Structural Analysis
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Module 11
Lecture No 56
Analysis of Indeterminate Structures by Displacement Methods (Contd)

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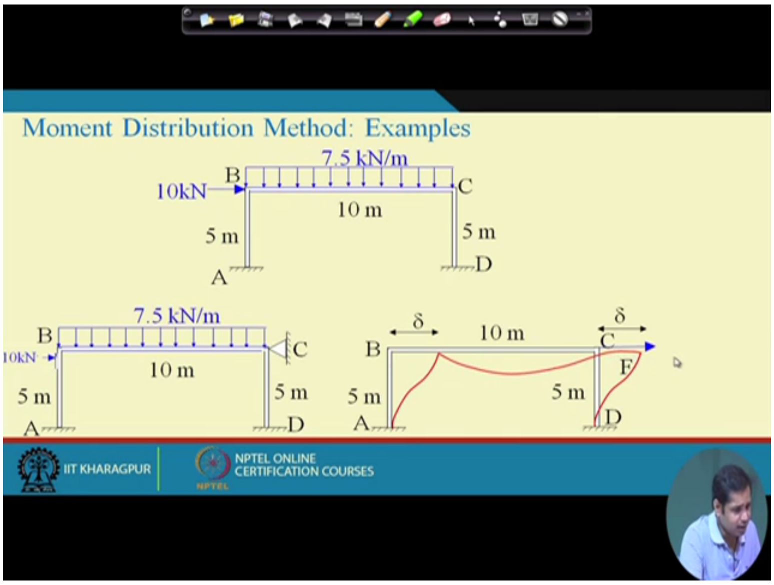

Hello everyone today what we do is, we will demonstrate moment distribution method through one more example it is a frame example which undergoes sidesway. So today is again demonstration of moments distribution method, the example that we will consider is this, again if you remember in slope detection method also we considered the similar example, now we will be analysing the same example through moment distribution method and then we will compare the steps and the final results okay.

If you remember in slope deflection method how we considered how we considered sidesway we use usual procedure of slope deflection method means writing the slope deflection equation than satisfying the equilibrium at every joints and then in addition to that we may needed one more equation and that equation we obtain by comparing the sheer force with the horizontal forces okay. Now we would do the same thing here but in some way the procedure that we follow in moment distribution method has a similarity the procedure that we use in slope deflection method for solving the same example okay, let us see what is that.

Now you see this problem can be divided into 2 sub problems; one problem is problem which has no sidesway so the sidesway is restricted by providing are huge providing a support at C which gives essentially horizontal reaction. Now since there is no actual deformation so providing only support at C will prevent the sidesway okay, so since the point C is not able to move in this direction and there is no actual formation in BC so point B will also remain at the same position. So there is no sway not in member AB and not in member CD, now then this is just considering this part, your only the vertical load only the vertical load because this vertical load will not cause sway in this structure okay.

Now then then what we do is then we allow the frame to undergo Displacement in this direction okay. Suppose this is the sway takes place at B and C and the amount of sway is Delta, we do not know the value of Delta okay. Now it is an integration method, and in slope deflection method what we did is, we directly use the equilibrium equation to get the equilibrium equation where we actually will be actually compared the all the forces in horizontal direction okay. Now again it is an iterative method moment distribution is an iterative method, what we do is, we give one arbitrary sway give some value of Delta okay.

Now we know that in this problem sway takes place because of some horizontal load right, but what is that horizontal load we immediate we do not know okay but we can derive an equation in which if you know this horizontal force we can obtain the sway right. Now what we will do here is we allow the member to sway allow the structure to sway by some arbitrary value and then knowing that sway deflection this Delta here, we determined that what is the force that may cause the frame to sway by Delta amount okay, now we calculate this force okay. Now what we do is, we compare this course with the reaction that we get from C x okay and then from this comparison we will determine what is the final moment final internal forces in the member, let us demonstrate that first okay.

Now you see the first example since there is so we have 2 sub problems, one problem is divide the entire problem into 2 parts; one is this part and another is this part. Now as far as this part is concerned we have already analysed this part in the previous in the last class, you remember portal frame subjected to uniformly distributed load there is no sway takes place no sway takes place but the same procedure what we have done in the last class we can confuse you with that, we will end up with the same results.

