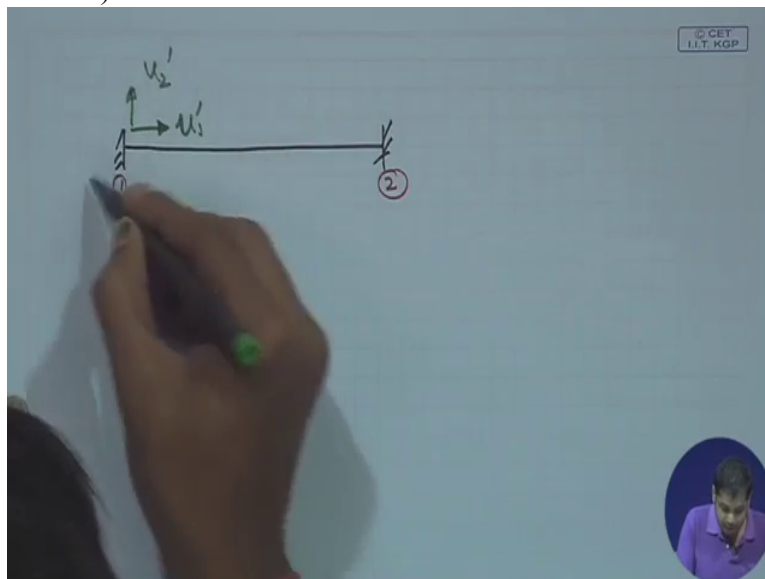
the axial deformation is also allowed. So what happens? This is the beam segment. This is the beam segment. And this is your point number 1, point number 2

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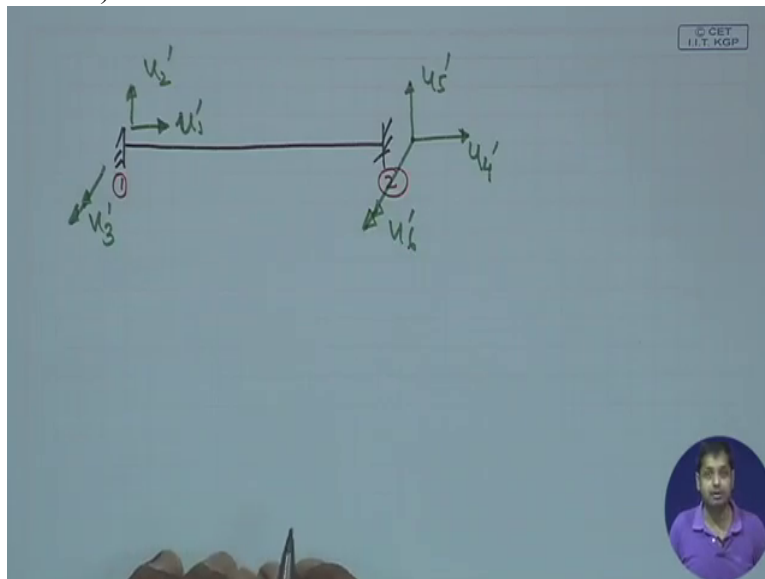
and degrees of freedom are, this is axial deformation, this is u_1 dash, and this is u_2 dash

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and this is u_3 dash. Similarly at this point you have u_4 dash, u_5 dash and then rotation here is u_6 dash,

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Ok.

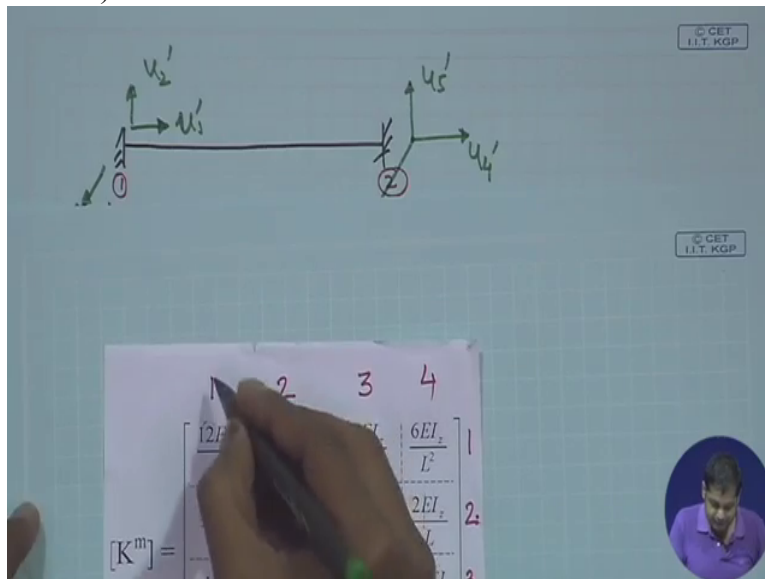
So when there is, u 1 dash and u 4 dash, there is no u 1 dash and u 4 dash, then stiffness matrix becomes, then stiffness matrix is this,

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$$[K^m] = \begin{bmatrix} \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix}$$

