Structural Analysis
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Module 10
Lecture No 51
Analysis of Indeterminate Structures by Displacement Methods (Contd)

Hello everyone welcome, what we have been doing is we have been demonstrating the concept of slope deflection method; we have already done so for beam problem. Now today we will demonstrate that through some problems and in the next class also we will continue with it. Now so today's topic is analysis of frames using displacement method or more particularly a slope deflection method.

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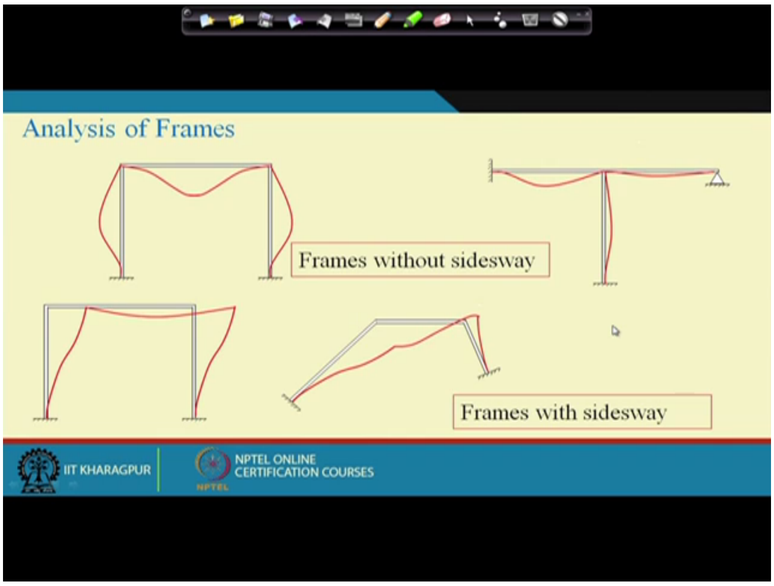

Now there are 2, the problems in frame can be divided into 2 parts, you see the first one the first 2 rather, these frames are called frame without sidesway. Without sides sideway means okay before that you see compare this, this is the deflected or the deformation behaviour of these 2 frames the first and second vis-a-vis third and fourth, in third and fourth what happened that due to the lateral load there is a sway takes place in a particular direction okay. For instance, in this case also in this case this is the amount of sway takes place. This point moves in this direction this sway takes place.

For this case also these points move in this direction this amount okay this point moves in this direction by this amount whereas, in this point all the joints they can rotate okay, rotation can take place for instance the rotation of these joints, there is a rotation at this joint, rotation at this joint, rotation at this joint, but all the joints they remain at the position so there is no sway in these joints. So what we do these are these kinds of frames is called the frames without sidesway and these frames are called frame with sidesway. So what we do is we today we will we will see with some example of frames which does not have any say and in the next class we will see if there is a sway in the frame then how it is to be considered in your analysis and how the amount of the sway to be determined okay.

So we will go with we will start with this kind of problem today. Okay now the problem let us take this problem, this is a portal frame which is subjected to uniformly distributed load okay. So this is the problem, we will apply slow reflection method for this problem okay. We will go with all the steps that we learned in the previous classes.

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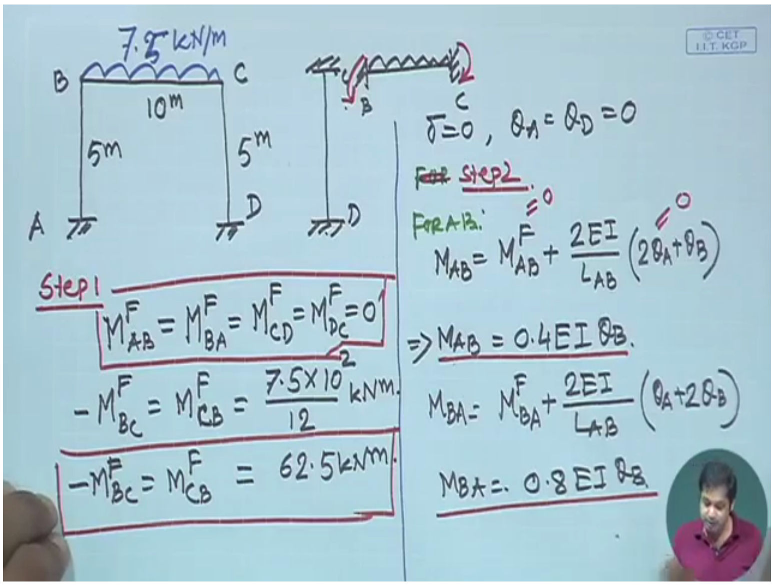

Now the first step in this frame has 3 parts, one is part AB, BC and CD okay. Now first is we need to determine the fixed end moment right this is the step 1. So step one is fixed end moments, now fixed end moment if you see for part AB, there is no external load applied on in this part so fixed end moment in part AB will be 0 so fixed end moment M F part AB at A that is equal to M BA fixed end moment at B in segment AB they are equal to 0. And similarly for CD, you check if you take CD separately and make and there is no, suppose what is the primary structure was CD, the primary structure for CD becomes this, this is C this is D so both ends are fixed.

