Course on Structural Analysis I
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No 57
Direct Stiffness Method

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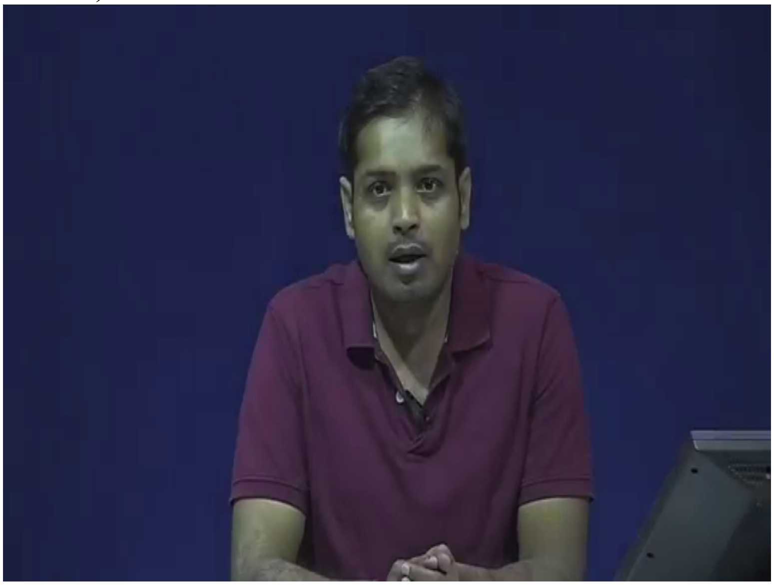

Hello everyone. Welcome to the last week of this course, week 12. You see this week, we are going to introduce a very important concept, just the introduction of that concept, and this week can be, can be, you can, you can, you can treat this week as, as a common link between, between this course. If you remember on the very first day I said this course, this Structure Analysis 1 is the basic level course, the first course on structural analysis and there will be several other courses, more advanced courses that you need to take in your curriculum and this, this week is essentially a common link between, between this course and the, and the advance level course.

This is the transition where, where with the knowledge whatever we have gained in this course Structural Analysis 1 can be used to, to, to, to derive more sophisticated method for analysis of structure that is your advanced structure analysis in subsequent semester. You see all the methods that you have learnt so far, all the methods, now up to, up to week 11, we have, we introduced many methods. We have solved several examples.

Now from this week onwards what we do is you need to look at whatever we have learnt so far as a concept, not as a method, Ok. By saying so what I actually meant is, you see all those methods we have discussed so far are developed 150 years back when there was no computer. Everything had to be done manually. And that's why the method had to be very simple, the calculation method should be such that the calculation, calculations are easier, but you know now hardly if you, if you go to industry and practical purposes the method that you have learnt so far, hardly they are used. Because there are more sophisticated methods developed over the years and those methods are used in the industry.

But what is important, what is take, what you can take from the first eleven week of this course is the concept that we learnt, moment distribution method we don't use nowadays but the concept of moment distribution is still being used, the concept of slope deflection, concept of indeterminacy, determinacy, concept of method of section, concept of method of joints, the concept of mechanics that we leant so far while discussing through method, discussing those methods. That concept is still relevant and those concepts are very essential to learn, to analyze any structure, whether you use any method.

Ok, so what is take from, from the first eleven week of this course is the concept that we learnt, various concepts related to structural mechanics. Ok. Now today we will learn,

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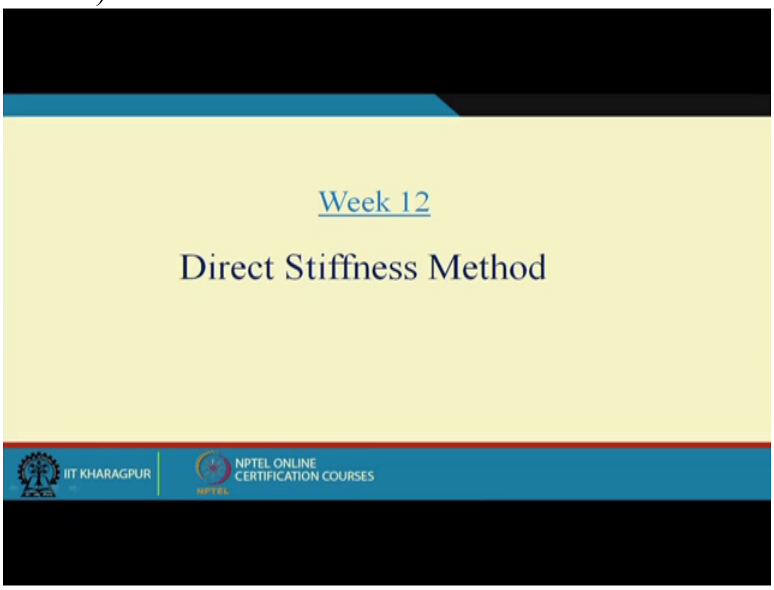

we will just introduce this week is direct stiffness method and more precisely today's class

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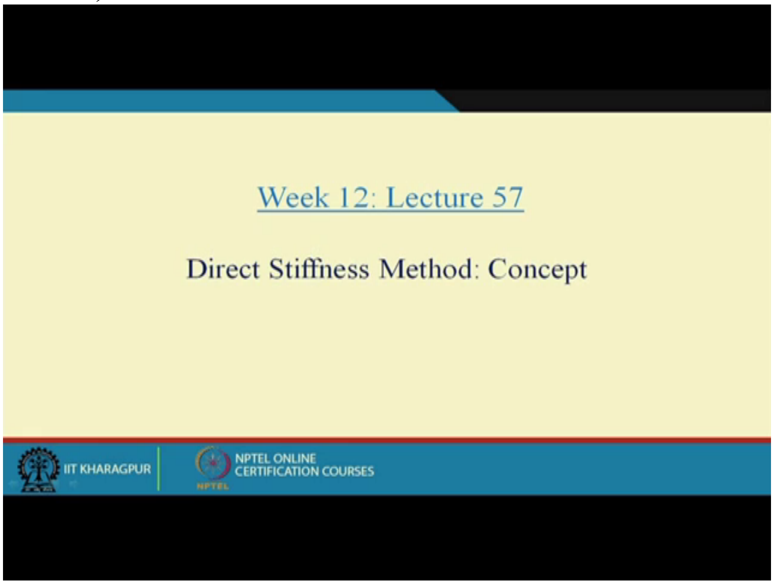

which is the lecture 57, we will just introduce the concept and see how

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this concept can be applied to analysis of truss and then subsequent, subsequent, subsequent lecture we will also see how this method can be applied to analysis of beams and frames and then some examples, Ok.

If you see this is the last;

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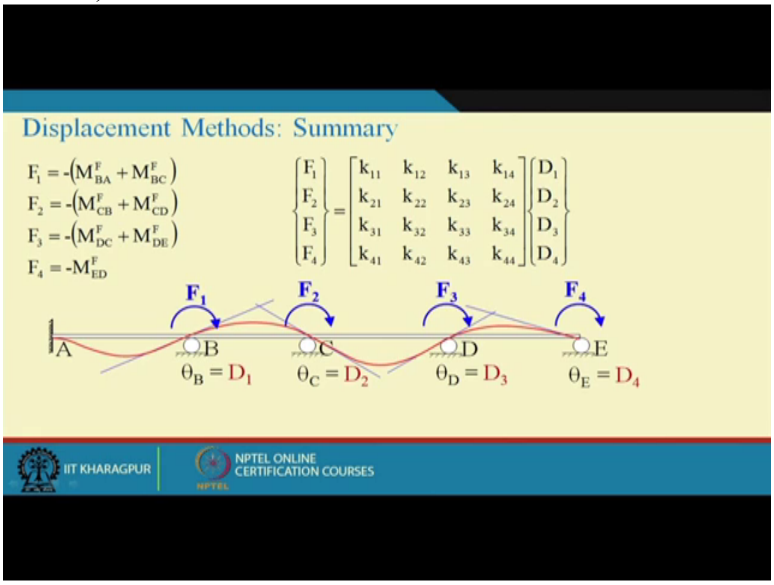

this is the summary of slope, summary of the displacement method that we have already, already learnt. And what it says is you have a, you have a, you have a beam. You have a structure here and this is true for any structure. Now the degrees of freedom that we are interested in this structure is four degrees of freedom, rotation at, rotation at B, rotation C, rotation D and rotation E. So theta, theta B, theta C, theta D and theta E are the degrees of freedom but depending on the structure you can have different, different

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other degrees of freedom of well.

But in this problem

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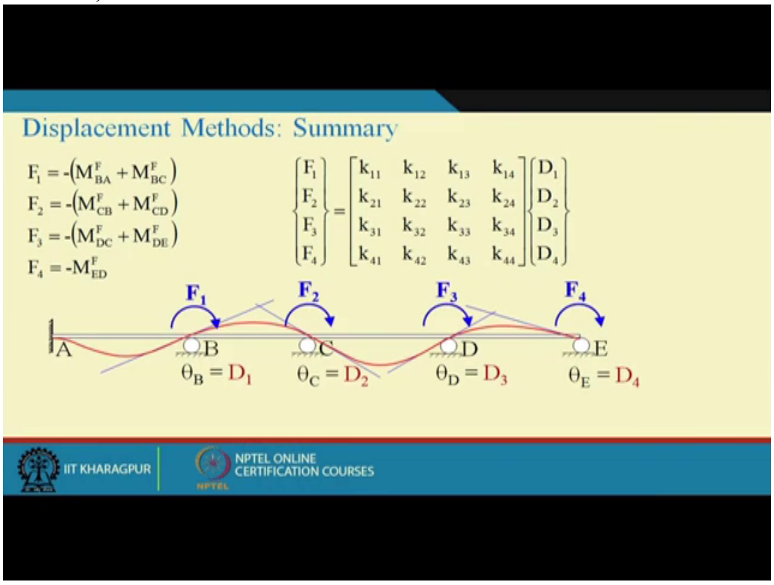

if we have only four degrees of freedom which are denoted as D 1, D 2, D 3 and D 4, and then corresponding forces at the corresponding joints, at B, C, D, E are F 1, F 2, F 3, F 4 which are essentially the, the, the, the, the total summation of the fixed end moments at, at, at a particular joint. Now, so these are the forces at these joints. These are the degrees of freedom at these joints that we are interested in. And then these forces and these degrees of freedom are related, related through a matrix which is called stiffness matrix.

And the k 1 1, k 1 2 these are the elements of the stiffness matrix. Now to get, now k 1 1 is essentially, is the, k 1 1 is essentially, is the, is essentially the relation how the F 1 and D 1 is related and similarly k 1 2 gives you how F 1 and D 2 is related. And similarly k 1 3 gives you F 1, how F 1 and D 3 are related and so on. Another important thing in this stiffness matrix is this stiffness matrix is symmetrical, symmetric, means the off-diagonal terms are, are same. k 1 2 is equal to k 2 1, k 1 3 is equal to k 3 1 and that we have already seen.

Now you see,

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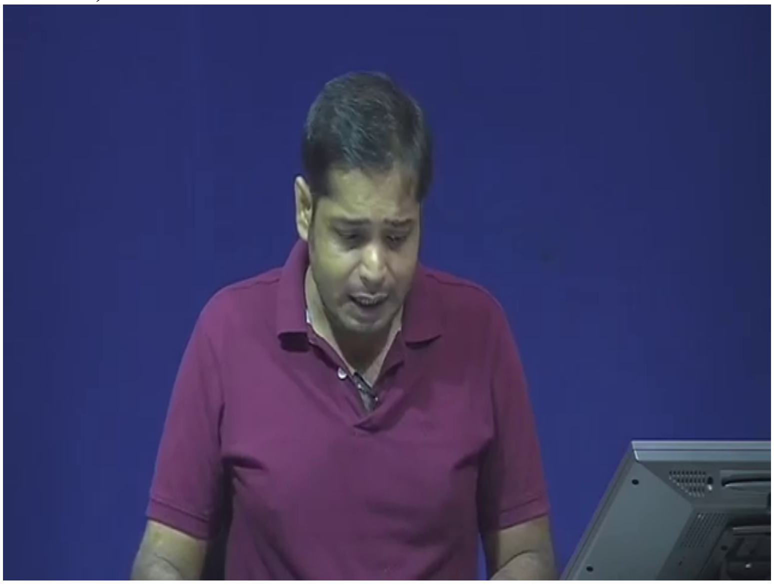

now direct stiffness method is or, or sometimes it is also called matrix, matrix method of structure analysis, Ok, now you see suppose, suppose I have, we know how to design, how to design, we know how to, Ok let's give you an example. Suppose I have

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a circle like this, Ok and if you are asked to determine what is the area of the circle we know it is pi r square where r is the, r is the

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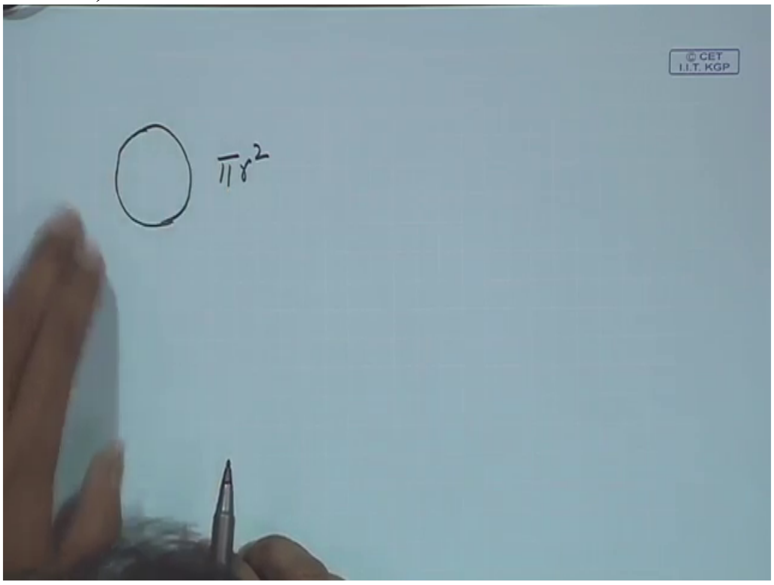

radius of the circle, right? Similarly if we have a square or a rectangle, rectangle which is side a and b, what is the area, area is equal to a into b. We can also determine the perimeter and

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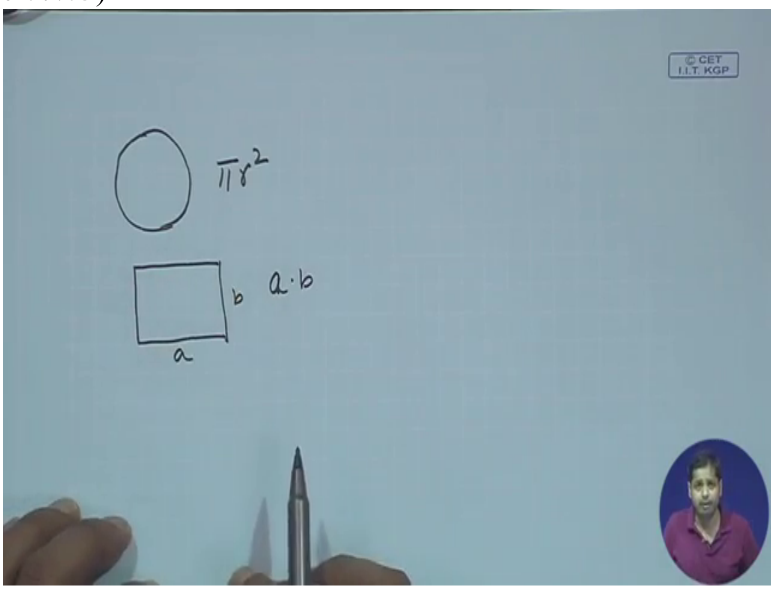

so on.

So for regular structure, regular shape we can calculate the area and the, and the perimeter. Now if I give you an arbitrary area,

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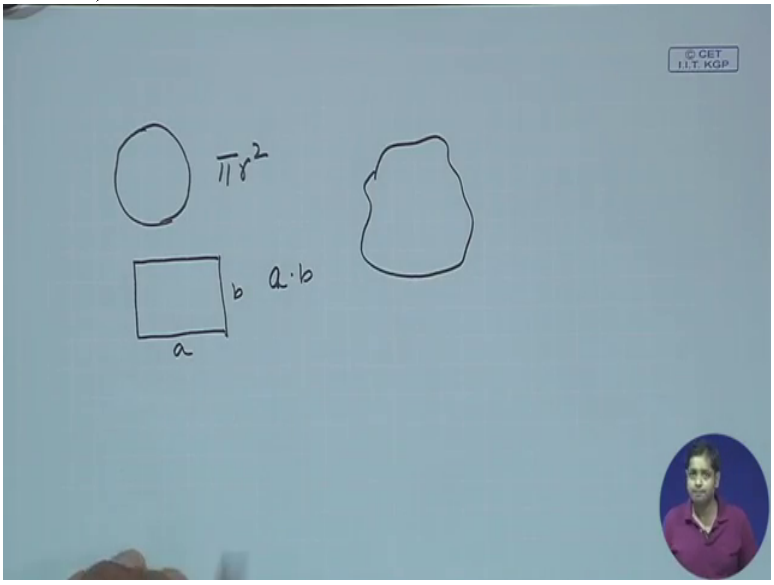

Ok. Now if I ask you to find out the, find out area of, of, of this shape. Now you see, now if we have to, if we need to bring some analogy, these are the, these are the in actual, in practice, our structures are like this. Structures are like this means these structures are just not one portal frame or structure is just not

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a, a cantilever beam or a singly supported beam, a structure is essentially a collection of many beams, many columns with different boundary conditions, with different shapes, different sizes, Ok and oriented in three dimension, there is no symmetry, Ok they may deform arbitrarily in any direction.

So structures are very complicated in real life, Ok. So but all the examples that we have done

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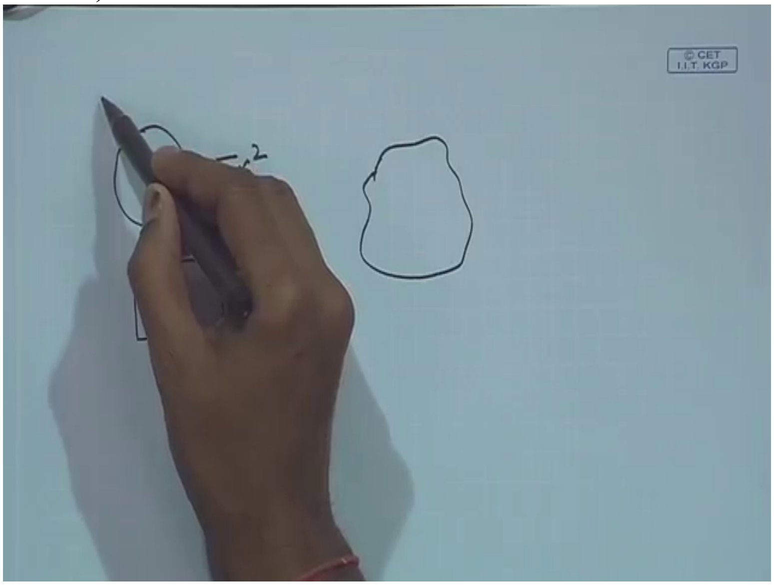

so far or probably in this class also will be, in this week also will be, the example we will be doing are, all structures are very similar to this kind of problem where it is very simple,

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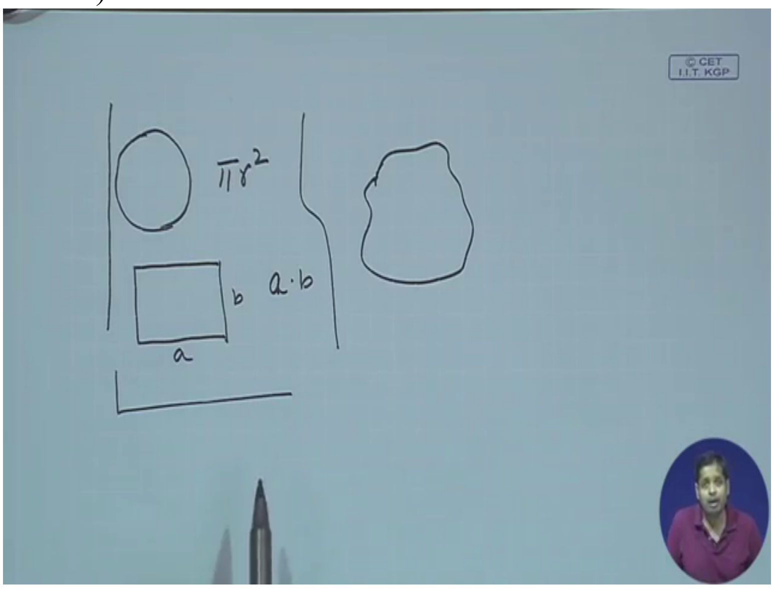

for which applications are very simple but in real life, the structures are actually like this, Ok. Now what we do, if we have to find out the area of this shape, we don't know any formula for that.

One thing we can do we all, I am sure you all, you all know this that we can divide this, the entire, entire area into smaller cells of, of regular rectangle or squares, like this Ok.

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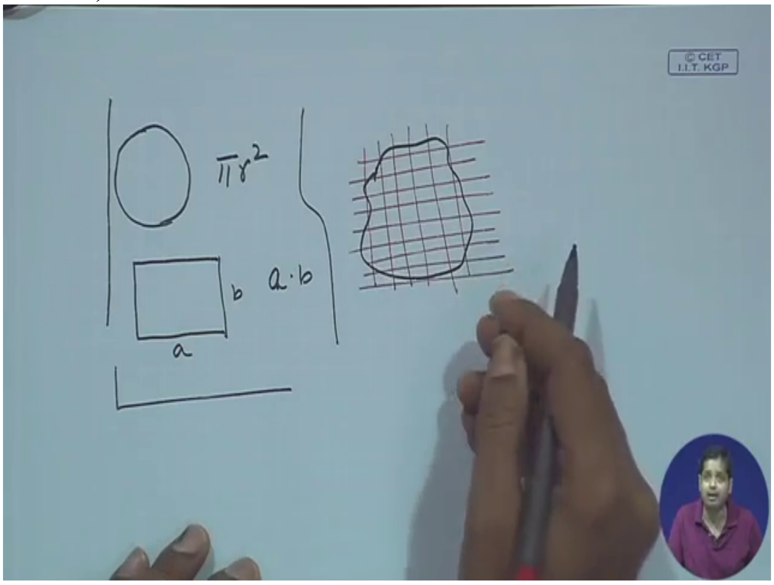

And we have done this probably many times in schools. Now we know the area of each square and then count the, how many squares we have and if the squares, now the boundary squares we can take half or, or 25% depending on the, depending on the how much, how much area is cut by this, by this line, so we can get the number of total squares and then knowing the area of each square, we can get the, we can get the total area of this, total area of this, this shape, Ok.

And similarly if we, now what it, what it tells you, it tells you that if you know, if you somehow, this is your actual problem, this is your actual problem, Ok. Now if you somehow decompose the problem, divide the problem into smaller sub-problems such that every sub-problem you have the information, you know

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the solution. For instance in this case the actual problem is the entire area, finding the area of this entire, entire loop and then, but which we cannot find out because we don't know the formula for that.

And then what we do is we divide the entire problem into smaller sub-problems and sub-problems are finding the area of each square, Ok. Now each sub-problem we know the result, we know the solution. We have the information. Now, and then what we do is we assemble that information to get the solution of the final structure, Ok or in this case, solving the final problem, final system.

Now in real life when we deal with any problem and believe me it is not particularly for structural analysis, it is true for, true for

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any system, Ok, any system if you see as an engineer at the end of the day, you need to find out solution of some real-life problem, right? You need to understand the laws of nature and mimic those laws to find out the solution. And when you actually, when you, in real life when you face a problem and for those problems doesn't come with a tagline that this problem belongs to civil engineering, this problem belongs to mechanical; this problem belongs to aerospace and so on.

The problem comes as a problem and as an engineer you have to deal with the problem. Ok, so now

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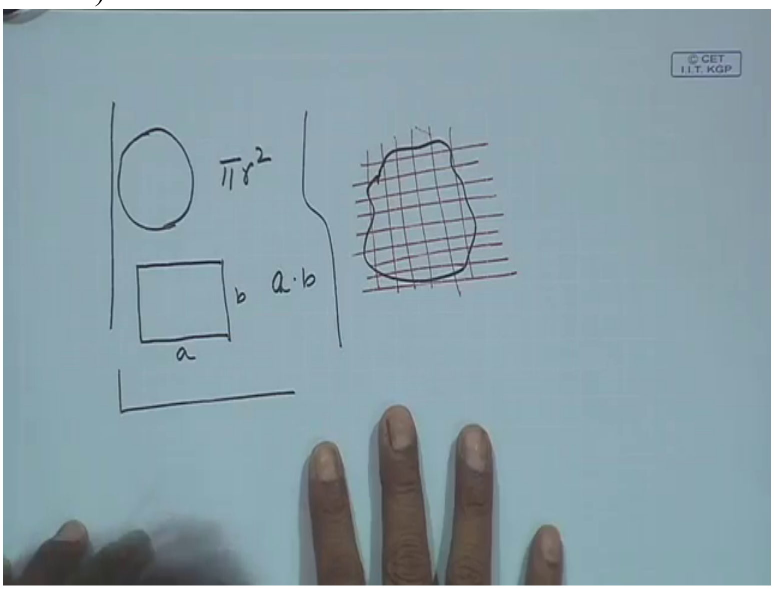

most of the time,

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almost all the time that we will see when you, when you deal with the problem, generally process, generally methodology that we use is divide the problem into small sub-problems and such that for all sub-problems we know the, we know the solution, we have some information and then we assemble those information to get the solution of the entire system.

Now direct stiffness matrix or matrix method of structural analysis is essentially the premise, one premise of this method is this. You divide the entire system into small, small systems. Get the information of all the systems and what information that we will see. Now we get the information of all the system and then assemble them. Now depending on the problem you are dealing with, as I said what information, that depends on the what problem you are dealing with.