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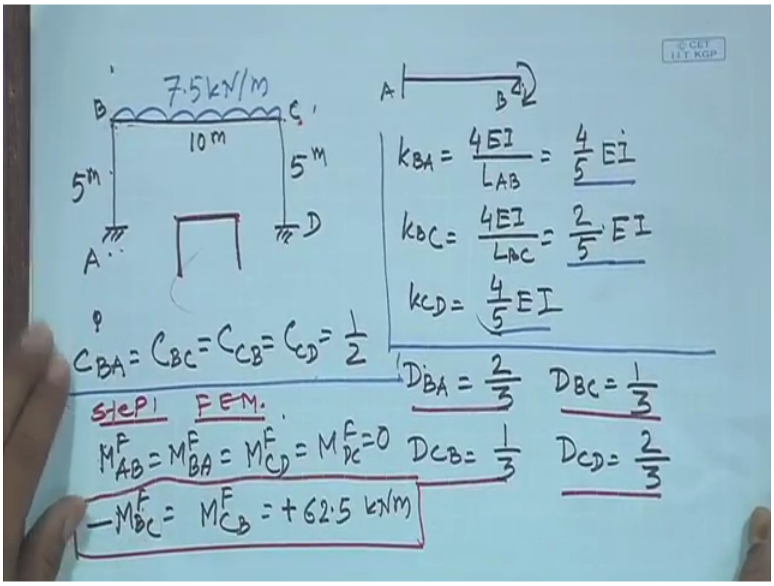

Just to just to refresh your memory, if you remember these are slides from last class, the same example we used and we calculated the carryover factors are half and then corresponding absolute stiffness is this and from this absolute stiffness we calculated the distribution factor for member BC and BA and then member CB and CD. And again from the distribution then the step 1 the fixed end moment we calculated for member BC, for member AB and CD fixed end moments are 0.

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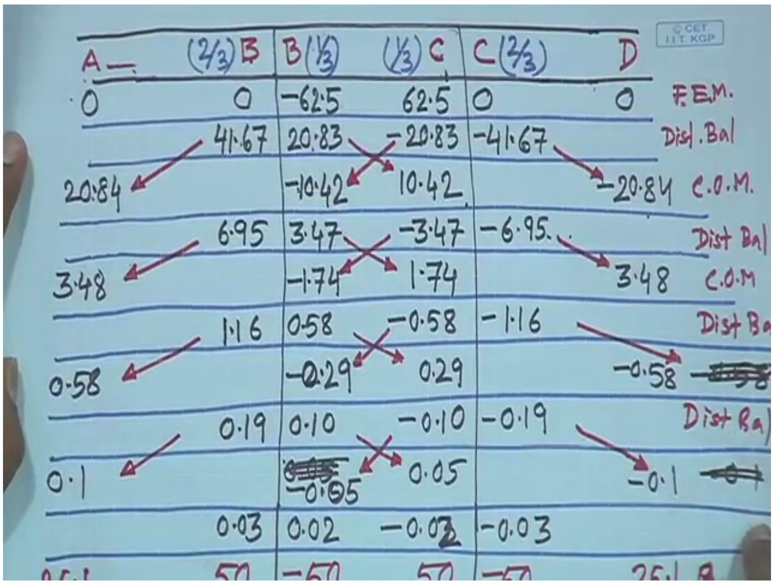

And then once we know the fixed end moments then we have the final this is the final iteration step okay, these are the fixed end moments and then we iteratively we apply the balancing moments, first we calculated the unbalanced moment at every joint and then applied the balancing moments which is equal and opposite to the unbalanced moment and then balancing moment is to be distributed over the members over the connecting members and then that distributed moments may get carried over to the other end depending on the support condition okay.

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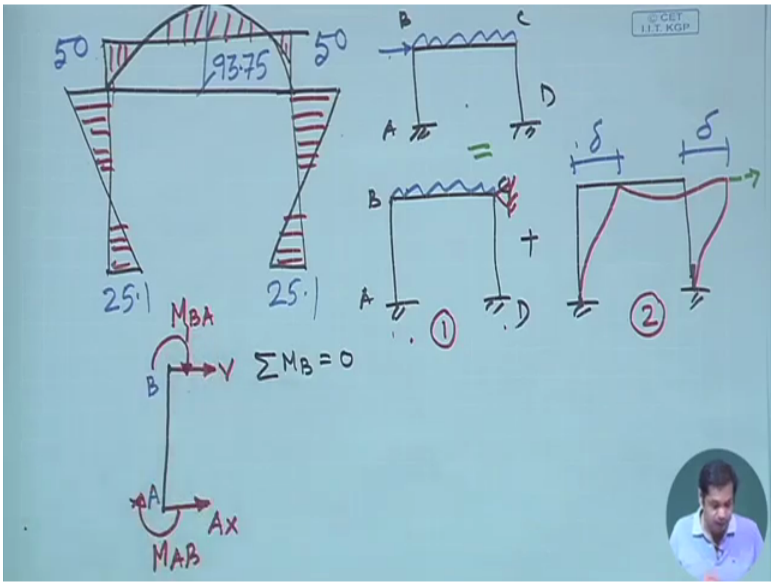

