Course on Structural Analysis I
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Module 11
Lecture No 53
Analysis of Indeterminate Structures by Displacement Methods (Contd.)

Hello everyone welcome we are going to start week 11 today last week what we did we introduce the displacement base method and then studied one such displacement base method that is called slope deflection method where structure (())(0:38) first we need to find out the we need to get the primary structures which are kinematically determinate and then calculate the fixed end moments and then write the slope deflection equation satisfy the equilibrium equation and from the equilibrium equation we get the unknown displacements and rotations and once we get the unknown displacements again those unknown displacements can be substituted into the slope deflection equation to get the unknown internal forces.

Right so and then we demonstrated that method through some example okay the method was efficient in any way but you see from the practical purpose what we need is as you remember in the very first week we said there are 3 aspects of design one is safety, one is serviceability and economy third one is economy. The safety says that in order to make a structure safe we need to understand the we need the know the internal forces in the member.

Okay see strength should the member should be such that it can carry those internal forces and then comes serviceability which essentially tells you that the deflection in a in a particular structure should not be more than a certain limit that is what the serviceability criteria we are concentrating in this course. So safety is the first criteria, so from practical point of view is the internal forces that we need first then the displacements, okay but in slope deflection methods question we need to find out the displacement and then from the displacement we could get the internal forces.

Now there is a method as but it would be easier it would be better if we get without computing the displacements if we can somehow get the internal forces that would be in some way at will reduce our effort. Anyway displacements we have to find out at without doing without actually calculating displacements if you can find out the internal forces.

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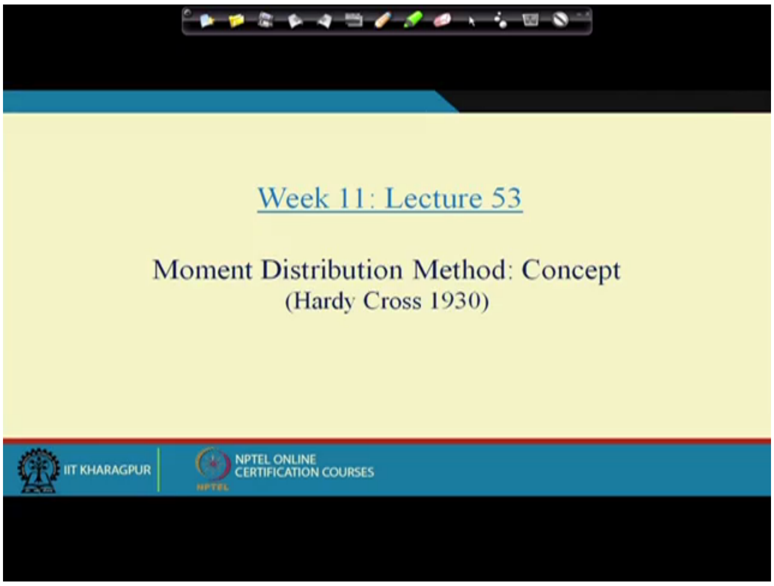

One such method is moment distribution method that we are going to start today. This is also known as Hardy Cross methods. Now as the name suggest the moment distribution method as it is it is the premise of the method is the distribution of moments okay and what we are going to discuss now, how the moments if we apply a moment in a given structure or (()) (3:08) how the moment gets distributed and that distribution process the constitute this method, okay. So let us first understand the concept of moment distribution method and then you will apply those concepts to various examples, okay.

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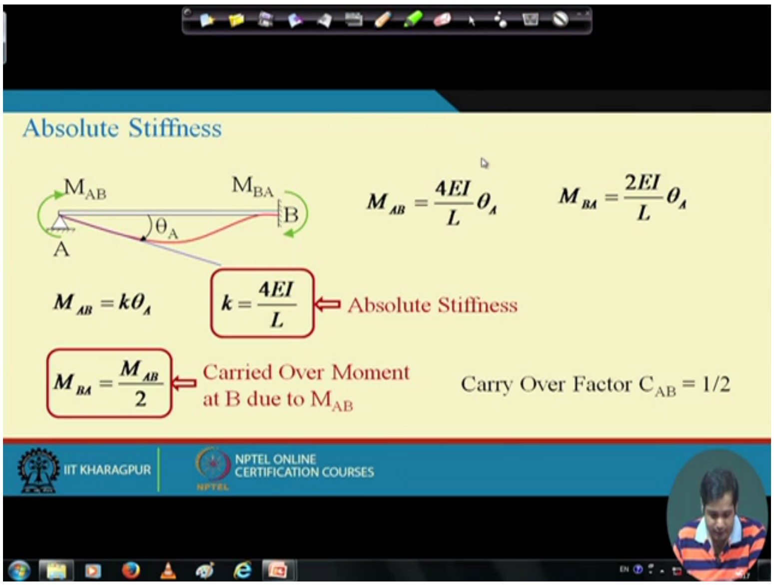

Now you see before we do that some revision of stiffness coefficient that we already discussed but since it is a very important important ingredient in moment distribution method let us revise that as well, okay. So if we take a propped cantilever beam A B and the suppose a moment moment AB applied at A, it means that moment at A in a segment AB.

Okay now because of this moment, the structure may undergo a (deform) a deformation like this we take clockwise moment as positive okay for all the calculation but when you draw the bending moment, we consider whether the moment is sagging or hogging moment so that is why all these moments are shown here and view as clockwise moment, Okay. So this is the and the corresponding deflection this corresponding rotation is considered as positive.