Now if we, if the problem is structure analysis problem like this, our information is very similar to

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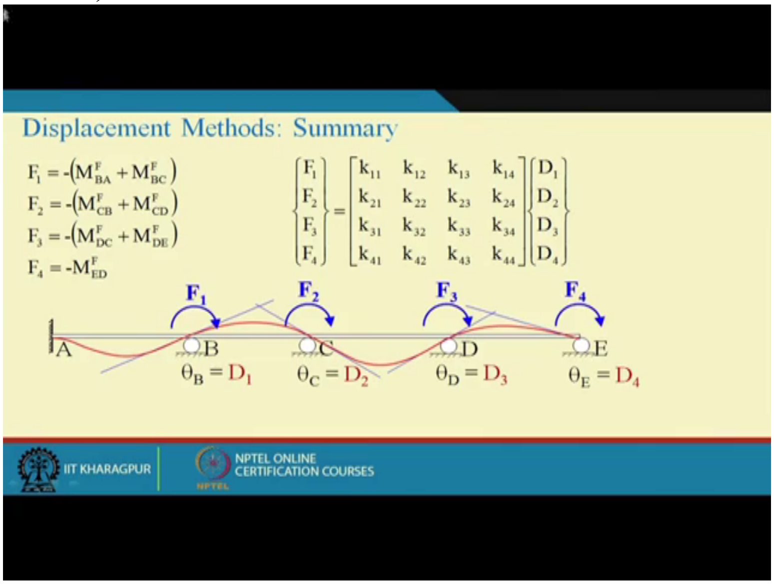

this, how the forces and displacements are related to each other, Ok. So, so means if we have, if we have a force,

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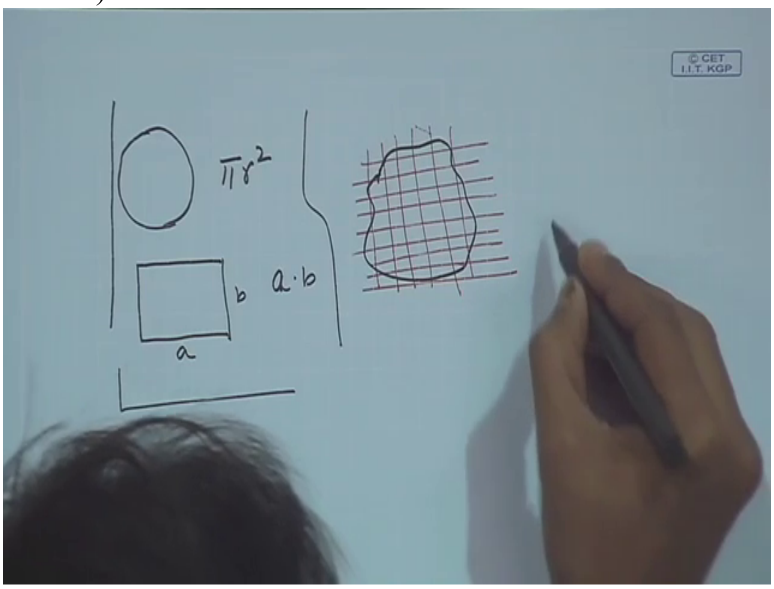

if we have a force this is the force and then this force is equal to some stiffness K and this U is the displacement. Ok and this K is the stiffness,

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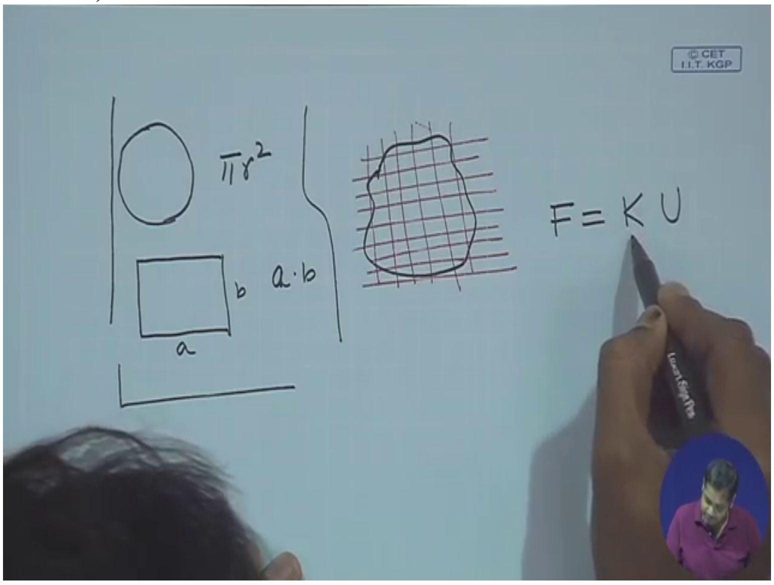

this is called stiffness. This is stiffness or deformation, and this is force, this is,

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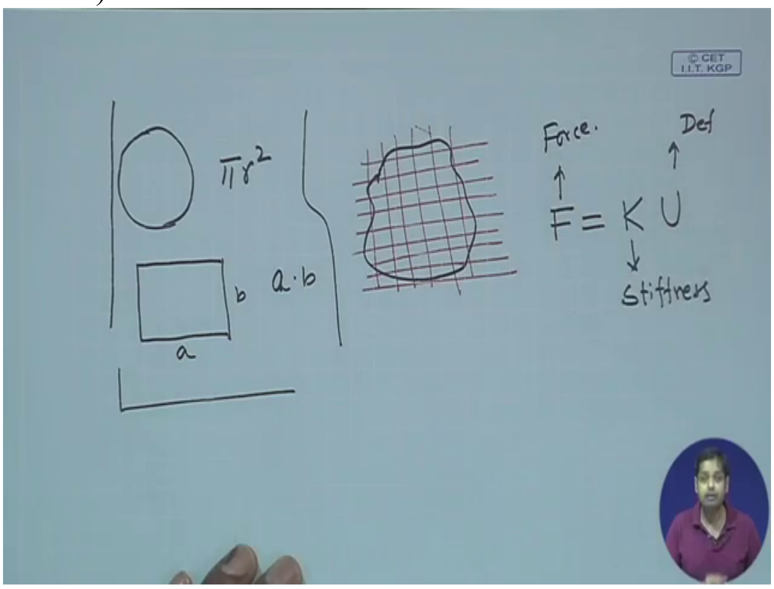

this is force, Ok.

Now if we, if we, if we know this, if we know this relation for given, given, given system, given structure, then what you have to do is we, if we, if we know the stiffness of the structure, then for given load, applied load on the structure, we can find out what is the deflection of the structure. Or for a given, given, given, given displacement we can find out what would be the corresponding forces caused by this deflection. And if you look at all the exa/examples, all the, all the

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methodology that we have learnt so far in first 11 week we actually did the same thing but in a different way, Ok.

Now, now the process that will be followed, that is, that is, that is, that, that concept can be translated in a way that it can be implemented in computer because most of the, all realized structure you cannot really do manually. Remember moment distribution method, you cannot, for a beam it is fine, for a portal frame it is fine, for, for but for any real life structure it is very difficult to do all these iterations manually. So you need to, the concept is fine but the translation of the concept needs to be in a language that can be, that can be, that can be used, that can be put in a computer code so that we can analyze these, we can analyze any,

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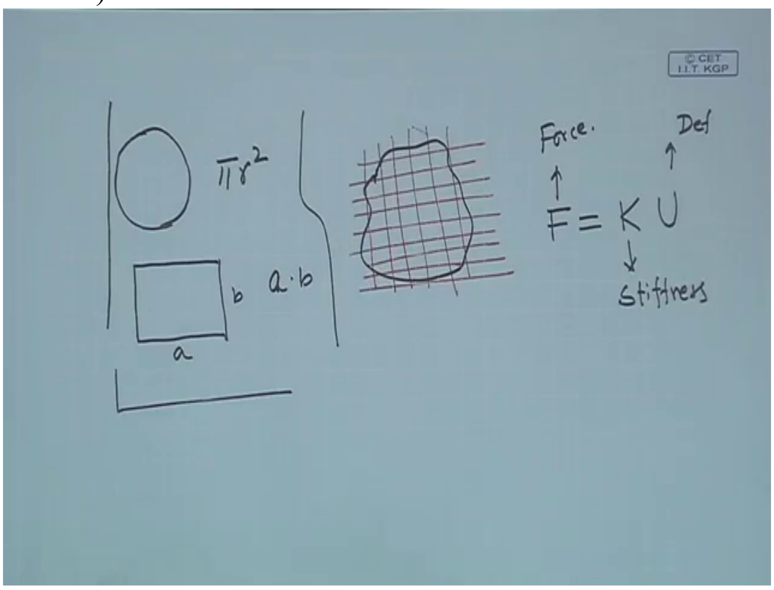

any,

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you can analyze the problem that actually you face in your life.

Now here

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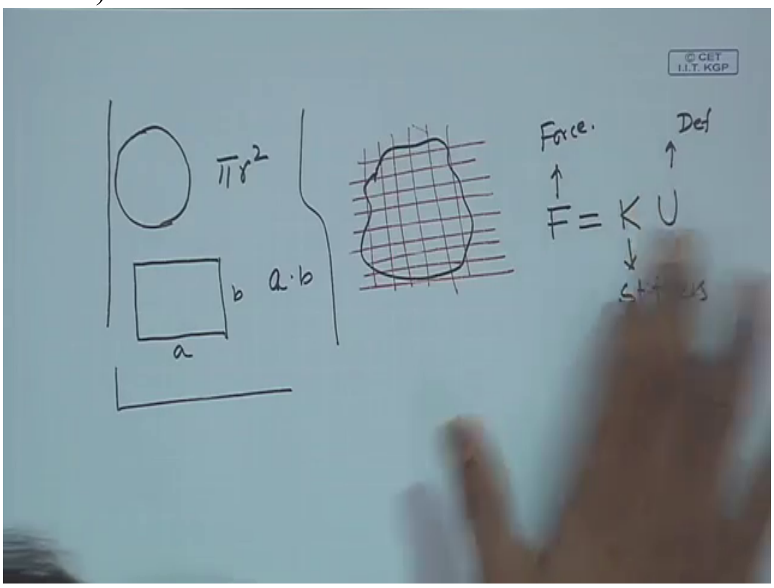

the force is equal to stiffness into displacement. This you have learnt again in previous classes as well but now the theme is, as I said, this

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concept needs to be translated in a language that can be put in a computational code, Ok. Now, so what we do is let me, see if we, for direct stiffness method, analysis of structure for direct stiffness method, actually we need a separate course, Ok. But idea of this week is again not to give you entire details of this method, idea is to, is to

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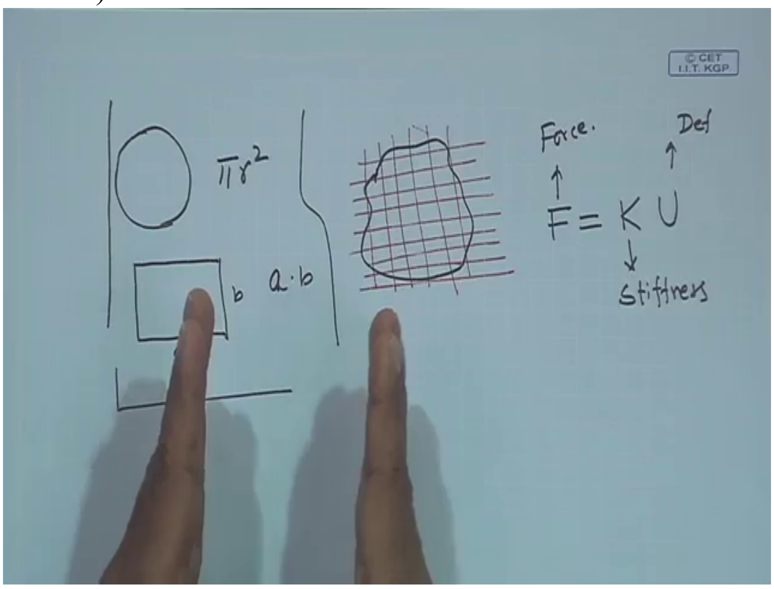

have a transition

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between your basic level course and more advance level course and this is the transition for that.

So here what we do is, what we will do is, we will just briefly try to understand the, the, the basic concepts and probably all these implementation issues where minor, minor, computational issues that probably we need different courses for that, Ok. Now

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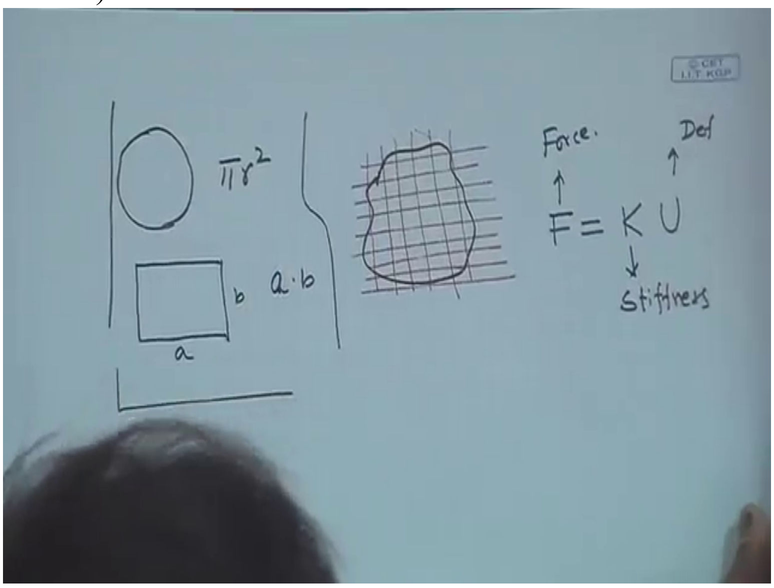

let us, today we will demonstrate this through, through a

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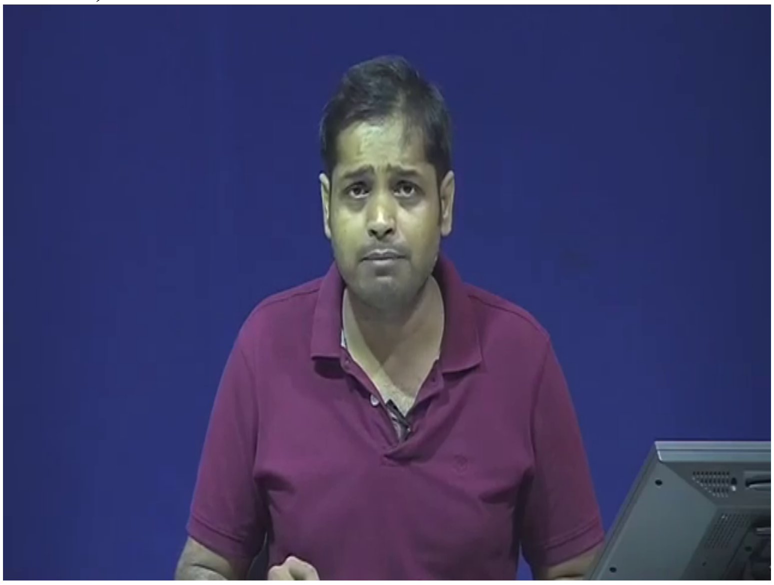

problem, a truss problem, just the concept, numerical demonstration we will be doing in the subsequent classes. In the next class we will see the similar concepts for beams and frames, Ok.

Now we will see, take,

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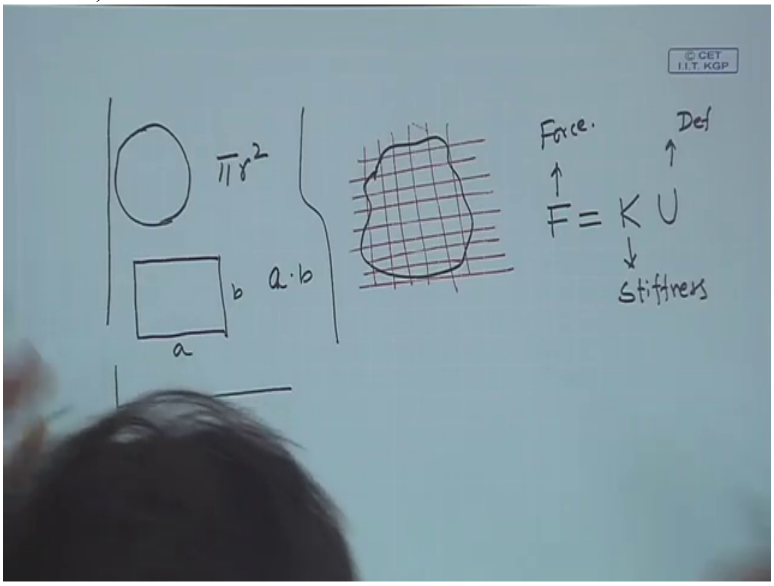

take a truss, any truss like this, suppose this truss, Ok. This is again, this is a deterministic truss and this is subjected to any load say P, any load say P, Ok. And

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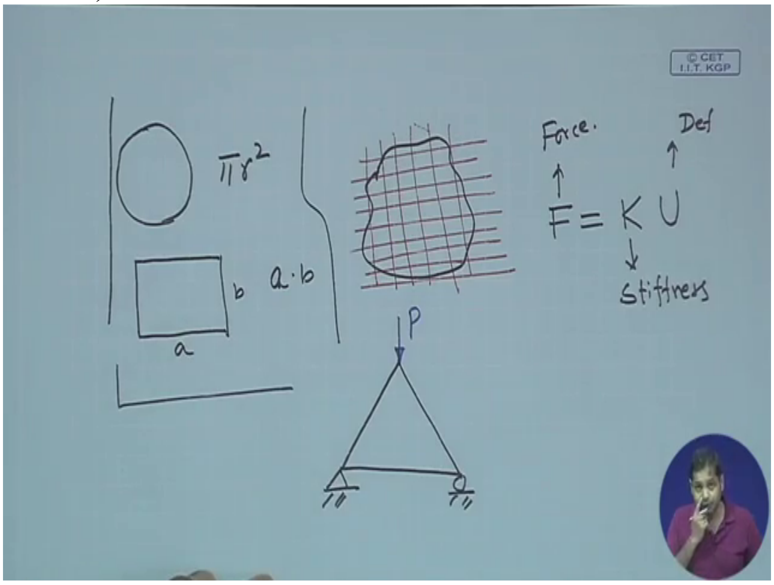

we need to solve this problem, we need to analyze the truss, we need to find out what are the member forces and corresponding displacement. Now this method that we are going to discuss that is, so the deterministic structure, indeterministic structure,

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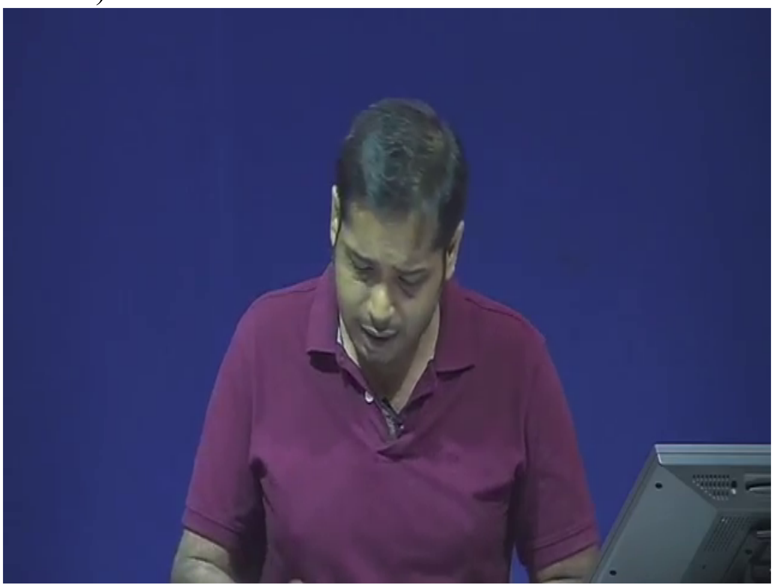

every cases you can find out, even you can apply it, Ok.

Now this is the real system, so

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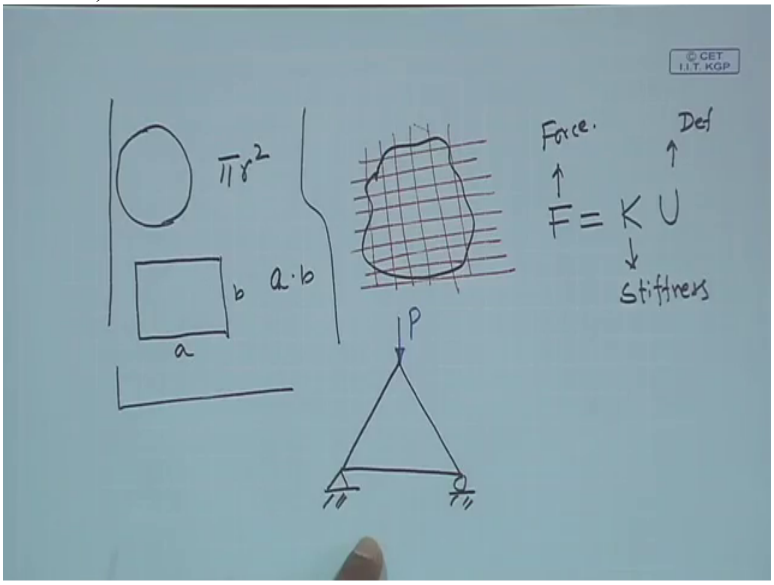

this system is this. This system is this, Ok. Now this cannot be, now what we do is we decompose the problem into small, several sub-problems, Ok.

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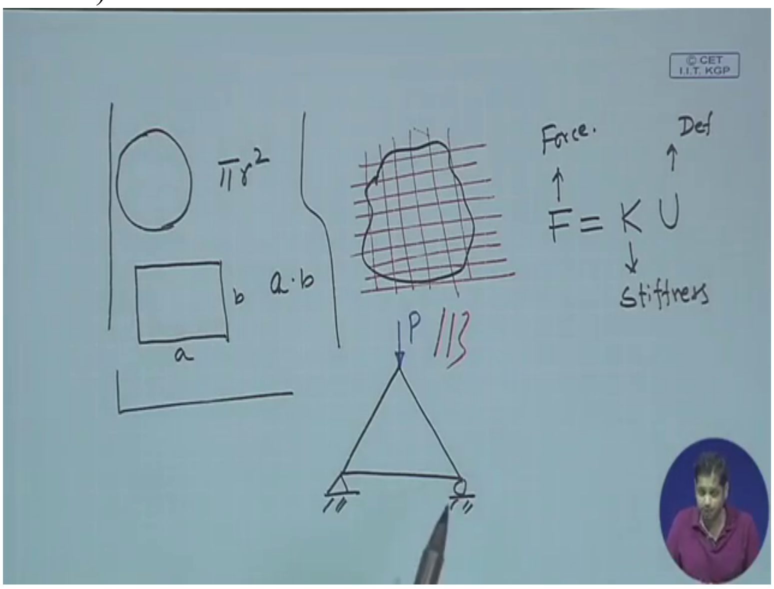

Now we decompose the problem into several sub-problems means let us take this is one member, this is another member and this is another member, Ok. So now we deal with each member separately, writing this relation for each member

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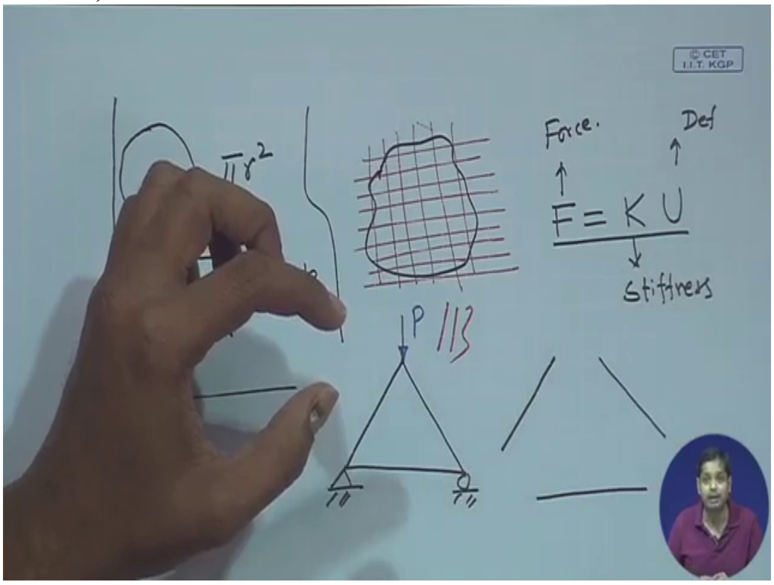

and then if you look at this example, this

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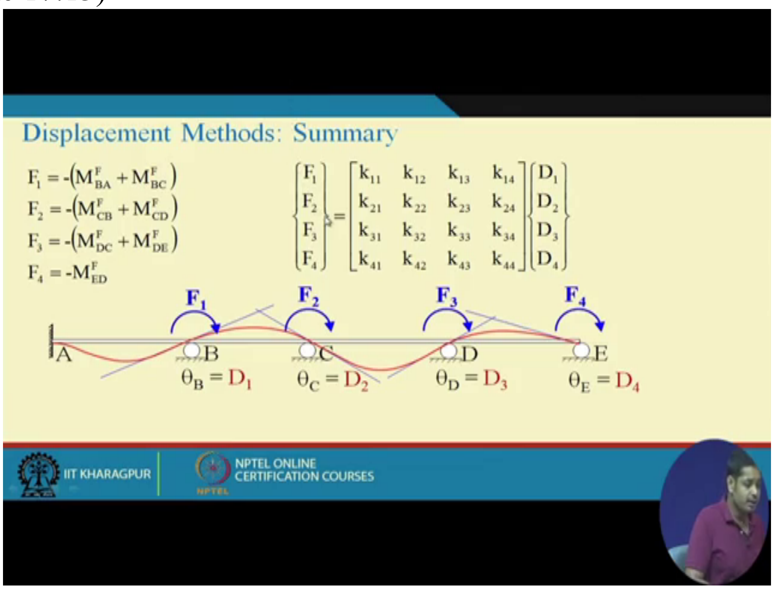

example, this is force, stiffness, displacement, this relation is for the entire beam, entire beam A B C D E, Ok.

Now what we, what we are going to do is, instead of writing this equation entire, for the entire beam, first we divide the beam into small, small beam and similar thing we did in slope deflection method as well, we treated each span as fixed span if you remember. Now, and then write this relation for

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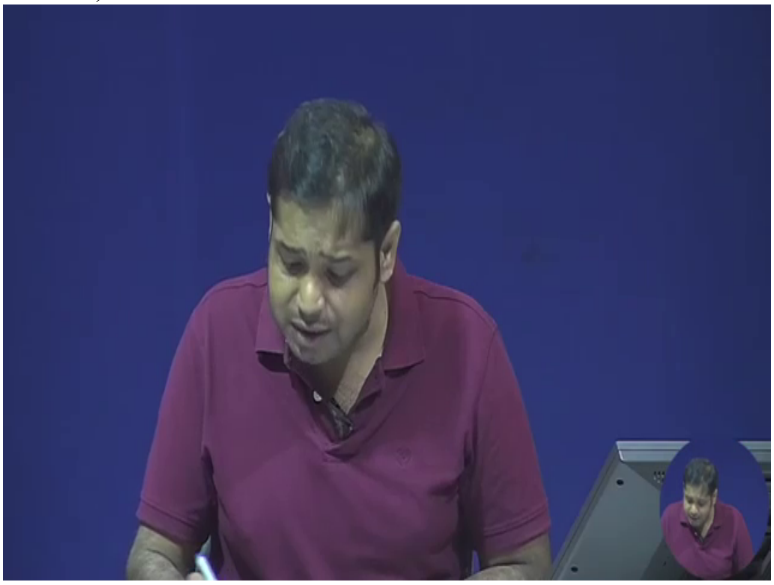

each member, Ok, here in this

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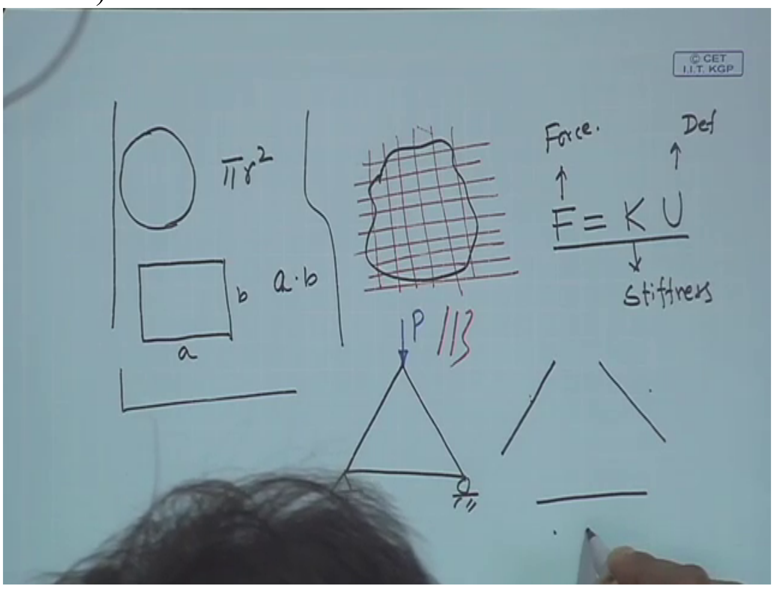

member and then assemble this relation of, of all the members to get the similar expression, force equal to stiffness into displacement, similar force displacement relation for the entire structure, Ok.

So we divide into smaller parts like this and then write the force displacement relation of these several parts and then assemble them to get the force displacement relation of the entire part, Ok. Now what we do is, why it is, Ok, now let's see how, how, how it is to be, how it can be, how it can be done, Ok.