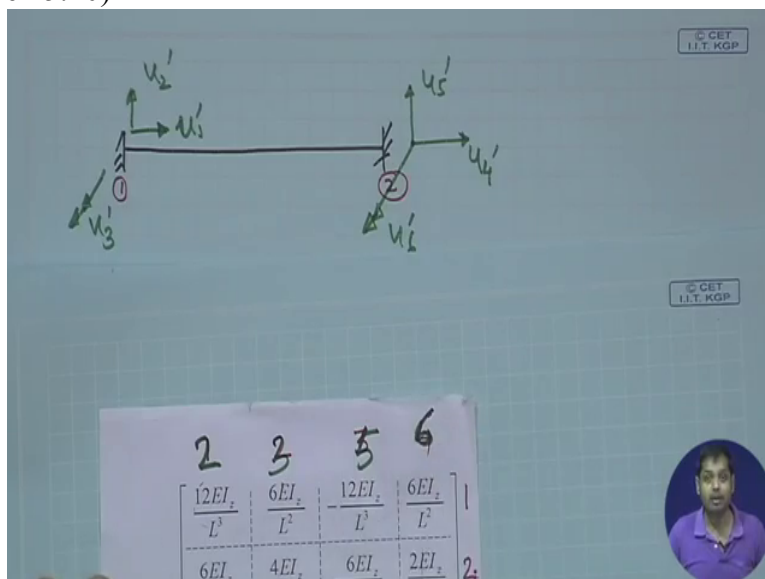
isn't it? So this stiffness matrix corresponds to, corresponds to this

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becomes, this is for, this is for translation, means second degrees of freedom, see now we are talking about, if we consider axial deformation and this is for rotation which is now 3 and this is for again translation at this point which is now 5 and this is rotation at this point which is now 6, Ok

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and similarly this will become 2, 3, 5 and 6,

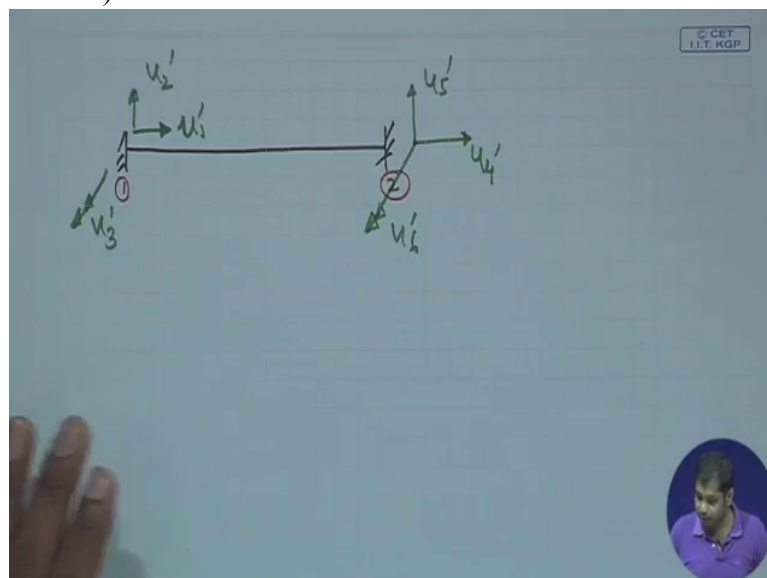
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$$[K^m] = \begin{bmatrix} \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix}$$

Ok.

So this is when we consider axial deformation. Now here we have to insert column number 1, like (1) corresponding column, rows and columns corresponding to first degrees of freedom and the fourth degrees of, fourth degree of freedom. Now we already know

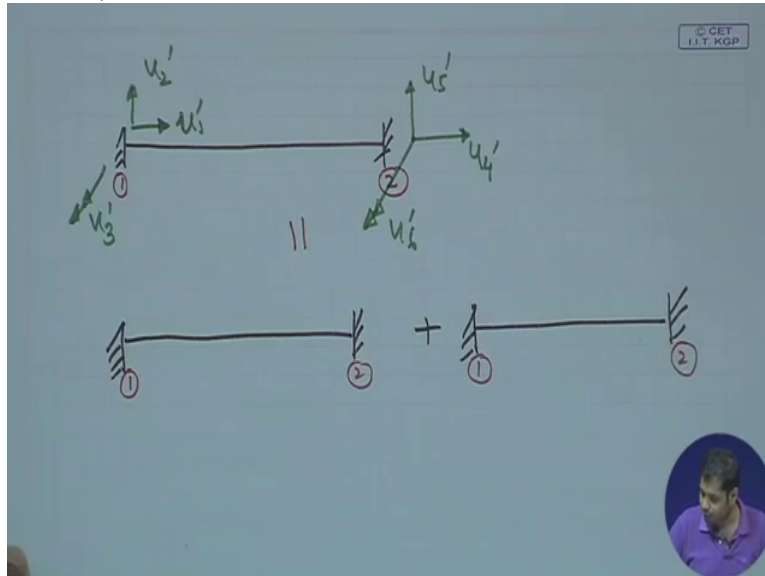
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from truss that if, now this we can decompose, this entire problem we can decompose like this.

