Structural Analysis
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Module 11
Lecture No 54
Analysis of Indeterminate Structures by Displacement Methods (Contd)

Hello everyone, welcome to the second lecture of this week. In the last class we introduced the moment distribution method, now what we will be doing today and in the subsequent classes, we will demonstrate moment distribution method, the steps involved in moment distribution method through some examples okay.

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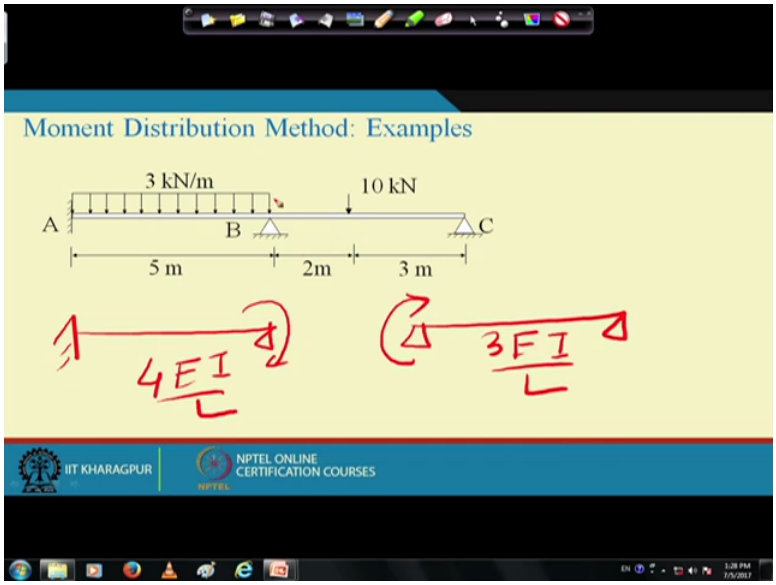

So now what we will do next, we will take this example, it is again a similar example that we used to demonstrate the steps, the only difference is now in this case these joints, it was fix joint if you remember in the class, now it is not fix it becomes it is hinge support at C. Now the first step is to find out what are the absolute stiffness then distribution factor carried over factor okay. Now let us see what is the stiffness for this member? Now if you remember, stiffness for this member is this member is an idealisation of this, this member is an idealisation of this moment and this member is an idealisation of this which is in support, and a moment is applied here.

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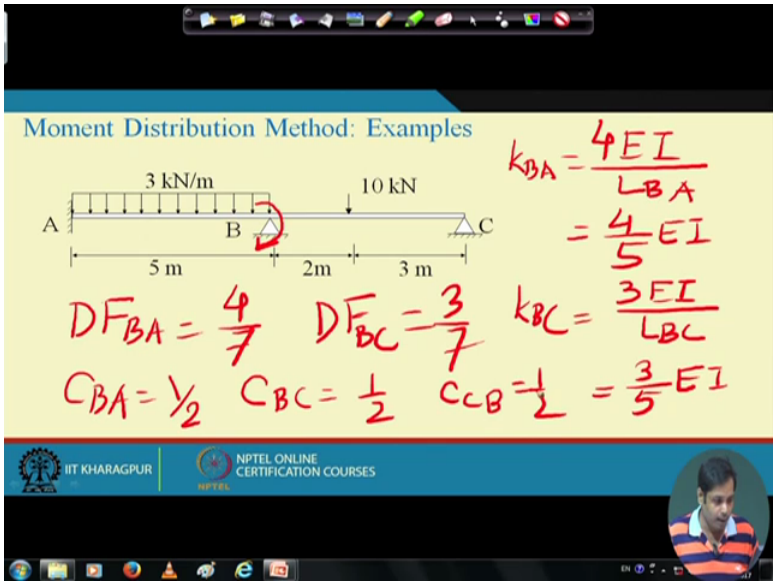

And for this we know the stiffness is 4 EI by L and for this we know the stiffness is 3 EI by L okay for joint B. So then what is K BC? K BC becomes K BA okay let us remove this first then what is K BA? K BA is K BA equals to 4 EI, EI is constant in both the cases so let us write EI and that is equal to L BA which is equal to 4 by 5 okay. And similarly K BC equals to 3 EI by L , it is 4 EI and this is 3 EI by length of BC L BC which is his equal to 3 by 5 EI okay. Now once we have this stiffness then the distribution factor distribution factor will be DF, distribution factor for BA will be this divided by this + this and this becomes 4 by 7, and similarly distribution factor for BC will be this stiffness divided by total this + this and this becomes 3 by 7, this is the distribution factor okay.