Now then finally we keep on doing it, finally we get the convergence reaches the tolerance limit and then finally this is the result that we obtain and this is the bending moment diagram if you remember this is the bending moment diagram for this part. So what we have is, we have 2 parts now, actual problem is this which is subjected to some which is subjected to vertical distributed load and horizontal load like this. I am not writing the dimensions in this case, this is A, B, C, D, now this is divided into 2 parts just now I have as I shown, these 2 parts are one part is this, one part is this, where it is subjected to vertical loading and BC A, B, C, D and then we have a support at C which will provide horizontal reaction and because of this horizontal reaction this will not move okay.

But as far as the member forces are concerned at A, B, moments are concerned at A, B, C, D, they can be obtained by the same procedure and then + we have another part which is with the same thing and then we give some arbitrary arbitrary sway so and then we give some arbitrary sway okay this value is Delta this value is Delta okay. Now we will have some value of Delta and then what we determine, we determine what is the force that might have caused what is the force, what is this force that might have caused this kind of deformation and then this is equal to this + this. Now as soon as analyses of this is concerned, analyses of this is exactly as this same.

So what we do, now is we do not repeat the process for the analyses of this, you can go back to the previous class and see the different steps but the final result is this. So for this we take the final result and then what we do is we will see how to analyse this part and then once we have the once we have the solution for this part then how to combine them to get the solution of the actual problem okay. Now yesterday the problem what we there is a small difference between the problem that we considered yesterday and this is problem 1, this is a sub problem 1 and this is sub problem 2 okay. Now the sub pro as I said as well as the internal forces, the moments are concerned at various joints A, B, C, D, the moments will be same.

But there is a small difference between between the problems that we consider in the previous class and this problem the difference is. Now here we have horizontal, we have we have a support at C okay and this support will provide some horizontal reaction okay but there was no horizontal reaction at C in this problem. But again as I said as far as the moments are concerned and if you apply moment distribution method in the forward step, you will see that as soon as moments are concerned, this will be same but in addition to that there will be horizontal reaction at C and let us first find out what is the horizontal reaction here considering, in order to calculate the horizontal reaction here we take the moments that we already obtained in the last class okay.

Now how do we proceed? First we take the free body diagram of this part, what would be the free body diagram of this part? This is C, this is D, and then we have we have a moment like this here and then a moment like this and then in addition to that what we have is, we have this is this end is supported fixed end so there will be a reaction force and suppose the reaction is A x okay. So this moment is M BA and this moment is M AB means moment at A in member AB and this is moment at B in member BA okay. And then we will also have shear force here so suppose that is is equal to V, so there is no other forces in this member okay. Now this is A and this is B and if you remember in slope reflection method also we calculate the horizontal forces in sway problem okay.

How we calculate the horizontal forces? By considering the free body diagram of 2 members okay. Now what we do is we apply the we apply the equilibrium condition, equilibrium condition at B and what it says that equilibrium condition at B will be submission of M B equals to 0, and what are the forces will contribute, A x will contribute, this will contribute and this will contribute, and what are forces values we have.

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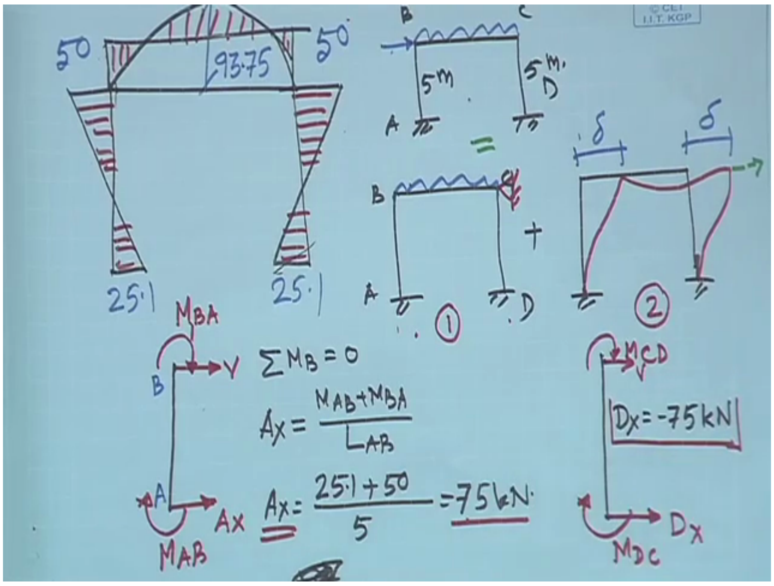

Again you see the values are at A it is 25 clockwise, at B it is 50 so these 2 are clockwise moment then it will be positive and then this will be this will be if you write the values we get A x equals to M AB + M BA divided by L AB, L AB was 5 meter, this is 5 meter, this is 5 meter so if you substitute then A x will be M Ab is 25.1 + M BA is 50 and divided by L AB is 5 and we get 75 we get 75 kilo Newton so this is A x okay this is A x. And similarly if we take the free body diagram of this part, free body diagram of CD and what would be the free body diagram of CD? Free body diagram of CD will be this and then there is a moment here which is M CD and then another moment here which is M DC then vertical then horizontal reaction which is D x and then the force V shall force V.