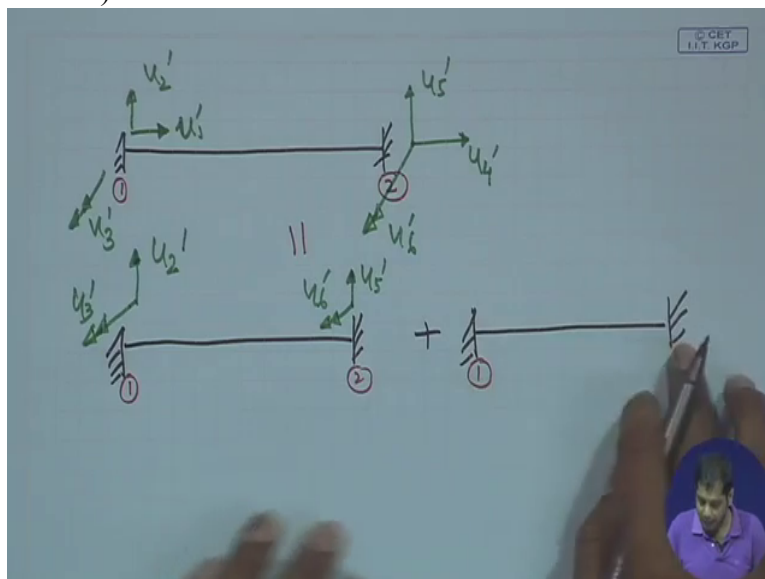
This can be decomposed into 2 problem, which one is this plus another problem this where this is node number 1, node number 2, this is node number 1, node number 2.

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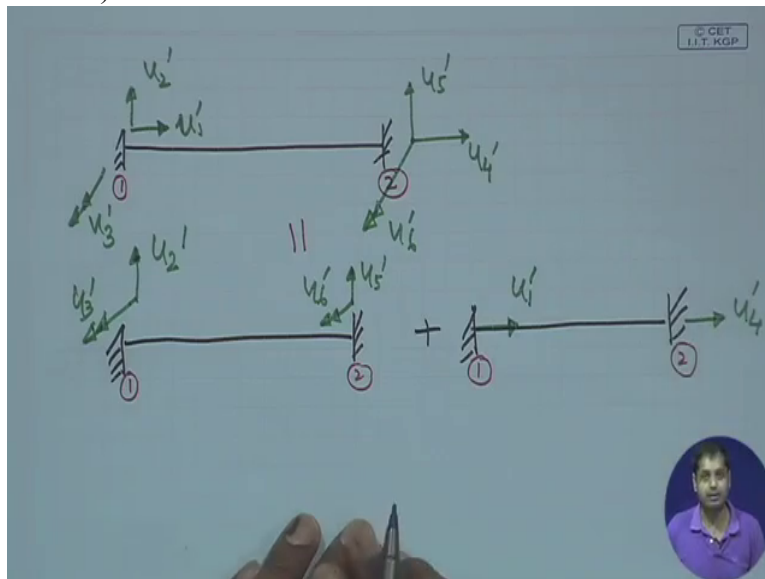
Now here you give only these degrees of freedom, u_2 dash and rotation u_3 dash, Ok and here you give only u_5 dash and rotation u_6 dash and here

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you give only translation u_1 dash and u_4 dash.

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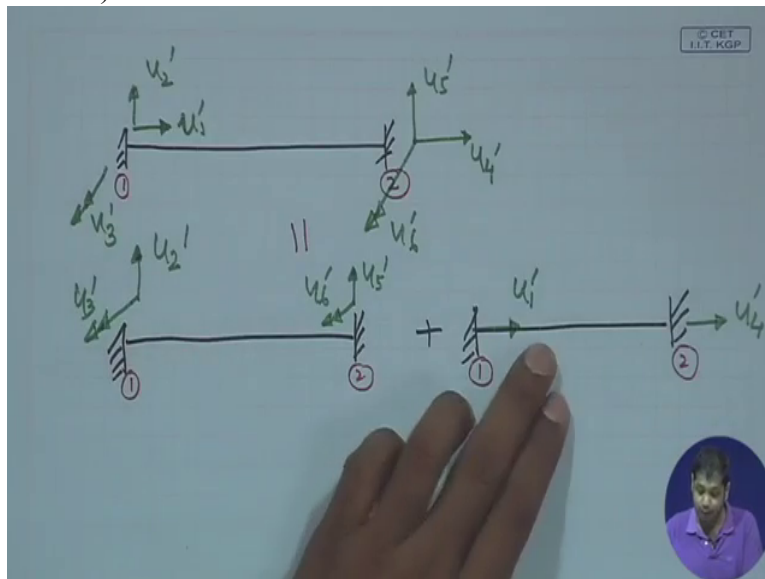
So since linear superposition, linear superposition is allowed here, so we have just now the beam is undergoing deformation this plus undergoing deformation this gives you the total deformation of this, Ok. Now for this we have just now obtained the stiffness matrix and for this it is very similar to truss. We know the stiffness, what is the stiffness matrix. For this stiffness matrix is this corresponding to degrees of

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	2	3	5	6
2	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$
3	$\frac{6EI_z}{L^2}$	$\frac{4EI_z}{L}$	$\frac{6EI_z}{L^2}$	$\frac{2EI_z}{L}$
5	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$	$\frac{12EI_z}{L^3}$	$\frac{6EI_z}{L^2}$
6	$\frac{6EI_z}{L^2}$	$\frac{2EI_z}{L}$	$\frac{6EI_z}{L^2}$	$\frac{4EI_z}{L}$

