Structural Analysis
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Module 10
Lecture No 52
Analysis of Indeterminate Structures by Displacement Methods (Contd)

Hello everyone will come, this is the last lecture of this week we will consider one more example on frame where the frame undergoes sway, so the example that we consider today is this.

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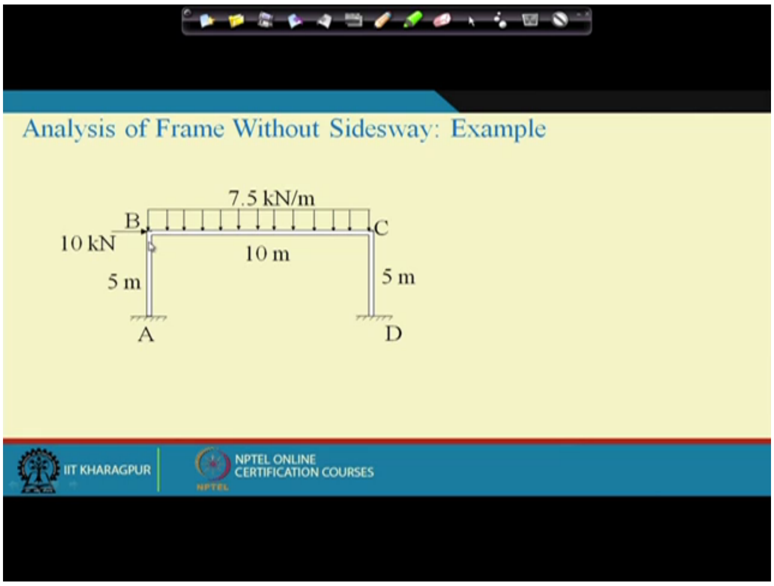

If you see the similar example of similar frame problem that all the dimensions and loadings are same, but in addition we have another 10 kilo Newton load horizontal load which will cause the frame to sway in this direction okay. Let us do dimension and loadings are taken same because some of the reasons that we have already obtained in the previous in the previous class we can directly use them okay, so this is the example okay.

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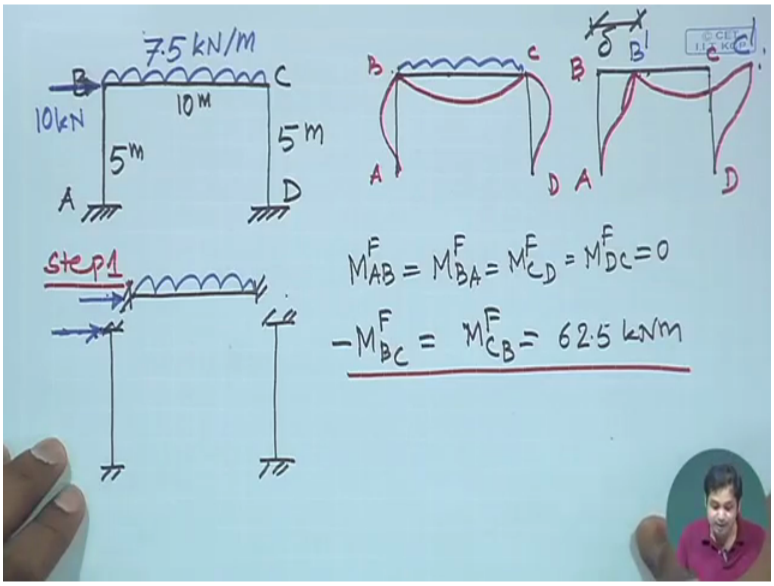

Now so this is the frame which is there is it is horizontal load, there is horizontal load here okay. Now because of this load what happens? If you remember if there is no load here it is just a problem like this okay then the beam may the frame may deform like this, if it is only subjected to particle load on BC there is no horizontal load then probably the beam may deform like this, this is the deformation of this and this is the deformation of this member and then similarly this will be deformation of this member okay, this is the probable deformation deforming behaviour of the structure when it is subjected to only vertical load on BC.

Now there is only vertical load on BC, now in addition to that if there is horizontal load as well then what happens the beam will undergo sway like this in this direction okay, then the beam may deform in addition to this deformation because of this horizontal load so this point this point B, this is A, B, C, D, point B moves in this direction, point also moves in this direction so this become something like this okay, your reform shape become something like this and this is A B C and D. Now point of B goes to B dash and point C goes to goes to C dash and this distance is Delta okay. Now since we assume there is no actual deformation in BC or in any member, so the amount of amount of sway takes place at B the same amount of sway will take place at C dash okay.