You take moment about this and what is the moment, here it is – 25 and here it is – 50, here it is – 25 okay, these are – not +, this – 25 and this is – 50, you substitute that and this length is 5 meters once again and we get D x equals to – 75 kilo newton okaynow this is the problem okay. Now what happens is then what would be the reaction at C? Reaction at C will be okay there is one thing you please correct it, the first problem will be like this, this is 10 kilo newton, first problem is like this and this will cause sway in this, this might have caused sway in the structure but the sway is restricted by this horizontal force okay.

And then the second sub problem is you allow the sway you allow some arbitrary Delta Delta amount of sway this problem, so this is a problem 1, this is sub problem 2 okay. So in sub problem 1 we have horizontal force and that the sway due to this horizontal force is restricted by this by this support. But this horizontal force will not change the moment because it is acting this for acting at the joints so it will not contribute anything to the moments at the joints okay. Therefore, for this problem you get the same same moments okay. Now we have horizontal force A x equals to 75, D x equals to – 75 now let us find out what is reaction what is the reaction C x here okay.

Now reaction C x will be then you apply the equilibrium condition, equilibrium condition will be again here itself we can do that equilibrium condition, equilibrium condition will be submission of F x submission of F x equals to 0 and what other forces we have in F x, we have force in reaction at A x then reaction A x reaction D x and then horizontal load 10 kilo newton and then reaction C x which is in this direction reaction C s which is in this direction okay. Now if we apply all this, what we get is we getC x equals to 10 kilo newton. Now actually without without doing all this without taking this part separately also we could have obtained the C x by drawing the free body diagram of the entire structure and take moment about this.

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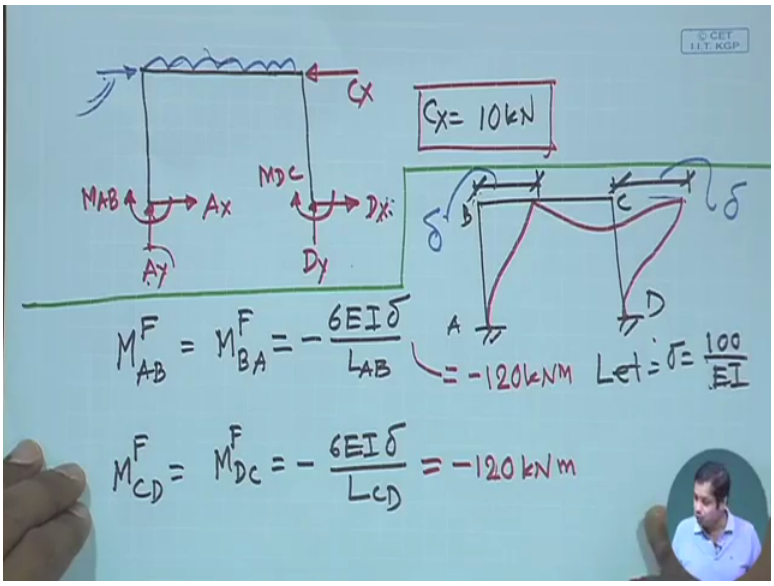

Let us do that we draw the free body diagram of the entire structure okay which is which is subjected to this load and then this load then in addition to that we have a moment here and we have a moment here and then reaction, here also vertical reaction, horizontal reaction and horizontal reaction, this is A y, A x, it is M AB and this is D y, D x and this is M DC now and then we have horizontal reaction here C x okay. Now instead of taking the free body diagram of this part separately, this part separately, we can take the moment about this and this D X and A x will not contribute and the contribution will be from A y from M Ab from DC from this load and from this load and then C x and you can compute the CX, but again for that C x you need to find out what is the value of A y okay.

So this is up to you how you find out the find out the support reaction, which part you take and which free body diagram you take and apply the equilibrium condition there are many ways you can do it, but the main point is we need to find out what is support reaction and support reaction is we have obtained C x equals to 10 kilo newton okay, so this is the solution for the first sub problem. Now let us do the let us now solve the second sub problem and what is the second sub problem? Second sub problem is this, second sub problem is we take we take this is the second sub problem, just now I showed you let us draw it once again, there is okay so this is Delta, this is and this is their Delta okay, so there is a sway Delta amount.