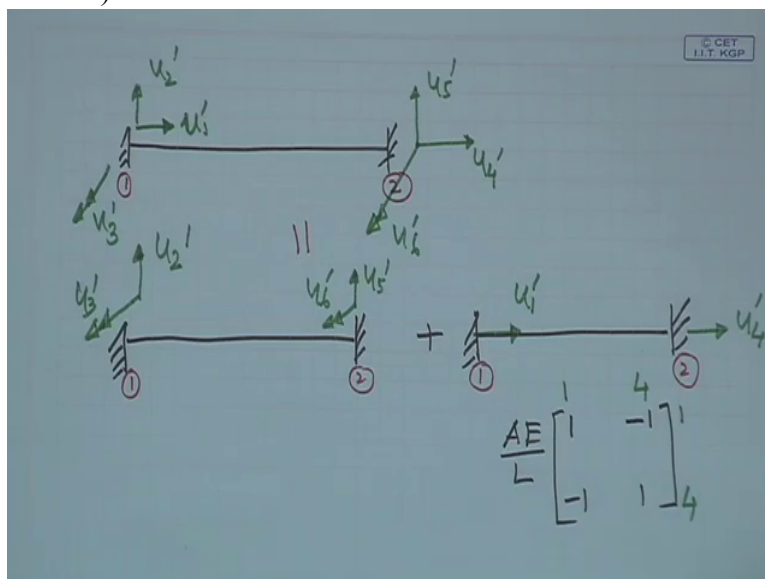
freedom, now degrees of freedom are, are identified as 2,3,5 and 6

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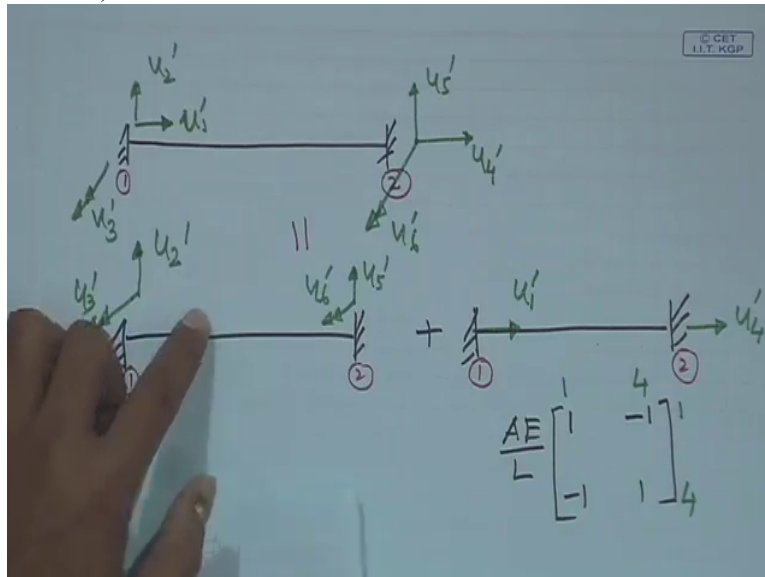
becomes 1 minus 1 minus 1 1. It is for truss and this is degrees of freedom 1, this is degrees of freedom 4, this is 1, this is 4, Ok.

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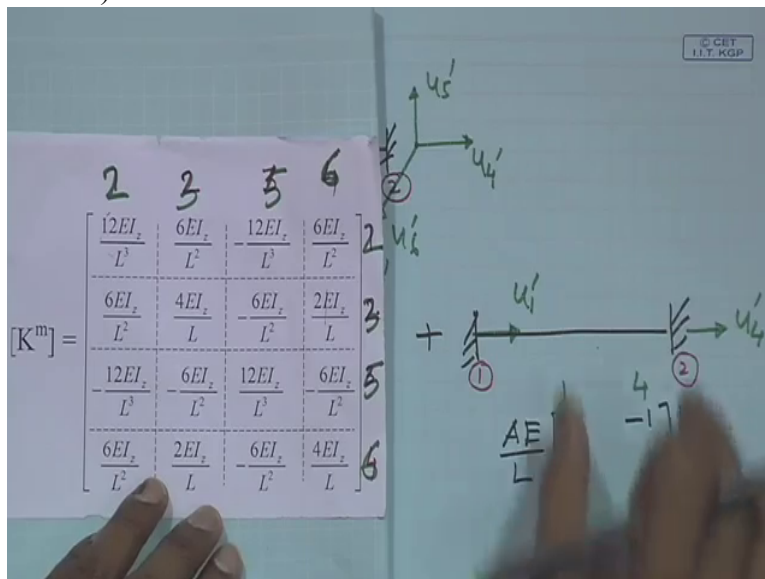
Now what we need to do is we have this, we have this stiffness matrix which is for this beam

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and we have this stiffness matrix which is for this beam

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and we have to combine them. How do I combine them? We will combine like this. We can, we,