So B B dash equals to Delta and similarly C dash equals to Delta okay. Now if we compare this problem with the previous problem, what are the changes we need to do? You see in the previous problem all the steps are again overall the steps are same, let us first start the steps and we will discuss what are the differences. Now the first was the step 1, step 1 was finding the fixed end moment okay, now again similarly since this joint is acting this force is acting and the joint, this force will not cause any moment in this member and in this member in the primary structure as well okay. For instance the primary structure becomes what? Primary structure for AB becomes this, for CD becomes this and for BC it becomes this okay.

And the loads are acting on it, there is no load on this, there is no load on this, there is a load externally load is acting here, external corresponding load is here but this will not induce any moment in this structure, this will not induce any moment in the structure because this is acting at the support okay. So what is the fixed end moment then? Fixed end moment M AB fixed end equals to M BA fixed end equals to M CD fixed end equals to M DC again fixed end; they all will be 0 okay. And then what is M BC? M BC fixed end moment equals to M CB fixed end moment, this will be – equals to w l square by 12 and w is 7.5; l is 10 so this becomes 62.5 kilo Newton meter okay.

Some of the first of the results, I suggest you to refer to our previous class because this we have already done in the previous class, we have already computed these values in the previous class okay so this is the fixed end moment we have, this was our first step. Now second step is applying the slope deflection equation for different members, member AB, BC and CD and the difference lies there okay. Let us see let us first apply the slope deflection equation for slope deflection equation for span AB okay.

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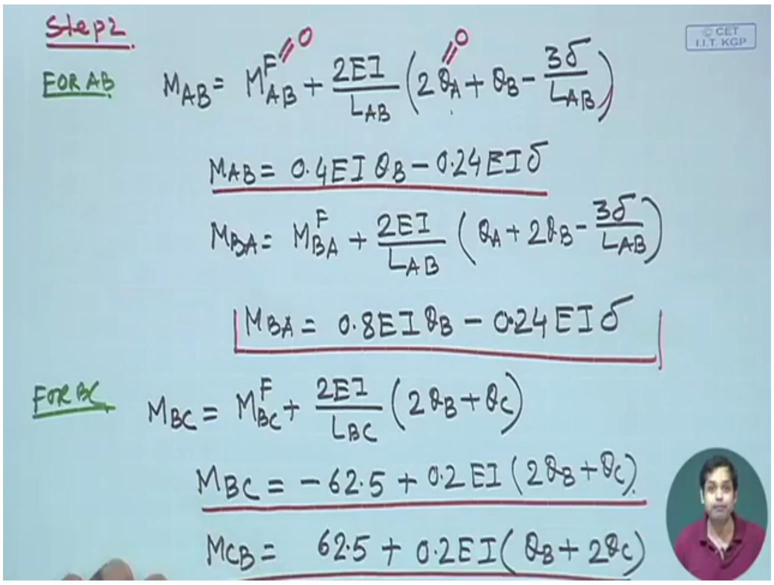

So step 2, slope deflection equation that is for span for AB okay, what is the slope deflection equation? M AB equals to M AB fixed end moment, let us write the general expression 2 EI by L AB and then 2 Theta A + Theta B – 3 Delta by L AB that is the general expression okay. Now Theta is 0 that is fine because it is Theta A is 0 we have, Theta B is nonzero then Delta is this amount. This is if we consider this is the beam AB is the separate beam okay separate member then joint rotation will induce rotation at B will induce some moments at A and B and then in addition to that the settlement of joint, in this case settlement is the translation of this sway takes place of joint.

This Delta will cause some moment at B dash, some moment B and some moment at A and so this Delta is not 0 that is the difference with the previous case where there was no sway there was no force horizontal force like this so there was your deflected shape was this so there was no sway at this the Delta was 0 point but in this case Delta was not 0. And this is also 0 this part is 0 fixed end moment is 0, now then this becomes M AB M AB becomes 0.4 EI Theta B – 0.24 EI Delta so this is the first slope deflection equation for member AB at A. So similarly we can write slope deflection equation for BA, M BA will be fixed end moment M BA fixed end moment + 2 EI by L AB then Theta A + 2 Theta B – 3 Delta by L AB okay.

