Structural Analysis
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Module 11
Lecture No 55
Analysis of Indeterminate Structures by Displacement Methods (Contd)

Hello everyone welcome once again, we have discussed the concept of moment distribution method and then that concept was also demonstrated through 1 example 1 beam example. What we will do today, we will give you an example of a rigid frame indeterminate rigid frame and then see the moment distribution, and how the moment distribution can be applied to get the unknown internal forces. Now again okay, so let us show you the example that we are going to do today.

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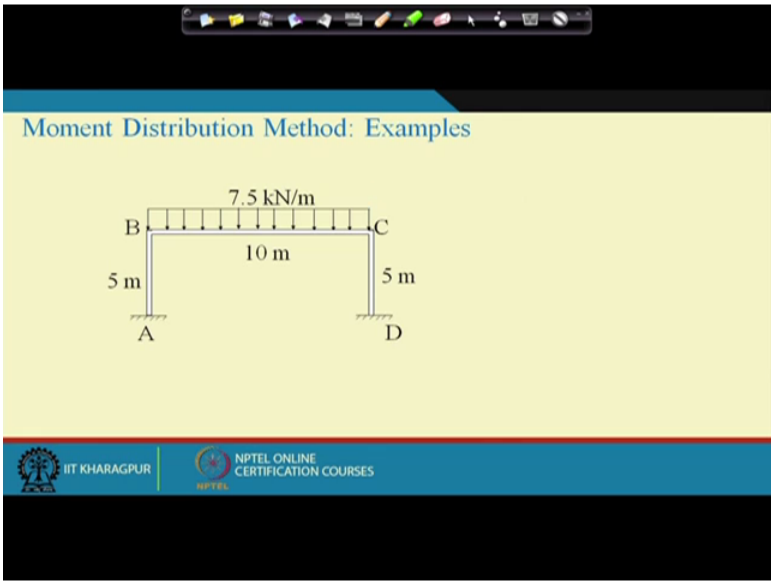

This is the example, if you remember the same example we considered, we analysed the slope deflection method, so what you can do is whatever results we get today using moment distribution method, you can compare that with the result obtained by slope deflection method and also you can compare the steps used in slope deflection method and the steps used in moment distribution method. And that is the precise that is the precise reason why as I said in the last class also that we have been using this similar example to demonstrate different methods okay.

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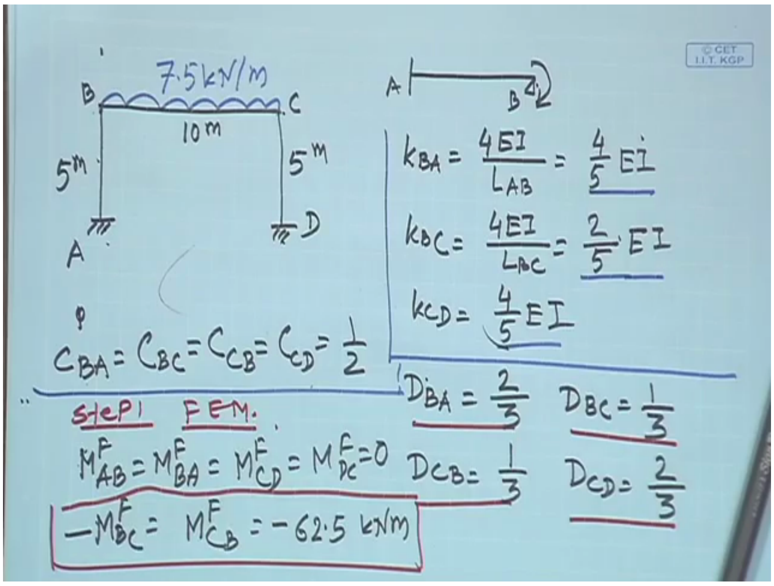

So this is the method, this is the example of a portal frame subjected to uniformly distributed load and BC. Now the first is we need to find out what are the distribution factor and carryover factor, in order to get the distribution factor we need to find out what are the stiffness we have okay. Now, all this is a fixed joint, this is a joint these are all fixed joint so member AB you can say that member AB is essentially an idealisation of this member AB is an idealisation of this or member BA can be idealised as this okay as this and you are applying a moment like this okay, now this is B and this is A. Similarly for member similarly for member CD and similarly for member BC. Now then what is the stiffness then?