Now let us find out what is the carry over factor, carry over factor is you see, if you remember that we discussed if we apply a moment here that moment will get carry over to this okay, but if we apply a moment at the fixed end that moment will not carry over to other end okay because that you can do than analysis and check if we apply the fixed end then this will not carry over. So in this case what happens, carry over factor will be C BA, CBA equals to half and then C BC equals to half and C CB equals to half, so carry over factors will be half, half, half okay.

Now you may ask that this end is C so if we apply a moment here then this moment will not carried over to C that is absolutely fine, but what happens is we do the analysis on the primary structure right and the primary structure we assume these joints are these joints are fixed joints. So if we apply, so when we actually carried over to the we do the iteration, we

assume initially we assume the primary structure fixed all the joints are fixed therefore there will be fixed end moment. Now once there is fixed end moment because there is this is hinge support there is there should not be any moment that moment needs to be balanced and in order to balance that these carry over factors will be required this will be clear when we actually do these iteration.

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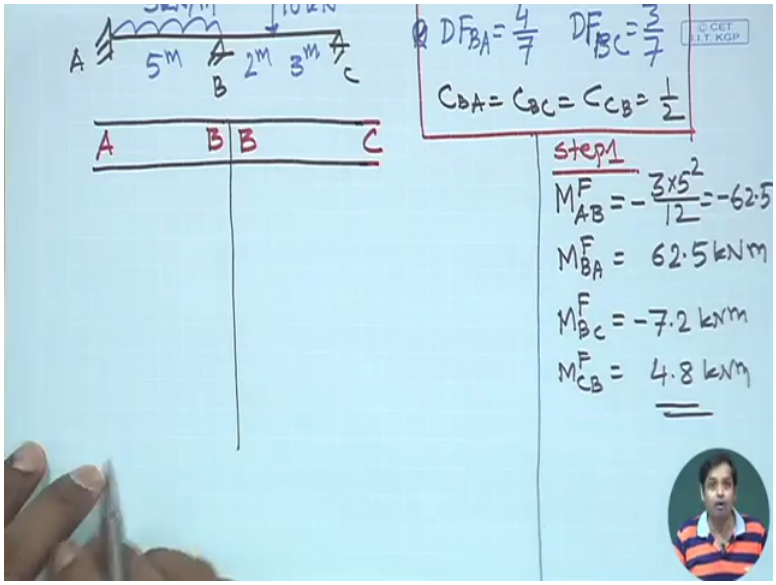

So these are the information that we have right okay, now let us start the let us start it, so what information we have this is the problem so this end is this is the fixed end, this is hinge and this is hinge and then we have an externally applied load and then load like this okay. Now this is 3 kilo Newton per meter and this is 10 kilo Newton, this is 5 meter, this is 2 meter and 3 meter okay. And then we have K or we have distribution factor DF BA equals to 4 by 7 and DF BC equals to 3 by 7 just now we have seen, this is the distribution factor we have, so if we apply a moment at B that comment will get distributed in A B C in AB BC, AB and BC as per this factor okay. Now, and then also we have carry over factor as C BA equals to C BC equals to C CB equals to half okay, this information we have this is the information that we have okay.

Now let us let us do the step, the first step is let us find the fixed end moment step 1 is fixed end moment okay. Step 1 let us write it here, let us divide it in two parts okay, now step 1 was fixed end moment okay, now is the fixed end moment? Fixed end moment will be M for AB fixed end moment that is equal to w l square by 12 which is – 3 into 5 square by 12 that is is equal to – 62.5 kilo Newton meters okay. And then we have M BA F this will be same value but 62.5 kilo Newton meter, this will be anticlockwise, this is clockwise okay.

Now similarly we can calculate M BC fixed end moment for M BC equals to – 7.2 kilo Newton meter and M CB fixed end moment equals to 4.8 kilo Newton meter, these are the same example similar examples same length and same load, these are the fixed end moments and how to get these values you can go back to the previous class and then see these values, verify these values okay. Now we have the fixed and moment so let us now start the process, start the step.