And again if we substitute all these values we get M BA equals to 0.8 EI Theta B – 0.24 EI Delta okay. Now in the previous example Delta was 0, only we had these 2 values okay, so this was for AB now let us do it for BC. Now for BC there is no for BC there is no sway , joint B still remains in this line joint C still remains in this line so there is no sway takes place for joint BC. When we consider joint AB the settlement is the joint settlement will be in this direction or in this direction, for CB also either in this direction or in this direction. But when we consider member BC, the joint settlement means either joint B goes to here or here or joint C moves in vertical direction but that is not happening here so for member BC the for member BC there is no settlement so contribution from the settlement will be 0.

So for member BC then slope deflection equation will be M BC F + 2 EI L BC into 2 Theta B + Theta C, Delta is equal to there is no Delta in this case, so this is the equation and if you substitute this value all the value corresponding value, M BC becomes – 62.5 + 0.2 EI 2 Theta B + Theta C this is for BC and similarly we can write for CB is equal to similar expression we will get 62.5 + 0.2 EI into Theta B +2 Theta C so this is for BC, this is for CB okay. So again let us do it for another for CD, for CD again there is joint settlement, joint C moves in this direction and that is this direction is Delta so contribution from the settlement of B or in this case sway at joint C that needs to be considered okay.

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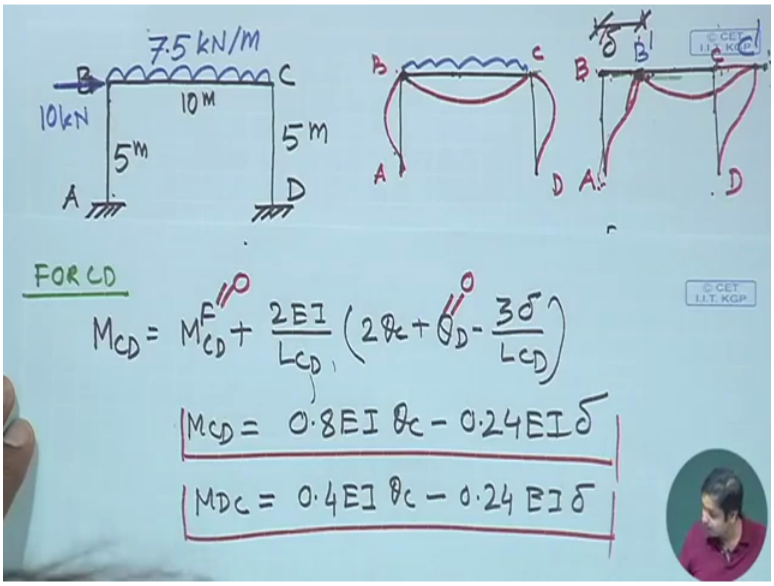

So the corresponding equation will become so for CD it becomes M CD equals to M CD fixed end moment + 2 EI by L CD, please note in these examples EI I have taken as constant for all these members, but if EI is not constant then you have to take EI for the corresponding member here. The way we are taking length different length for different member similarly, different EI for different member needs to be taken okay. So this becomes 2 Theta C + Theta D – 3 Delta by L CD, again Theta D equals to 0 because it is fixed end so this becomes 0 and then final expression becomes M CD is equal to and this is also 0, this is also 0 so M CD equals to 0.8 EI Theta C – 0.24 EI Delta okay, so this is for M CD and similarly we can write for M DC and that will be 0.4 EI Theta C – 0.24 EI Delta okay so this is for DC okay.

Now we have already applied the slope deflection equation for each segment and the next part is next step is apply the equilibrium condition and equilibrium condition is we have equilibrium condition is that joint at B… M BA + M BC equals to 0 and M C + M D equals to 0 so let us apply second step second let us apply the equilibrium condition step 3.

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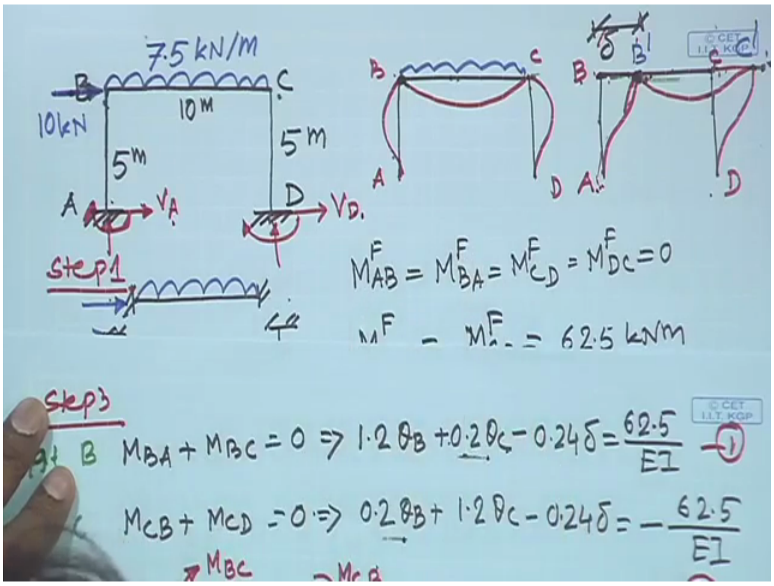