Stiffness K BA will be 4 EI, EI is constant that is why I am not explicitly writing the length, L AB now L AB is 5 so this becomes 4 by 5 EI okay. And then what is K BC? K BC similarly 4 EI by length of BC, length of BC is 10 so this become 2 by 5 EI okay. Then what happens? Then for length CD for member CD K CD, K CD is also same as K BA so K CD is also 4 by 5 EI so these are the stiffness these are absolute stiffness for different members okay. Now once we have the actual stiffness, then what you need to find out the distribution factor, so distribution factor we need to find out at B and at C because these are the intermediate joins.

Now distribution at B, two members are involved BA and BC so you need to find out distribution for BA and distribution for BC and similarly at joint C, two members are involved CB and CD so what is the distribution for CB and what is the distribution for CD okay. So distribution for so distribution factor D so this is this is 1 part okay, so now distribution factor D BA, D BA will be what? D BA will be stiffness of this + this stiffness +

this stiffness, so 4 by 5 divided by this by this, and if you do that then what you get is 2 by 3 you get 2 by 3. And similarly for distribution for BC distribution for BC will be 1 by 3, and similarly distribution factor for CB which is equal to which will be this + this, this divided by this + this which is is equal to one third and distribution for factor CD will be two third okay.

So these are the distribution factor for different for different members, now we need to have the carryover factor. Now member joint if we apply a force member moment at B, this moment will carried over to because this is an this idealisation so this will carried over to this. Similarly if you apply moment at C, it will carry over to this, now moment at B will be carried over to C, moment of C will be carried over to B so the carryover factors are and in this case since it is the this idealisation this idealisation, the carryover factor will be half so all the carryover factors so C BA equals to C BC equals to C CB equals to C CD those will be all will be half. So we have all the all the parameters that we need with us, one is distribution factor for different members and then carry over factor for different members.

Let us now first start the process okay, before we start the process let us also find out the find out the fixed end moments. Now find out the fixed end moment because that was our step 1, step 1 was fixed end moments okay. Now fixed end moment for AB you see there is load in AB so fixed end moment for AB will be 0, fixed end moment for CD will be 0, only fixed end moments we have is for BC so M AB fixed end moment equals to M BA fixed end moment equals to M CD fixed end moment equals to M DC again fixed end moment they all will be 0. And M BC M BC fixed end moment will be w l square q l square by 12 we have seen it and q equals to 7.5, l equals to 10 meter and this will be in the here it will be anticlockwise and here it will be clockwise.

So – M BC M CB F that will be is equal to – 62.5 kilo Newton meter okay, so only non-zero fixed and moments we have for member BC, for all other members the fixed end moment is 0 okay. This fixed end moment again we have done it for slope deflection method please please go back to that lecture, we have to see what is the formula for this 62.5 kilo Newton per meter okay great. Now let us start the process, we have all the ingredients ready let us start the process okay. Now what we do is, we have 3 members here AB, BC and CD, how we will write it, we will write it like this.

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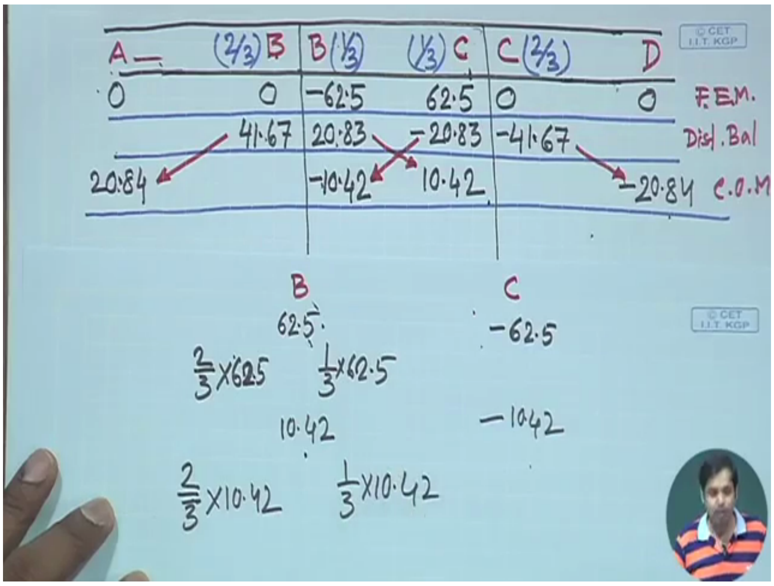