Now see so this is the rotation at joint A and joint B is fixed therefore there is no rotation there is no slope at joint B0. Now because of this moment there will be some there will be some moment generated at visual which has the support reaction at B say that is MBA, So MBA essentially moment at B in segment AB okay. Now we have already seen this MBA this this this moment is related to this rotation theta A as this, where 4EI by L this term is the this term is the stiffness term, right and similarly MBA is equal to which is the reaction at B is equal to because of this moment applied at A is 2EI by L, okay that we have already seen.

Now so now these we can write this MAB we can write as MAB into K theta where K is equal to 4EI by L 4EI by this value, okay and K is called stiffness of K is called absolute stiffness, okay. Now so if we apply a moment and then the rotation is rotation at that point is theta A and that is represent by MA into K into theta where K is called the absolute stiffness, okay sometimes you can say stiffness.

Now next is we have also seen that MBA is equal to this so therefore MBA is equal to half of MAB, okay. Now this moment is called this MAB, MAB the moment MAB this moment is called carried over moment and B due to MAB means this this when you apply a moment at one joint and that will induce moment and another joint and that moment is called carried over moment, okay. So now and then this this MBA we can rise MBA as MBA MBA as CAB into MAB, okay. MAB is the moment at A and CAB is the carry over factor.

So in this case carry over factor is half okay so carry over factor is carry over factor is essentially now CAB is equal to it tells you that it is MBA divided by MAB. Right so carry over factor tells you what fraction of moment gets transferred to the other joints when you apply moment at any joint, okay. So these 2 is very important one is this absolute stiffness term k and the carry over factor and also the concept of carried over moment. These 3 are very important and we will see that they constitute their main ingredients of moment distribution methods.

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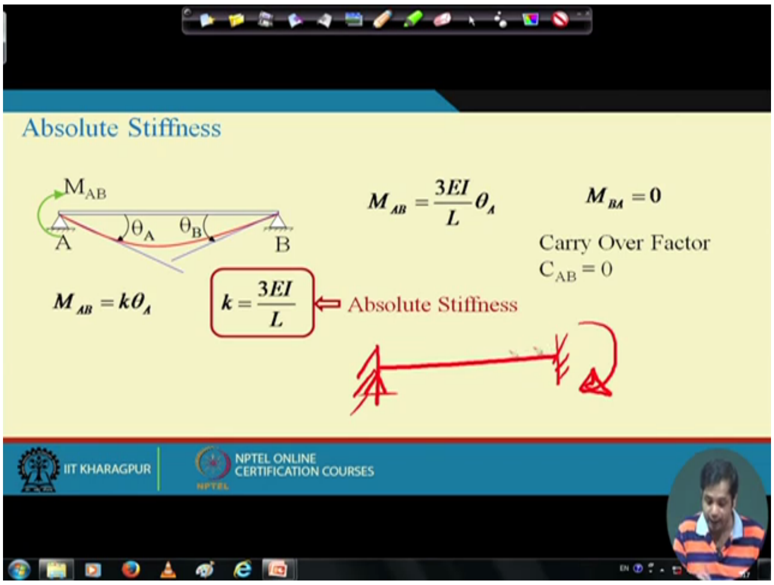

Okay now see one more example it is now suppose this initially the last example the support was fixed support now let us make it hinged support, both support are hinged support okay and another moment MBA applied as theta A. Now what is MAB, MAB is equal to we have already seen it MAB is equal 3EI by L and MBA is equal to 0 because it is now hinged support okay now so MAB is equal to K into theta A and K is equal to this value and this is called absolute stiffness, okay and what is the carry over factor, carry over factor will be 0 because when we apply a moment at this point there will be no moment generated at B okay so carry over factor in this case is 0.

Okay now another example if we can see suppose you take a being which is fixed support here and we apply a moment at the fix support and this is hinged say for instance this is hinged now we apply the moment at the support. Now this moment will not cause any induce moment at this point therefore the carry over factor will be 0. So even if it is fixed support and if the moment is applied at the it is the fixed (())(9:35) moment is applied at this point and this will not induce a moment here and therefore what happens your the carry over factor will be 0.

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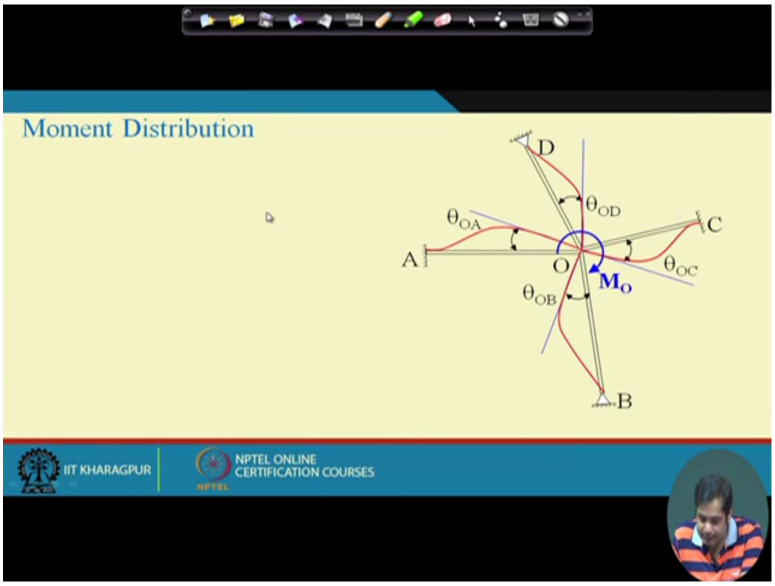

Okay now next is moment distribution now suppose consider a rigid frame which has 4 members and then after they are connected at O and then one concentrated moment is applied at O. Now all these support conditions are at A and C it is fixed support and at D and B it is hinged support okay now what essentially we are going to see is once we use apply a moment at this joint the some portion of this moment will get transfer to OC some will go to OD and some will go to OA and some will go to OB.