Step 3 is M BA + M BC equals to 0 okay, this is one equation and then equilibrium condition at C is M CB + M CD equals to 0 so this is at B and this is at C the equilibrium condition at 0. Now there is an interesting point, how many unknown we have here? Rotation at B, rotation at C and this sway Delta, we have 3 unknowns, and how many equation we have? We have just to equilibrium equations, is not it? The equilibrium at B and equilibrium at C, it means we need one more equation okay and let us see that is the that is the main difference that you have in problems between the problems which does not have any sway and the problems which have sway, means frames without sway and frames with sway.

Frame without sway this equilibrium conditions was enough to give you the sufficient information and to solve the unknowns, but in this case we need one more equation and what is that equation? Before we come to that let us write what is this expression, let us substitute the expression for M BA and M BC and get the corresponding expression. Now this will give the expression becomes at B is like this, 1.2 Theta B and then + 0.2 Theta C – 0.24 Delta equals to 62.5 by EI, so this is the first equation this is equation number 1 and then then second equation becomes this becomes 0.2 Theta B + 1.2 Theta C – 0.24 Delta equals to 62.5 by EI so this is the second equation this is equation number 2.

Please again once again you please check so this 0.2 is what? 0.2 is the contribution of Theta C at B and this part is contribution of Theta C at B and this part is contribution of Theta B at C and associated stiffness coefficients are, in this case stiffness coefficient is 0.2, in this case 0.2 into EI, in this case stiffness coefficient is 0.2 EI, again the symmetry can be observed okay. So this is the equilibrium the 2 equation that we obtained from equation equilibrium equation. Now we need one more equation, another equation that we get that let us see what is that equation.

Now let us draw the each draw all the members separately okay, now this is the member and this is another member and this is another member okay and this is member A, this is A, B, C and D okay. Now what are the forces we have? This force, this is the corresponding moment, this is M CB, M at C in member CB, this is M BC okay and then we have this M BA means moment at B in member A and then all are shown in clockwise direction, this is M AB and then again this is M CD and this is M DC okay. Now you see then what we have is, in addition to that we have shear forces as well okay. Now what is what if we take moment about in addition to that say these are the supports, we have force at VA, this is the sheer force at VA this is the sheer force VA and similarly we have sheer force at VD okay.

We can we have sheer force at this point as well, at this point as well corresponding point say this is as well, this is as well okay. And we have a sheer force at this member and this member as well but since we are only concentrating on the moment, let us now we are not buying it here okay. Now the reason why this sheer force is shown here, now if you take the free body diagram of this part and moment take moment about this okay and then what we have? We have that VA is equal to this length is L AB and this length is L CD okay. Now if we take moment about A moment about this point about B then what is VA? VA will be VA will be this moment + this moment and then divided by L AB okay.

So M BA clockwise then M AB + M BA divided by L AB okay, this is this is BA okay. And similarly if we take moment about this point and then what are the moment what the forces that we contribute are? This will contribute, this will contribute and V D will contribute, they will not contribute because this is this is passing through this point so similarly VD will be M CD + M DC divided by L CD okay. Now this is, now what is VA essentially? VA is the this is we have a support at this point right we have a support at this point, so VA essentially which is sheer force at A, it is essentially the horizontal reaction at A. And similarly VD is sheer force at D which is essentially horizontal reaction at B.

Now if we look at the equilibrium of the equilibrium of the entire structure means if we look at the if we look at the equilibrium of the entire structure supposed this was the structure, this is the structure, now and draw the free body diagram of this of the entire structure then what are the free body diagrams we have? This will be a force like this and then reaction here and similarly of moment and reaction here, reaction here and then reaction here right. Now so this is essentially we have obtained this is VA, this is VD right. Now so equilibrium says that total VA + VD + this horizontal force that should be equal to 0 that is the mission of FX equals to 0 that is the equilibrium condition right.