We will take, so this is for say joint A and this is for joint A is this is for joint B and then suppose this is for joint C okay, so what we have is this and then this, this we are doing in a tabular form. So this is joint A, this is joint B but when B belongs to BA and this is again joint B when B belongs to BC and then it is C when C belongs to CD and this is D okay. So essentially this was our structure, this was this was the portal frame, we took it like this we just this member is this member, and then this member is this member and this member is this member okay. Now first then now let us write all the solution factor and carryover factors, now when at B the carryover factor is 1 by 3 for BC and for BA it was 2 by 3 that is the distribution factor.

And again for C for CD it was 2 by 3 and for C it is 1 by 3 okay we have already calculated, and all the carryover factors are half okay, there will be no distribution factor for A and D okay. Now next is we have to write the fixed end moments, what is the value of fixed end moments? Fixed end moments only we have in member BC, in member AB and CD fixed end moments are 0 and what is the value of fixed end moments? this is 62.5 this is 62.5, for this member it is negative, for this it is positive okay. So this is positive + 62.5 please correct it okay. Now next is right the fixed end moments though fixed end moments are so this is 0 here for this it is 0 for member AB this is 0 and this is – 62.5 and this is 62.5, again this is 0 and this is 0, so this is fixed end moment right.

Now, next is we need to check what is the status, what is the status means whether the equilibrium is satisfied or not. Now equilibrium is so this is your first line, so naturally equilibrium is not satisfied at joint B and at joint C because we have an unbalanced moment of 62.5 and at joint C we have unbalanced moment of 62.5. What we have to do? We have to apply a balancing moment of 62.5 at B and – 62.5 at C and that balancing moment will get transferred to will get distributed in different members. So what you have is so at this at this point we have to apply + 62 point + at b so this is this is say B, this is C we need to apply + 62.5 here and at joint C we have to apply – 62.5.

Now this 62.5 will distributed among BA and BC as per this distribution factor and again this will transfer in distributed in CB and CD as per this distribution factor. So this part will be two third of two third into 62.5 and this will be one third of 62.5, similarly this part will be one third of – 62.5 and this part will be two third of 62.5 and that value will be 40, here it will be 41.67 and it is 20.83 both are positive because we are applying your balancing moment is positive and then it is – 20.83 and this value will be – 41.67 so this is distributed balancing moment distributed balancing moment okay this is distributed balancing moment, so this was our second step second line.

Now what next? Now whatever distribution that whatever moments you have externally applied to balance the balance the joints to achieve equilibrium that moment will will be carried over to the other end and carryover factor in this case is half, so half of this will carry over to this, half of this will go to this and then half of this will go to this. So what will what will be the carryover factor for this? It will be 20.84 this is half of this and carryover factor from this will be – 10.42 and from this it is 10.42 and again from this it will be – 20.84, so these are all carryover moment these are all carryover moment, this is carryover moment okay, this arrow is used to show that this is these are carryover moments so this is carried over moment right.

Now next is, once you have this carryover moment again you change the equilibrium, whether the equilibrium is achieved or not. Now at B we have unbalanced moment of – 10.42 at we have unbalanced moment of + 10.42 so in order to achieve the equilibrium at the joint B and C, we need to apply + 10.42 at B and – 10.42 at C and this balancing moment, this is this is unbalanced moment and equal and opposite moment we need to apply for equilibrium. Now this is the balancing moment and this balancing moment again will get distributed among the members as per the distribution factor. So here it will be two third of 10 into 4.2 and it will be one third of 10 into 4.2 and similarly for this case.