So and so being moments in all these members if you sum them the total moments is equal to that has to be M0 that is equilibrium condition at O. Now what we are going to see here is in what basis this moment the concentrated moment applied at O will get distributed among the different connecting member, different member connecting at O, okay. Now see first suppose this is the deflected shape of the frame where because of this load.

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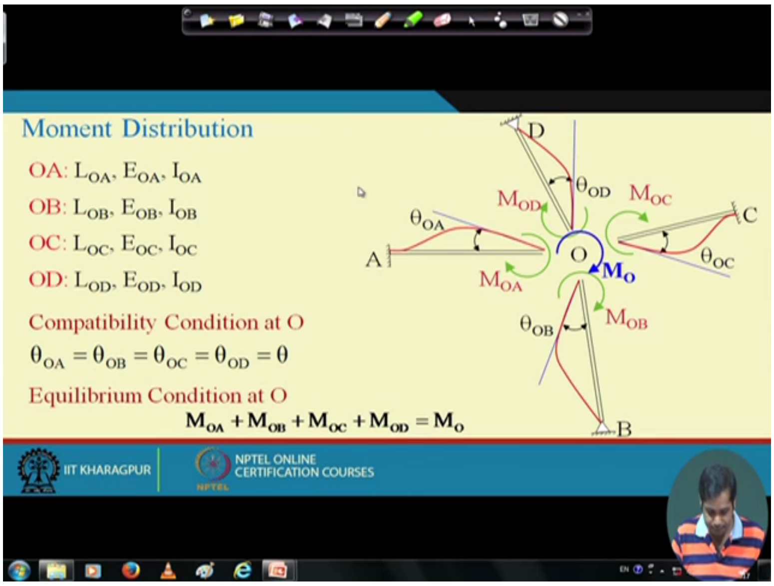

Now what we can do is next is suppose let us derive a very general formulation, so all these members they have different length they have different Young's modulus and different 2nd moment of area. And that is denoted by this subscript okay so all the members are different length Young's modulus and 2nd moment of area. Now what compatibility since this is a rigid frame now what compatibility condition at O is at this point O compatibility condition O say is that, that all the rotation at this joint rotation in all the members at this joint, they should be same.

So this theta A, theta OC, theta OD and theta OA they all should be same and suppose they are the value of this rotation is theta, okay that is the compatibility condition. Now suppose now what we do is, take all these members that we have that we take all the members separately treat them separately okay so this is all separate members, okay. Now this is the concentrated moment applied at A and the corresponding and the sheer of OC this suppose this one MOC is sheer of OC. Similarly OD is the portion of the moment that gets transferred to OD and similarly MOA is the moment in OA and MOB is the moment OB okay.

Now it is just seems that we are only concentrating on the distribution of moment in this diagram only the moments have shown all the sheer force and the actual forces not shown in this diagram. Okay now what is the equilibrium condition at O? The equilibrium condition at O say is that all these moments this MO all these the summation of all these contribution all these shared, they should be equal to M0.

Okay that is the equilibrium condition at M, so what we have we have 2 conditions here one is compatibility condition this one and one is equilibrium condition this one. Now we are going to use both this conditions compatibility and equilibrium condition to find out what would be the values of these moments, moments in each in different members.

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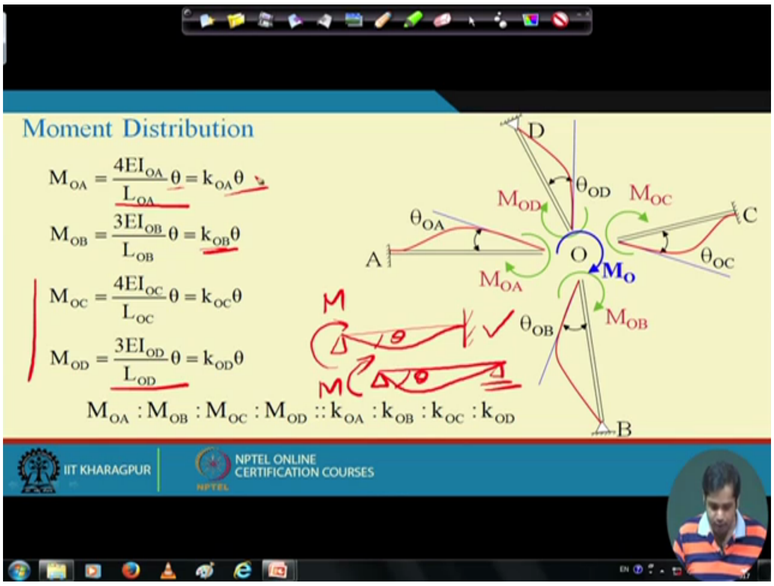

Okay now let us do that now this member this can be treated as this one just now we have seen that if we apply this is a fixed hinge and if we apply a moment like this and then what is the corresponding if it deflects like this and it is theta we have seen that this moment and this theta how they are related? Right similarly we also have seen that if it is both end is hinged support and then if you apply a moment like this and suppose your deflection is like and this is theta and how this moment and this theta is related okay.