Now again this we solve using moment distribution method and what is the first step in moment distribution method? We know first step is to find out the absolute stiffness then distribution factor, but since we are talking about the absolute stiffness and distribution factor, they depend on the geometry of the geometry and the material property of this structure. Since geometry and the material properties are same, distribution factor is carryover factor support conditions are also same, distribution factor carryover factor all will remain same. So whatever distribution factor and carryover factors we calculated, the same thing we can use here.

Now the next step is to find out what is the fixed an moment. In the previous example in this case the fixed end moment we have because of the external load okay. Now this load will not cause any fixed end moment because it is applied on the on the fixed end on the support on the joint itself. Now but this joint displacement this you can use as a settlement of joint B and this can be treated as the settlement of joint C so these settlements of joints will cause fixed end moment and we have already seen if the joint undergo settlement then what is the value of fixed end moment. So what is the value of fixed an moment? Fixed end moments are M fixed end moment M AB fixed end moment equals to M BA which is again fixed end moment if you consider this joint this member and in this member 1 dismember this joint undergoes displacement and settlement of Delta amount.

And both the cases if you remember the fixed end moment will be – 6 EI Delta by L AB, this is the fixed end moment and if you if you and this is the fixed and moment. And similarly what would be the fixed end moment for this? For this the fixed end moment will be M CD of F, fixed end moment equals to M DC again fixed end moment equals to – 6 EI Delta by L

CD. The fixed end moment in the fixed in moment BC there will be no fixed end moment in BC the fixed end moments only we have in member AB and member CD.

Now Delta is unknown here is not it? We do not know how much the sway takes place, now what we do is we give some amount of sway some arbitrary amount of sway. Let us is the let us give let Delta is equal to let give Delta equals to 100 by EI okay, 100 by the IE is a very arbitrary value, you could have taken you may take 10 by EI, 1 by EI or any arbitrary value by EI, the reason of taking EI because then this EI and this EI will cancel each other, so you will not be having any EI in the expression okay. So if we take 100 EI as Delta then fixed end moment becomes, this becomes – 120 kilo newton meter and then this also becomes – 120 kilo newton meter okay. So fixed end moment at A, at B, at C and at D will be – 120 kilo newton meter for members AB and member CD okay.

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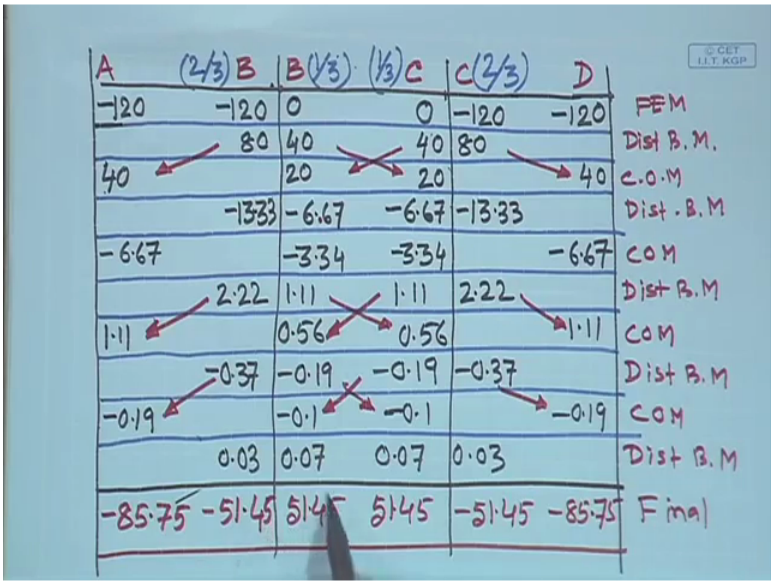

Now once we have the fixed end moment, next is to do the iterations, now let us do the iterations. The iterations in moment distribution method the same thing we have to do, so first is let us let us take we have 3 members the same procedure we will follow suppose this is A and okay and draw a line, this is the table for moment distribution method and okay. Now this is B when B belongs to AB and this is B when B belongs to BC and this is C, this is C, this is D. And then we have distribution factor, distribution factor here it is 2 by 3, it is 1 by 3, you can check the previous class where we have obtained the distribution factor and this is 2 buy 3 right, these are the distribution factors at B and C.