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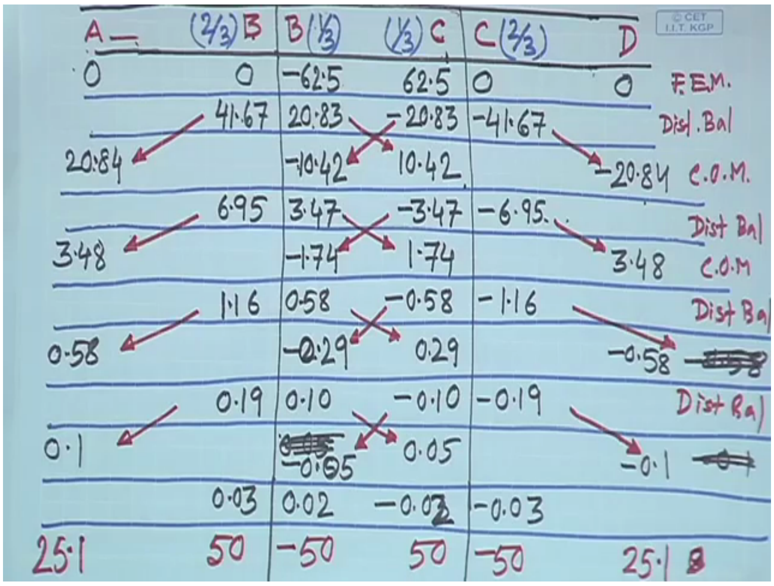

So what will be the values here? So distribution factor will be here it is 6.95, 6.95 and this will be 3.47 and similarly here it will be – 3.47, the problem is symmetric that is why we are getting same values 6.95 okay. Now this is the, now these values are now what is these values are again distributed balancing moment, this is distributed balancing moment distributed balancing moment okay. Now what next? Once we know the distributed balancing moment then that moment needs to be that distributed balancing moment okay. Once you know the distributed balancing moment that moment will carryover right.

So half of this will go here so this becomes 3.48 so half of this will go here and similarly half of this will go here, this becomes 3.48 this is again carried over moment and half will go here and this become – 1.74 and half will go here 1.74, these are again distributed carried over moments okay. Now so this is carried over moment, now we want to check the equilibrium now unbalanced moment at joint B is – 1.74 , it is 1.74, we need to repeat the process, we need to apply the equilibrium we need to apply the balancing moment of 1.74 at B and – 1.74 at C and that balancing moment will get distributed in B and again and CB and CD as per this distribution factor and that value so we need to so we need to repeat this process.

This process we need to repeat but in this case 10.42 it will be 1.74. Now, that distribution moments will be here it is 1.16 and this value will be 0.58 and similarly it will be – 0.58 and this is – 1.16 okay, so this is again distributed balancing moment okay. Now still we need to continue the iteration because we have not achieved the convergence okay not convergence, but the tolerance is not achieved, so what next? Next is then again carryover moment, this will get carried over to this, corresponding carry over moment here will be half of this, this will be 0.58 and then this will go to here 0.29 and this will go to – 0.29 and this will be – 0.58, these are all carryover moments these are all carryover moments this is carryover moment you write it here – 0.58 okay.

So this is then again check what is the status, status is again it is not yet balanced, equilibrium is not yet achieved here we have an unbalanced moment of – 0.29 again unbalanced moment of 0.29, apply the balancing moment equal and opposite which will get distributed as per this distribution factor. And if you do that so this becomes 0.19, this becomes 0.10, it is – 0.19 and this is again distributed balancing moment okay. Now these distributed balancing moment will get, now you can observe that slowly these values are becoming very small continuously it is becoming very small and we are already reaching the final solution.

So this will get distributed how much it will so half will get here so this becomes 0.05 and half will get here this becomes – 0.05, please correct it – 0.05 and this will get 0.05 and this will get half of this which is 0.1 and this will get – 0.1 progressive distributed moment this is distributed moment this is right it here – 0.1 okay this is distributed moment. Now again check the status, check the status so again we have – 0.05 unbalanced moment, apply 0.05 balancing moment and get this which will distributed and the corresponding distribution will be here it is 0.03 here it is 0.02, – 0.03, – 0.02 and 0.03 and this is your this is the distributed moment okay balancing moment okay.

Now you see we have already reached 0..02, again you can continue the way how much you want to continue, but at least if you look at the fixed end moment compared to fixed end moment, these values are very small okay so there is not much increase in the corresponding values. So let us stop it here okay, let us this is let assume that this is we have already achieved the equilibrium state, and if we stop it here then the final tally is we need to add all of them and if you add them so this become 25.1 and then this become 50, this become – 50, again this become 50, this is – 50 and again 25.1 okay. So this is the final result, if you compare this result with your with the slope deflection result and if I recall if I remember correctly, the values you obtained is the 50, at joint B it was 50 – 501 but joint A it was 25.0 and at joint D was 25 point 0.