Now

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take any truss member, Ok and we already know truss members are two force

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members. Either they have, they are subjected to compression or they are subjected to tension. So either it is force is like this, force is

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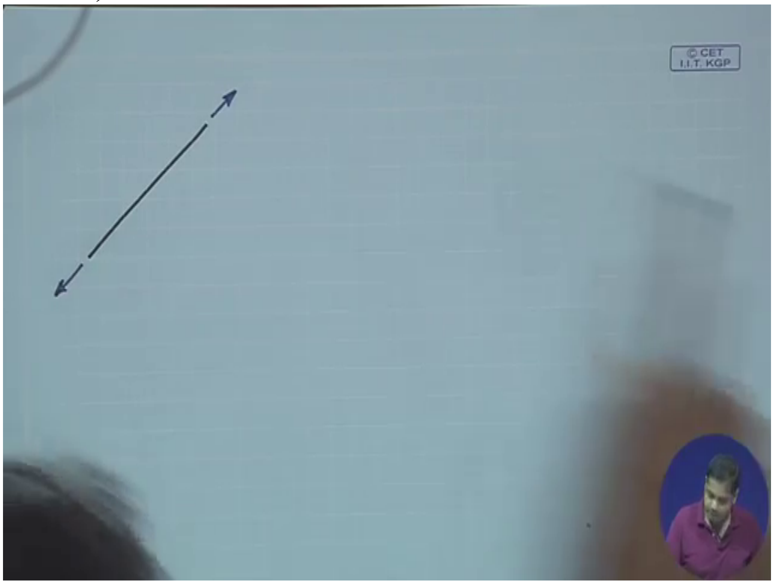

like this or member is this, force is like this, Ok and as per our sign convention and this

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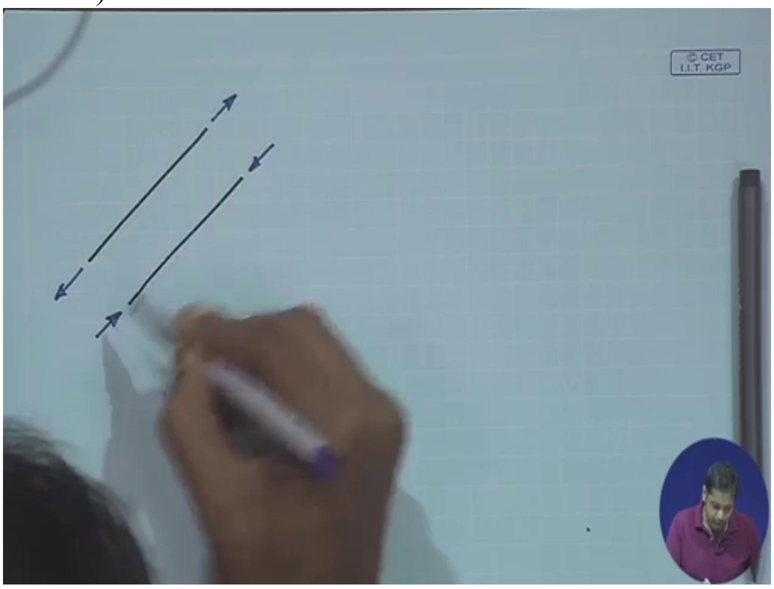

is tension, this is tension, tension and this is compression. These are

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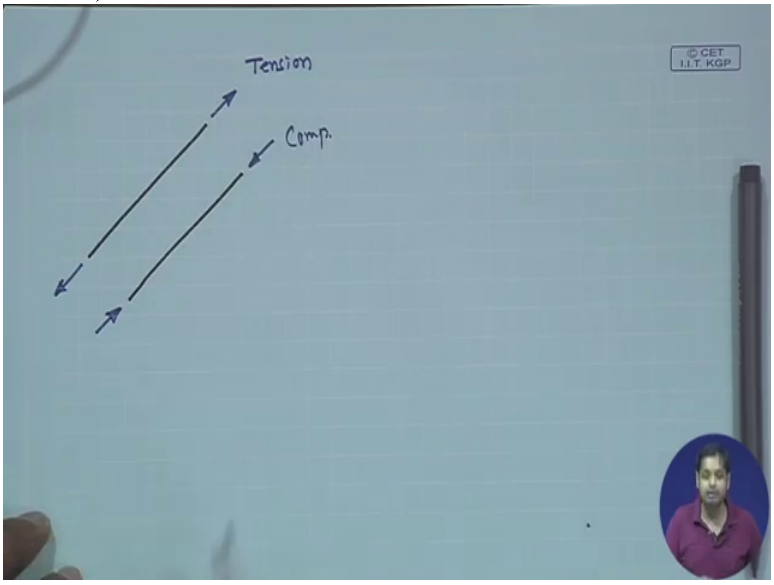

sign conventions, so any truss member.

Now you see, now if you remember, Ok, now since this member, these members are along the axis of the truss, they are either compression or tension, now what happens, this force will cause deformation in the, deformation in this truss member but that deformation will be always in axial direction. So either for this, this will cause, this will cause elongation of the truss, elongation of this truss member in this direction and this will cause reduction of length in the truss member in this direction.

So either these members, whatever deformation takes place in this member, that deformation is always, always axial deformation, either they, either the deformation causes increase in length of the truss or truss member or causes decreases in the length of the truss. So deformation always in this direction, Ok. Now let us find out, let us, now suppose

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that deformation is, suppose in this member,

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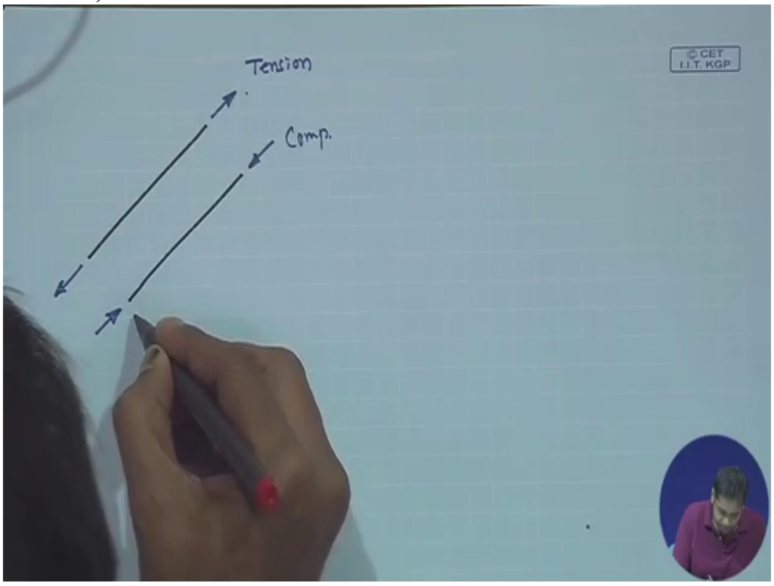

this member is, this is, this is point number 1, or joint number 1 we say, say this is joint number 1 and this is joint number 2, Ok. Or this is joint number 1 and this is joint number 2, Ok.

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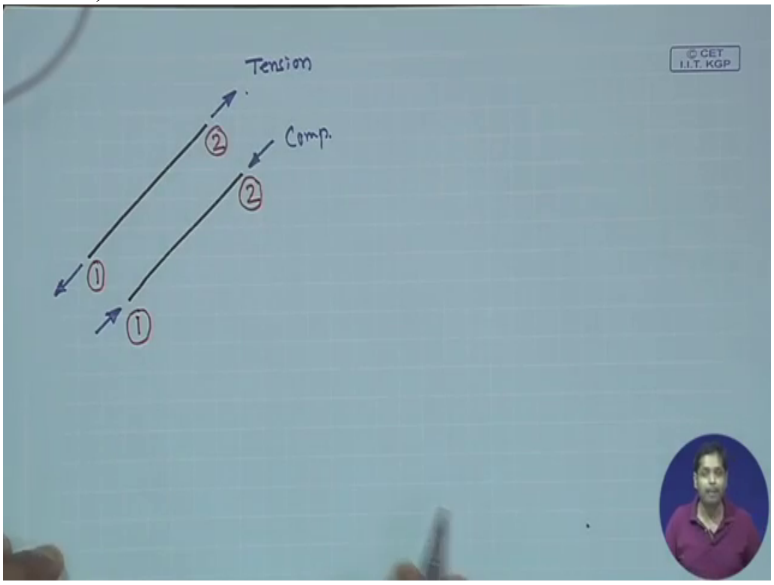

Now we take tension as positive and compression as negative, Ok. Now we, if you remember the Hooke's Law, Hooke's Law said that your how this stress and strain are related. We know that sigma is equal to Young's Modulus into strain,

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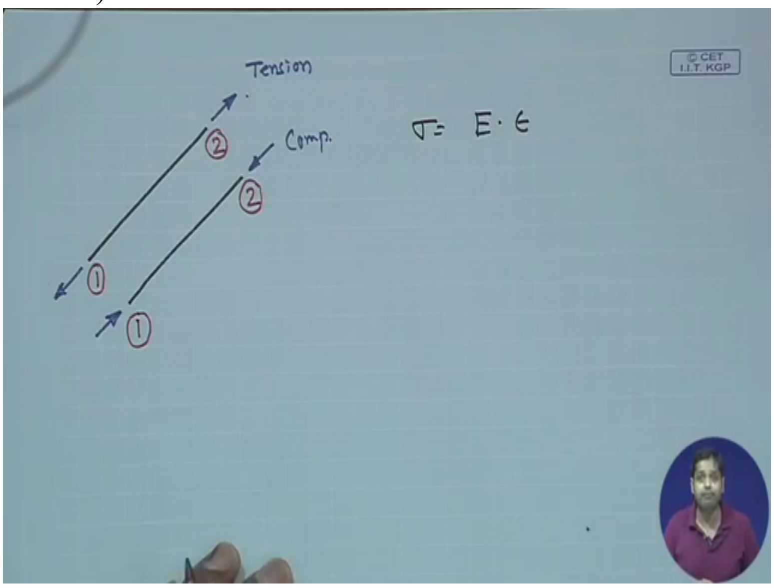

Ok. Now if suppose the cross-section area of these trusses, this truss member is A. Cross-section is A and suppose this truss is subjected to a force called F and again, suppose this Young's Modulus of this truss member is E.

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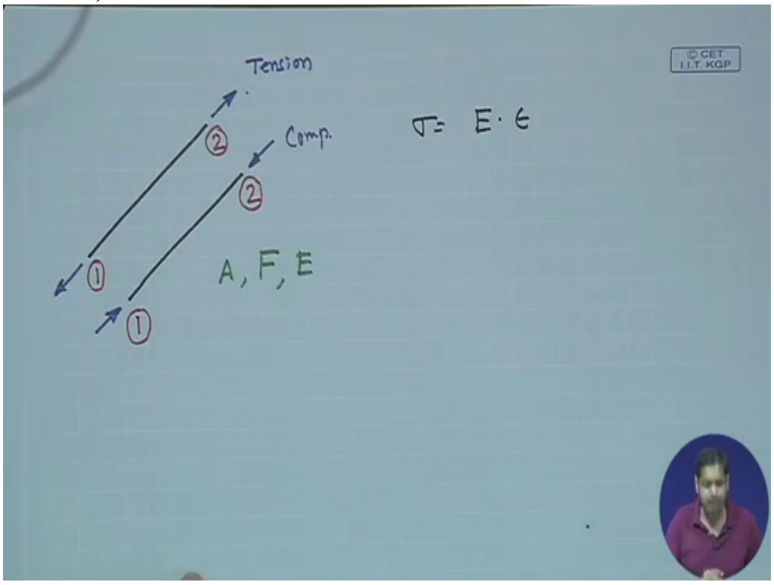

So Young, cross-sectional area is A, Young's modulus is E, and this truss member is subjected to a force F. That force could be compression, that force could be tension. But this force is along this, along this, along this longitudinal direction of the truss member.

And suppose, Ok let us, let us,

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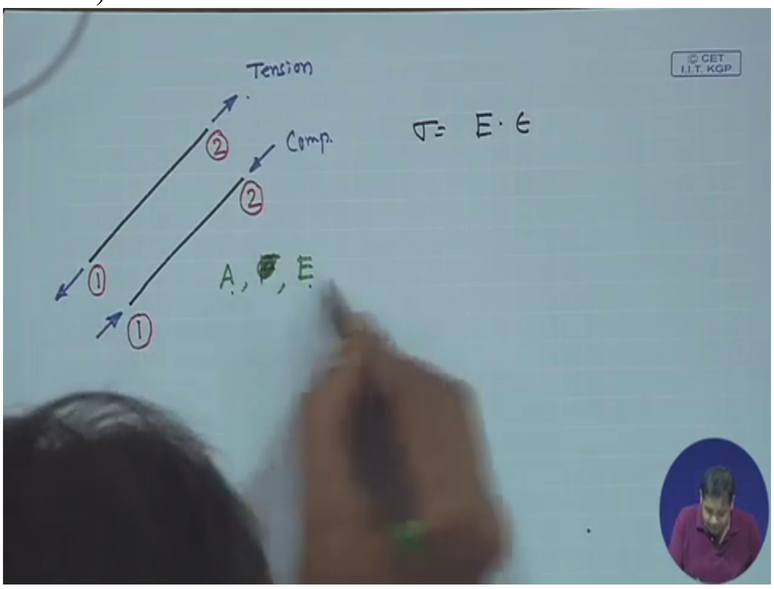

A and E , suppose this is subjected to the force F,

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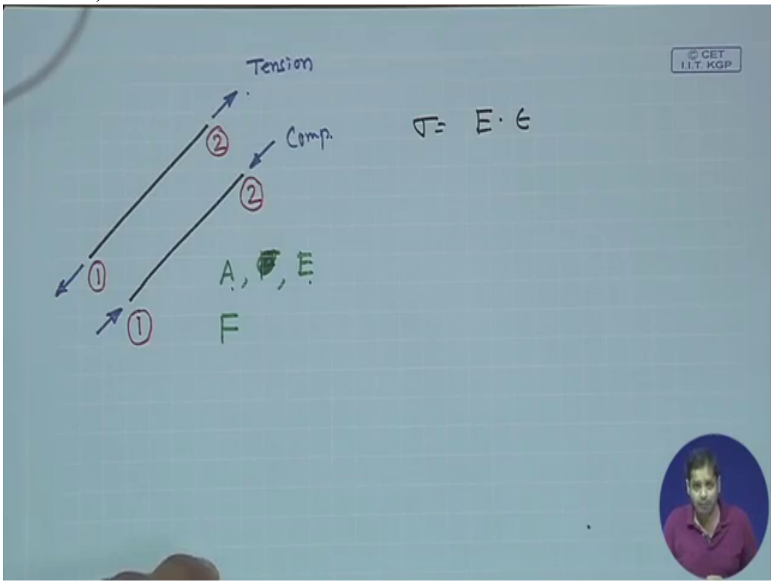

Ok and the properties of the truss member is area is A and Young's Modulus E. Now when it is subjected to the force F, suppose it causes, the force causes displacement u, displacement is u.

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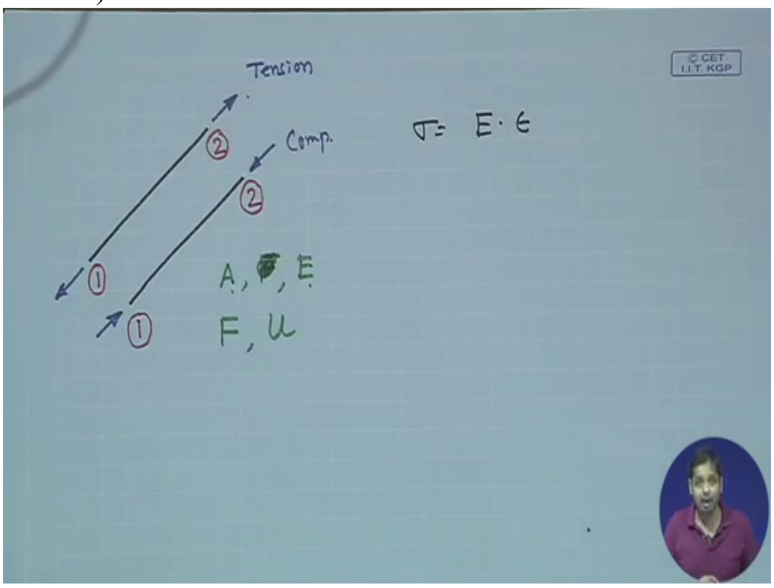

Again that displacement is in this direction. Either, it is in the longitudinal direction, Ok.

Let us find out how these all parameters, the area, Young's Modulus, force and displacement, how these parameters are related to each other and that relation we get from Hooke's Law,

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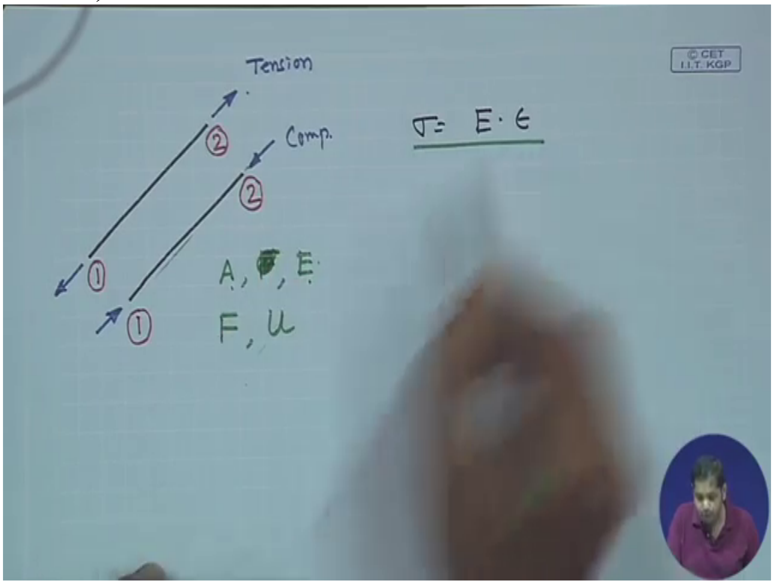

this, Ok. Now, so sigma is equal to, sigma is equal to stress, sigma is equal to force by, force by area Ok. Now you can put a star mark here, Ok, star mark here, I am just crudely defining stress as force by area but as you take courses on Continuum Mechanics and, and Solid Mechanics, many of you might have already taken it, you will see there are different measures of stress, Ok.

And that is not the, that may not be the most, it is not the better way we define stress but for, for the time being in this context, this definition is absolutely fine for us, Ok. Ok, now sigma can be represented by F by A, is equal to Young's Modulus is E and then strain, strain will be displace/displacement, Ok, less another thing, another important thing; suppose length of this truss member is L,

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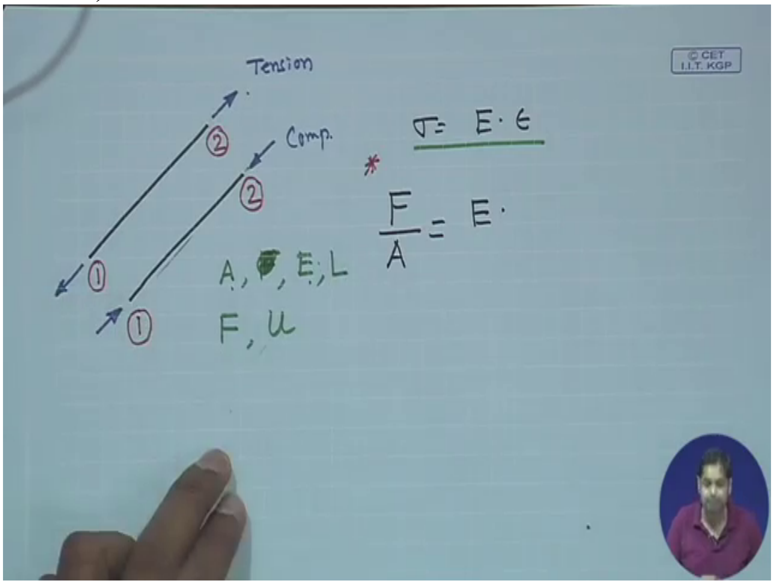

Ok. Initial length of this truss member is L. Now so, therefore what will be the strain? Strain is defined as elongation by initial length, U by L. Again there also you can put a star here. There are different measures

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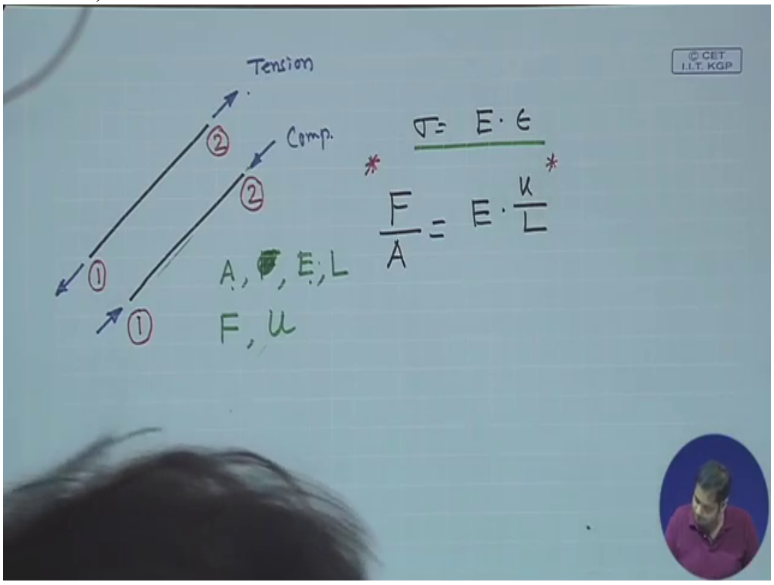

of strain, Ok and you will definitely see all the different measures of strain in Solid Mechanics and Continuum Mechanics course but again for the time being, to demonstrate the theory concepts this demonstration is absolutely fine for us.

Ok, now if we have this, then next let us write out what is U then? So this directly comes from Hooke's Law. Now from that we can write U is equal to, U is equal to, we have L by A into E into F, Ok

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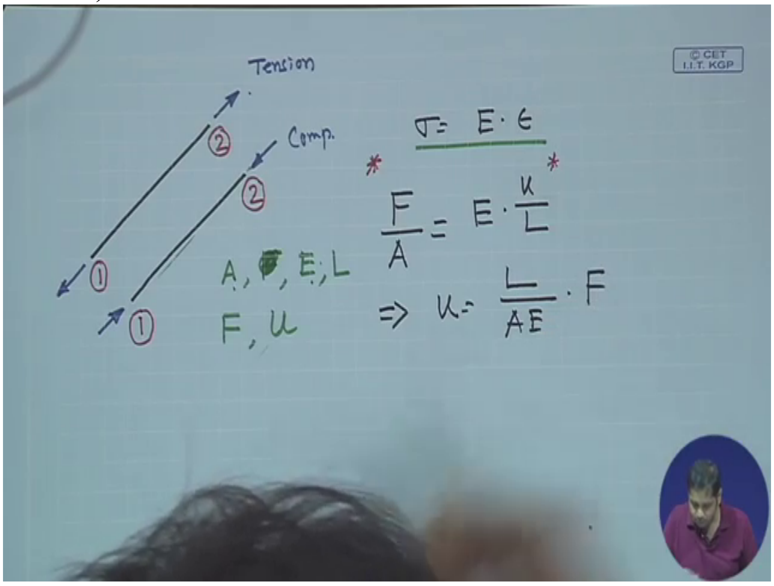

and another thing, we have F is equal to A E, A E by, A E by L into U. Two relations are,

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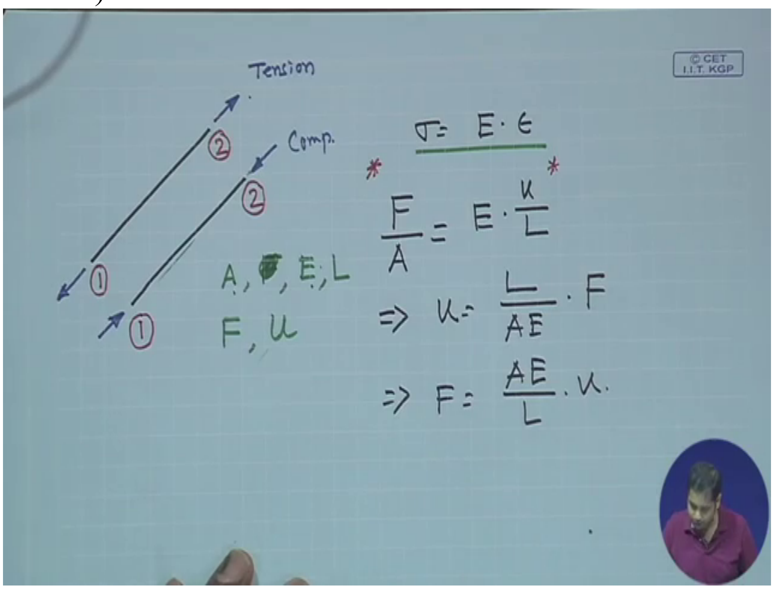

Ok, now you see this was the relation we had,

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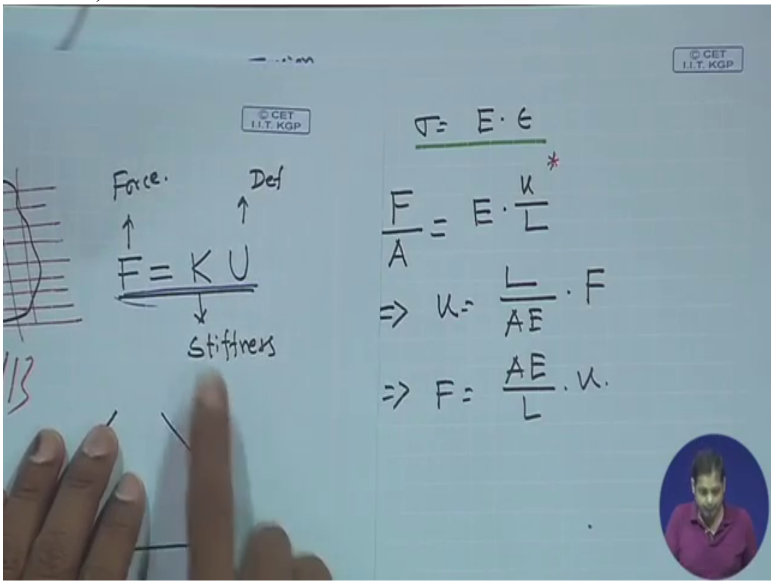

that force is equal to stiffness into displacement, you see here also you have the similar thing, force is equal to something into displacement, and this something is essentially this stiffness, Ok

Now but again this stiffness is associated with only for

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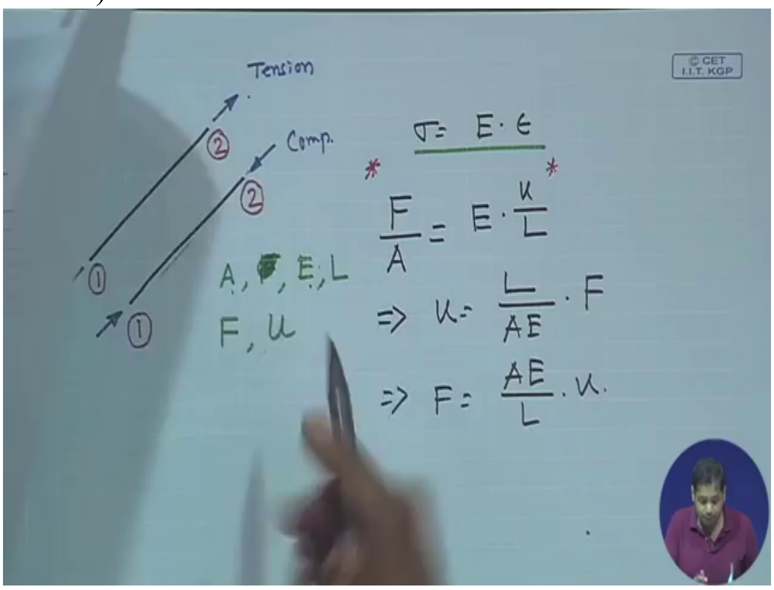

truss member, this member where your member forces are only axial and they are subjected to only axial deformation, Ok but the value, the expression for stiffness for different forces and different kind of degrees of freedom are different, that we will see subsequently. But for axial deformation and axial force, axial force causes axial deformation; this stiffness is this, Ok.