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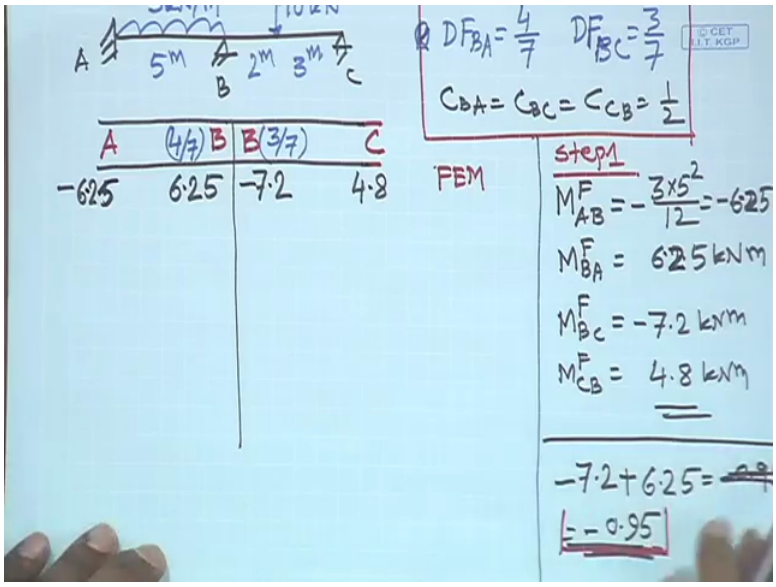

Now the steps are, first we take we will do the steps in a tabular form. Now first suppose this is end A and let us take this is end B but when B belongs to AB and again this is also M B, B belongs to BC and this is M C okay. Now first is we write the then we know the carry over factors, carry over factors for BC is 3 by 7 we have seen it and carry over factor for BA is 4 by 7 okay. Now then and the carryover okay now this is the distribution factor okay and carry over factor for all these cases is 1, for this to this, this to this they all are half the all are half okay. Now first we write the fixed end moment, what is the fixed end moment at A? This is – 62.5, now when we again write the values give the sign whether based on whether they are clockwise or anticlockwise and 6.25, this is 6.25 not 62.5 please correct it 6.25 okay so this is 6.25 not 62.5 okay.

6.25 and this is 6.25, this is – and this is + okay, and then this is – 7.2 and this is 4.8, so this is these are all fixed end moments fixed end moments okay, which is step 1 okay. Now next step is we need to see what is the unbalanced moment here what is the unbalanced moment here? You see unbalanced moments are unbalanced moments are 0.95 in the previous example we have seen and that unbalanced moment needs be distributed in this and this is as per this

factor okay, so unbalanced moment in this case is –7.2 + 6.25 so unbalanced moment is in this case – 0.95 okay this is the unbalanced moment now we have unbalanced moment of 0.95 at joint B.

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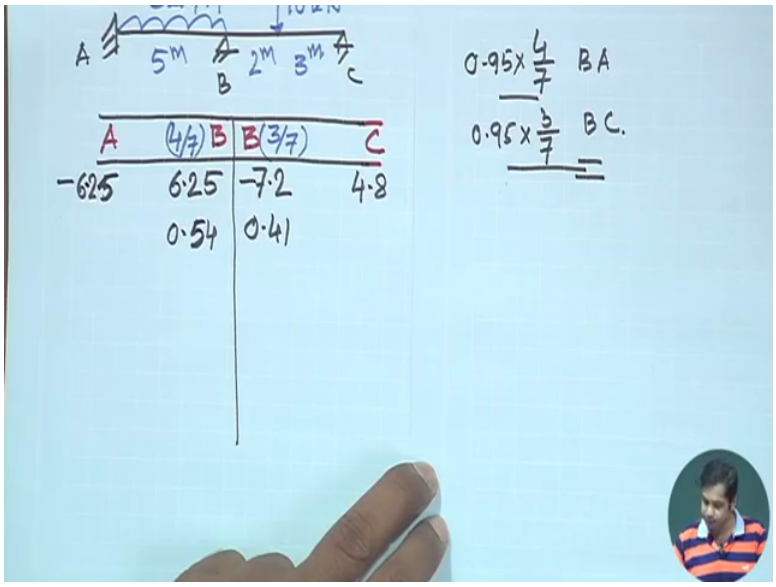

Now in order to then what is in order to balance in order to get an equilibrium in equilibrium state equilibrium state at joint B we need to apply A 0.95 moment, this is negative moment unbalanced moment then we need to apply 0.95 moment at joint B okay and positive 0.95. Now this 0.95 the balancing moment, this is the unbalanced moment, now the balancing moment will get distribute to BA and BC and then how that balancing moment will get distribute in BA and BC, based on their distribution factor okay. So unbalanced moment is 0.95, when you write in the tabular form, you do not have to explicitly show how much is the unbalanced moment and then how much is the balancing moment that calculation you can do separately.