They are related through this stiffness and the values of stiffness for this is, this is the value of stiffness for this is this and values of stiffness for this case is this, okay. Now take member AB, member OA, member OA member OA is essentially this is the idolisation for member OA, so the moment OA is related to theta OA as this and this is the stiffness term. Suppose the stiffness is K of OA, okay.

Now again it is actually theta OA but just in the previous slide we have seen that all theta OA, theta OD, theta OC and theta OB are same and we assume that suppose we assume the value of all this rotation is theta so that is why it is theta, okay. Now KOA is the stiffness, stiffness at it is due to the rotation at A rotation at O in member OA, okay. Now so this is how the moment OA is related to theta and the stiffness.

Now similarly take member OB, now member OB this joint hinge joint and the idolisation of this is this, okay. Now therefore we have seen it is the stiffness is 3EI by L and (())(16:33) and this is the corresponding stiffness. Similarly for MOC and OM OD we can find out depending on their support condition. Now in this the concept is being demonstrated through only 2 support condition hinge support and fix support.

The extreme other ends are hinge and fix support but you can have any different kinds of support the other end and based on the support condition you have to choose the you have to get the stiffness coefficients, okay. Now then what we can do is now if we know the expression of all the moment in terms of their stiffnesses. Now therefore if we take the ratio of all these moments at can be written as the ratio of their corresponding stiffness, okay.

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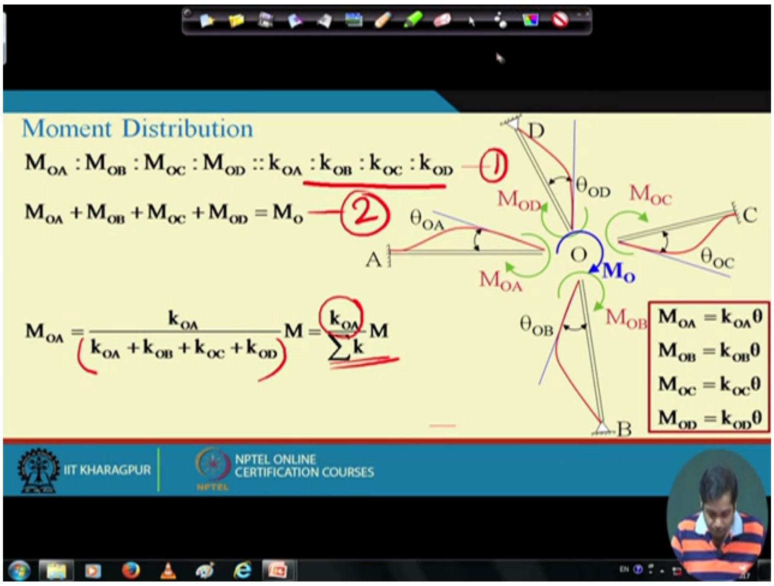

Now if it is then just now we have seen all the ratio of their moment they are essentially the ratio of their stiffnesses, okay. Now and then we have equilibrium condition this okay so these essentially we obtain this is from the compatibility condition was we could obtain this duration because this theta for all these members theta is same. So this is essentially the compatibility the outcome of the compatibility condition at O and then this is the equilibrium condition, right.

Now next if we if we use these 2 conditions these equations this is equation number 1 and this is equation number 2 then we can express MOA as this, okay. So this is the total the summation of all these stiffnesses of all the members and KOA is the stiffness of that particular member and the M is the total applied moment.

So this is K0 by K okay so now if you remember we discuss absolute stiffness, this is the absolute stiffness this you can say this is stiffness related to the total stiffness, okay. Now similarly so these things we can do for these expression what we have obtain for MOA that we can obtain for other members as well, so for other members also we can have similar these expressions for all the members, okay.

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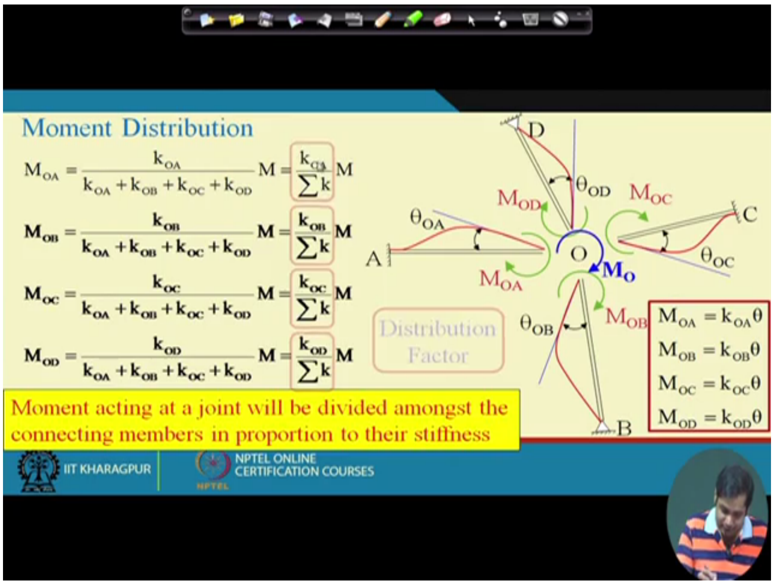

Now these so this says, this says that moment if we take a rigid frame and apply a moment at any joint, then that moment will be distributed among the connecting members and in what way they will distribute, these equations these says that they will distribute in proportion to their stiffnesses, okay. So based on their stiffness they will get distribute.