So this relation is equal to essentially force; force

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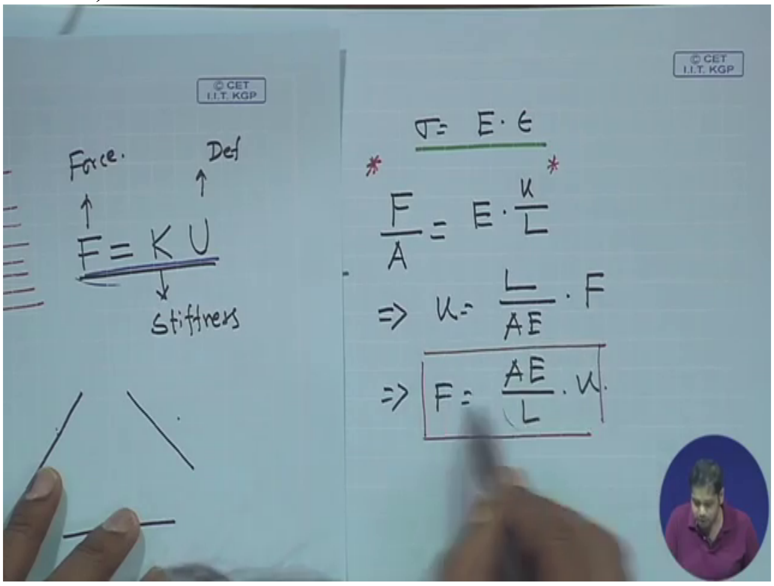

is equal to stiffness into displacement. Another reason, again I wrote this expression as well. This expression I wrote because here the displacement is equal to inverse of stiffness into force and if you recall, recall we discussed in the force method, while discussing

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the force method and displacement method,

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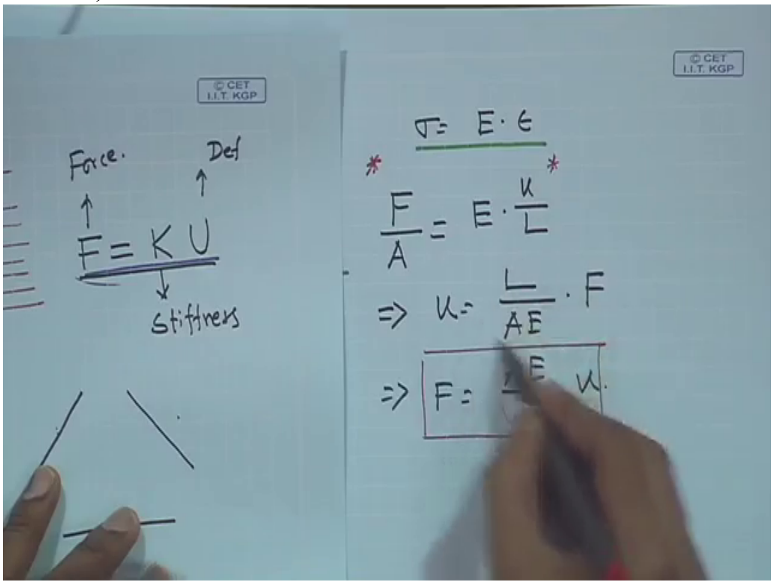

we say that inverse of stiffness is equal to flexibility. So it is that your displacement is equal to flexibility into force, Ok. Ok, but we don't need this expression right now.

So this is the, this is how the force and displacement are related for a truss member, Ok.

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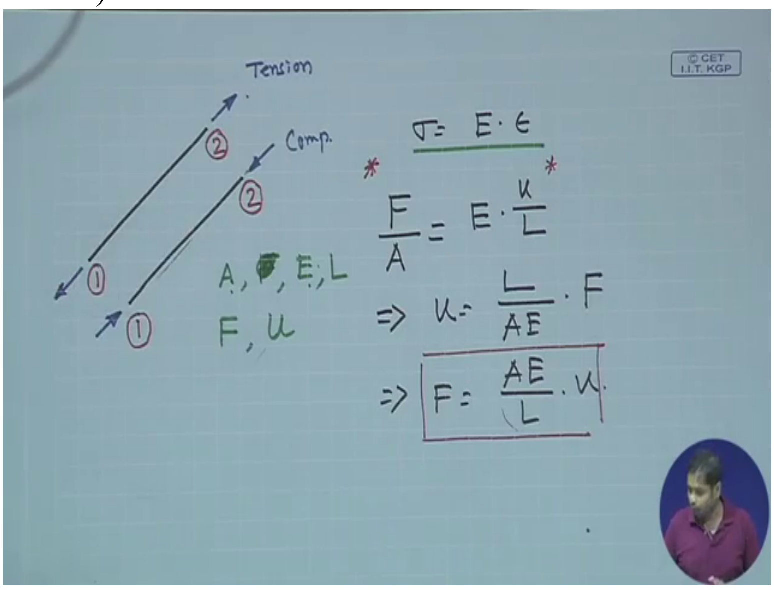

Now here we introduce a very important concept of local coordinate and global coordinate.

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Now local coordinate and global coordinates are like this. You see, when we write this

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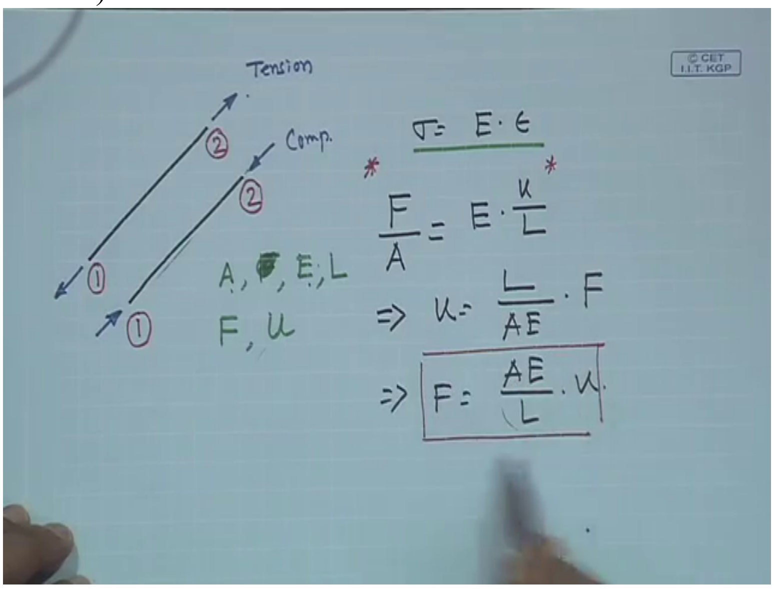

expression, this expression we assume the forces are in these direction and displacements are also in this direction. But in actual structure, suppose

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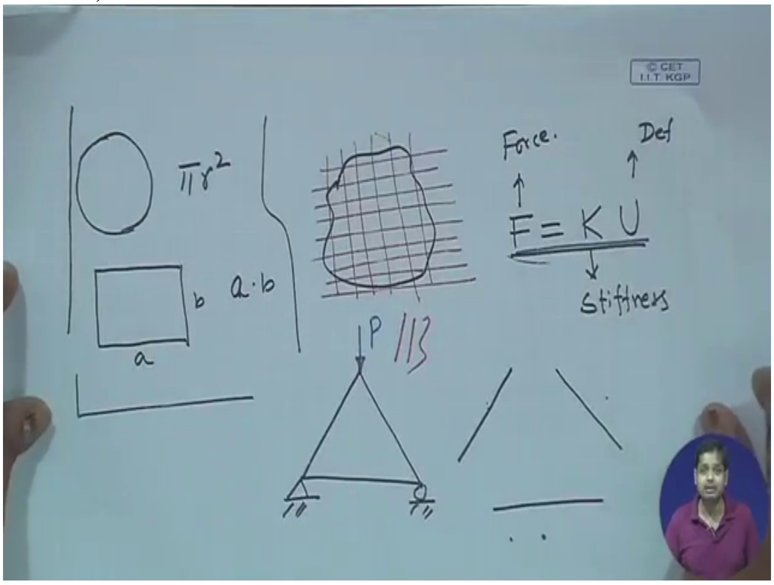

this is the structure, this is the actual structure, this is the actual, truss if you recall, now we are, we are, if we are interested in displacement in these directions.

We want displacement in, what is the displacement

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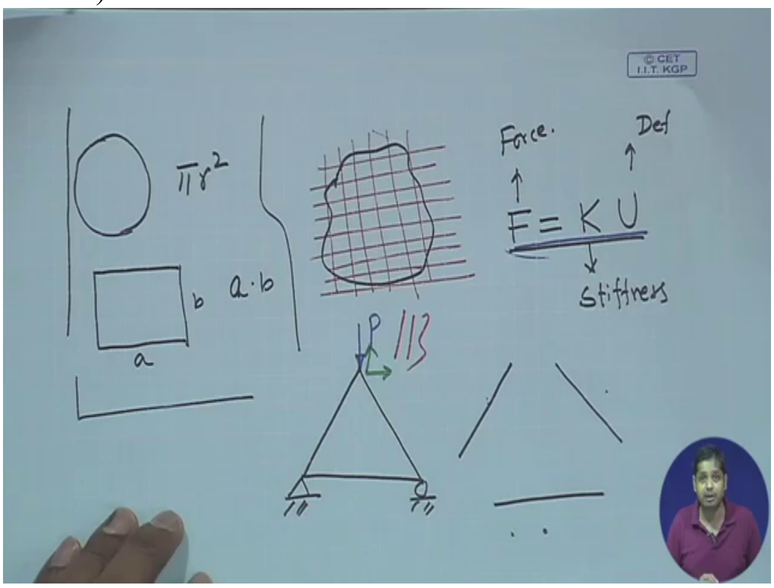

in this direction, what is the displacement in this direction, means if it is my x coordinate and this is y coordinate, so

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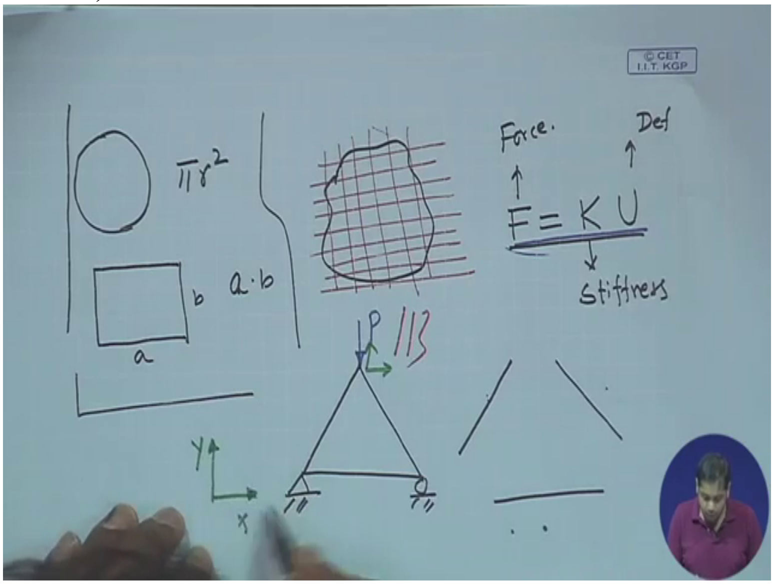

we want what is the displacement in x direction, take any joint here, what will be the corresponding this, what is the displacement at joint in x direction and what is the displacement of that joint in y direction, Ok. But these displacements,

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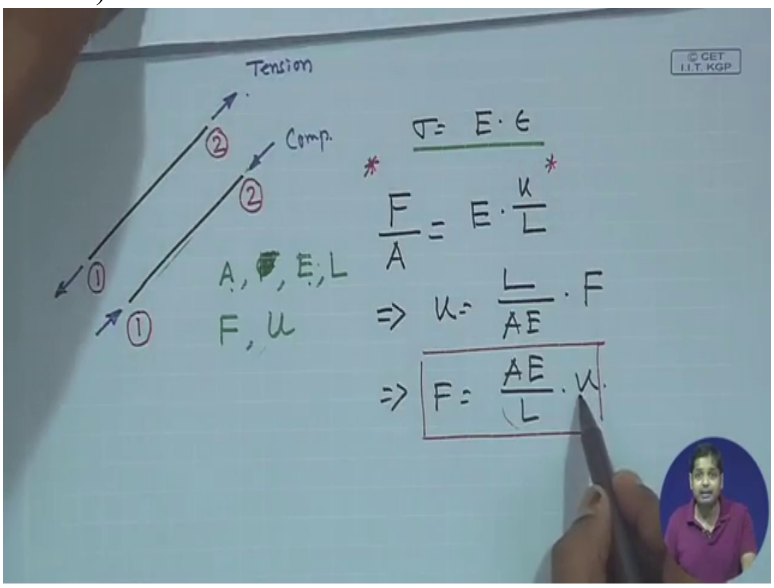

when I write these displacement u,

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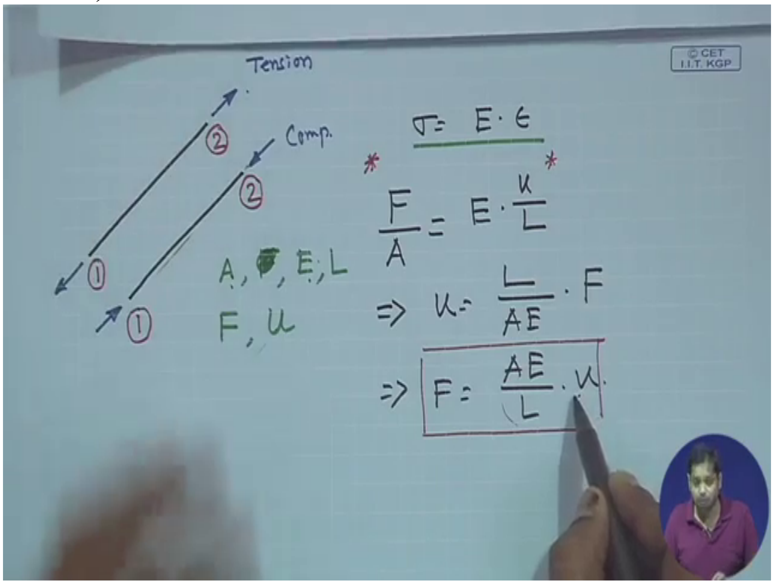

deformation u, that deformation is not in x and y, with respect to that coordinate system.

This is along the, along this deformation and this is local coordinate system. And local coordinate system we defined like this, Ok. We define local coordinate system like this. For instance in this case, instance this case, the local coordinate system is, this is, this is, this is x, say x and this is y. And in order to differentiate between the global coordinate system and local coordinate system, let us write this x dash

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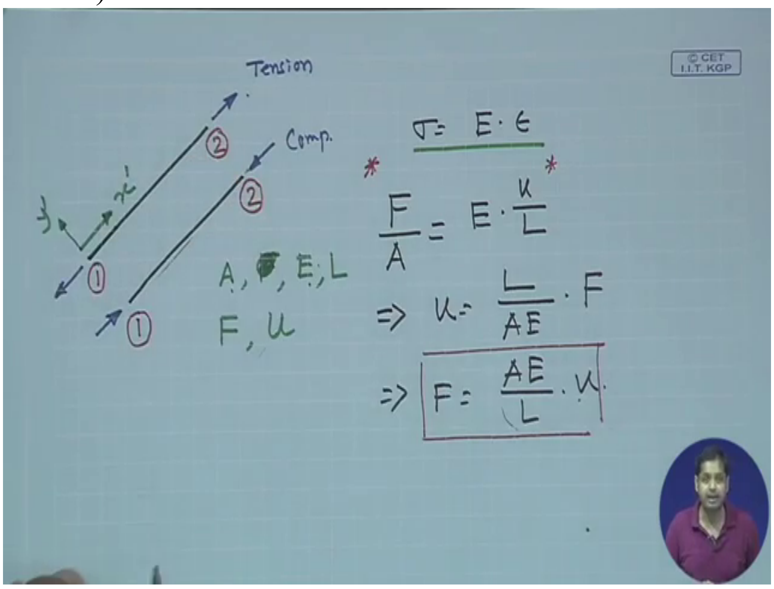

y dash, Ok.

So x dash y dash, x dash is along the length of the, length of the truss and, and which the origin of this coordinate system is at point 1, and y dash is this, and this is local coordinate system where as the global coordinate system is like this. This is x and this is y, Ok.

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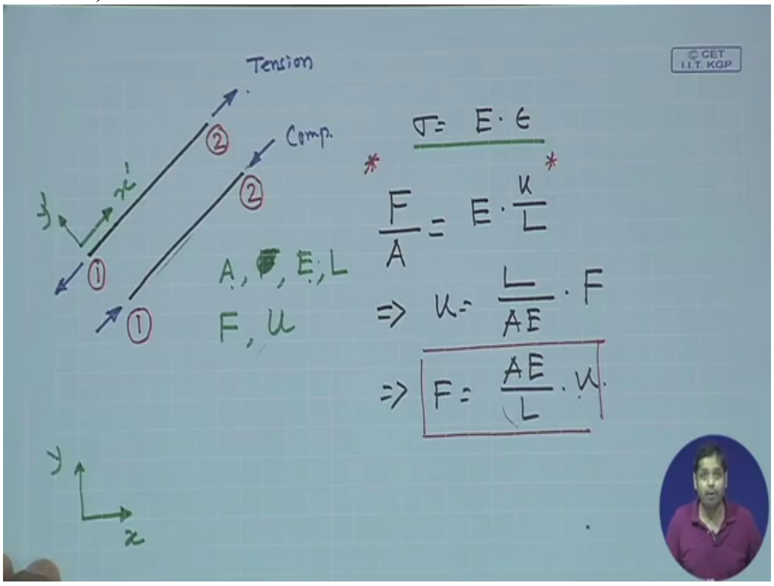

So if the truss is like this, truss is like this, truss is like this, this is my global coordinate system.

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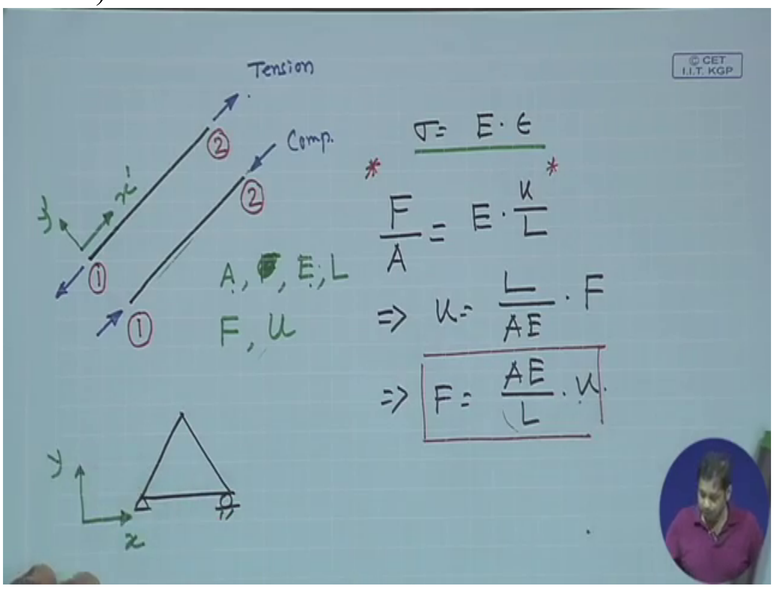

For this member local coordinate system is this. Again if you take for this member, for this member say this member is oriented like this, oriented like this, and for this member suppose this is 1, this is 2, and then we have global coordinate, local coordinate system is this is x dash and this is y dash, Ok.

Similarly for this member, this member,

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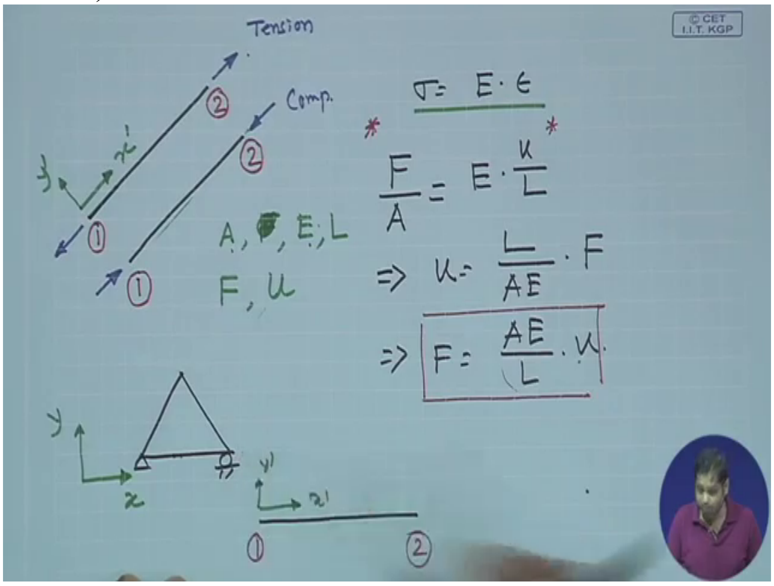

the local coordinate, the member is oriented like this and suppose this is 1, and this is 2, there is nothing

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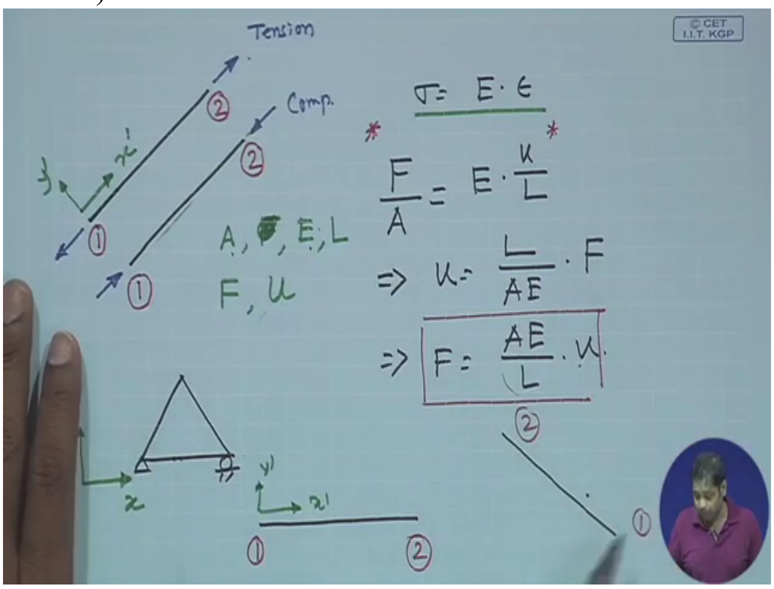

that you have to take always this as 1 and this as 2, you can take this as 1, this as 2 as well. In that case the local coordinate system origin will become here. Ok, so this becomes like this. This is your, this becomes your global coordinate system, local coordinate system. This is x dash and this is y dash, Ok

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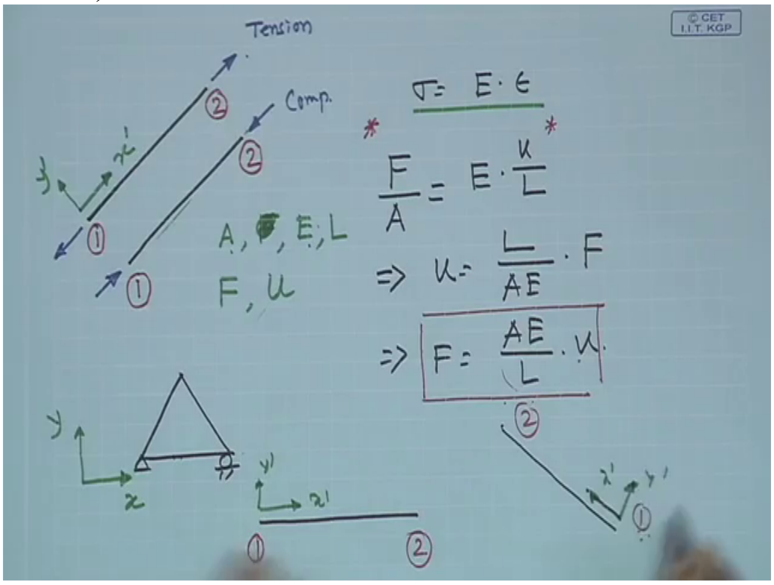

these become local coordinate system. But global coordinate system always remains same. So when we write this relation, this relation can be written for this member, this member, this member, this member, when we write this relation, this relation is always with respect to the local coordinate system because u is essentially the deformation, in axial deformation, u is essentially axial deformation. But if you remember the methodologies, you divide the problem into smaller part,

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smaller sub-system and then assemble them. When you divide into smaller sub-system, for each sub-system the equation is written with respect to their local coordinate system.

Once you have all the information, once you have the information of all the system with respect to their corresponding local coordinate system, what we need to do is we have to then assemble them. But when we assemble them, we need to transfer all this information to global coordinate and then assemble them, Ok. So this is a very important step. We get this relation, first stiffness matrix; we have not defined the stiffness matrix yet. We have just defined how the force and displacement can be related.

Now,

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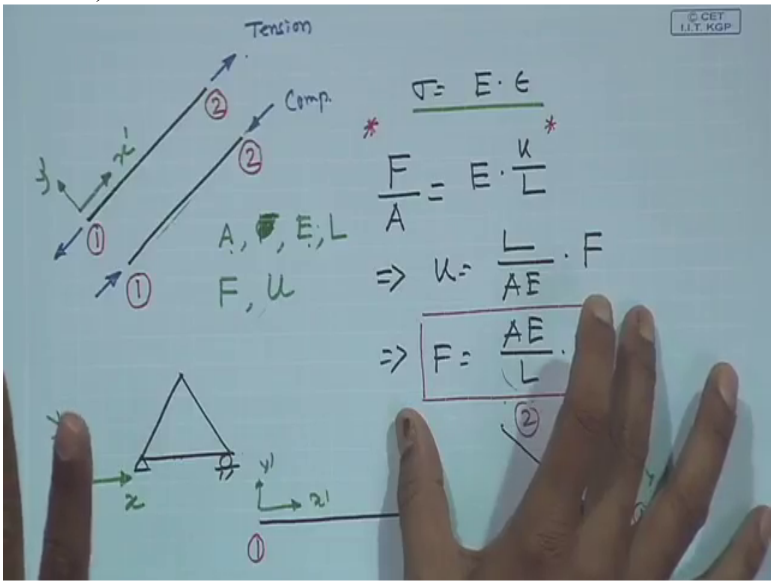

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we are going to do, we are going to write this expression now, this expression

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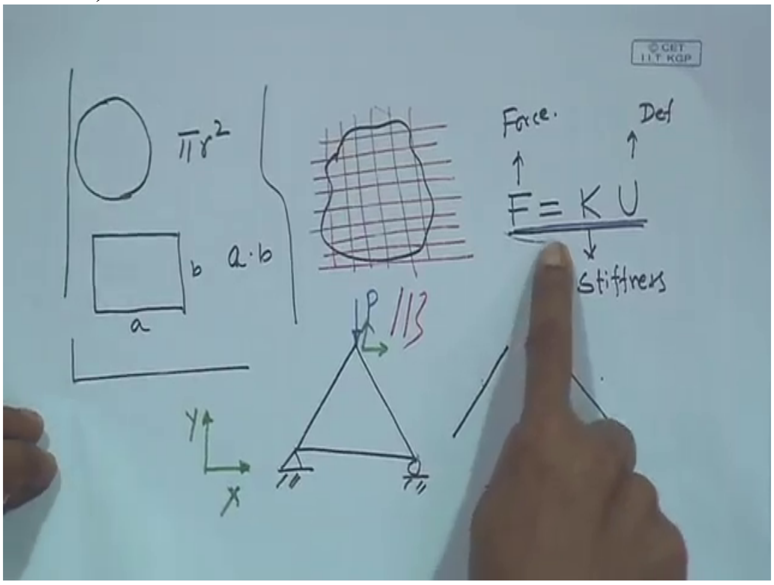

force is equal to K into U for each member with respect to local coordinate system and then we will see how this equation, how this equation can be transferred from local to global

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coordinate system and then we have to finally assemble them, Ok. Now let us find out quickly what is the, what is the, what is the, now suppose you apply the force like this.