Similarly for primary structure for AB becomes this so there is no load so M at this point and this point the fixed end moment should be 0 so this CD fixed end moment is equal to M DC fixed end moment they all are all are 0 okay. Now what is the primary structure for member BC? Primary structure for member BC becomes this and this is uniformly distributed uniformly distributed load, this is B, and this is C. So what will be fixed end moment we are saying if it is a fixed beam subjected to uniformly distributed load then the fixed end moments are w or q l square by 12 right. So then fixed end moment BC M BC F and what will be the direction of fixed end moment, direction for fixed end moment is this in this case okay.

So at B it is anticlockwise and at C it is clockwise, so M BC – M BC equals to M CB F equals to q l square by 12 okay, so q is 7.5 into l square is 10 divided by 12 that is kilo Newton meter and this value is you get 62.5 kilo Newton meter, this is – M BC F equals to M CB F okay, so this is the fixed end moment and this is fixed end moment for other members okay. So this means at BC it is – 62.5, CB it is 62.5 so this is step 1. Now step 2 was apply the slope deflection equation, now we need to apply slope deflection equation slope deflection equation for AB, BC and CD okay, now let us see what information we have okay.

Now there is as I say there is no sway in any members so Delta equals to 0, so if you recall the equation for slope deflection is a general equation which has rotation and translation as well suppose settlement or sway as well so in this case Delta equals to 0 this is the information we have. Then also for support A and support D, they are fixed supports right so Theta A equals to Theta D that should be equal to 0, is not it? So only unknown that we have is rotation at B and rotation at C okay. So let us apply the slope deflection equation for span AB this is step 2 so for AB okay. Now what is, what would be the slope deflection equation for AB? That will be M AB will be M AB fixed end moment + 2 EI by L AB 2 theta A + theta B + Delta, there is no Delta equals to 0.

Now M AB this gives you, now what we have is we have this is equal to 0, this is equal to 0 and then theta A equals to 0, L AB equals to 5 so this becomes 0.4 EI theta B so that is M AB. Similarly we can do it for BA, M BA M BA will be M BA fixed end moment + 2 EI by L AB into theta A + 2 theta B Delta equals to 0 then this gives you M BA this is fixed end moment, this is again 0 here so this is 0 again theta equals to 0 and this becomes 0.8 EI theta B, so this is for B okay. Now let us do the similar exercise for other members, member BC and member CD okay.

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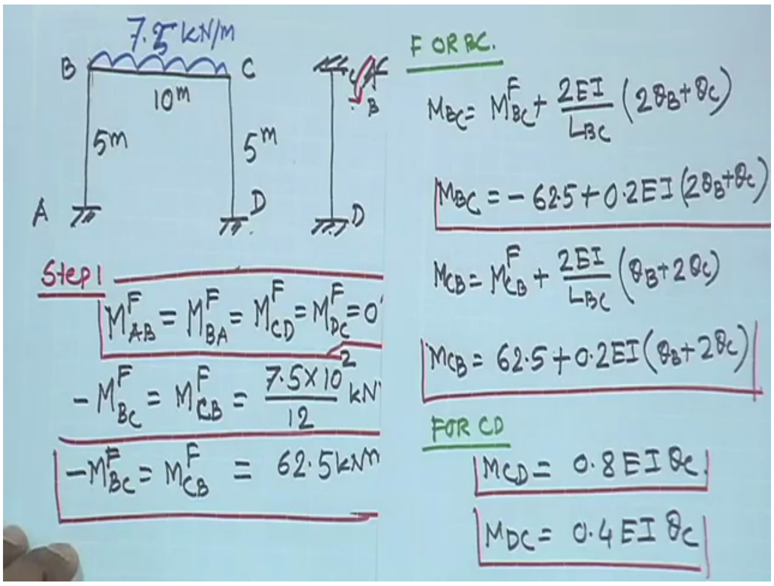