Now then this is called these factors of called distribution factors, why are they called distribution factors? Because these factors essentially give you how the moment will get distributed and the summation of this all these factors should be equal to 1 which is obvious from their expression, okay so this is distribution factor.

okay now next is what we have learned so far we there are a few terms we know now what is scary your moments and we also know what is carryover factor, we know what is absolute stiffness and then we know what is distribution factor, okay. Now what all these ingredients are now ready with us, now we can start cooking. Now let us discuss what is moment distribution method and all the terms what we have learned so far that terms which have been introduced those terms would be used in this method, okay. Great now let us demonstrate the method through an example okay.

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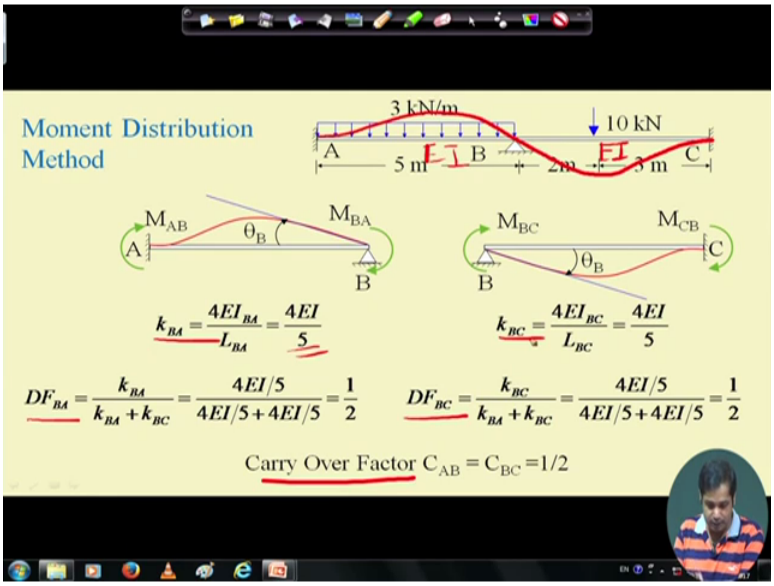

Now you see in this is if you compare this with slope deflection method are slope deflection method we get there was no iteration in the slope deflection method right once we get the equation, the system of the equation then it is the solution of this equation and the theta that we get that is the exact value with respect to the assumption that we met. Okay now but moment distribution method, since we are bypassing that part, the calculation of displacement, directly we are going to evaluate the forces, internal forces.

Now these methods is the iterative methods you have to do it in a iterative manner but that iteration is also in a very standard way that the duration can be done and that is we are going to demonstrate here. Consider this is the beam it is a 2 spam beam where A and C are fixed and 2 loading conditions one at AB one at BC. Similarly example what will be doing you will see that almost we will be using similar example in order to demonstrate different methods.

Slope deflection, moment distribution and direct stiffness will be using more or less similar example. The reason behind using similar example is then you can really compare those methods compared the values he will get almost similar values but the steps you can compare between the methods, okay. Now so there are 2 spans here span AB and span BC. Now let us separate these 2 spans okay now because of this loading okay the span AB may deflect and span BC will deflect.

Now take 2 spans separately okay now suppose this is spans AB and this is span BC and these angle is theta and theta B rotation at B and this angle is also rotation at B to satisfactory compatibility condition, okay. Now so essentially we assume that deflect the shape of DB something like this okay defective shape of B we assume something like this and then we take AB separately and BC separately, the reason deflective shape is assumed, you may not get these deflective shapes but the reason why we took this deflective shape because it is consistent with the sign convention that we are using for the moments.

Now if you are deflect shape is different then you will get a different negative values. Okay now so this is 2 parts part AB and part BC, now you see this is an idolisation. These 2 idolisation we have seen that a propped cantilever beam then the moment is applied at the propped end, now for this what we know that KBA is equal to this 4EI by now assume EI is constant for both the cases, it is also EI it is EI and EI. EI is constant for both the cases okay.

Then what is K absolute stiffness for this and joint B this becomes this and similarly for this member similarly for this these are an idolisation of (())(24:54). So KBC is in again this between is that if you (())(25:01) moment at B then and there is rotation at B, then this moment and this rotation they are related to their stiffness and their stiffness is this and similarly if you apply moment at BC it will undergo rotation like this and these moments at this rotation will be related to this stiffness coefficient KBC right.

Now then what is the carryover factor, now what is the distribution factor, distribution factor we have seen that the situation factor for this will be KBA plus the total stiffness and this becomes half here and similarly distribution factor for this will be half for this member as well and then what is the carryover factor, carryover factor will be half carryover factor is, if we apply MBA moment here, the moment generated at the other end is MAB.

Similarly if it is MBC moment generated at the other end is MCB and MCB will be half of this MAB will be half of BA that is carryover factor so we have already discussed this okay so we have already discussed carryover factor then we have already discussed distribution factor and we have already discuss what is stiffnesses as I said these 3 are the ingredients of this methods, so this is how they are required to use this method.