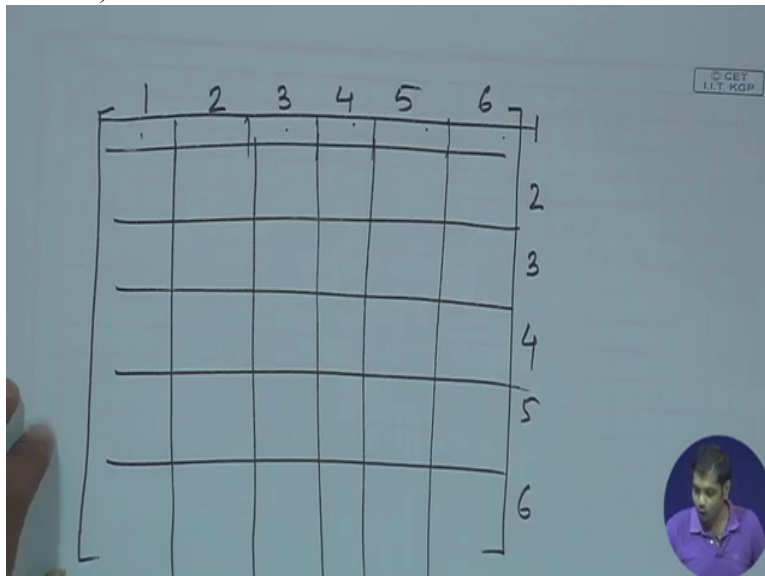
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we, we, we write the degrees of, 2, 3, 4, 5, 6 and then 1, 2, 3, 4, 5, 6 Ok.

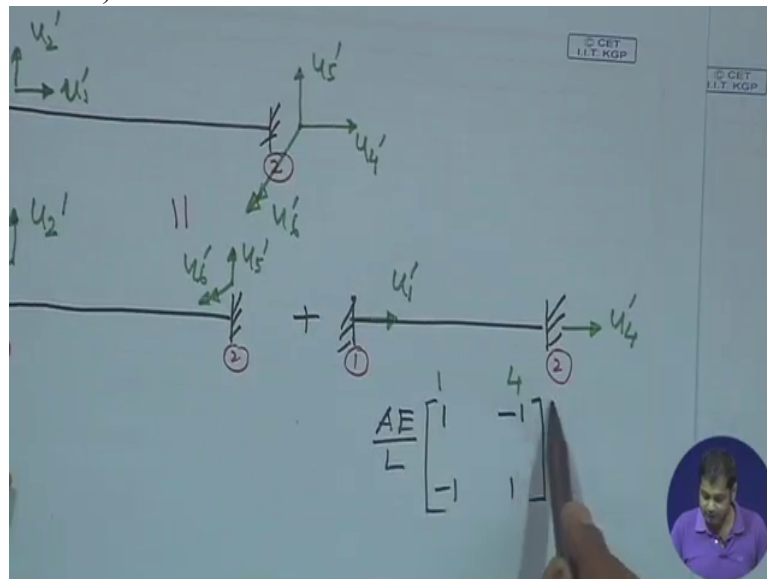
And then this becomes this. This becomes this, Ok. Now so, Ok, this is 1, 1 this is 1, 2 this is 1, 3. 1 4, 1 5, 1 6, Ok. Now

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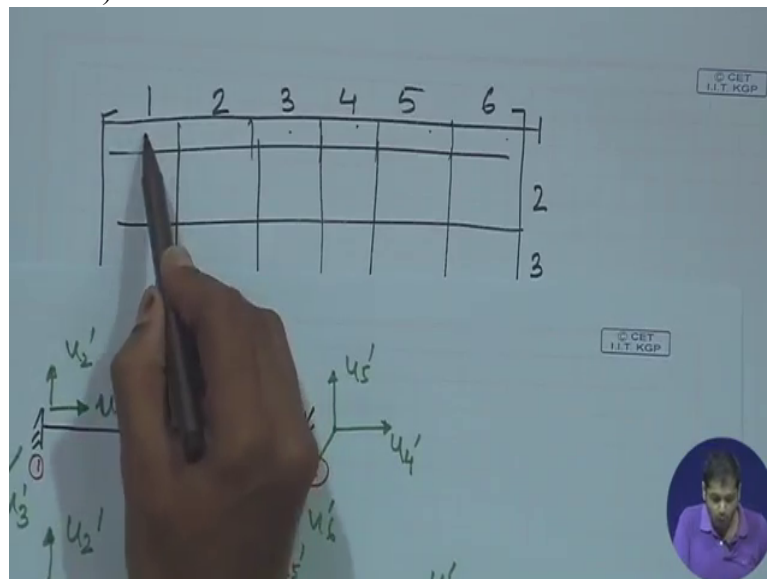
what is 1 1, 1 1 is

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A E by L and 1 4 is minus A E by L. So