Now what is the fixed end? First step is to write the fixed end moments, the fixed end moment here it is – 120 and here it is again – 120, here it is 0, for BC it is 0 and it is – 120 and again it is – 120 okay so this is fixed end moment. Now once we have the fixed end moment next part is next part is we need to check what is the status equilibrium status at every joint. Now at joint B what happens, there is no it is not in equilibrium so there is an unbalanced moment of – 120 and again joint C that is an unbalanced moment of – 120, so in order to balance it we have to apply moment fictitious moment of 120 at joint B and 120 of joint C. And once we apply that moment is called balancing moment and when you apply this balancing moments at the joint then that balancing moment will get distributed with the connecting members as per the distribution factor.

So what happens, had joined B we need to apply moment of 120 kilo Newton, at joint C also need to apply moment of 120 kilo Newton then this – 120 and this 120 they will balance each other and – 120, 120 they will balance each other. This 120 will get distributed in BA and BC and the contribution of BO will be to third of 120 and contribution in BA will be one third of 120, similarly this will be 80 and this will be this will be 40. So contribution will be this will be 80 and this will be 40, similarly this will be 40 and this will be 80 okay. Now this is the balancing this is the distributed balancing moment this is distributed balancing moment right.

Now once you have the distributed balancing moment then this moment there will be a carryover moment at A because of the moment at B there will be carryover moment at D because of the moment at C and from B to C and from C to C and carryover factors we have calculated carryover factors are half here. So this 80 so moment 40 will carried over to will be carried over to A, this is the this is how we show the carryover moment and then similarly from 80 it is 40 here and from 40 it is 20 here, these are all carryover moments these are all carryover moments okay. Now once we have the carryover moments and then again we need to check the status of equilibrium, now let us see what is the status of equilibrium?

At B there is an unbalanced moment of 20 and at C also there is an unbalanced moment of 20, we need to apply – 20 at B and – 20 at C and this – 20 will get distributed as per this. So if we need to apply – 20 here and then we need to apply – 20 here, and once the 20 get distributed, two third of 20 will be here and one third of 20 will be here so those values are – 13.33 this is for BA and – 6.67 for at B and similarly it is – 6.67 then this is – 13.33. Now this will, this is the distributed balancing moment so this is carryover moment, this is distributed balancing distributed balancing moment.

Now this moment will also be carried over to here, half is the carryover factor so half of this will go here – 6.67 will go here and then – 3.34 will go here, – 3.34 will go here and – 6.67 will go here and this is the carried over moment okay. And again we need to check the status of equilibrium, we need to apply +3.34 here, + 3.34 here balancing moment which will get distributed and the corresponding distribution will be 2.22 and 1.11, it is 1.11, 2.22 and this is balancing distributed balancing moment, distributed balancing moment, again that will be carried over and this become 1.11 will carryover from here, it is 1.11 will carryover and from here it is 0.56 will carryover, so this is carryover this is carryover moment, these are all carried over moment okay.

Now again once we have the carry over moment, again check the equilibrium status so this is carryover moment. Once again check the equilibrium status, we need to this is unbalanced moment of 0.56, we need to apply – 0.56 here and that will get distributed according through the distribution factor and those of corresponding values will be – 0.37, – 0.19 and this will be – 0.19 and – 0.37 okay and this is the this is the balancing moments distributed balancing moment distributed balancing moment, you always put a remark column at the end where you can write what is the step that you have corresponding line you have performed.

Now again there will be a balance carryover moment, carryover moment is – 0.19 and then this is from there it is – 0.1, – 0.1 and then it is – 0.19, we are approaching towards the converge towards the convergence these the values are becoming very smaller and smaller and then let us see the last step you need to last step always be the balancing the balancing moment the distributed balancing moment, otherwise there will always be an unbalanced moment at the joints okay. So this 0.1, we need to apply 0.1 balancing moment that will get distributed and the distribution is 0.03 here and 0.07 here and it is 0.07 here 0.032 here, so this is the last step, this step is the carryover moment and this is distributed balancing moment.

So let us make this is the last step because the values are very small, you can continue 0.07 is the unbalanced moment then again you can apply some similar again you can carry this you can get the carryover moment and then check what is the again for the you can continue till you get value 0 but it is reasonably smaller value you have obtained compared to fixed end moments we have started with. So final results are now – 85.75 and then it is -51.45, it is 51.45 and then it is also 51.45 and then – 51.45 and – 85.75 and this is your final okay. Now you check this is also an equilibrium, this is an equilibrium so this is my final result, final results for which? This is the final this is the this is the these are the moments at various joints when Delta is equal to for Delta is equal to 100 by EI right.