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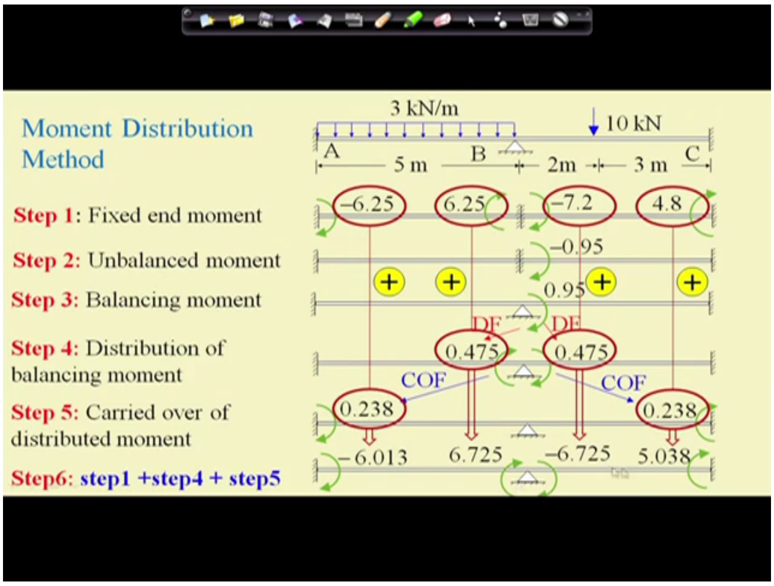

Okay next is let us do all the steps okay for studies step 1 is, it is very similar to slope deflection method step one is we the essence of whether we apply a displacement method or moment distribution method or any method they all are displacement base method and the essence of displacement base method the general steps in this placement base method will remain same and how we are doing that step depending on that they give different methods.

Here the steps are being performed iteratively the first step if you remember we discuss in the previous last week that we get the primary (())(27:07) which are kinematically determinate right and how do you get that primary structure we know what is the kinematic indeterminacy of the problem and then we need to constraint that corresponding degrees of freedom. So now therefore all the joints we have to make fixed joint so these becomes fixed joint so this is a kinematically determinate structure for this is the primary structure, okay.

Now primary structure is subjected to the external load and because of this external load will be having some moments at the end and those moments are called fixed end moments that we discussed last week, so what will be the fixed end moment so this fixed in moment for this will be in this case it is minus 62.5 and these are the fixed end moments and we know how to calculate fixed end moments for this case it will be (())(28:07) by 12 for in this case also we know the value for different loading conditions we know how to calculate fixed end moment.

So this is the fixed end moment, right for these the fixed end moments will be actually hogging moment but here when we write the moment, we write the sign of the moment based on whether the moment as clockwise or anticlockwise, clockwise moments are positive okay.

So we show all the moments as clockwise moment but when we write the value for anticlockwise it becomes negative, okay. So these are the fixed end moments so once we know the moments, now we will see what is the what is the there is no external moment here, right.

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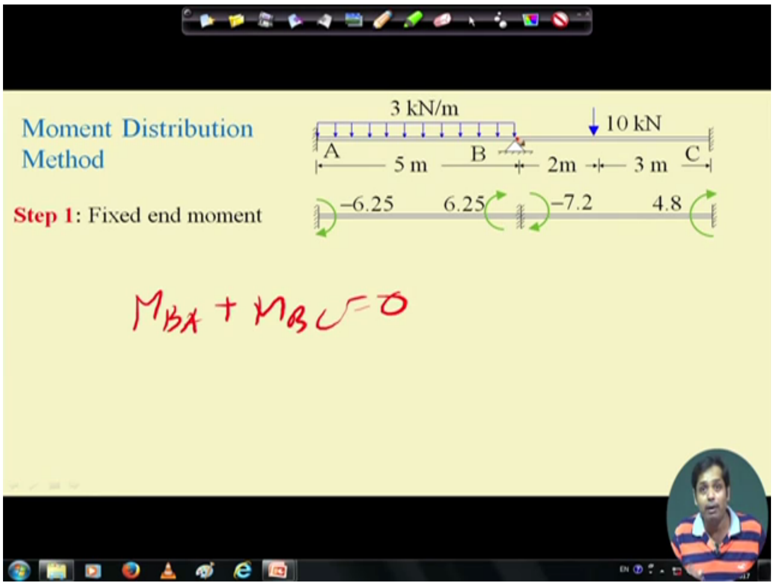

If there is no external moment here then what happens, the member MBA, if you remember slope deflection equation, slope deflection we wrote that MBA plus NBC that has to be 0, is not it? That is the equilibrium condition at B, now let us see whether that equilibrium condition is satisfied not. Now here this equilibrium condition is not satisfied so there is some unbalanced moment, right? Now step 2 is calculate the unbalanced moment, now what is the unbalanced moment here, unbalanced moment at this joint will be this plus this and in this case it is minus 0.95.

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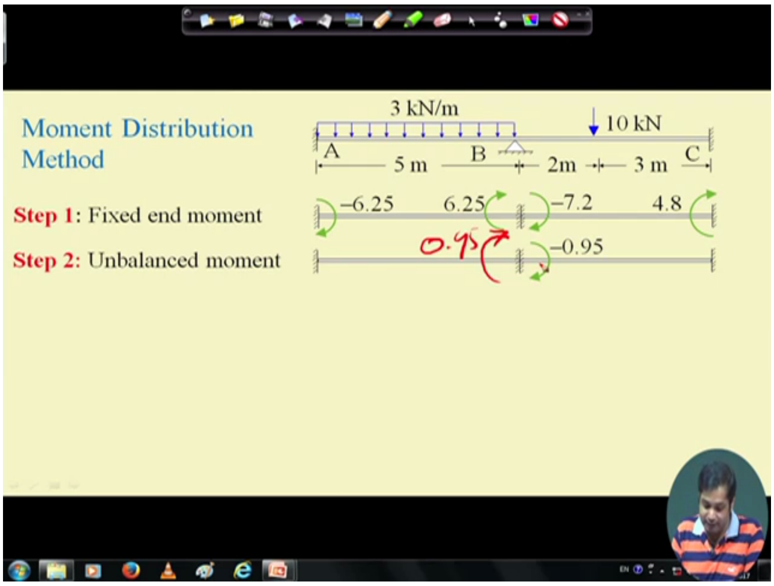

Now these unbalanced moment needs to be met 0, right because there is no externally concentrated moment here, so moment and B both the segment they should cancel each other. Now next it is we have to make it 0, how do I make it 0? Just by if we apply and if it is minus point if we apply an additional moment here which is 0.95 suppose fictitious, you apply an additional moment which will balance each other, right. Now then this equilibrium is satisfied, we are going to do that.