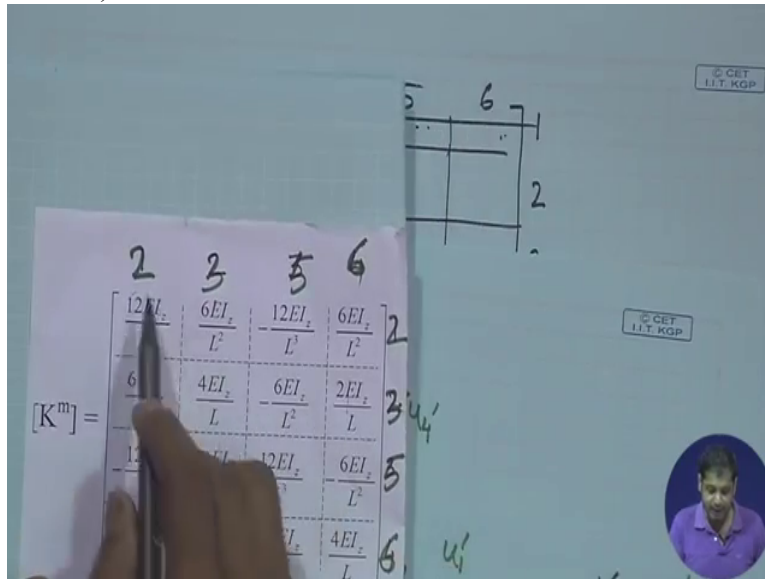
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1 1 will be A E by L, 1 4 will be A E by L.

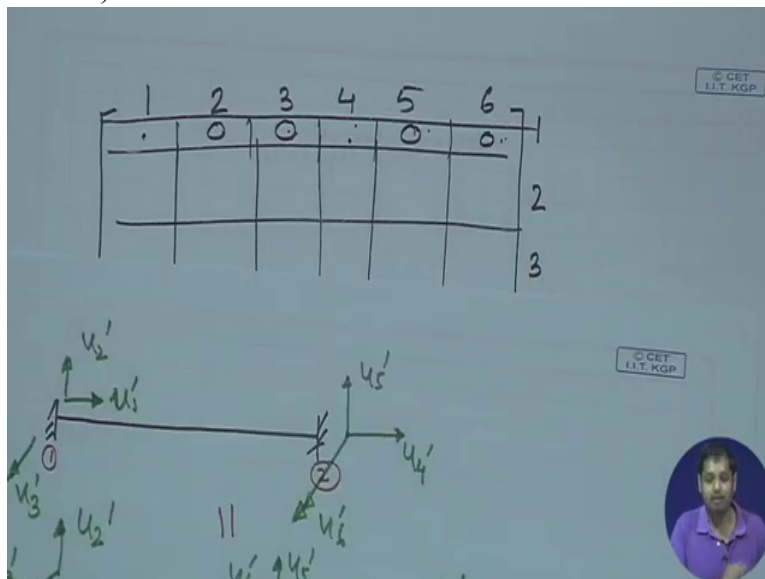
And what is 1 2, 1 3, 1 5, 1 6? You see 1 3,

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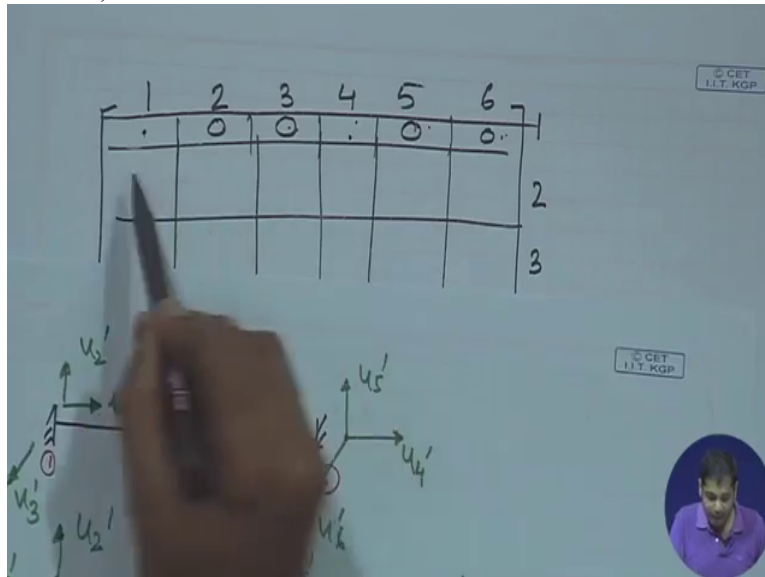
these are all 2 3 5 6, 2 3 5 6, there is no 1 here, there is no 2 3 5 6 here so these all become zero,

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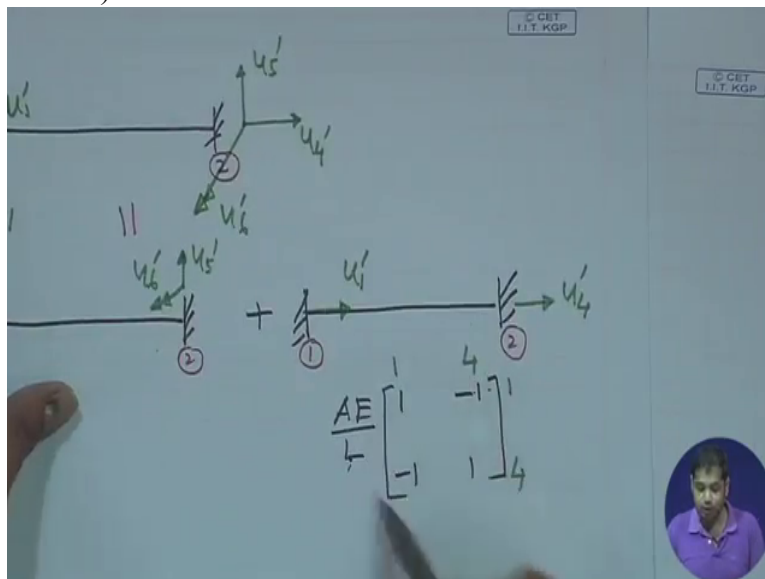
Ok. Similarly