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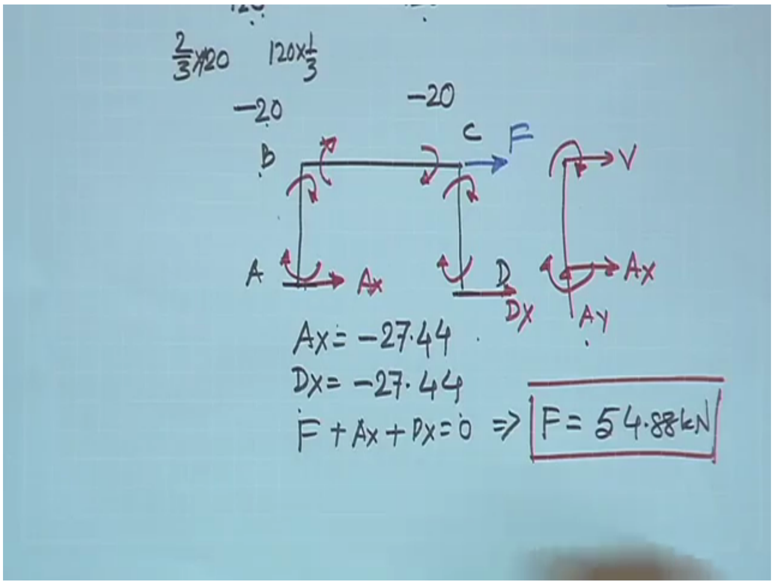

But Delta we do not know that in an arbitrary value right, Delta may not be 100 by EI, now how to get the final value okay. Now you see the final value will be let us now find out for the same problem let us now find out what is the support reactions. For the same problem same problem we have now obtained what is the moment, this is A, B, C, D okay we know what are the moments these are the corresponding moments so we know what is this moment, this moment is -85.75, this moment is – 51 and then we have this moment, all are shown clockwise and we have this moment okay.

Now we can take separately this, we can take separately this, apply the equilibrium we can take separately AB, the free body diagram of AB will B like this and then this, there will be this is A x and then A y, there will be V here and then you take free body you get moment about this point and this will not contribute, we can get the A x. Similarly we do the free body diagram of C exactly the way we have done for the previous sub problem 1, we can calculate what is the value of A x what is the value of D x and then we now what we do now is, what we do now is say A x equals to – 27.44 and D x equals to – 27.44 okay. Now you see means what?

Now A x is equal to in this A x is equal to this and D x is equal to this right and total force in this direction is A x + D x means because of this Delta okay, this moment is calculated these moments are calculated based on all the steps we have done. These are based on the sway of amount Delta, but actually what? And there is some force that make cause this amount of sway right then what will be the force which may cause this amount of sway? The force will be this + this so this suppose this is the force that might have caused this Delta and this F will be then this F will satisfy the equilibrium condition equilibrium with A x with A x and D x and that equilibrium condition will be F + A x + D x equals to 0 and from there we will get F equals to 54.88 kilo Newton okay.

So if we apply 54.88 so instead of there are 2 things instead of applying Delta amount of sway Delta by 100 Del 100 by EI sway at B and C, if we apply 54.88 at C, they both have the same effect, is not it? So the Delta sway is caused due to this amount of force this Delta by the 100 by EI sway that we started with, that that is caused because of this amount of force, instead of 100 by EI if you take 50 by EI, 25 by EI or 330 by EI then you get different F value but that F value will be consistent with the Delta that you take. Now we are almost at the final stage, now what we have here?

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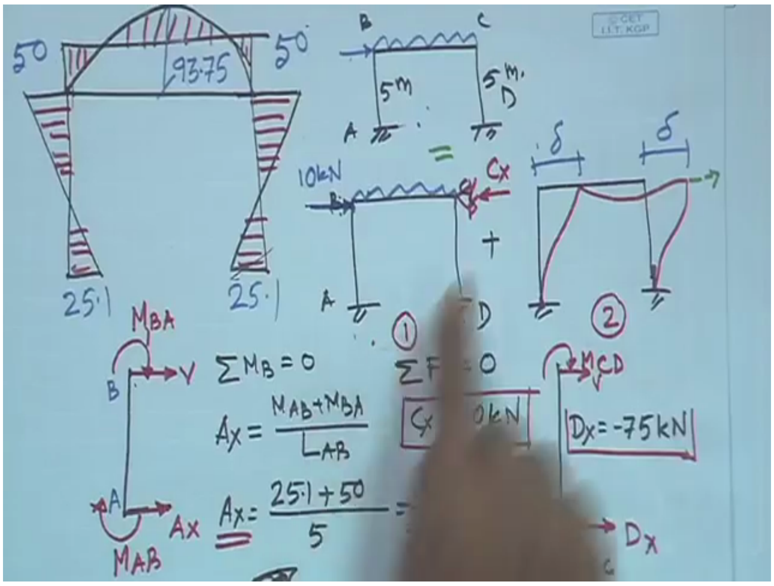