So then balancing moment, how do I balance the moment? We apply 0.95 moment here so it is unbalanced moment in order to balance that same value but in opposite direction we need to apply. Now when we apply this moment at this joint they will that moment will it is the moment applied at joint but what we are interested now in, how much of this moment will get transfer to this member and how much of this moment will get transfer to this member? And just now we discussed moments will get transferred based on the situation factor based on the their stiffnesses and what is the distribution factor for this case?

We just now we have seen the distribution factor is half and half, so half of this moment that 0.95 that will get transfer to this member and half of this moment will get transfer to this member. So the next step is distribution of the balancing moment, so this is the balancing moment, now that moment is to be distributed, so distribute that moment this half of this moment goes here and half of this moment goes to this member.

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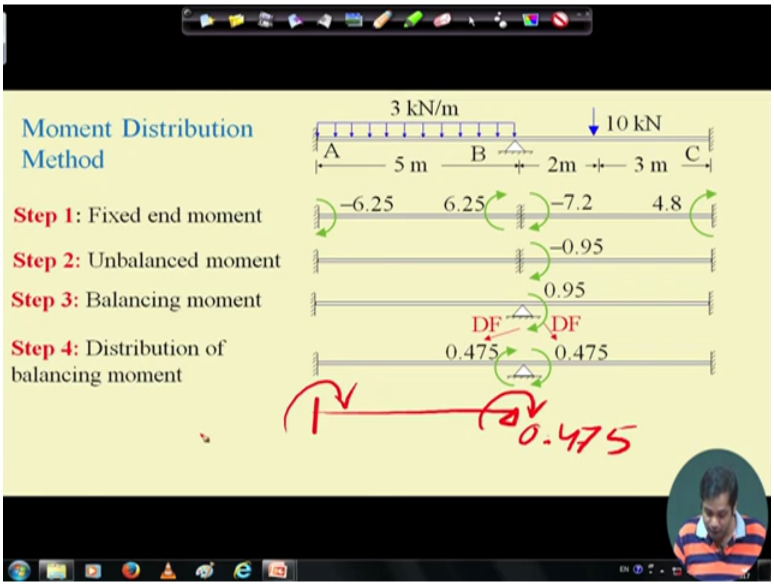

Since this is positive all these moments are positive. Now then once this moments goes here, then this will generate okay, this will generate some carried over moment at this point this will generate some error moment at this point, right. Now so 0.475 you are applying on this essentially what we are doing, this is beam and we are applying 0.475 and this 0.475 is the balancing moment, that moment will generate some carried over moment here, okay and what is the carryover factor just now we have seen for this carry over factor is half.

Similarly for this member so let us do that, so next it was carried over the distributed moment, so this is the distributed moment that will carry over to the other end and what is the carried over moment, carried over moment will be since the distribution carryover factor is half, half of this moment will go there and similarly half of this moment will go there, okay. Now step 6 is then once it goes there let us see what is the presence what is the status now?

The status is this now lets you what is the total moment we have at this joint, at this joint and at this joint. The step 6 is let us, so this is carryover factor so this is through carryover factor. Step 6 is step 1, so this step, this step plus step 4 and then step 5, okay so sum them, so what is the total moment now we have at different joint, joint A, joint B and joint C. Now total moment will be so at this point the total moment will be, so you sum them it is this value and then at this point the total moment is this value and at this point the total moment is this and this point the total moment is this.

Now this is a complete cycle, okay now after the cycle let us see what is the presence status? So this is essentially now this is this step is essentially for the next cycle this step is essentially next (())(33:43) this step. We started with this when you saw that there is an unbalanced moment at this joint, then we balance this unbalanced moment and correspondingly distributed and carried over moment we will have.

Now the after step 6 C what is the presence status whether we have any unbalanced moment or not, right. Now if we have any unbalanced moment then again we have to do the similar steps, step 1 to 5 on this, okay. Now again obviously I choose a problem where we get balancing moment where we get perfectly balanced moment after the end of step one because the objective here to demonstrate the step.

We will be doing some examples where even after one step one cycle we will see that there are some unbalanced moment and we have to repeat the cycle we have to get the balanced or equilibrium position we have to do the iteration to repeat the iteration until we get the equilibrium position. Now it is now in equilibrium position you see this is positive 6.725 this is negative 6.725 and they can balance each other.

So this is the final status at least for this problem but it may so happen that after one cycle you may not get the equilibrium states and in that case you need to repeat the process that will be demonstrated to some example. Now let us so for this now in general for this example I showed you in a very with all this beams and supports drawn but when you actually do it that can be do it in a tabular form will demonstrate that.