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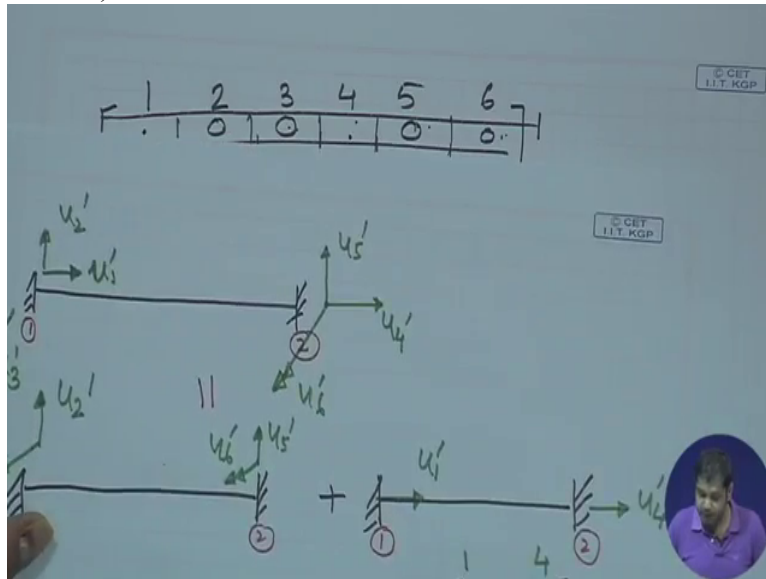
this will be 2 1, 2 3, 2 4, 2 5, 2 6. What is 2 1? 2 1 is, there is

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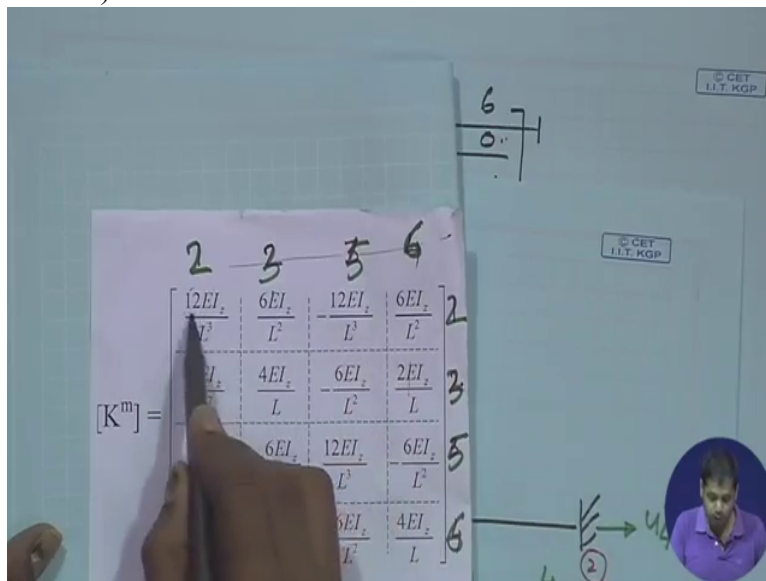
no 2 1 here, there is no 2 1 here, so 2 1 becomes zero. What is 2 2? 2 2 becomes, there is no

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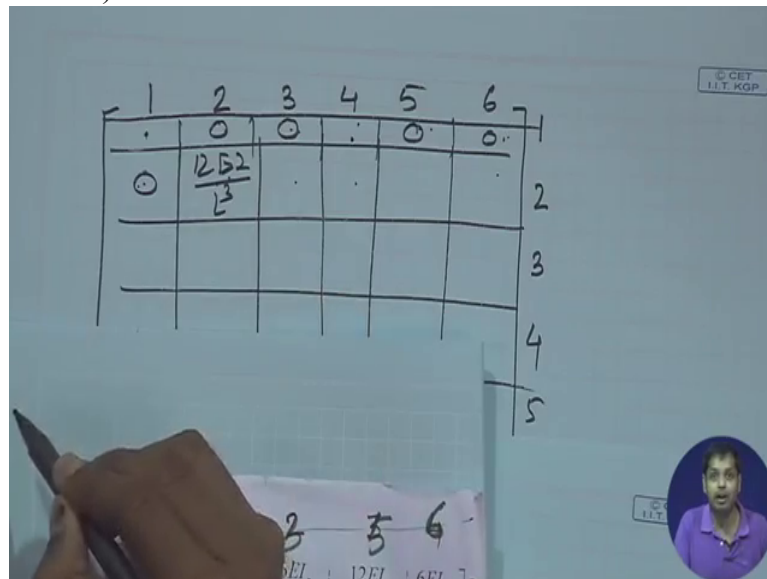
2 2 here, there is 2 2 here, 2 2 is