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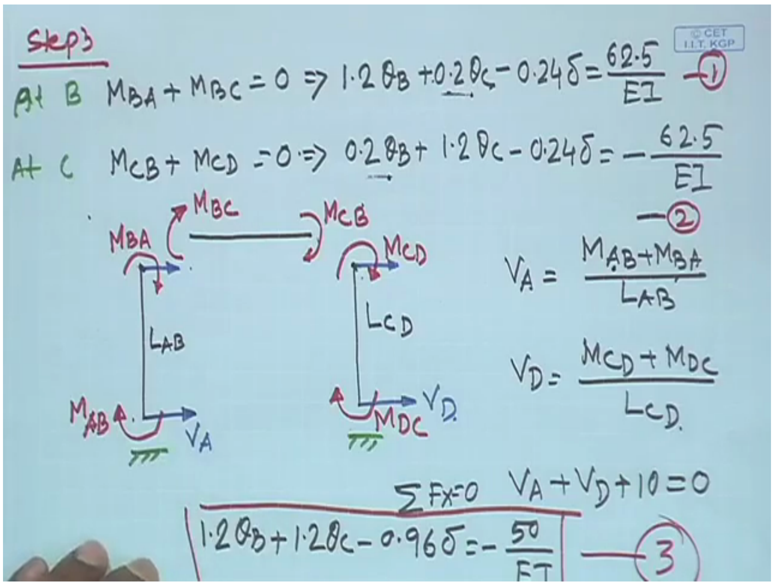

So if we if it is then can we say that then submission of FX equals to 0 gives us submission of FX equals to 0 gives us that VA + VD + externally applied load 10 kilo Newton that should be equal to 0, is not it? Now, now VA is equal to this and VD is equal to this, we have already obtained the expression for M AB and expression for M BA, we have already obtained the expression for M AB and then corresponding expression for M BA and then we have already expression for M CD and we have expression for M CD and corresponding expression for M DC. What now we have to do is we have to substitute these expressions in this equation, if we do that we get one more equation that is equation number 3 okay. And this is we do that the equation that the equation finally what we get is this.

1.2 Theta B + 1.2 Theta C – 0.96 Delta that is equal to – 50 by EI that is the additional equation we get by considering the sheer force okay, this is the additional equation you can check this equation, this is the additional equation we get okay. And how we obtain this equation? We obtain this equation by the shear okay. Now again as I say this there will be some other forces, this is not the complete free body diagram of each member separately because you have some other forces here, all the forces I did not show here because if I show all the forces that this diagram will not be clear. So all the forces which is contributing to this third equation is shown here okay. Now so this will be equal to – VA, this will be equal to okay forget it.

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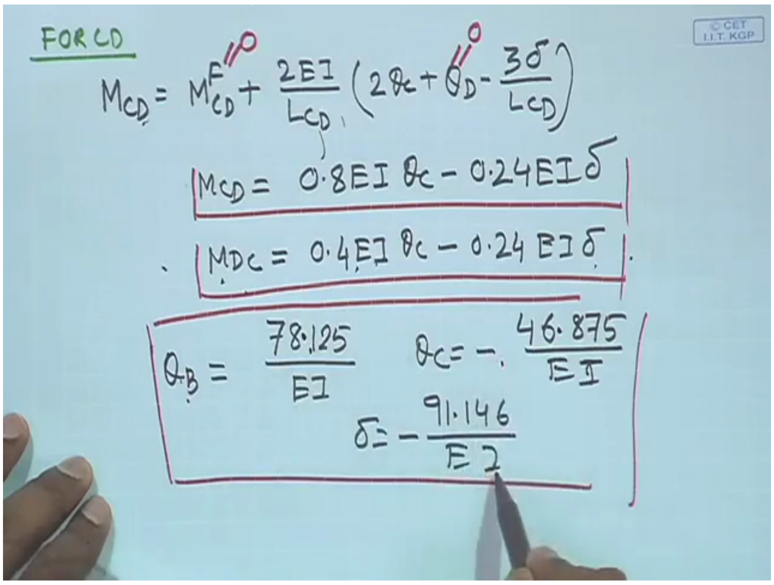

Now this is we have 3 equations, now one two equations are 1 equation, 2 equation, which is equilibrium at B and C and another equation by which is obtained from equating the total shear force with the externally applied load okay. Now next step is again straightforward next step is we need to solve these equations okay, and if we do that then the solution becomes Theta B equals to 78.125 by EI and then Theta C equals to – 46.875 EI and Delta equals to – 91.146 by EI, 78.125, 46.875 this is negative and Delta is going to – 91.145. This is the final solution, now one interesting thing is in the previous example where there was no sidesway Theta B and Theta C were same but in this case Theta B and Theta C is they are not same okay.