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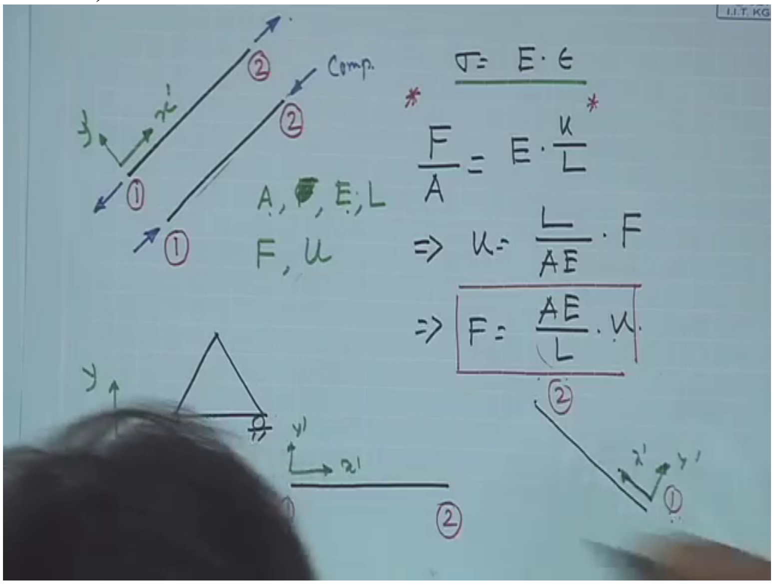

Suppose u 1, Ok

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suppose u 1, u 1 dash, u 1 dash and u 2 dash,

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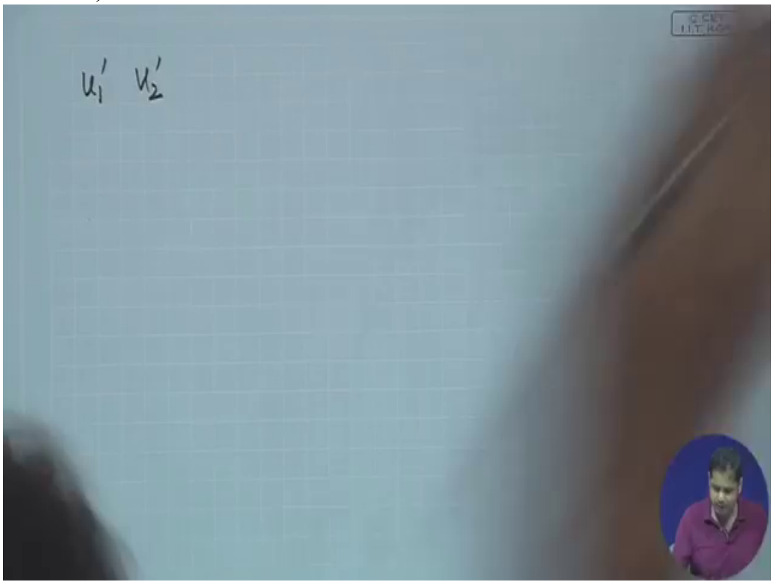

u 1 dash and u 2 dash are the displacement, deformation at

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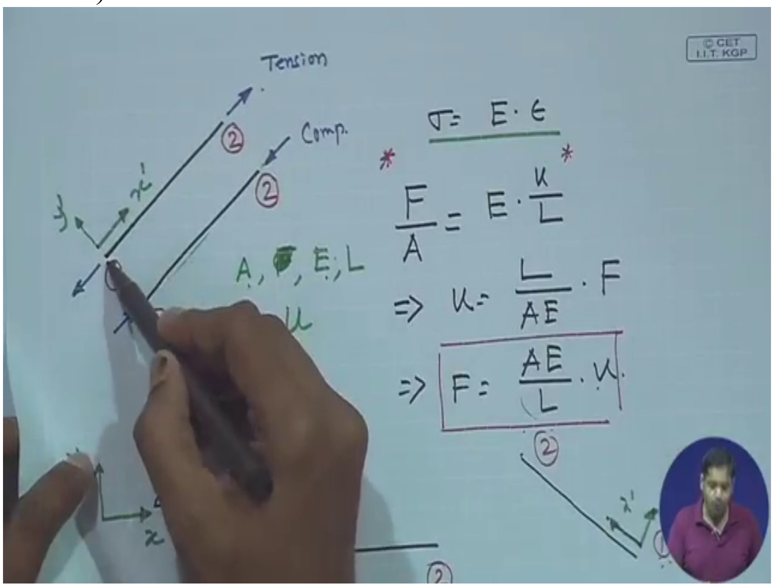

point number 1 and point number 2, Ok. u 1 dash is the deformation in this direction and u 2 dash is the deformation in this direction, Ok and similarly corresponding,

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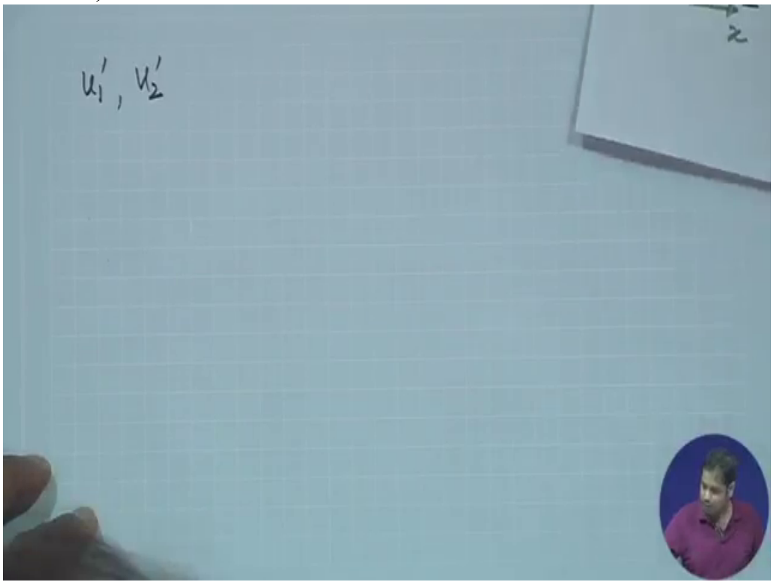

corresponding forces are P 1 dash and P 2 dash. They are all written in dash because it is with

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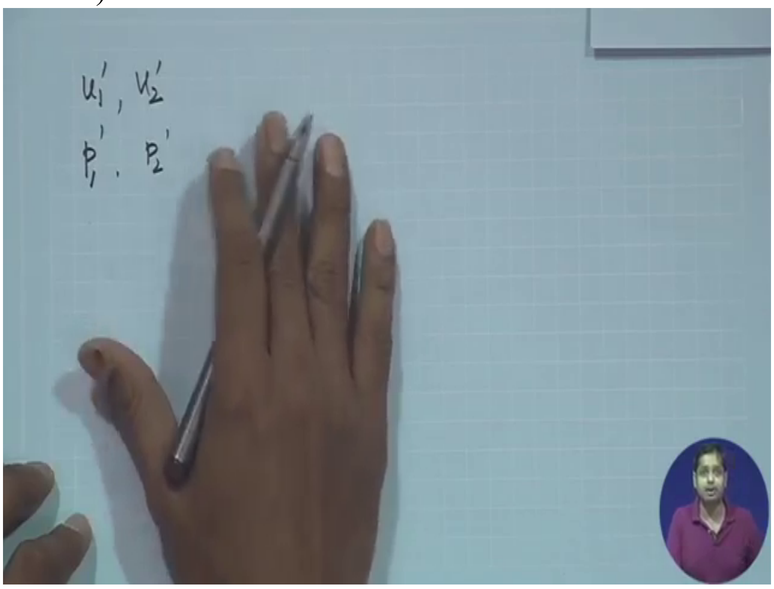

respect to global, with respect to local coordinate, local coordinate system, Ok.

Now let's, the member is now only subjected to u 1 and, and, and u 1 and, u 1 and P 1, Ok. Now this is the member, if you remember this is joint number 1, this is joint number 2

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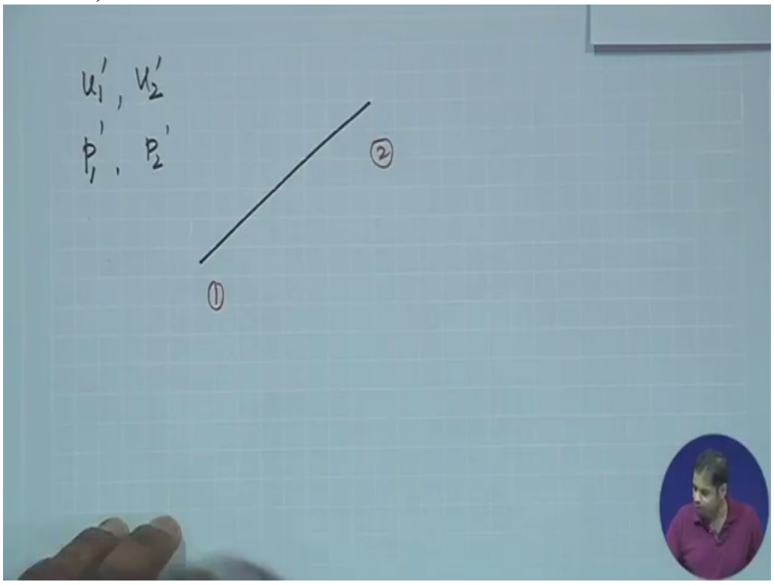

and suppose this member has, this member is subjected to displacement u 1 and also force P 1 and this end is fixed. This end is fixed means there is no deformation here,

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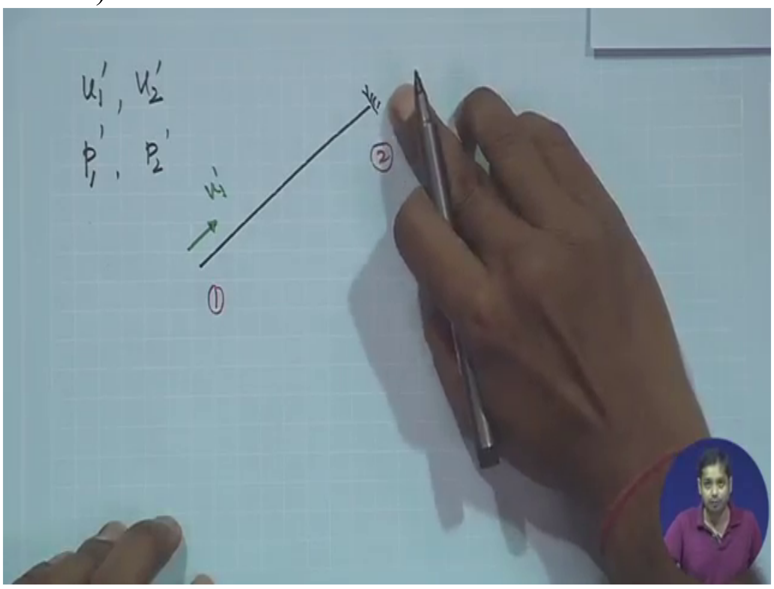

Ok.

Now when you apply a force, when you apply displacement here and, or when you apply a force here, say, say, say it is, say force P, if you apply a force P, P 1 here, suppose this force is P 1, suppose this force is

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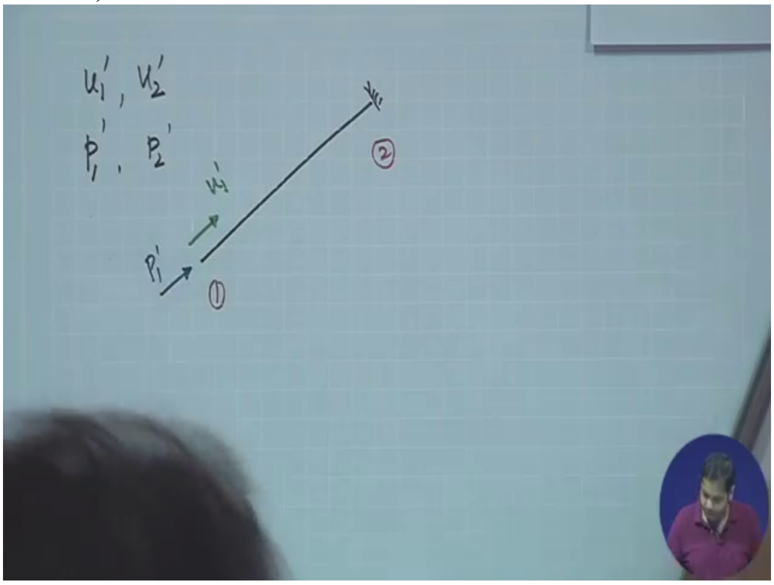

P 1 dash, then this P 1 dash and u 1 dash is related as, P 1 dash, P 1 dash is equal to A E by L into u 1

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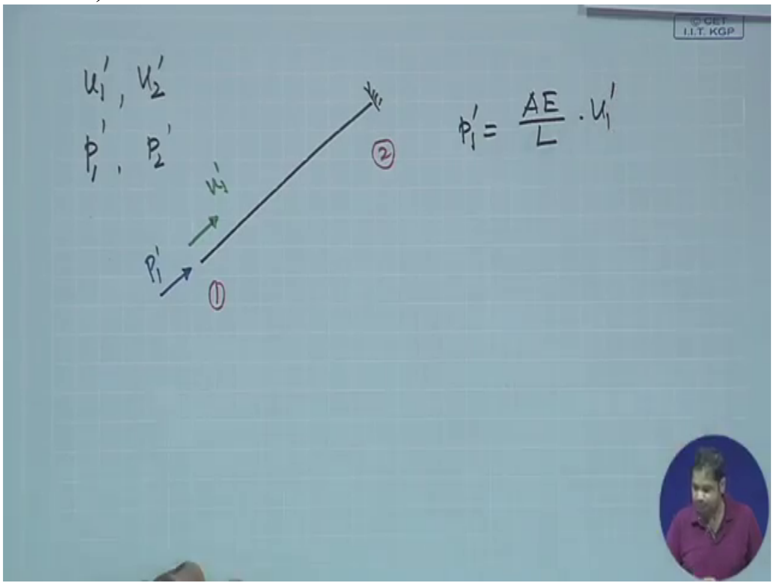

into u 1 dash, Ok, into u 1 dash. This is how just now we defined,

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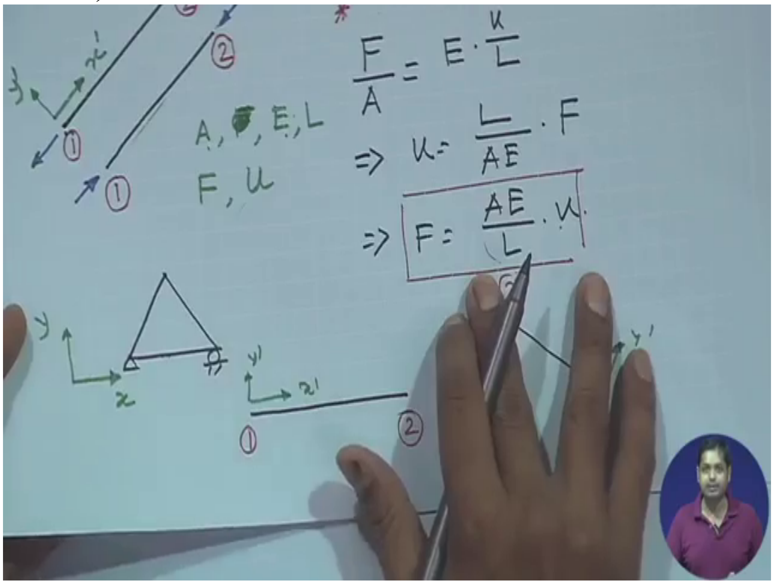

Ok.

Now you see, if you apply

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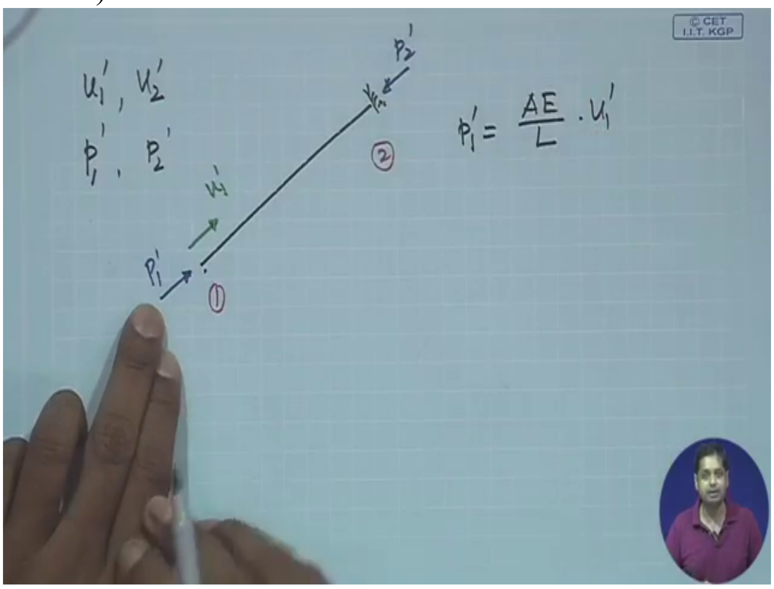

a load P 1 dash at this point, there would be a reaction at this point, isn't it? And that reaction is, corresponding reaction is suppose, this reaction is P 2 dash, Ok. Now so P 2 dash would be minus P 1 dash to maintain the equilibrium. So this P 2 dash will be, P 2 dash, P 2 dash will be then minus A E by L, A E by L into u 1 u 1 dash

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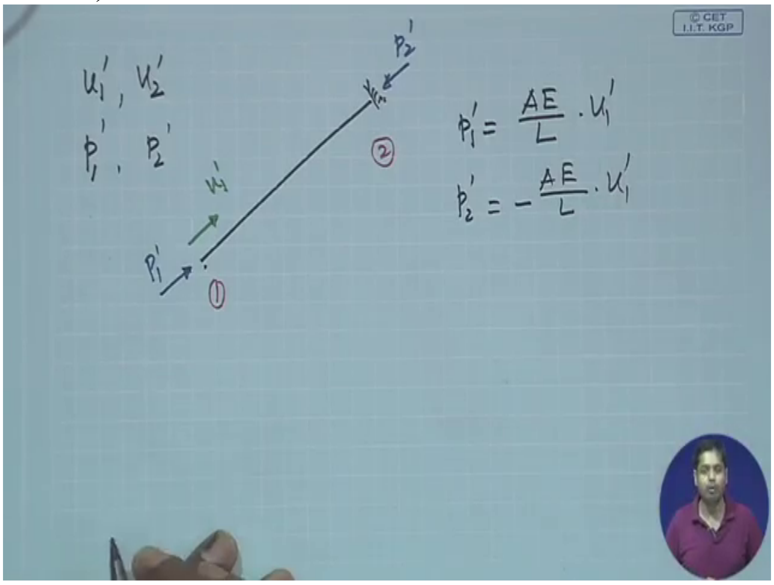

isn't it? So you try to understand this.

Actually what we are going to do is we have a member

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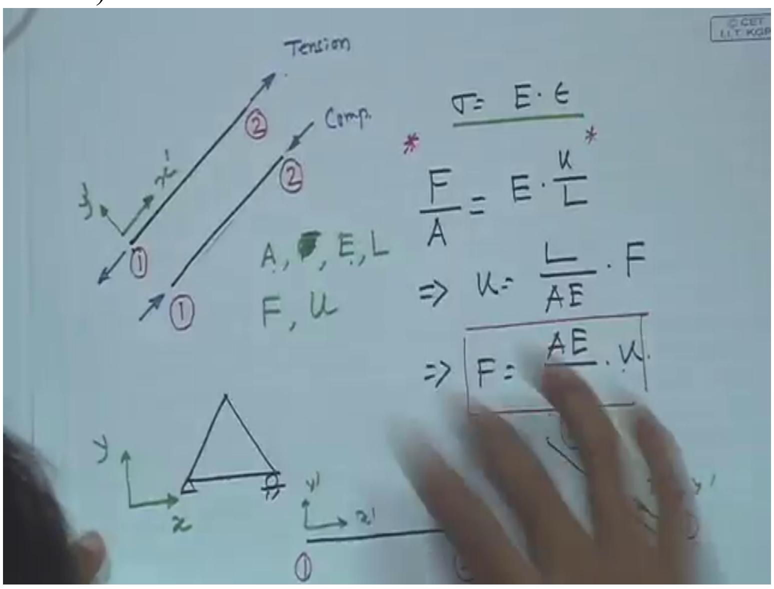

like this, where we have displacement at this point and this point both. But we are dealing with, we are considering first deformation only at A 1, only at point 1 and then, and then see how we have the displacement and force relation and then similarly we will do keeping point 1 fixed and allowing only point 2 to, point 2 to deform,

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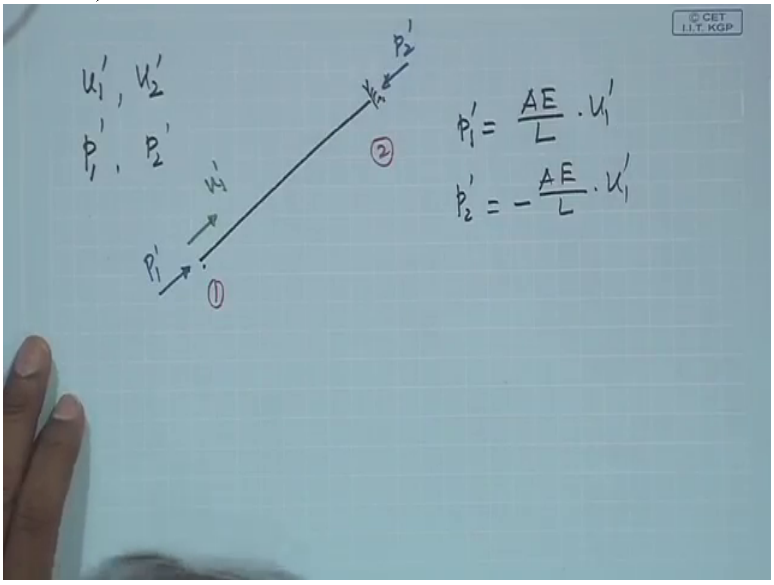

Ok.

So similarly we have, take, take this. So B C is fixed and we have, we have, we have similarly we have deformation, deformation, this, this will be u 2 deformation, and corresponding

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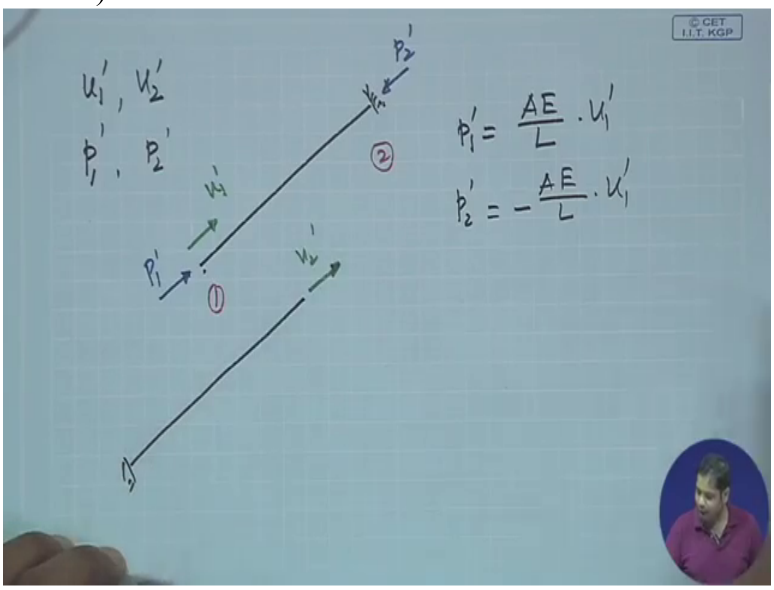

load will be P 2 and then P 2 dash and due to P 2 dash the reaction here will be P 1 dash. So then, they will be related as P 2 dash. Similarly A E by L into u 2 dash

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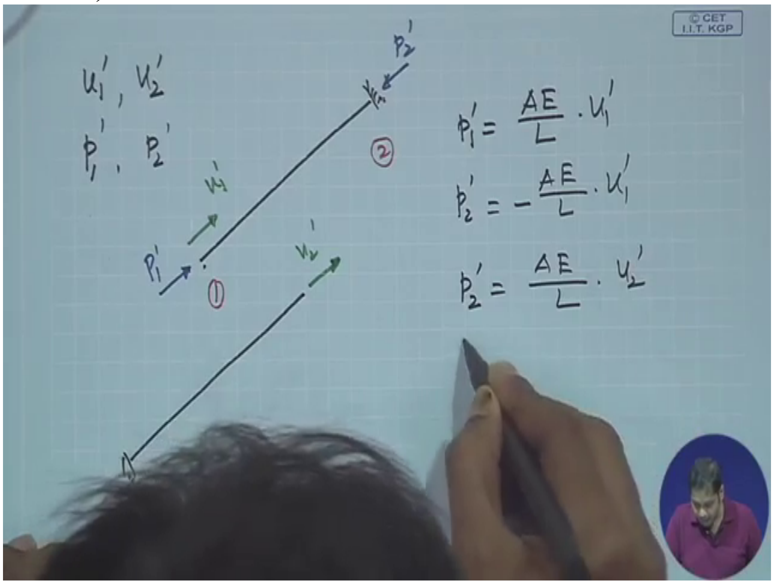

and which will cause reaction P 1 dash which is minus P 2 will be minus A E by L into u 1, u 1 what is called this is u 2 dash, Ok, this

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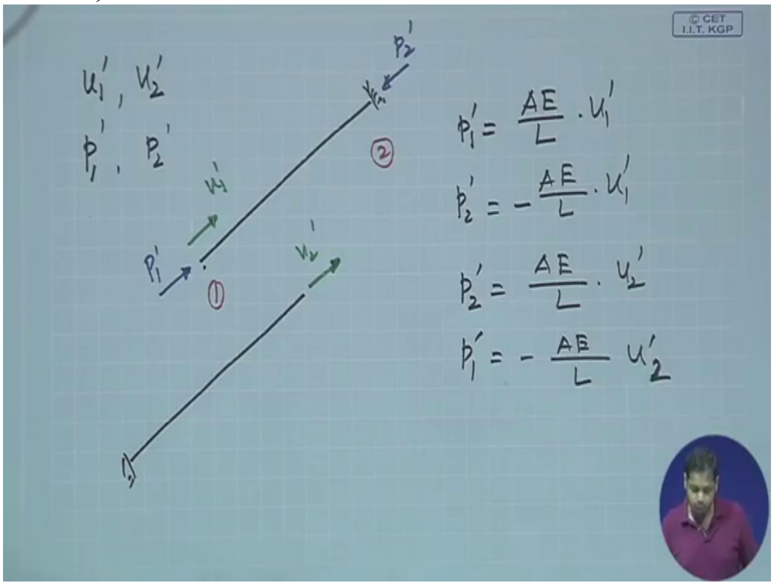

is u 2 dash, Ok.

Now you see, Ok, let's, let's write these as different, different, different name if we can give, if we can give these as, these as, what name can we give, Ok this P 2 and this P 2 are not same, that's what, Ok P 2 double dash,

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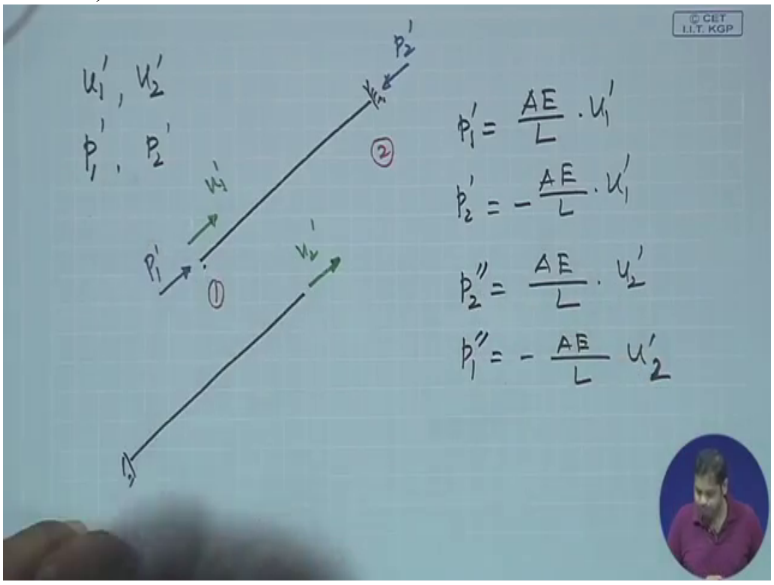

P 1 double dash, this is fine absolutely. Ok, so now this is due to the, due to, this is due to the displacement at 1, and this is due to displacement at 2. This is the force P 1 dash applying and corresponding displacement at u 1 dash and on the other hand, your reaction will be minus P 1 dash which is equal to this.

Now in this case, P 2 dash force is applying, P 2 double dash force is applying here. Corresponding displacement is u 2 dash and the relation is this, and then what happened, now this will cause a reaction at this end and reaction will be P 1 dash, which is P 1 double dash, which is negative of this because they are opposite in direction, to maintain the equilibrium, this will be this.