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12 E i by L cube, so this become 1 2 E i by L cube, Ok. So

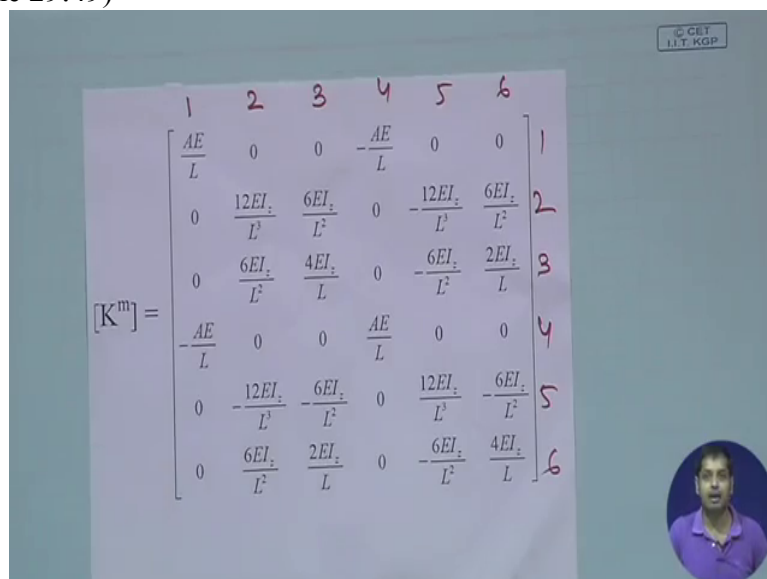
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similarly if we, if we, if we put every element in their corresponding, corresponding cell, the stiffness matrix that we get is this, Ok.

So this will be for 1, 2, 3, 4, 5, 6. 1, 2, 3, 4 5, 6.

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This is element stiffness matrix which is 6 by 6 stiffness matrix because you have total 6 degrees of freedom per element. Now once we have similar stiffness matrices, member stiffness matrices for all the members then we assemble them and get the final, global stiffness matrix which and then we can write $K u = F$ in this form. And then we have to solve

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$$[K^m] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \quad KU = F$$

these equations.

And how to assemble them and how to solve them, how to calculate the loader, this, the load vector F that we can discuss when we actually

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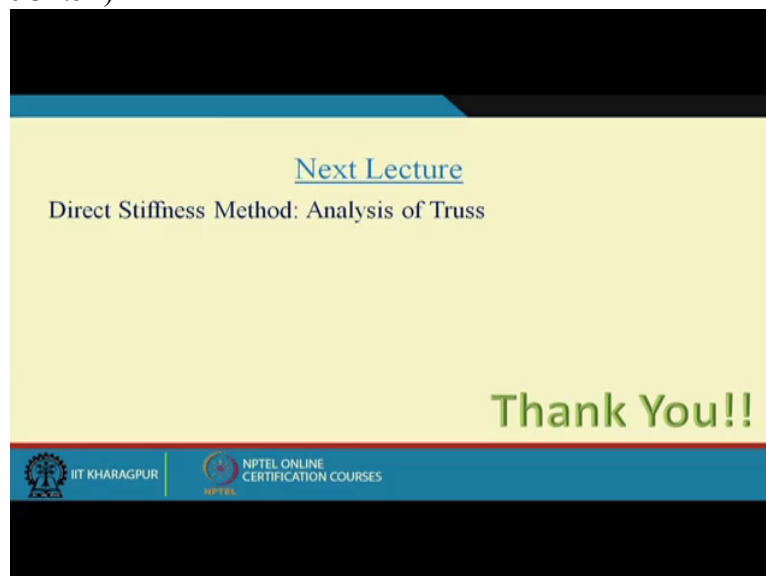
solve some examples, Ok. So as far as the theory is concerned, it is a, I mean, it is not even, not even a brief introduction what you have done so far in last week, last class and this class. It is not even, it is not even, you can see it is a, it is a, it is a small trailer of the, of the entire movie which is direct stiffness method.

Ok, the idea here has been to tell you that there is a method which is, whose, which is based on the concept that we have learnt in the first 11 lectures but the method is not, method is, method is, it is, method can be translated into, into computational code and from there, from there is more advanced, advanced method for analysis of structure can be developed. The idea has been that.

If we really want to learn direct stiffness method then there is a separate course and you have, there is a book on direct stiffness method and you have to take other course and learn it properly. This week, this week is just to take you to the, it is the transition from your basic level course to the more advanced level course and in that perspective, in that perspective whatever we have discussed in last, in last, in the last class and this class, probably it, it, it, it, it can motivate you to learn the method in a more detailed way.

So next, next few classes what we do is whatever the concept that you learnt, matrix method of analysis or direct stiffness method, we will solve three problems, one is for truss, one is beam, and one is, one is, one is frame. Truss is where only axial deformation takes place. In the beam we will neglect the axial deformation. We will assume that there are 4 degrees of freedom, 2 degrees of freedom per joint, the transverse direction and the rotation. And then frame where we will combine both, where axis deformation also is degrees of freedom. In that case, there would be, the stiffness matrix would be 6 by 6 per member. Ok, so next class will be direct stiffness method, analysis of

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truss Ok, see you in the next class,

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thank you.