We have what we have obtained? In this, this is our first sub problem, this is our second sub problem, but these if we add them that should give the the same they should give these values okay. Now it is again very, instead of 100 by instead of hundred by EI you could have given unit displacement, unit sway here okay and get what is the F value. Now but actual reaction, actual reaction will be C x right, actual reaction see these 100 kilo Newton force will cause this sway, 100 kilo Newton force is the actual structure but that sway is restricted by this reaction C x. So essentially if we remove this reaction then sway is allowed right? So the sway is actually taking sway is actually taking place because of this 100 kilo Newton force.

Now the amount of sway we assume i 100 by EI and that is so easy to 54.88 kilo Newton force, means if we apply 54.88 kilo Newton force in the actual structure, in the actual structure if we apply 54.88 kilo Newton load then what will happen, then we get a moment or sub moment then we get moment which is equal to the which is equal to the moment in sub problem 1 + okay.

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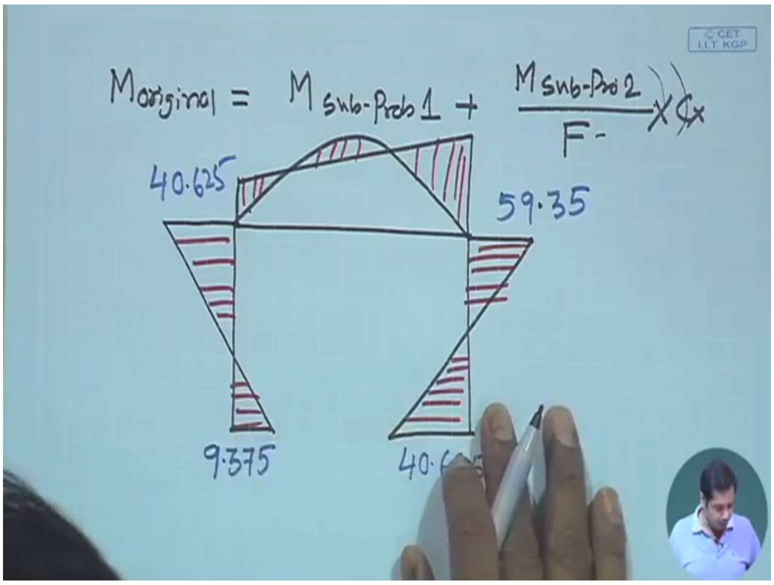

Let us write it so actual moment in sub problem 1 will be then M original will be original will be M sub problem sub problem 1 + M sub problem 2 divided by F, divided by F means this is when you apply unit force here, when you apply unit force here okay. Now if we the moments that you get, the moments that we get that is because of because of the force F and if we apply unit force, this will be the total moment but actual reaction you have into C x okay. So total moment in original you need moment sub problem 1 + moment in sub problem 2 by F into x, it is again very similar to your apply unit load and get the get the corresponding moment.

So these values we have already opted in, these values are this and for sub problem 2 the values are this, we substitute that and for and F equals to we have obtained F equals to 54.88 and we also obtained C x equals to 10 kilo Newton. And if we substitute this, the values that we get this let us finally draw the bending moment diagram and write the values okay, you get the bending moment diagram like this and this is the final bending moment diagram that you get, in the previous case it was symmetric but now it will not be symmetric okay. The values are you get this is 9.375, this is 40.625, this is 59.35 and this is 40.625, this will be the bending moment diagram for the corresponding bending moment okay.

So essentially if you have a sway problem then that needs to be distributed that needs to be divided by 2 parts, one is the structure without sway and the structure where only sway takes place and that because of the sway whatever moments you have that need to be calculated and then that moment need to be normalised based on the reaction that you get in sub problem 1 and then finally you add the moment in sub problem 1 and + the normalised reaction in sub problem 2 to get the original moments in the original structure okay.

Again the steps will be clear if you if you do some examples, if you do some example there are gay many examples given in the book, please do that because without doing them the steps will not be clear, you may have some confusion even after all this discussion but believe me those conclusions will be clear once you start actually applying the concept to several problems. Okay, so we will end here we will end here today so what we will do, next week if we will discuss direct stiffness method as I said that next week would be the last week of course and direct stiffness method as I said in the beginning this is a very basic structural analysis course, this is the first course in structural analysis and you have, you will be having advance course in structural analysis in your subsequent semester.

And the next the last week when we talk about direct stiffness mating that will be the transition between your the first course in structural analysis and advance structural analysis course okay, see you in the next week thank you.