Now what is for BC, for BC is for BC M BC equals to M fixed end moment BC + 2 EI by L BC then 2 theta B + theta C okay and this becomes M BC is equal to fixed end moment is – 62.5 so this is – 62.5 and length of BC is 10 further becomes + 0.2 EI into 2 theta B + theta C okay this is + theta C okay. Now so this is and similarly M CB becomes fixed end moment + 2 EI by L BC and this is theta B + 2 theta C and this becomes M CB equals to 62.5 which is the fixed end moment and then 0.2 EI theta B + 2 theta C okay so this is M CB. And then we can do it for for CD, now again CD rotation at D equals to 0 and the fixed end moments at C and D both are 0 similar to AB so the expression that you get is M CB equals to 0.8 EI theta C and then M DC equals to 0.4, the expression will be very similar to AB which is obvious because the structure if you look at the structure this is a symmetric structure, is not it?

So whatever you get for AB you get the similar expression for CD, so we have done the step 2 is now completed, so slope deflection equation for all these members we have. Next step is to apply the equilibrium conditions at various joints equilibrium at A, equilibrium at B, equilibrium at C and equilibrium at D okay. Now step 3 equilibrium, let us apply the equilibrium condition now.

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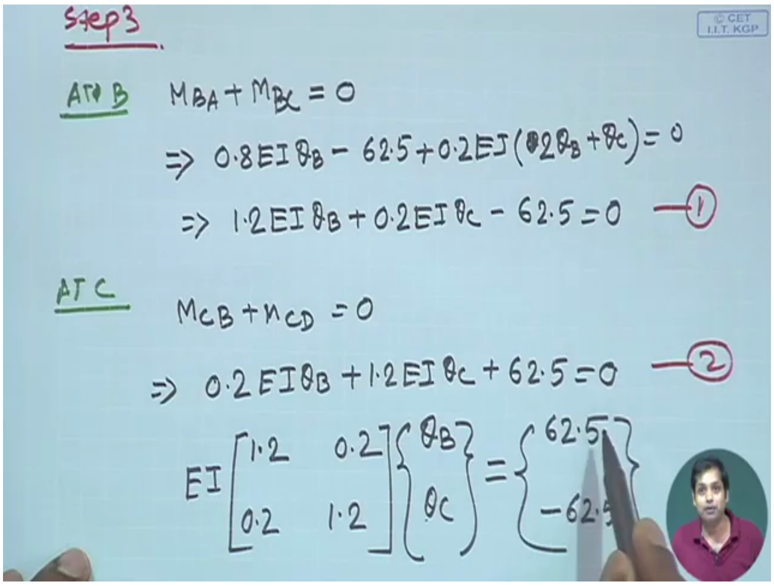

So step 3, now we have equilibrium for first at B, you apply equilibrium at B, equilibrium at B will be M BA, we know that M BA + M BC that that should be equal to 0 okay. Now we have already obtained the expression for M BA, M BC and if we substitute that this is 0.8 EI theta B – 62.5 + 0.2 EI 2 theta B + theta C that is equal to 0 and finally what to get is 1.2 EI theta B please check these values 0.2 EI theta C – 62.5 equals to 0, so this is your equation number 1 okay. And similarly we can have at C and what is equilibrium condition, M CB + M CD equals to 0 that is equilibrium condition. Similarly, if you substitute the expression for M CB and M CD, the equation that we have a 0.2 EI theta B + 1.2 EI theta C + 62.5 that is is equal to 0 for this is your equation number 2 okay.

Now if we write this equation in a matrix form then what we have? We have you see if you take EI common this becomes 1.2 and then 0.2, 0.2 and then 1.2, this is theta B theta C and is equal to 62.5 – 62.5 okay. Again this becomes K, this is K BB, this is K BC, this is K CB and this is K CC and these are all stiffness coefficients and this is associated displacement and this is equivalent joint load okay, so this becomes FB and this becomes FC. And if you write in 1-2 numbers so this becomes F 2, this becomes F 3, so this is the expression that we have seen in the definition of slope deflection equation and the discussion on displacement method is old. Now what we need to do? We need to solve this equation, we need to solve this equation to get the theta B and theta C and if you do that, the expression that we get is this.