Ok now this is when displacement allowed at A 1 and this is when displacement allowed at 2. Now when displacements are allowed at both the ends, then what happens, the total forces at 1 will be this plus this, this plus this and total force at 2 will be this plus this, isn't it? So suppose they are q 1, so q 1 dash and q 2 dash, suppose the total force at 1, at point number 1

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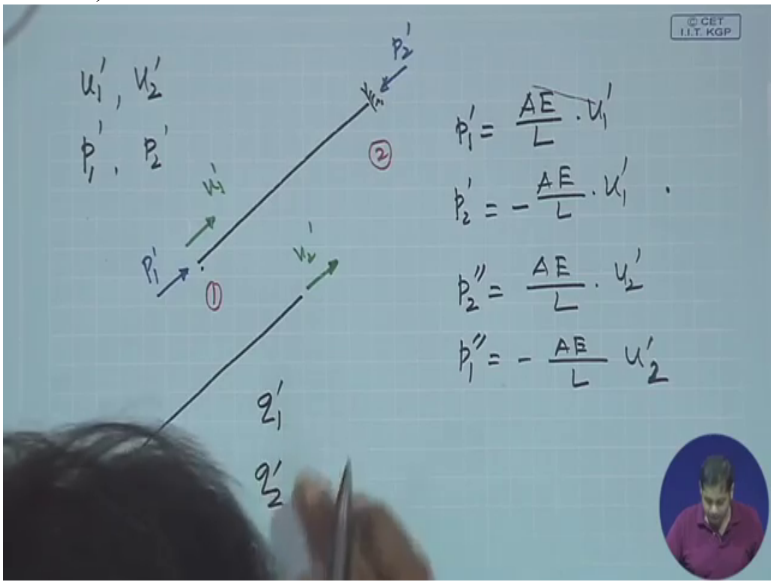

and q 2 at the point number 2. So q 1 will be this plus this plus this and q 2 dash will be this plus this.

So we can write q 1 dash is equal to A E by L u 1 dash minus A E by L u 2 dash.

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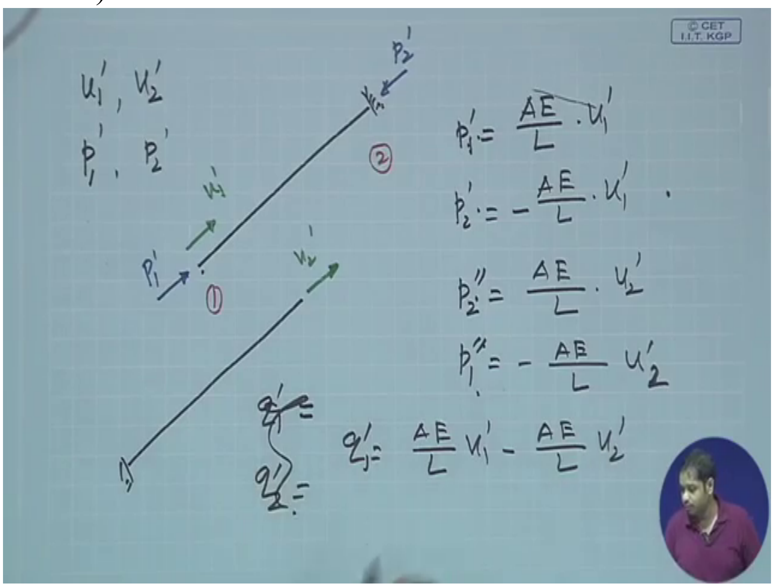

Similarly q 2 dash will be A E by L minus u 2 dash and then plus A E by L u 1 u 1 dash,

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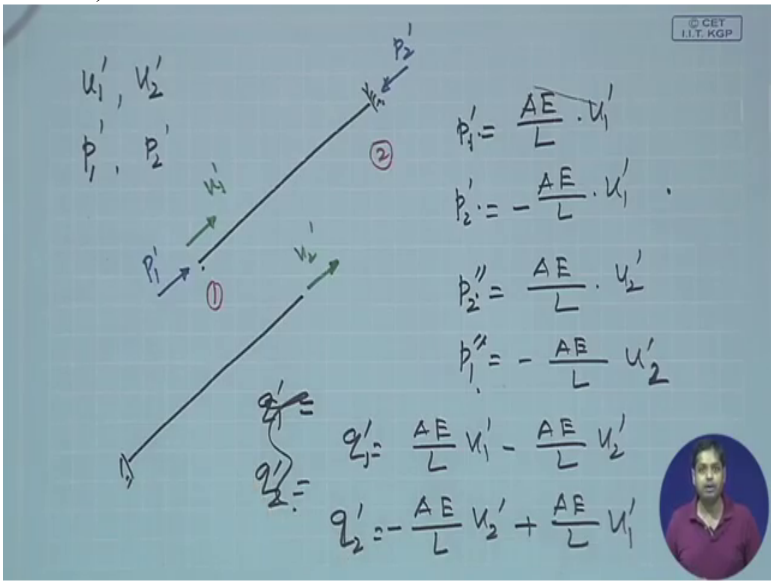

Ok, now if we write that in the matrix form what we have is we have this. q 1 dash q 2 dash is equal to A E by L, if we can take common, A E by L this 1 minus 1 minus 1, 1 u 1 dash and u 2, u 2 dash Ok.

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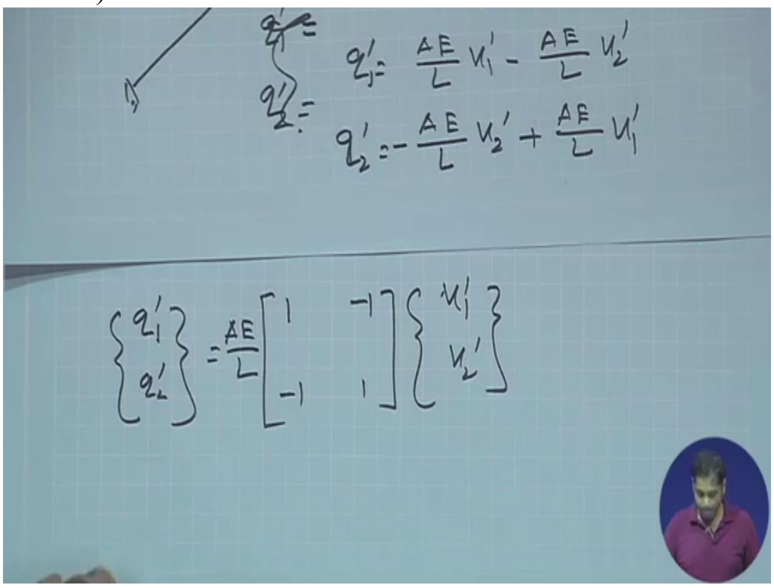

So this is the force, force at point number 1, node number 1, point number 2 and this is displacement at point number 1, displacement at point number 2. Now how these are related to each other? And this is, this relation is exclusively for

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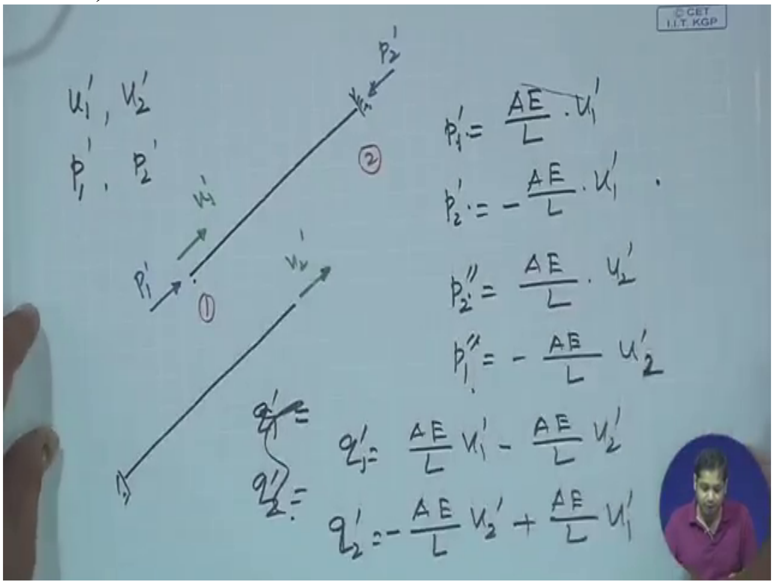

this member, Ok. This member has two joints and this member has two degrees of freedom in local coordinate system remember and these two degrees of freedom is displacement at this point and displacement at this point, axial

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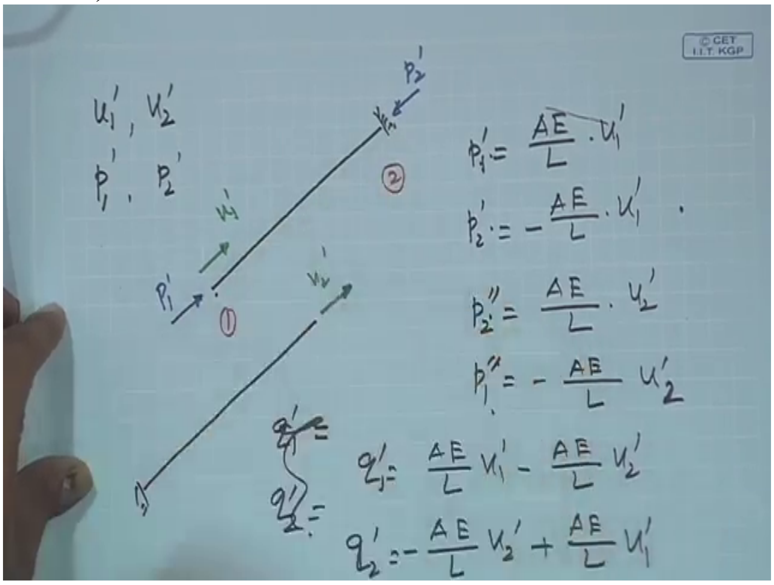

deformation

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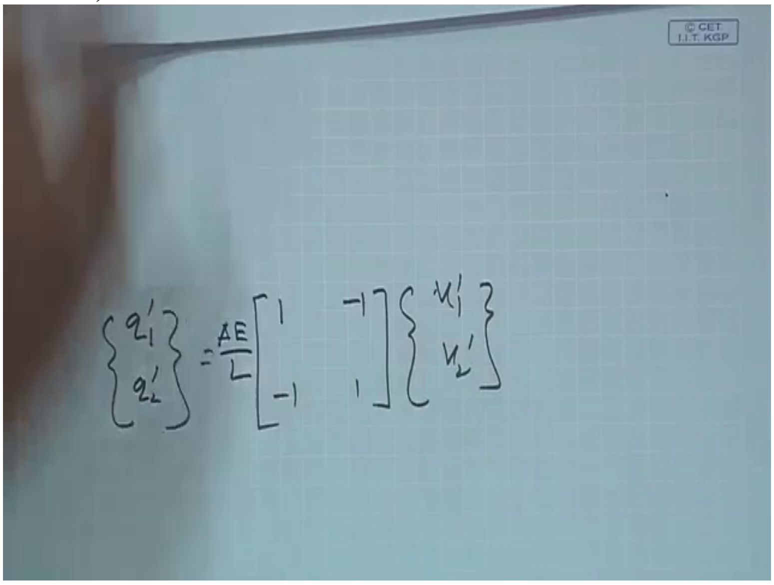

and these are, these are related to each other.

This is called, so this is, this is stiffness, this is stiffness. Now this stiffness is called member stiffness matrix, member stiffness. This is called member, member stiffness

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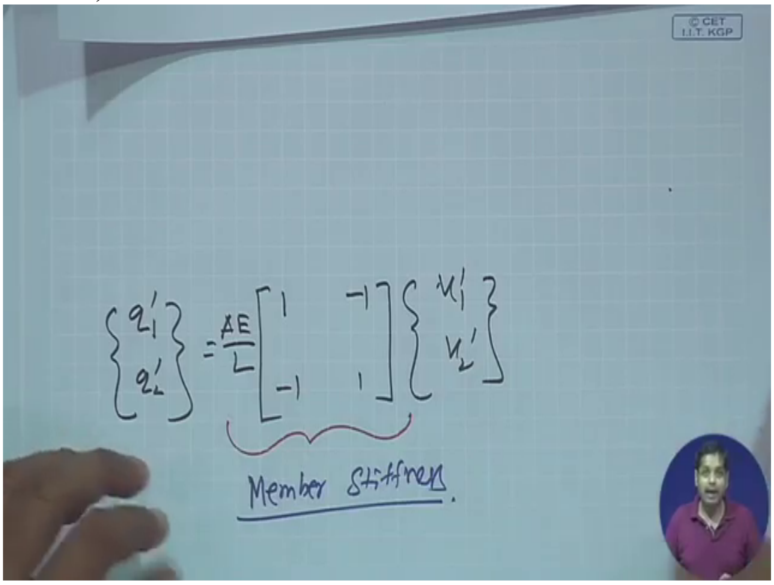

and similarly you can get the stiffness of each member separately, Ok. So you have, you have

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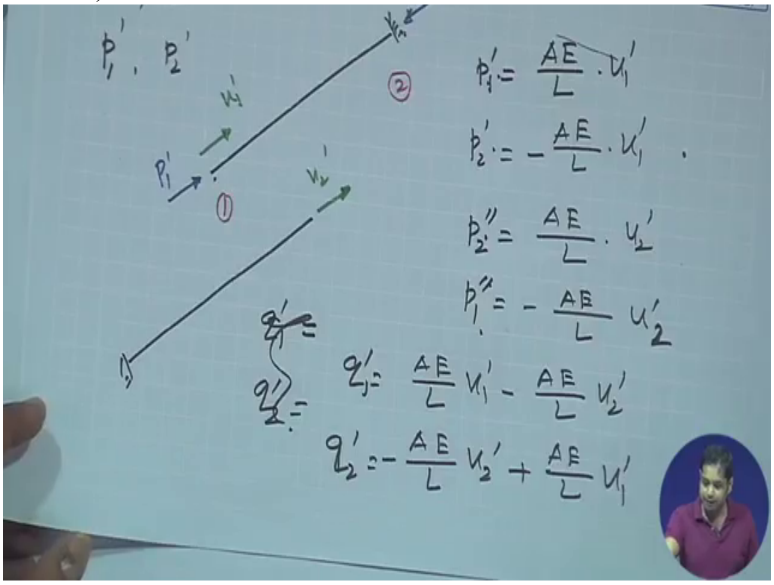

this member,

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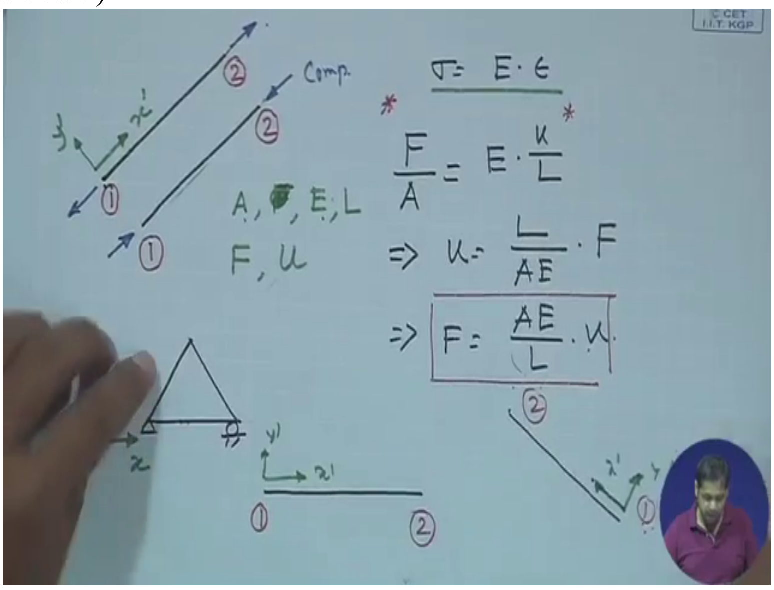

we have determined the stiffness of this member, relation, we can have similar relation for this, similar relation for this but only difference will be your definition of

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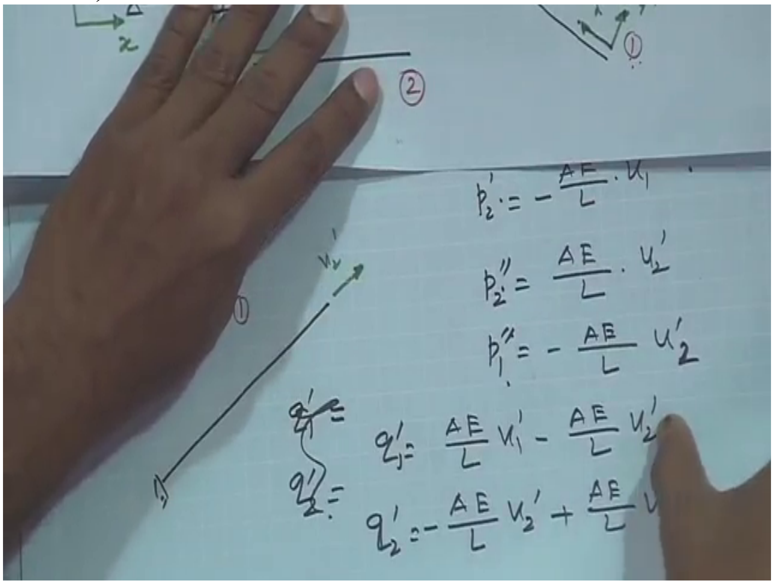

displacement and the forces will be different, because

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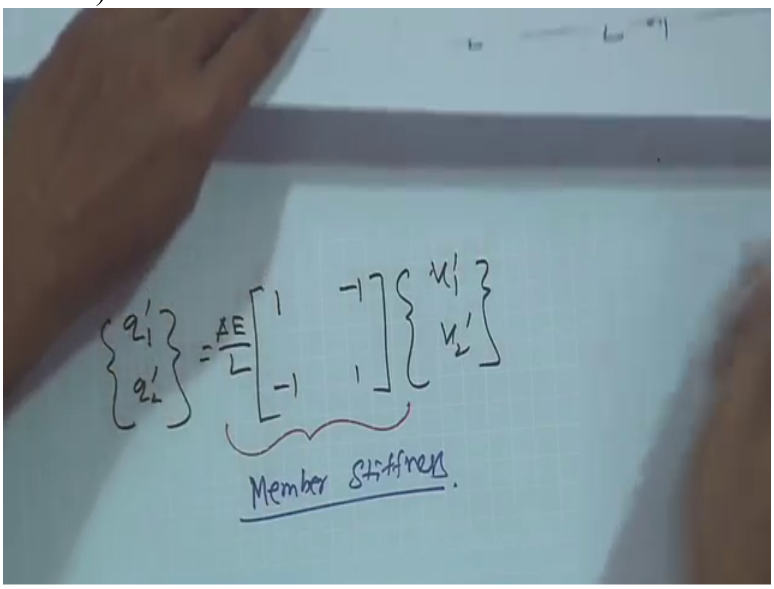

those forces and displacement are defined at a particular joint and joints are different for different members.

So

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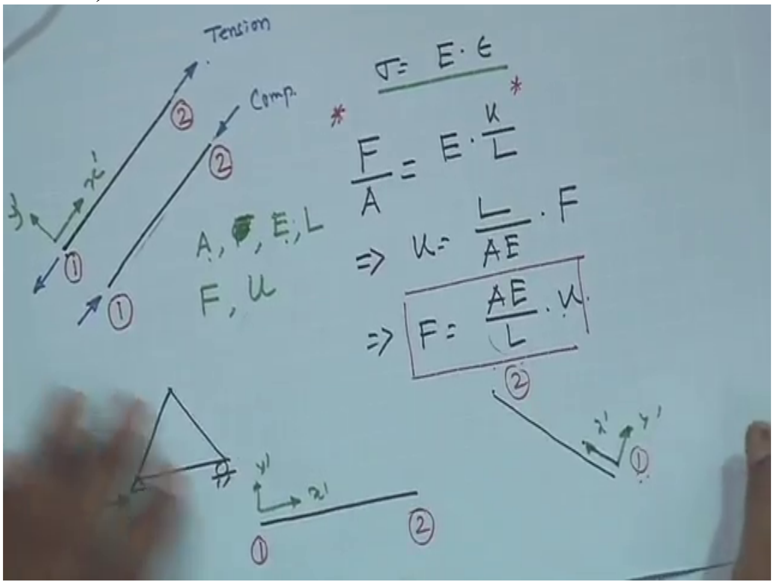

we have member stiffness relation, stiffness relation for each member separately. So we have the member stiffness matrix. Sometimes the member stiffness matrix is also denoted

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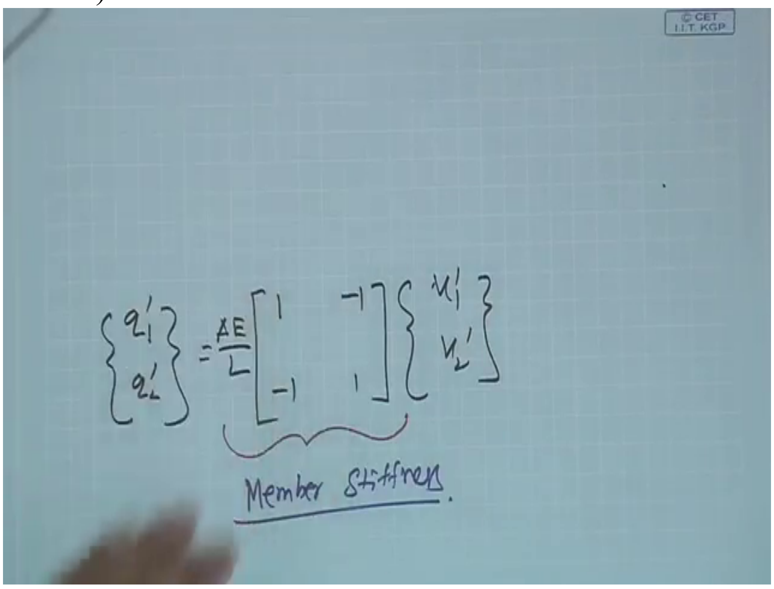

as small k e, k e for

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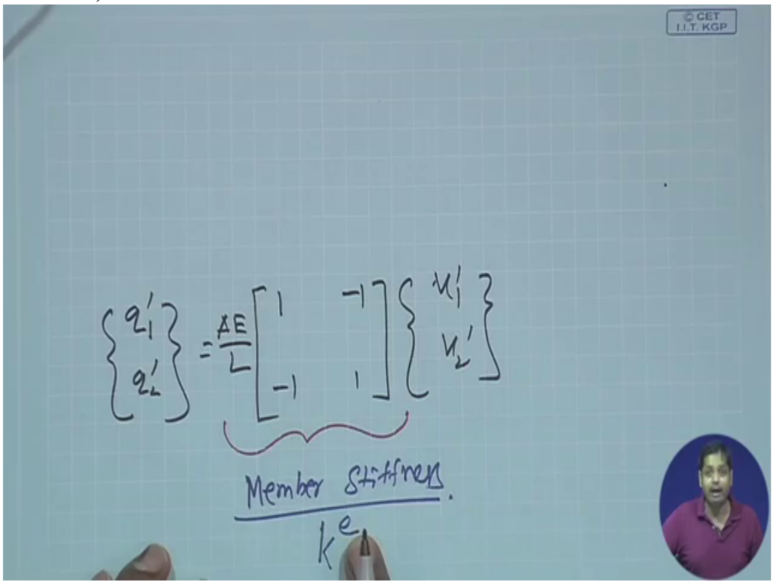

element I am not introducing, I am not using the term element because you see there is, there is, again in subsequent semester there will be another course on

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finite element method which is a more refined version of, of this, this, this direct stiffness method. And there these are the terms are used. There each, here we are saying that you decompose the entire structure into small, small sub-structures, there we say you decompose the entire system into small, small, entire structure into small, small elements, Ok but let's not use the term element right now.

So this is something, use as k e. Now this is called local

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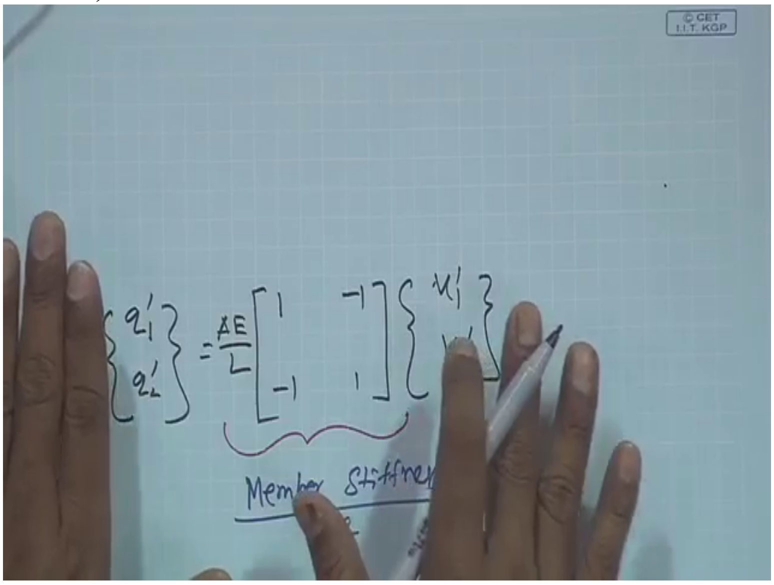

stiffness

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matrix, local stiffness matrix or member stiffness matrix. Ok what we do, this stiffness matrix we cannot, we have a stiffness relation for this,

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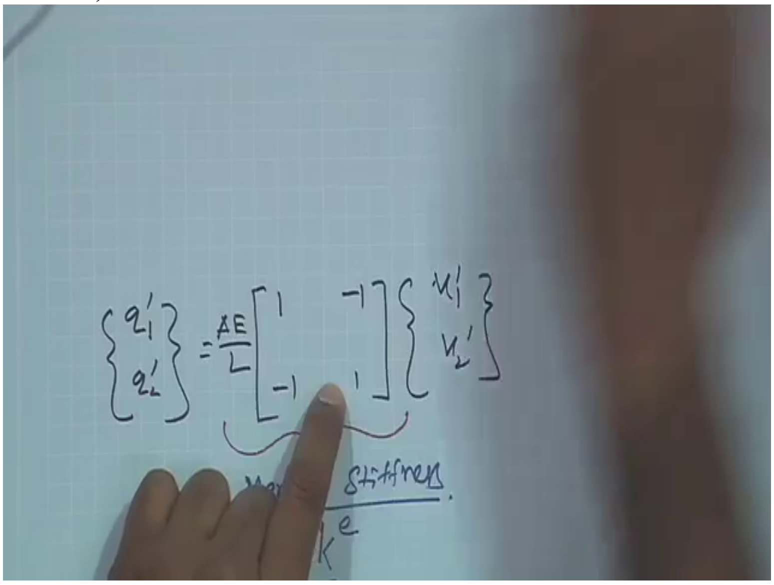

this similar stiffness relation for all the members but we cannot assemble, we cannot add them, add them as it is

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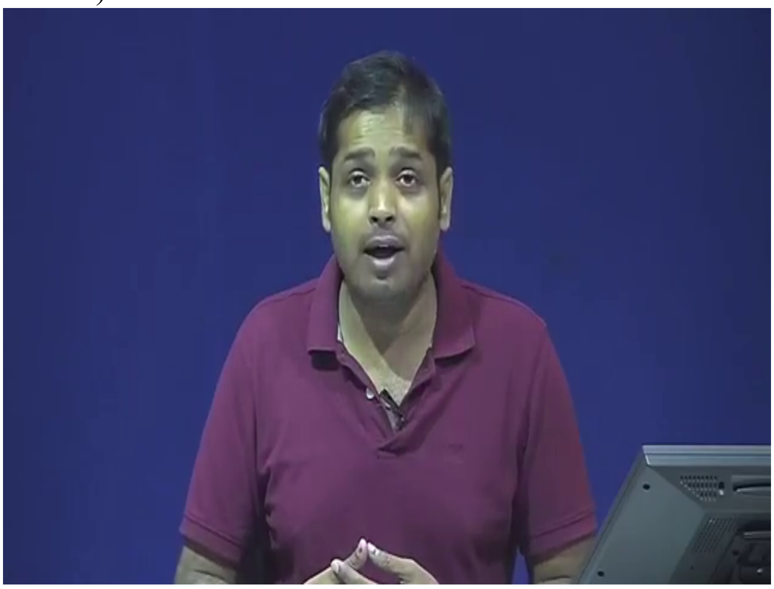

because we have to transfer them on the same plane with respect to same coordinate system then only algebraic, all these addition and subtraction can take place.