Now we need to apply 0.95 balancing moment at point B, now if we apply 0.95 balancing moment what would be the distribution of distribution at B. Distribution at B will be distribution at B will be 0.95 into 4 by 7, it is in BA and similarly 0.95 into 3 by 7 it is in BC. And if you do that then this value that distributed value will be here it is 0.54 and then and this value is 0.41, so 0.41 is essentially this value and 0.54 is essentially this value so this is distributed, so this is let us write it here explicitly, this is distributed balancing moment, so this is distributed balancing moment this balance.

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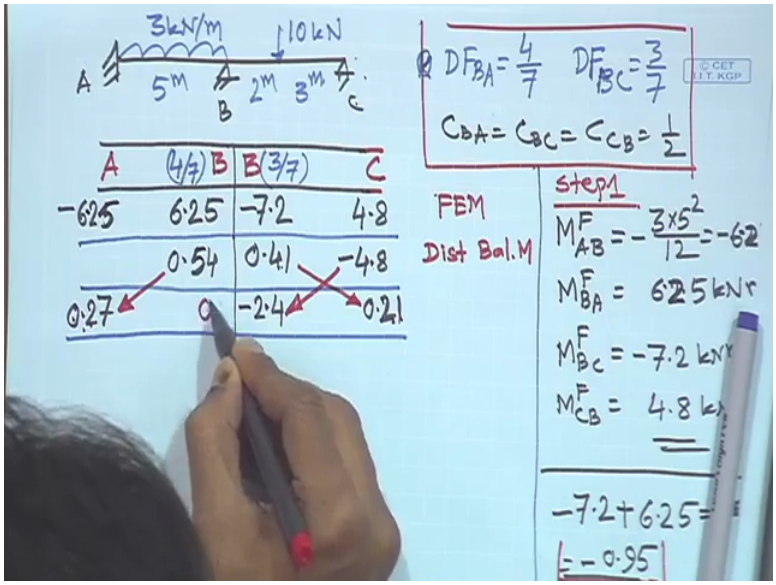

Now once you have this, you can actually separate it in a different line do it in a different line. Once we have this, now again this is also the separate line okay, now once we have this now this moment we have this some portion this moment from this will be carried over to A and some there will be carried over moment to C and carryover factor you have seen because the carryover factors are half in this case. So how much will get carryover, half of this and then here also how much will carry over half of this. So carryover moment will be here, 0.27 and in this case the carryover moment will be 0.21, it is 0.405, 0.205 but just you can write it in a 2 decimal place okay. So this is carryover moment and carryover moments are shown like this using an arrow, so this is a carryover moment.

Now there is an interesting step here, you see this is the fixed end right, so this is the moment goes to fixed end there is no problem but this and this end C it is simply supported right there is a hinge support here, so there should not be any moment there should not be any moment here right. Now, so equilibrium condition at B in order to satisfy the equilibrium condition at B we apply clockwise moment of 0.95 here at B and that 0.95 moment get distributed in BA and BC. Now what is the equilibrium condition at joint C? Ideally at joint C there should not be any moment, the fixed end moment is 4.8 is fine but in the actual structure there should not be any moment.

Now in order to satisfy the equilibrium condition at C we need to apply a – 0.48 balancing moment okay, we need to apply – 0.48 in the balancing moment okay. Now whatever moment we apply at C, the same moment will be there so balancing moment and distribution

moment at C will be same because there is no other members here where this balancing moment will get distributed, whatever moment you apply at C that will be the moment at C in CB right, so in this case balancing moment and distributed moment will be same okay.

Now once we have this, now this moment will also get carried over to this because this idealisation if you take this if you take this idealisation, we consider this is the fixed end so this moment will also carried over to this and how much will be the carried over moment? Carried over factor is half then this is – 2.4 so this is also carried over moment okay. But we do not really have to apply a balancing moment at A because the end A is fixed end, okay in the actual structure also it is fixed end so whatever so because the equilibrium we are satisfying equilibrium at C and B and consequently whatever we are moment we are getting at A that is the consequence of that okay, so this is now next we need to check what is the status present status.

Now again you can draw a line you can draw line okay, now the present status is and what is what is this value, so there will be no distribution from this so this value will be 0, so let us write the 0 value in different colour, this value is 0 here okay. Now what is the present status? The present status is present status is we have unbalanced moment of – 2.4 at joint B and that needs to be that needs to be in order to satisfy the equilibrium condition at B we need to apply a positive moment positive moment at B and that positive value that positive moment is 2.4. And that external moment this is carryover moment right it right this is carried over moment this is carryover moment.