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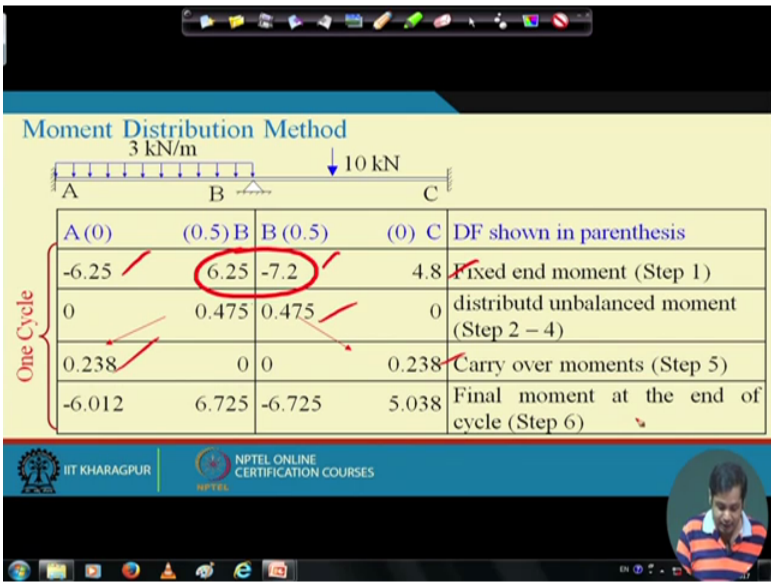

So this is the same thing but written in tabular form so this is one axis this is we started with this where we have an unbalanced, these are all fixed end moments where we have an unbalanced moment at this point then we distributed the moment and their distributed moments were carried over to other end and then we sum them this and this and this and this and this is the final status, so this is this is complete cycle and we are lucky in this case that after one cycle we got the equilibrium stage.

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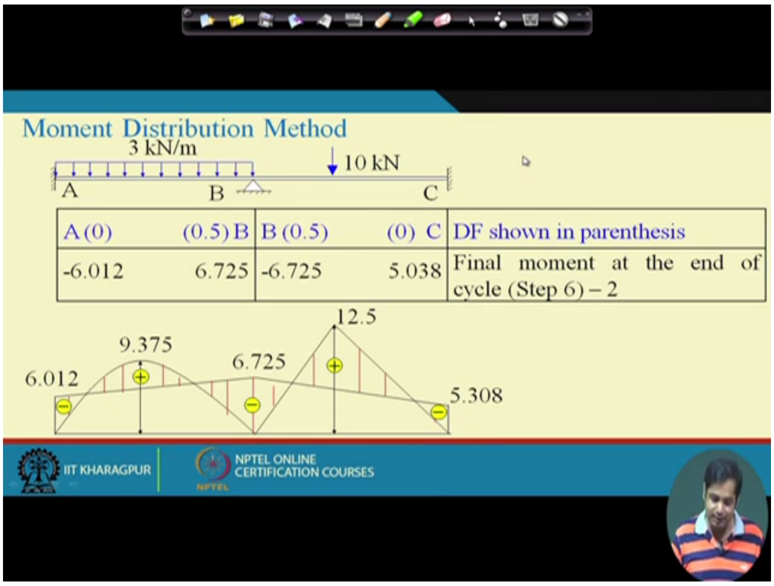

Now finally once we this is the final moment at different joints. Now if you have to if you are to draw the bending moment diagram, let us draw the bending moment diagram, the bending moment diagram will be bending moment due to the external load and then plus bending moment due to this moments at different joints. So because of the external load this is the moment on ABS this and this value is (())(36:22) we know this and because of the external load and BC this is the bending moment diagram for BC this is 12 point 5.

Now on this we need to super pose the bending moment for this for this at the joints and these bending moments will be for this joint for this member it is 6.012 and 6.725 then for this it is 6.725 and 5.035 but when we draw the bending moment diagram please make sure to not to not draw the bending moment diagram based on clockwise or anticlockwise moment, see whether the moment is hogging moment or sagging moment and that is the sign convention we use for bending moment diagram and then this is the bending moment, the red lines that is the bending moment and this so these are the positive moments and these are the negative moments, so this is the final solution of this problem, okay.

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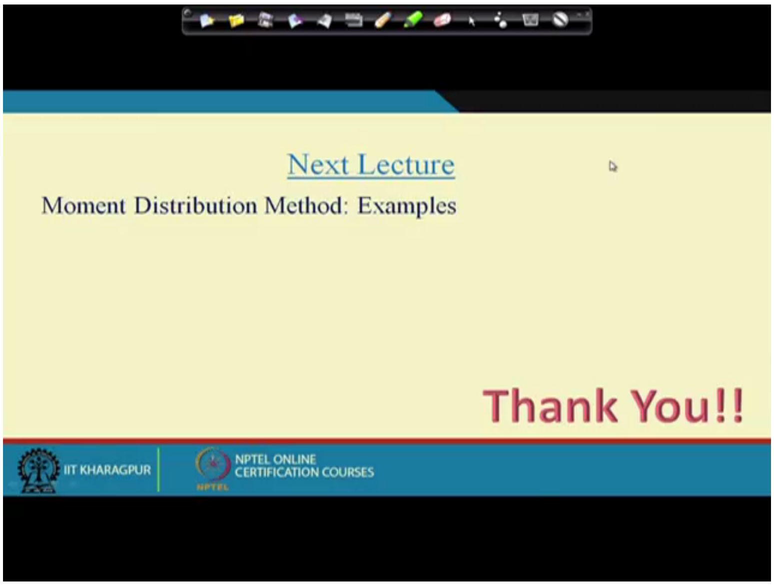

Now what we will do next is, the idea the objective of today's class has been to just to introduced the moment distribution method and introduce the steps involving moment distribution methods that will be demonstrated through some example in the subsequent classes, okay. So next class we will see next class onwards we will see some examples where indeterminate structures are analysed using moment distribution method, see the next class. Thank you.