So now we will see how this stiffness matrix can be transferred to its global coordinate system, Ok. Steps will be clear if we, if we, if we actually solve some examples, Ok. Now quickly, now suppose this is

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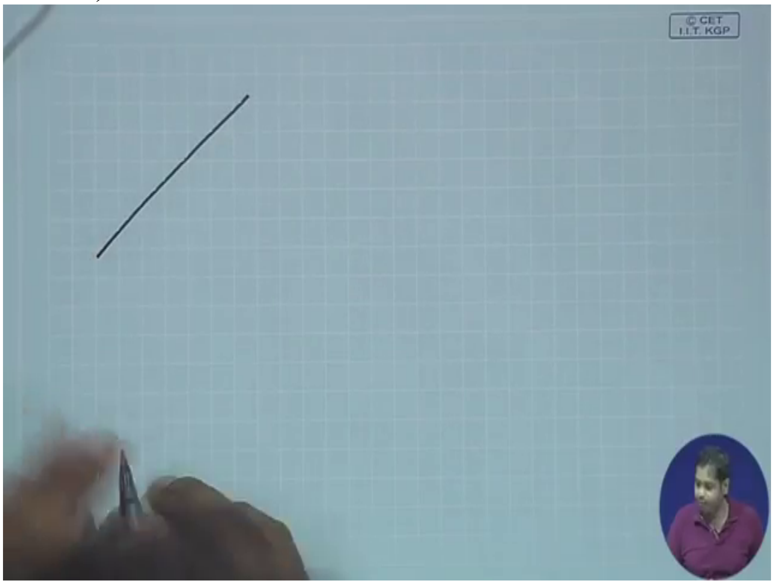

the member, Ok and this is your x 1, this is, this is point number 1 and this is point number 2. This is point number 1, point number 2 and this, it is u 1, this is u 1 dash and this is, here it is u 2 dash, Ok and this direction is x 1 dash and this direction is x y 1 dash, Ok.

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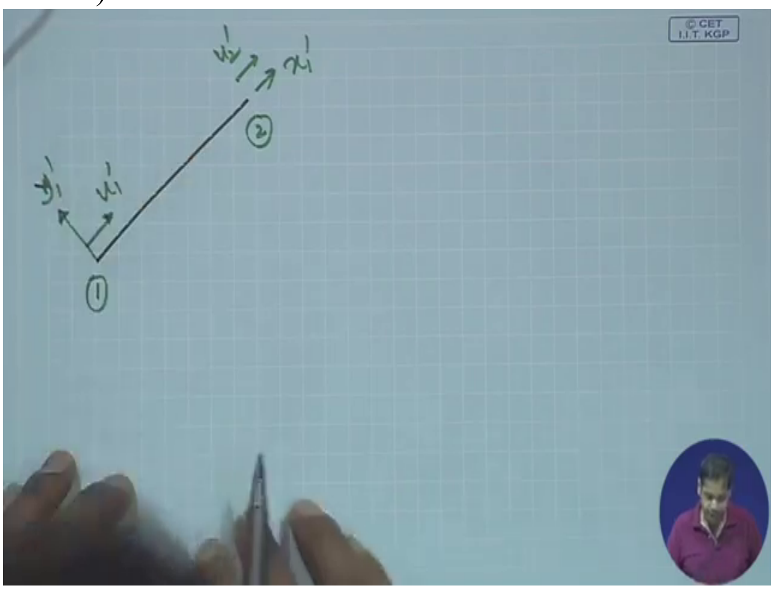

So this is all local system.

Now similarly the global system, corresponding global system will be like this. This is u 1 and say it is v 1, v 1, this is u 2 and this is v 2 Ok and suppose this

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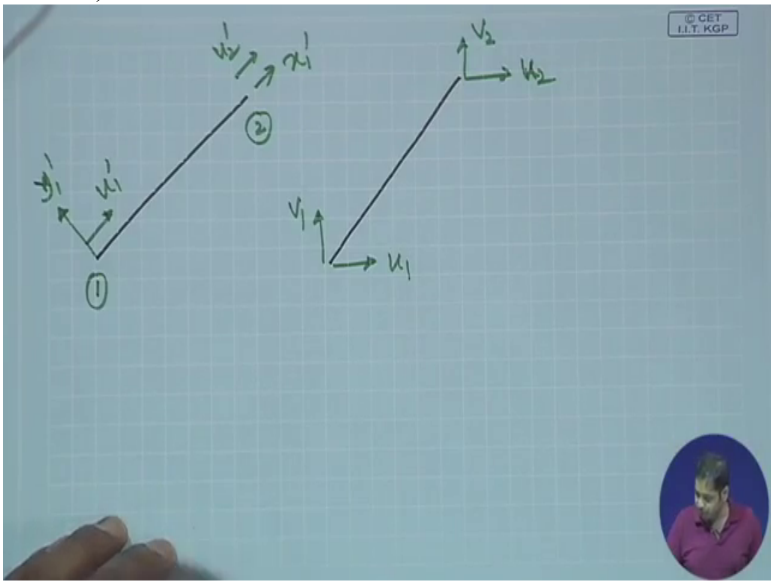

angle is theta. This is, this angle is, this angle is theta,

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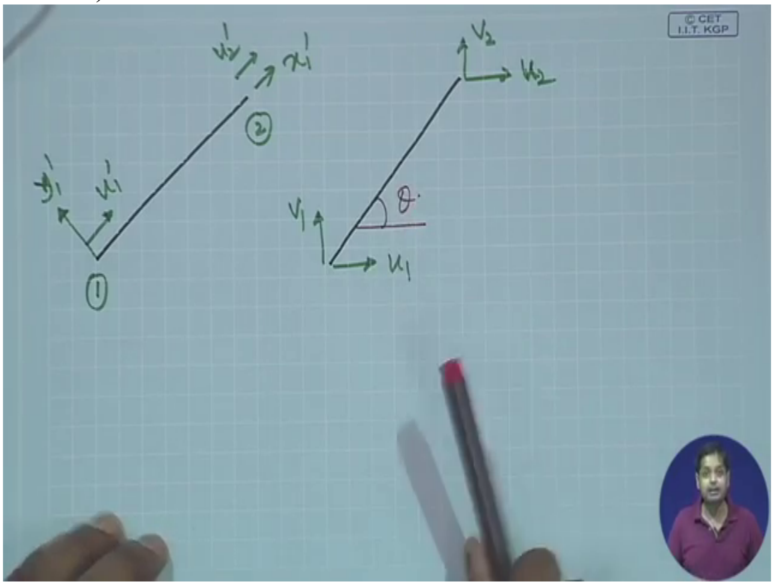

Ok. Now what we have to transfer, we have to write this u 1 dash and u 2 dash in terms of u 1, u 2, u 1 u 2, v 1 v 2, Ok. And if this angle is theta then we can easily write that u 1 dash is equal to, u 1 dash is equal to u 1 cos theta plus v 1 sin theta

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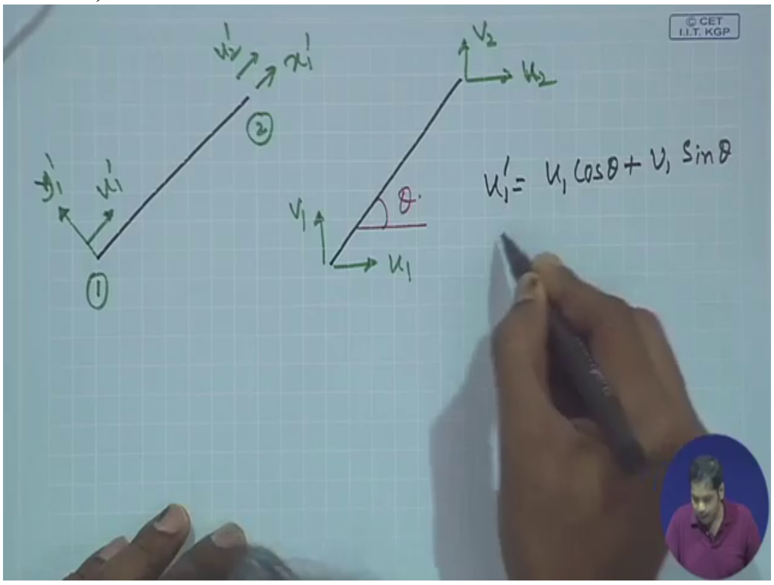

and that you can, this thing you can check easily and I am just writing the final expression cos theta minus, plus plus v 2 sin theta, Ok and if we write that in a matrix form, then you have u 1 dash u 2 dash is equal to cos theta, cos theta then sin theta zero, zero, and then zero, zero, cos theta and sin theta and then you can write this as u 1 v 1 and u 2 v 2, Ok.

The same thing written in

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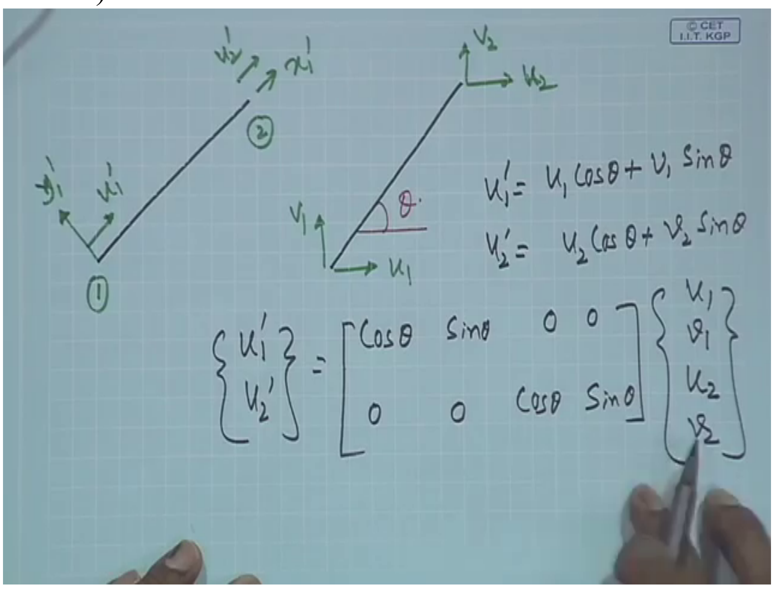

this way and suppose lambda x is cos theta and lambda y is equal to sin theta, Ok. So next time onwards

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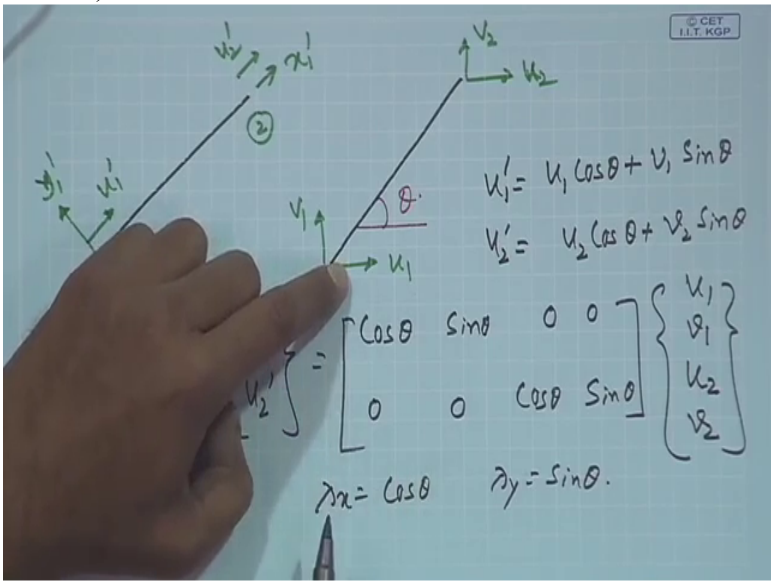

we won't write cos theta as cos theta, instead of cos theta we will write lambda x and sin theta we will write lambda y, Ok. Now Ok, now next what we can do is so,we can write that u 1 dash, u 1 dash u 2 dash in terms of that is equal to lambda x lambda y zero zero zero zero lambda x lambda y and then this is u 1 v 1 u 2 and v 2, Ok.

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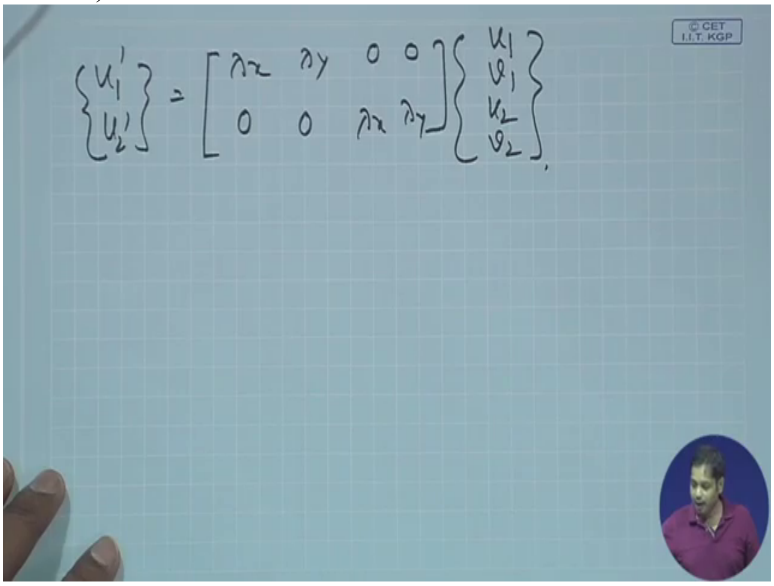

Now next thing what we have to do, next thing we need to, suppose this is T. Suppose this is T. This is T. This, this expression is written as T.

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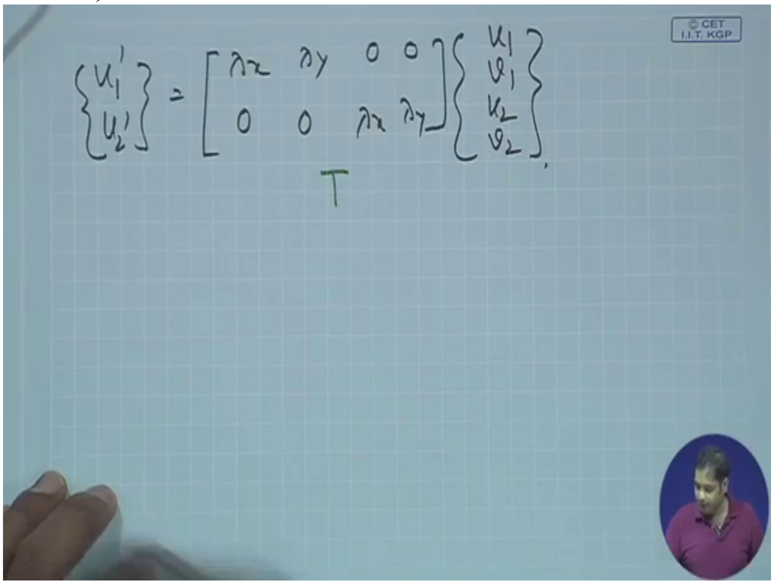

Now similarly I leave it to you, what we can do is, here we transferred the, here we transferred what,

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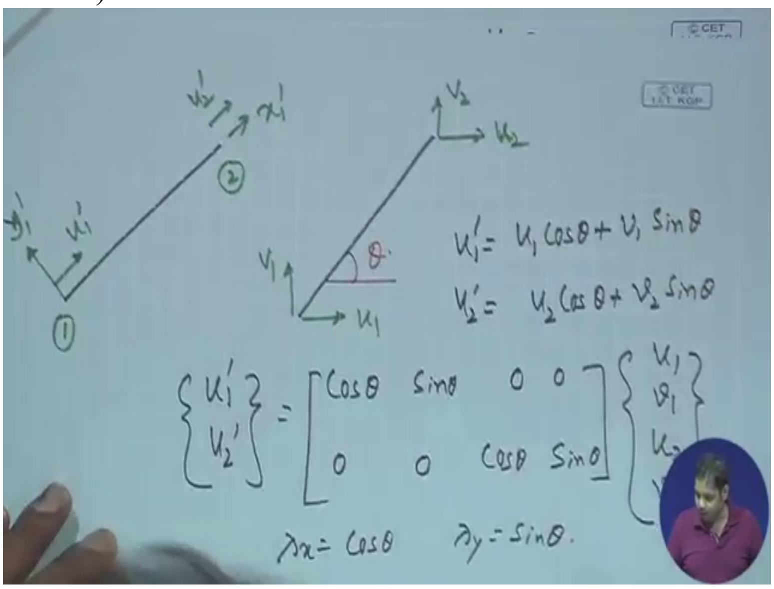

here we transferred only the displacement, Ok. Similarly we can transfer forces as well. Forces means again,

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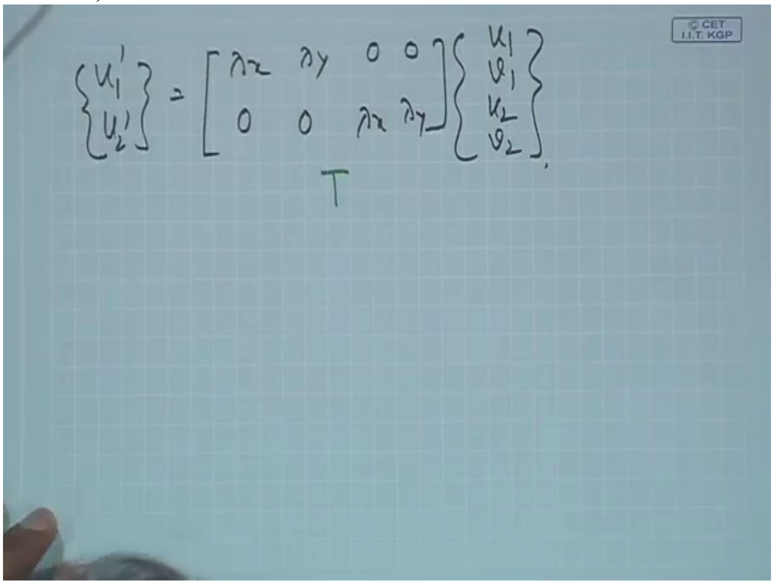

we can, we have the same thing. It is, it is joint number 1, this is joint number 2 where we have force, it is P 2, this is P 1 and it is in local coordinate system P 1 dash, sorry this is q 1 dash and this is q 2 dash and in actual

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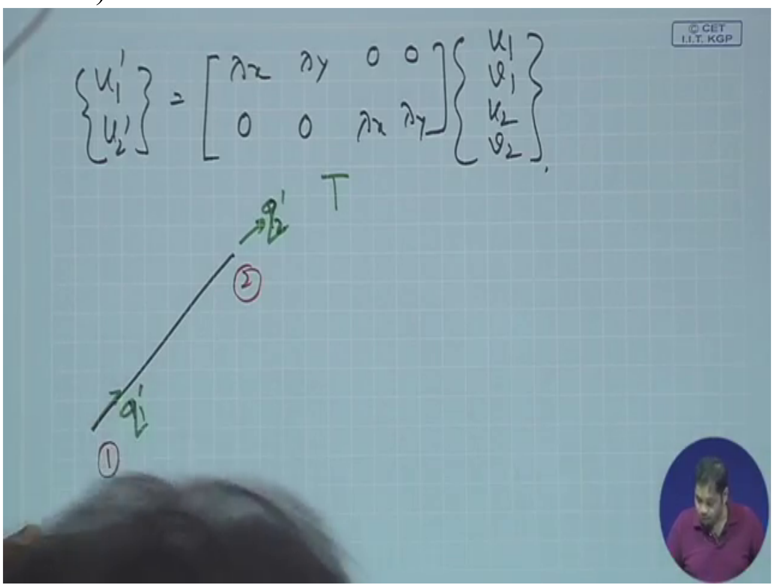

coordinate system we have, suppose this is, we have force like this, we have force like this. Suppose this is, this is P 1, P 2, P 3, P 4, suppose this is P 1 and then this is P 2, this is P 2 and then this is

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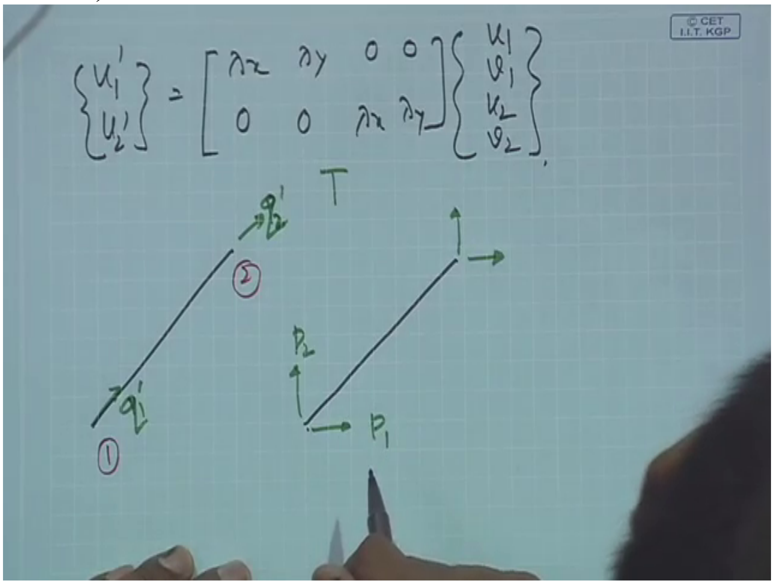

P 2 and then this is P 3 and this is P 4,

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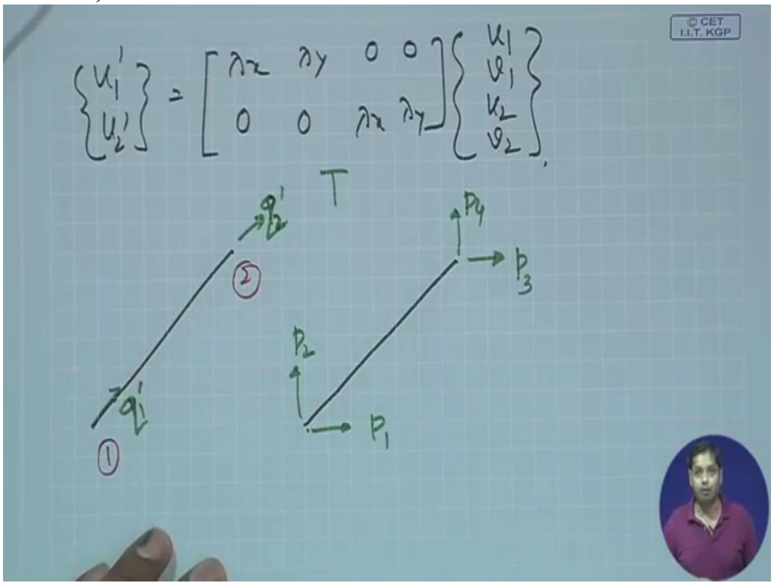

Ok. Now what we have to do is we have to similarly, we have to, instead of, here it was, here it was

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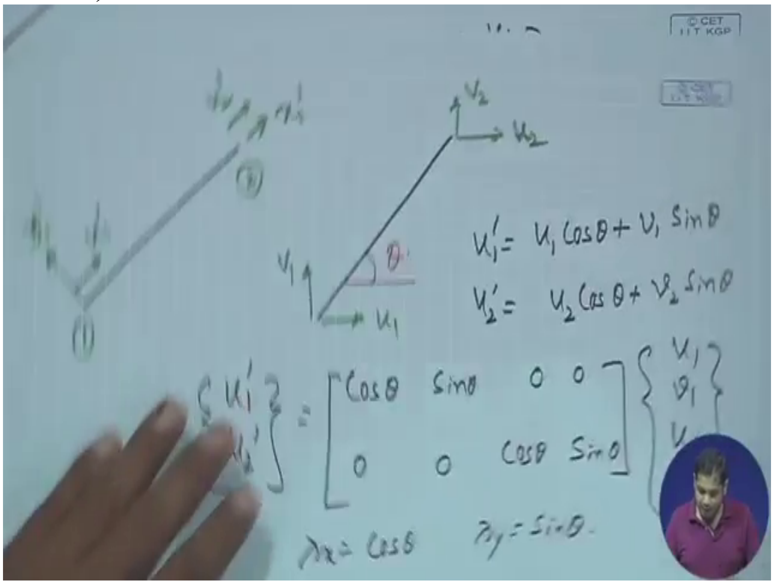

u 2 v 2 and all this, and instead of that here we have

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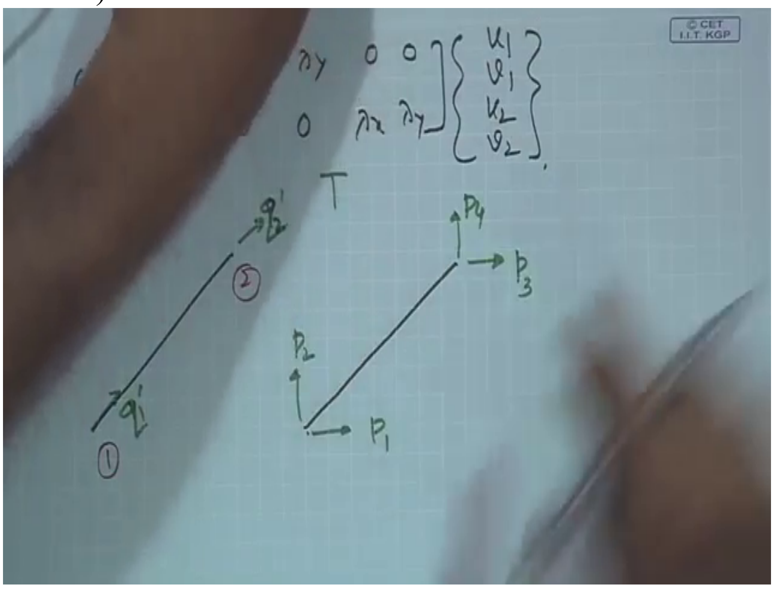

P 1, P 2 P 3 P 4,what we have to do is, what we have to do is, we have to just trans, write P 1, P 2, P 3, P 4 in terms of q 1 dash and q 2 dash and if we do that I am not writing the final expression, I am writing just the final expression, I leave it to you, you have P 1, P 2, P 3 and P 4 this entire thing will be equal to, it is the, transfer is same way, zero, then sin theta, zero, zero, cos theta, zero, sin theta, Ok.