Now if you remember the in the last class, this was the complete cycle the first part okay. Now what we have is this, now this is the unbalanced moment at B we have and that needs to be balanced, in order to balance it we need to apply a clockwise moment of 2.4 at B and that moment will get transferred to BA and BC as per their distribution factor. So we need to apply a clockwise moment of 2.4 kilo Newton meter and how much will go to BA? BA will go to 2.4 into 4 by 7 for BA, and how much will go to BC? It is 2.4 into 3 by 7 goes to BC, so this will be the distributed balancing moment, 2.4 is the total balancing comment at B and these values the distributed balancing moment at BA and BC. So what will be the distributed moment? The distributed moment will be this becomes 1.03 and this becomes 1.37 okay, so this is the distributed moment balancing distributed moment.

Similarly, the balancing distributed moment at point C will be how much, this will be -0.21. You see initially fixed end moment was 4.8, we applied – 4.8 so point C the equilibrium at

point C is satisfied. But again because of the moment balancing moment at B, we have some carried over moment of 0.2 at C but that carried over moment again we need to balance. So now we apply – 0.21 balancing moment at C and again the equilibrium will be satisfied okay so this is distributed so again can you can draw this, this is distributed balancing moment okay.

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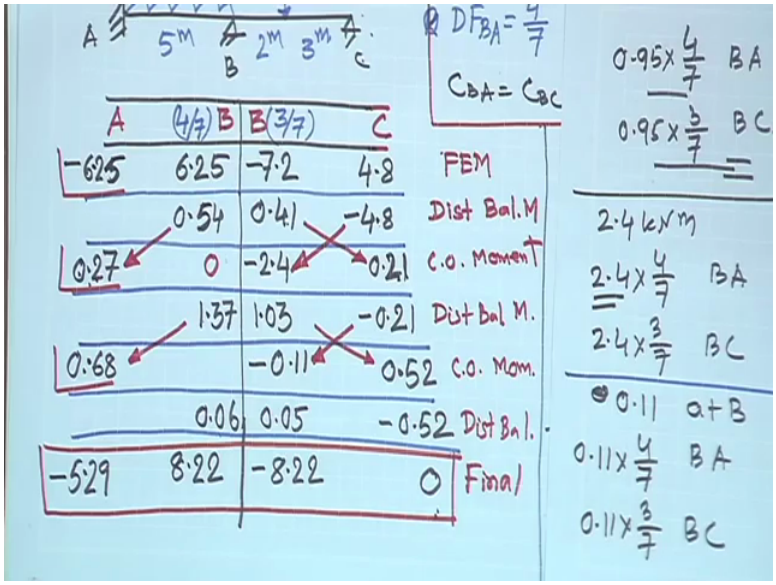

Now once we have the distributed balancing moments then that moment will generate some carryover moment to the other end and how much will be the carryover moment? Now carryover factor is 1.37 so 1.37 half of this will go here and this becomes 0.68 and similarly 1.03 half of will go here and this becomes 0.52 and half of this will go here, this becomes – 0.11 so these are all distributed moment these are all distributed moment sorry these are all carried over moment so these are all carried over moment okay. Now once we have the carried over moment the C the status present status, the status is now we have unbalanced moment of – 0.11 at joint B and we have 0.52 unbalanced 0.52 moment at point C okay.

Again we in order to satisfy the equilibrium at joint B we need to apply a positive 0.11 at b and negative 0.52 moment at C. And this 0.11 moment at B will get distributed as per their distribution factor. So again 0.11 balancing moment we need to apply at B and out of this 0.11 into 4 by 7 will go to BA and 0.11 into 3 by 7 will go to BC okay. Now what is, how much is this value then? Then this becomes 0.05 and this becomes 0.06 so this is this is carryover moment this is carryover moment okay. Now again here also we need to apply – 0.52. Now this is this all this all these moments are all these moments are again distributed balancing moments distributed balancing moments okay.

Now again let us you what is the present status, present status is what is the present status is now again you can keep on doing this until you get balance is, now you have to define a tolerance that how fast shall we go, any iteration method when to stop that iteration that stopping criteria you have to give. Now stopping criteria one stopping criteria when you see that the final the moments that the increment in the moments that you are getting or the change in moments you are getting that is very small as compared to the initial fixed end moments okay.