Now once we know Theta B, Theta C next step is just to apply Theta B the value of Theta B and Theta C in this expression, value of Theta B, Theta C and Delta in these expressions in all these expressions to get the moments at the joints and if you apply the moment at the joints the final value that you get is this.

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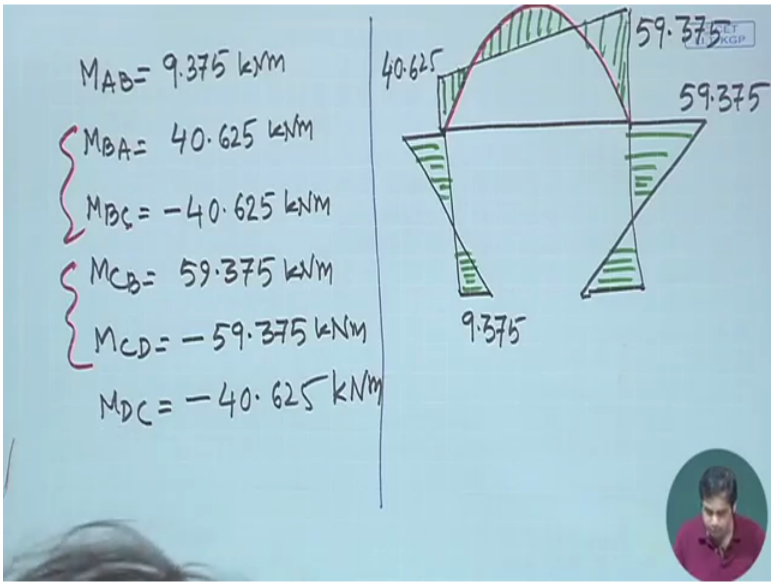

M AB equals to 9.375 kilo Newton meter, M BA equals to 40.625 kilo Newton meter, M BC equals to – 40.625 kilo Newton meter, M CB equals to 59.375 kilo Newton meter and then M CD equals to – 59.375, always write the unit because I have seen many places this is the common mistake we do 40.625 you write just like this M DCequals to 40.625, this does not carry any meaning okay, when you write it is kilo Newton meter then it carries some meaning okay. So anything you write, always write the corresponding unit with it, without unit any data is anything is meaningless okay. Now let us draw the bending moment diagram, the bending moment diagram becomes like this very similar to the previous case but the only difference here will be, now again you can check this equilibrium is satisfied, M BA + M CB equals to 0, M CB + M CD equals to 0 okay.

Now the equilibrium the bending moment diagram becomes at this point it is at B it is 40.625 and at C it is 59 point something so this bending, to get from this is the bending moment you get from the joint and then corresponding bending moment corresponding bending moments from the external load, this is this okay and then for so bending moment between bending moment become this okay, this part is negative, this is negative and this is positive. And similarly bending moment for this is 40.625, this is 59.375 okay and again it is 40.625 and this value is 9.375 and this and again here it is 59.375 and this value is M Dc 40.625 this is 40.625 and finally this becomes this and then your bending moment is this, your bending moment finally your bending moment become this okay.

Please so please check these values once you once you have this okay, now next is what we do we stop here, now let us just summarise what we have learned in this week okay. We discussed the overall concept of displacement method, and then that concept is translated I mean is written there are 3 kinds of displacement methods we learn we discussed in this course, one we have discussed is slope deflection method. Now another 2 method which is moment distribution method and then (())(30:57) stiffness matrix that we will discuss in the subsequent weeks okay.

Now please do some exercise that concept, once you understand the concept that is fine that is absolutely necessary part just by understanding the concept is not enough, you need to apply that concept in order to appreciate that concept in a better way in order to comprehend the concept in a better way, we need to really apply them through different problems and do some exercise some of the concept we have demonstrated here but those problems are related to simpler in terms of number of unknowns and terms of loading conditions and the boundary conditions. But the idea has been not to as I said idea has not been to to solve any particular structural analysis problem, the idea is to demonstrate the concept through some problem through some relatively easier problem.

Okay, so next week what we will do is, next week we will see what is moment distribution method okay then so see you in the next week thank you.