Or

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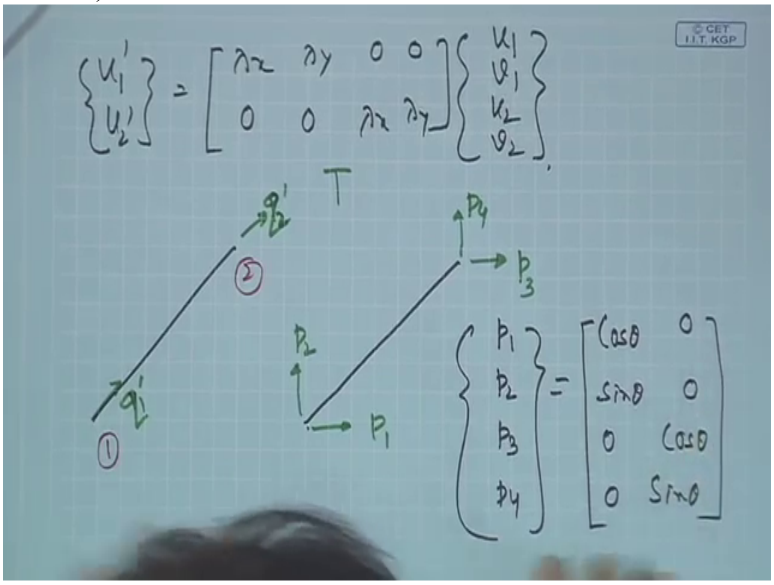

we can write this as well, P 1, P 2, P 3 and P 4 is equal to

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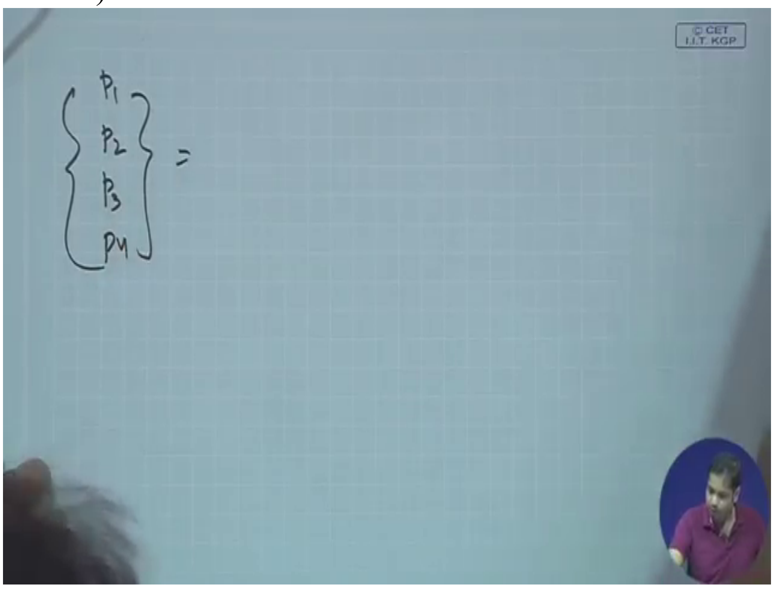

and here we have q 1 dash and q 2 dash Ok

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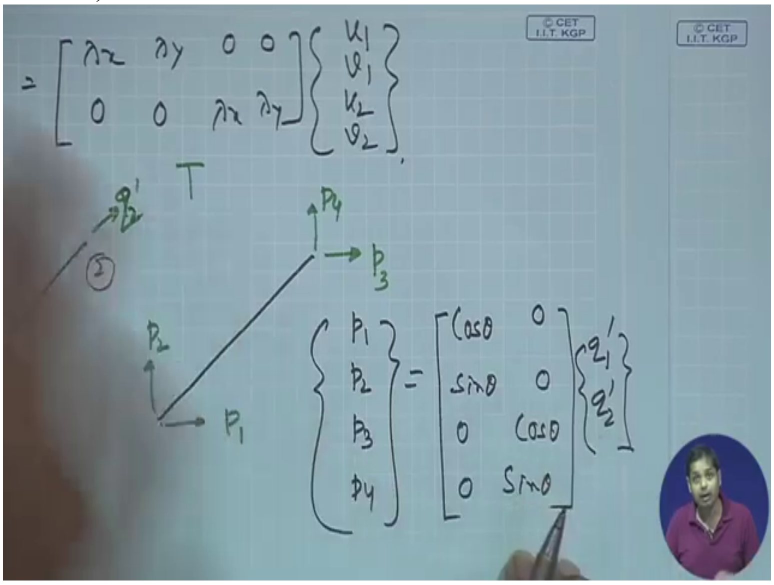

same thing we can write, substituting cos theta as lambda x, zero then lambda y, zero, zero, lambda x and zero, lambda y, zero lambda y and this is q 1 dash and q 2 dash, Ok. And if you compare this with

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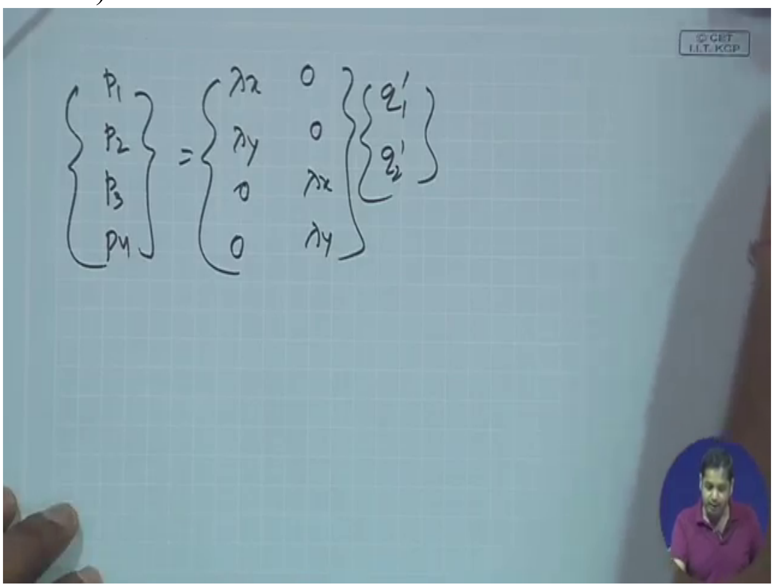

this

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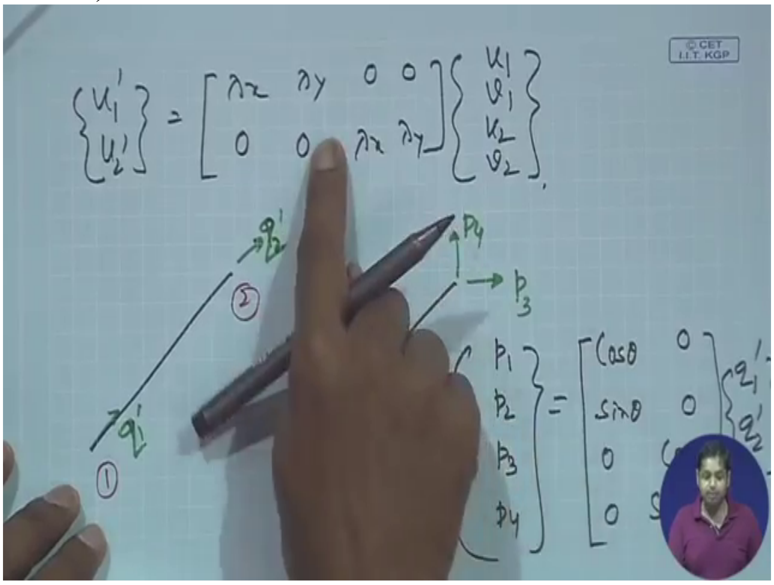

expression then it is essentially T dash.

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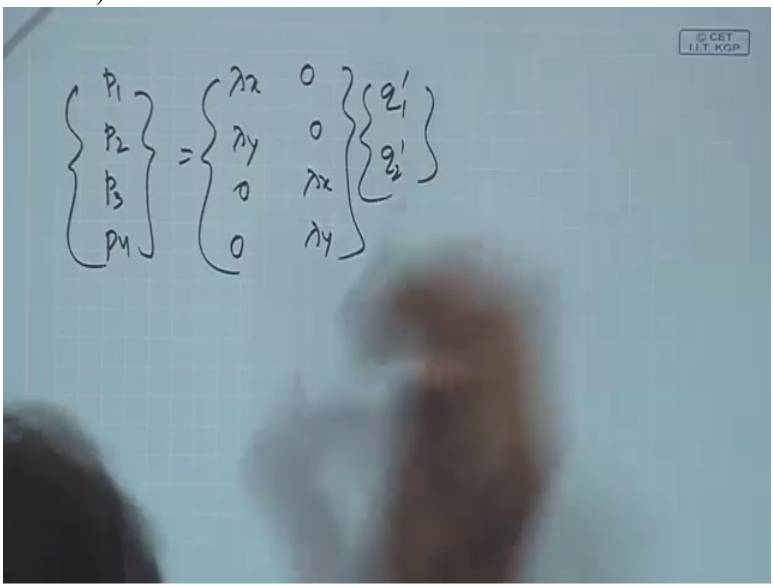

So this is T T dash.

Ok

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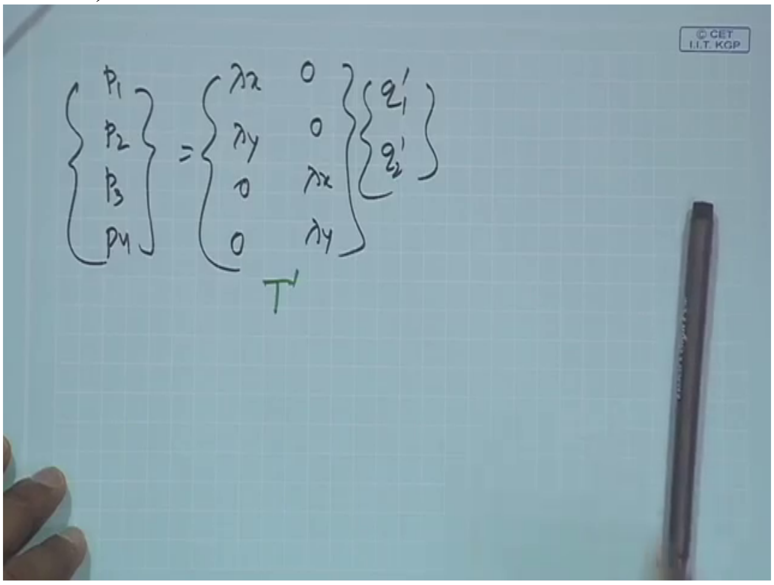

now we have a relation between, we have a relation

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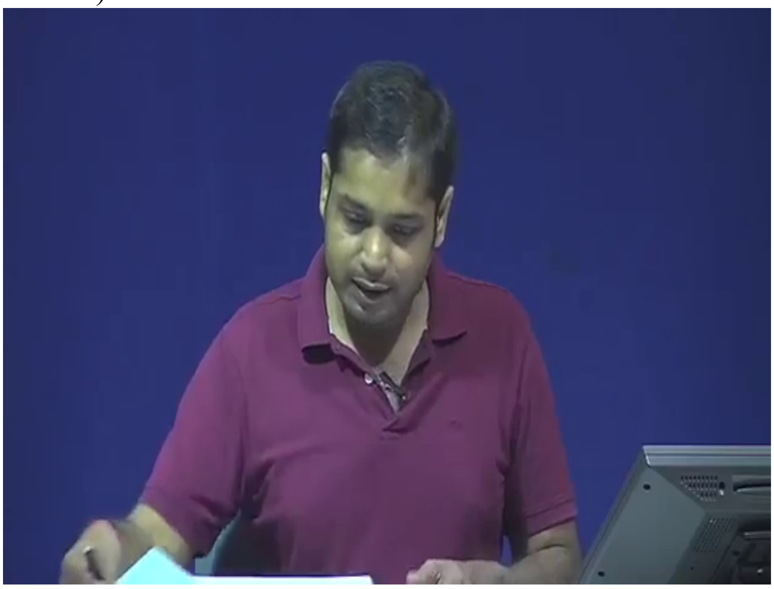

we have a relation between u 1,

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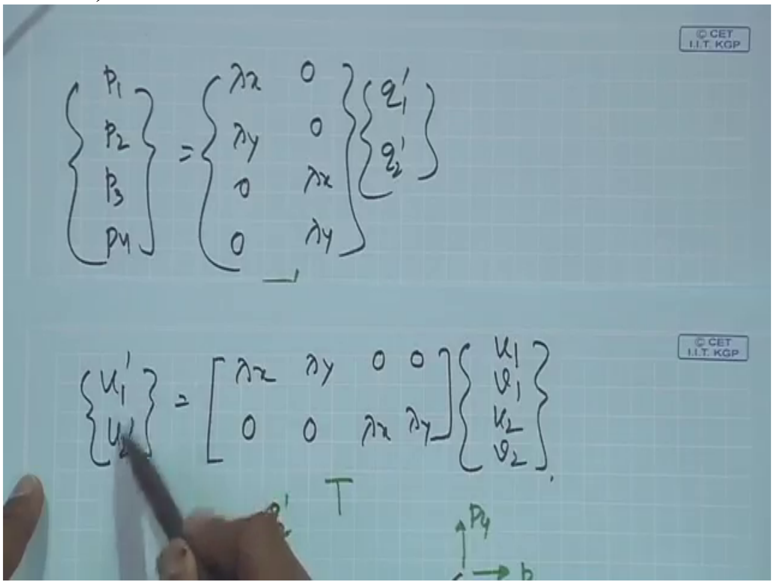

between u 1, displacement, local coordinate displacement and global coordinate displacement, we have a relation between global coordinate load and the local coordinate load and in addition to that, we know the relation between, we also know this relation, we also know this relation, Ok this relation.

So what we can do is now, so we also know q 1 dash, q 2 dash, that is, is equal to k which is the, which is your element stiffness matrix k e into u 1 dash and u 2 dash, What you have to do is now this u 1 dash u 2 dash is replaced by this and this q 1 dash and q 2 dash and that is replaced by, that is replaced by these. If we do that, or we can do is, in this expression this q 1 dash q 2 dash is replaced by this. And then this u 1 dash u 2 dash is replaced by this.

And if we do that, and what

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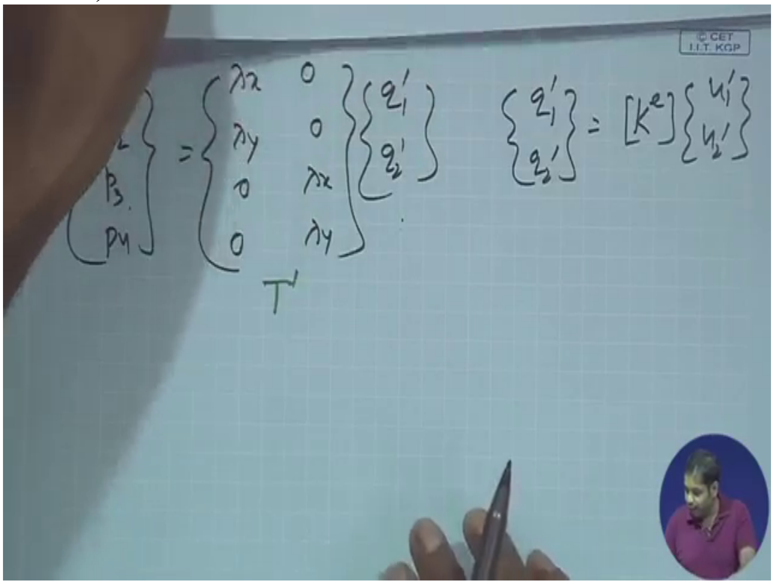

we get is we get P 1, P 2, P 3 and P 4, which is your load vector that is equal to, that is equal to T, which is the transformation, T transpose into k e which is the element stiffness matrix, and element stiffness matrix, and finally, and then we get T. We get T and this becomes u 1, u 2, u 3 and u 4, u 4 Ok

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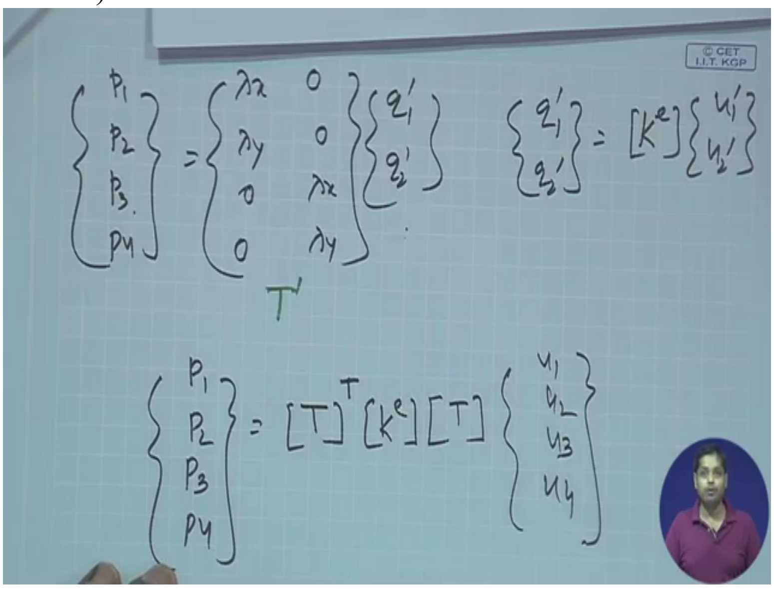

and you see what is the size of t transpose, size of t, size of t transpose is 4 cross 2 and this is 2 cross 2 and this is 2 cross 4. So resultant of this will be 4 cross 4.

And this is essentially k,

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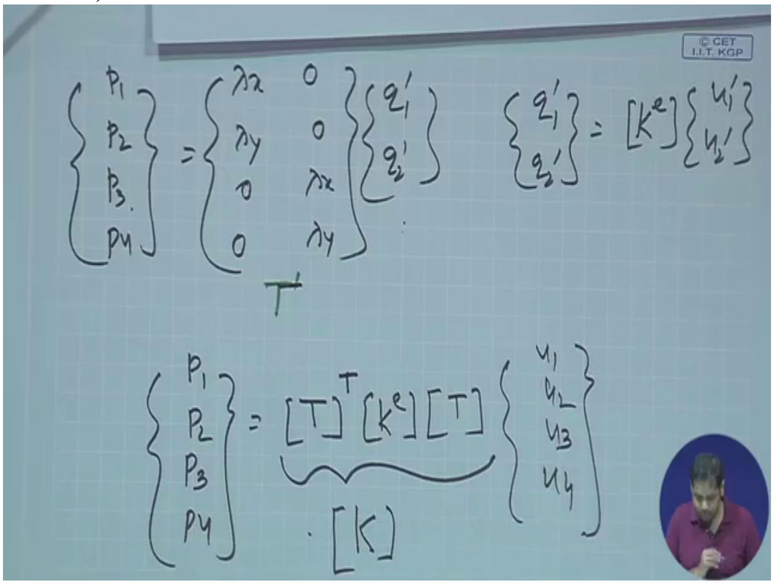

global stiffness; it is essentially stiffness matrix, element stiffness matrix written with respect to global coordinate system. Why I am saying it is written

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with respect to global coordinate system because this stiffness matrix relates

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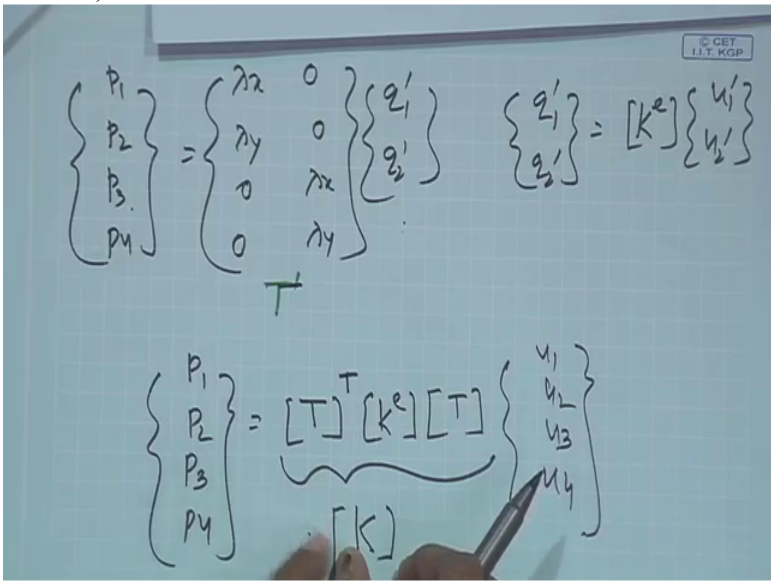

global load vector with global displacement where as this stiffness matrix, this stiffness matrix,

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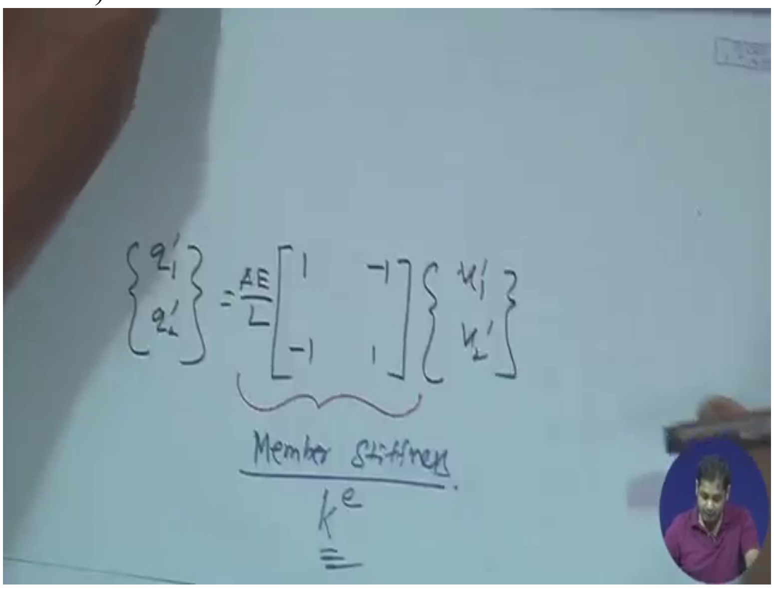

this stiffness matrix relate local load vector and the local, local deformation,

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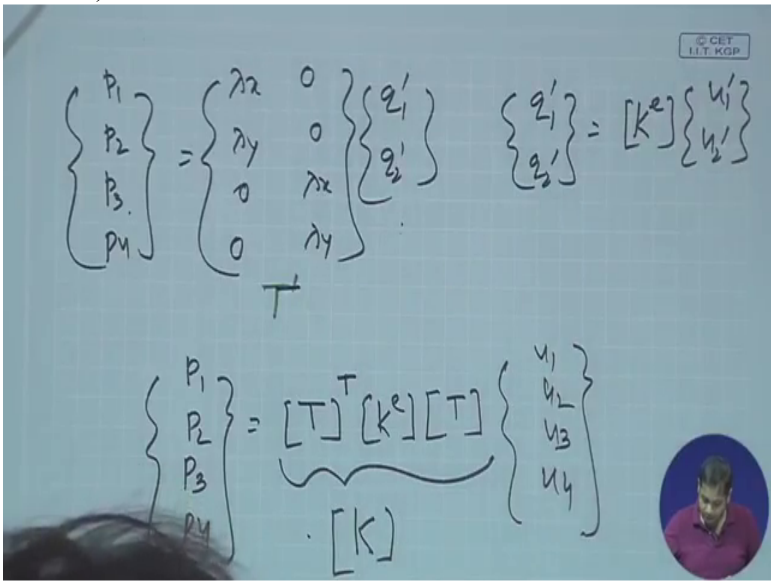

Ok.

So if you write that, finally, finally if this is the, if this is the global, if this is the global, this the element stiffness matrix written in global coordinate system and if you substitute t by this and k e by, k e by, we have already determined what is the value of, expression of k e, then the final expression of k we get, which is the global stiffness which will be the element stiffness matrix written in global form will be lambda x square, lambda x is going to be cos theta, lambda x lambda y and then minus lambda x square, you can please verify this, lambda x and lambda y and then this is lambda x lambda y, lambda y square, minus lambda x lambda y and then minus lambda y square. This is minus lambda x square, minus lambda x lambda y, lambda x square and then lambda x lambda y and the final row is minus lambda x lambda y, minus lambda y square then lambda x lambda y and lambda y square. You see this matrix is symmetric, Ok.

And of course we have another A E by L term. Ok

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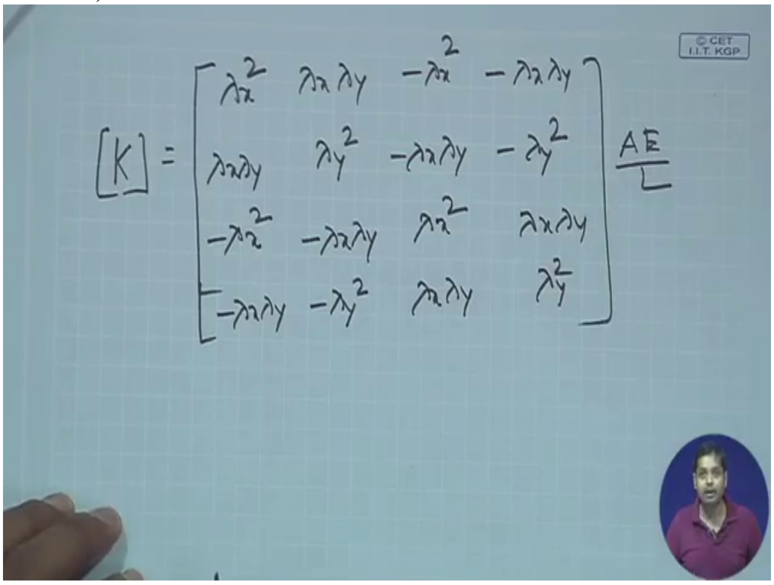

so once you know what is the properties of this member and this, in order to get this, we only needed the config/configuration, orientation of this because lambda x, we only needed is the orientation, right? If we, if we, if we,

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if we see this, yeah, if we see this, we only

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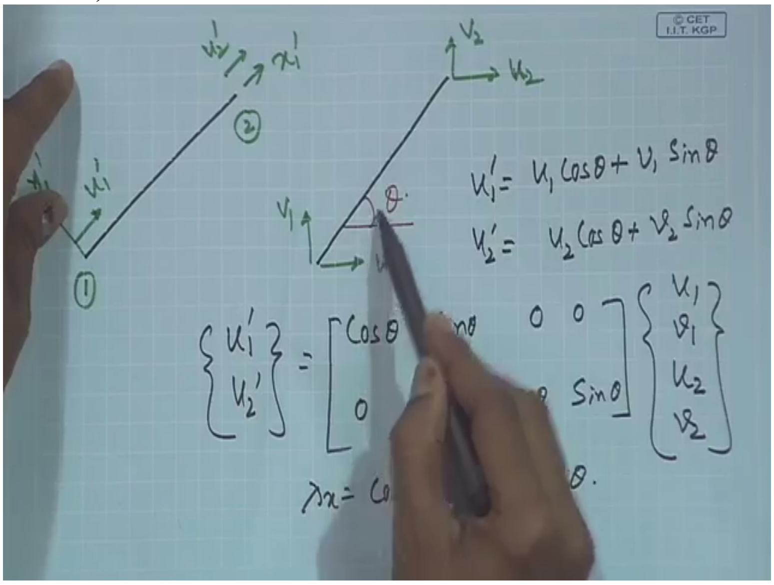

needed the orientation of this member, Ok, this angle theta, Ok. Now so if we know the property and the orientation of this,

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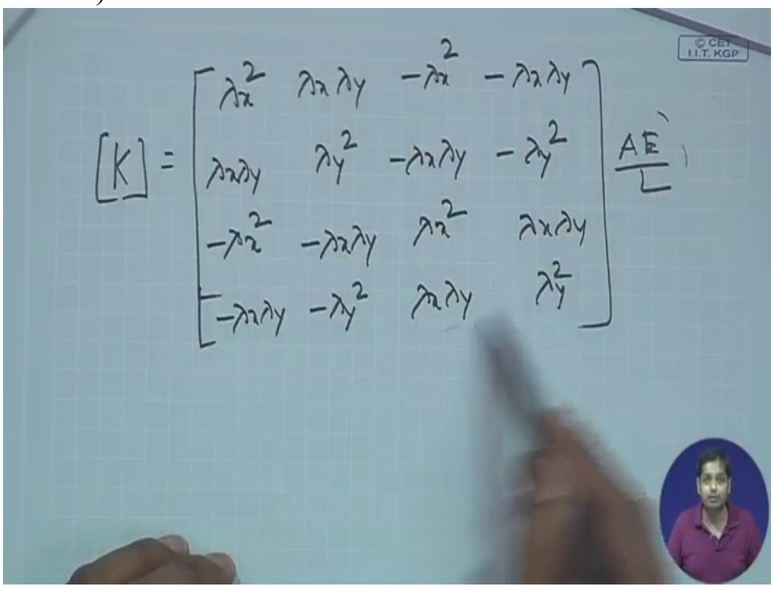

we can get the element stiffness matrix for the member with respect to global coordinate system.

How to get the, how to get the,

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how to assemble them, all the element stiffness matrix that we will see when we actually solve some problem. So what we have learnt so far, how to determine element stiffness matrix for class member and similarly we can determine the element stiffness matrix for other members as well and then we need to assemble them to get the solution and then subsequently to get the solution and assembling and all we will discuss when we actually solve some problems.

Ok, these are very, very brief; I mean a very, very brief introduction to analysis of trusses and direct stiffness matrix but as I said, direct stiffness matrix is essentially a new course. It is a separate course altogether, idea of whatever we are discussing in this week is, is just the transition, just to take you, whatever you have learnt in the first 11week, with that information to prepare yourself for the next level course which is advanced structure analysis course, Ok.

Next day what we do is, next day we will similarly derive stiffness matrixes for beams and frames and then we will soon, and subsequent, subsequent day lecture we will see how to actually use them for solving different structures, Ok. Ok, so see you in next class, thank you.