Now if you stop it here because these values are very small now 0.06 and 0.05 these values are very small here and again we can go on because now this will get distributed here and this will get distributed here, we can keep on going that. Suppose if we stop it here then what happens then what is the final tally. Final tally will be we need to add at point A the final moment will be this + this + this that will be the final moment and this value is -5.29 okay. As I said you can continue with this and you continue up to you get tolerance, the tolerance that you need to specify, but suppose now if we stop it then we will see what is the value we get.

This becomes – 5.29 and then this becomes 8.22, and this also becomes – 8.22 and this becomes 0 okay. Now the reason we stopped here because this is equilibrium at B and equilibrium at C is now satisfied okay. So the final moment at B is – 5.29, moment at B is 8.22, moment at B in BC is this and moment at C is C equals to 0. But if you see that after adding them that it is not the equilibrium is not the equilibrium again is not being satisfied then again you have to continue the process okay so this is our final tally so this is our final tally received our final moment so this is called final this is final moment okay. So now what we have?

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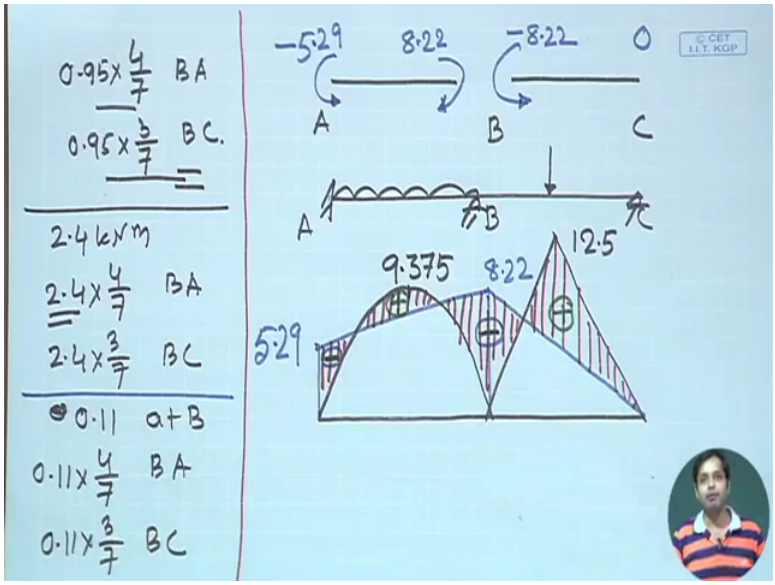

Now if you draw this in the structure, this is A, this is B and this is C, at point A the moment is – 5.29 – means it is anticlockwise so it is anticlockwise, this is – 5.29 and then at point B in AB it is 8.22 means clockwise and – 8.22 means it is anticlockwise, so it is clockwise which is 8.22 and then here it is – 8.22 which is anticlockwise and then here it is 0, so these are the moments at joint okay. Now and what was the actual structure? The actual structure was actual structure was this actual structure was an actual structure something was this okay. And this was this was this is A, this is B, I am not writing the values of this.

Now we will be drawing the bending moment diagram, the bending moment diagram will be the bending moment of this + bending moment due to this + bending moment due to this okay. Now bending moment due to this will be in this case if we if we release this moment end, it becomes a simply supported and this is also simply supported, so this will be this and for bending moment for this will be this okay and this value is this the total value is 9.375 and this value is 12.5 okay. And then bending moment for this again you have to see whether it is hogging or sagging moment, bending moment for this will be it is 5.29, this is 5.29 and this is 8.22 and this point is 0.

So this you joined them and then joined them so this value is 5.29, this value is 8.22 okay and then what will be the bending moment? Bending moment for this bending moment will be this part is the bending moment on the beam. And this is one way of drying bending moment, another way of drawing bending moment you draw the negative part at the bottom in the compression side and then from that you draw the positive part and then you get depending

on your diagram for this. Now this is the bending moment, again when you write bending moment like this as I said you need to be careful about which one is positive, so this is positive and this is positive right and this is negative and this is negative and this is negative, this is also negative okay.

So this is the bending moment diagram for this and this is the step that we, this is the step for moment distribution method okay. So now again as I said the idea has been to demonstrate the steps through some examples, but the steps will be clear the calculations will be clear once you once you solve some more examples that you have to do. There are many exercise given in the book, please follow them, do them and be comfortable with this method okay. In the next class we will give we will show you the steps to someone examples okay we will stop here thank you, see you in the next class.