But in moment distribution method what value obtained, the value obtained is 25.1 here and 25.1 here which was which is obvious because this is an iterative procedure, every step we have to do some approximation. If you continue with that probably, you may get 25.0 but ,

but this is a very close result to the actual moment is 25, so the point is the reason why I am saying is you take any problem, solve it using moment distribution method and solving using any other method which gives you accurate value. Then what happens is you see the moment there may be a very slight deviation in the result there is slight deviation because the process is iterative, it depends on at what point you stop the process okay.

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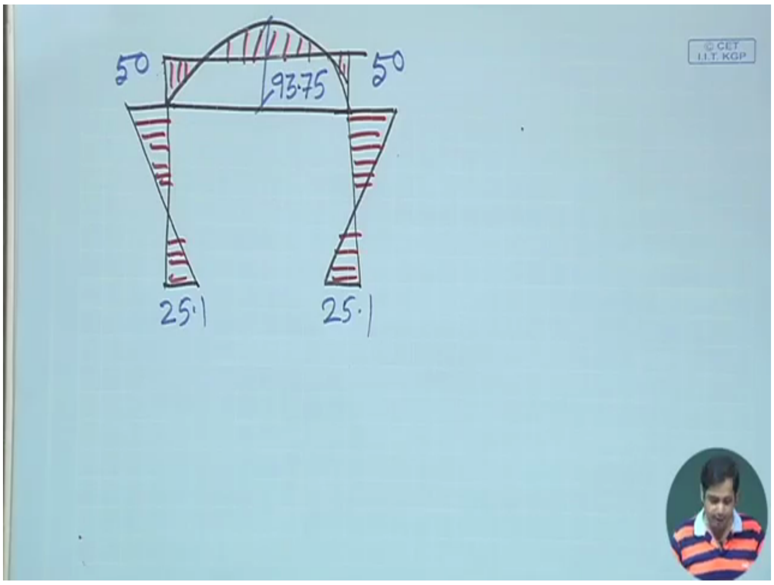

At every step you do some rounding of the values and incorporate some some approximation in the every step, so but this gives you a reasonably not reasonably I mean quite close result. So this is the final tally of this problem, now if you have to draw the bending moment diagram you can, the bending moment diagram below like this, we have we have used it for in slope deflection method, it will be for your external load which is parabolic and then for the joint it is 50 here and again 50 here then 50 here, it is 25.1 here and again 50 here, 25.1 here and this will be your bending moment diagram will be your bending moment diagram this will be your bending moment diagram and these values are 50, 50 and this is 93.75 and this is 25.1, 25.1 okay.

So this is this is the these are the steps that you can use for analyses of frame using moment distribution method. Again the idea has been to demonstrate the method and that is the reason why we chose a problem where at least few iterations will give you will take you to the solution but any problem any loading condition any boundary condition the essence of the method remains same, only difference will be some problem it may take more iteration and in some problem it may take less iteration that depends on the problem right. So again in the beam example it was only 2 members, now in frame example it is a three-member, in another problem you can have many members but again similarly you can do it okay.

So once you understand the concept the process, the next thing is to next step is to understand the application of that concept and process and again I am telling you this is not enough, just we demonstrated the concept through one example in the class, this is not enough. To understand the concept in a very comprehensive way and to understand the process and to be comfortable with the process there is only one way is to practice and you take any book there are many examples given in the book, many exercise, you choose those examples, address those exercise problems and can only you will be comfortable with the concept and as well as the steps steps that involved with the process okay.

So we will stop here, now in this example is a symmetric problem there is no sidesway so this is a Portal frame, this is a frame problem but without sidesway, we will do one more example where in addition to the load we have an addition to the whatever load we have, even horizontal load and will that horizontal load will cause sway in the structure and we will see how to how that sway to be taken care of the analysis. And if you remember the slope deflection method if there is a sway, then only slope deflection equation will not give you the sufficient information, we needed one more equation that is the equation based on calculate the sheer force and comparing the sheer force with the external load.

So let us see what process we have in moment distribution method if a frame undergoes sway in a particular direction. Okay, we will stop it here see you in the next class thank you