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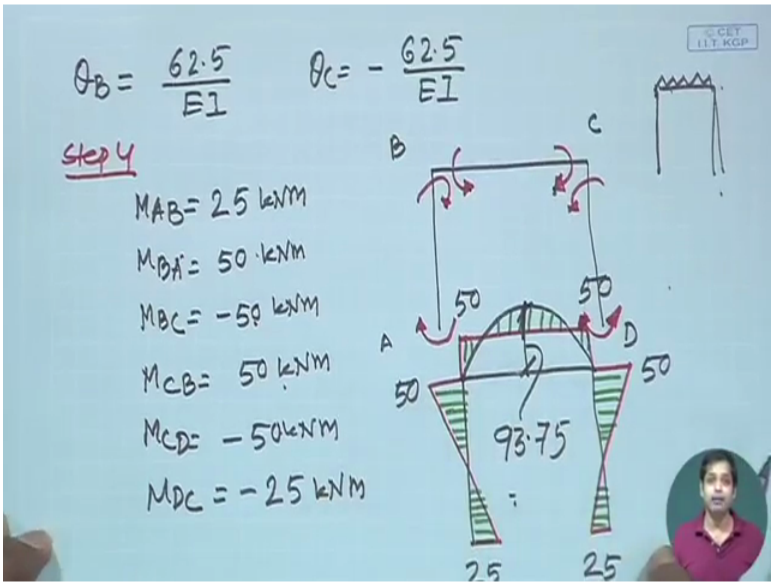

The final expression that we get this is theta B we get theta B equals to 62.5 by EI and theta c equals to – 62.5 by EI, again which is which is again expected because your symmetric structure so theta B rotation at B and rotation at C will be in opposite direction, the value will be same in opposite direction. So this was step 3 and final step 4 is now we substitute this equation in all these all the slope deflection equation that we have obtained for instance this is the slope deflection equation for AB and this is for BA and similarly this is the slope deflection equation for BC and this is CB. Now what we have to do is, we have to substitute the values of theta B and theta C to get the corresponding moment and if you do that then the corresponding moments will be either slightly final values.

M AB equals to 25 kilo newton meter, M BA equals to 50 kilo newton meter, M BC equals to – 50 kilo newton meter, M CB equals to 50 kilo newton meter, M CD equals to – 50 kilo newton meter and finally M DC equals to – 25 kilo newton meter. Now you see at what is the condition for adjoint B, M BA + M BC equals to 0, 50 – 50, what is the equilibrium condition at joint C, M CB + M CB equals to 0, 50 – 50 okay now let us draw the bending moment diagram, bending moment diagram is this. Again as I say this sign gives you whether the bending moment is positive or negative sorry whether the bending moment is clockwise or anticlockwise but when you draw the bending moment diagram we need to consider whether the bending moments is sagging or hogging bending moment.

For instance for (())(18:45) A B C D, in AB bending moment is 25, 25 means it is clockwise right it is clockwise and for BA it is 50, again it is clockwise it is clockwise, for BC it is – 50 means it is anticlockwise and for CB it is 50 means it is clockwise and again for CD it is – 50, – 50 means it is anticlockwise and again for DC it is – 25, – 25 means it is anticlockwise, it is anticlockwise okay. Now M BC and M BA is okay fine. So now what you have to do is, you know this is clockwise and this is clockwise and this is anticlockwise and this becomes clockwise, this is like this is this is like this okay. Now this is bending moment, these are the bending moments.

Now when you draw the bending moment diagram we need to see whether they are hogging moment or sagging moment or we draw the bending moment diagram and the and the bending moments are drawn in the compression side at least as per our sign convention okay. Now so what would be the bending moment diagram? The bending moment diagram will be something like this okay, so the first is the total bending moment will be the bending moment due to the external load and the bending moment due to these internal these moments okay. Now your loading was something like this, so for external load there is no bending moment in this, there is no bending moment in this, so for external load bending moment will be only on BC and we know it is a parabolic bending moment diagram.

So this is positive bending moment okay, now you see this is hogging bending moment as per our sign convention okay, so this bending moment will be negative bending moment so this is always 50, so this bending moment will be negative bending moment okay and this is negative bending moment okay. Now again this is this value is M BA is 50 so this is and this is and then we have value like this, similarly here and similar here, so you draw it, now this becomes the bending moment diagram. Now what is the bending moment for this? These are the bending moment this is the bending moment diagram and the values are this is 25, this is 25 and then this is 50, this is 50, this is also 50, this is also 50 and this becomes and this total becomes 93.75 selection moment at this point will be 9.375 sagging and then – 50 which is hogging so the moment at the main span will be only this part and this part will be 43.75.

So this is the bending moment diagram, now again another way you can draw the bending moment diagram is as per you first draw it when you draw the bending moment diagram another way is you always draw the bending moment in the compression side okay depending on whether it is hogging moment or sagging moment.

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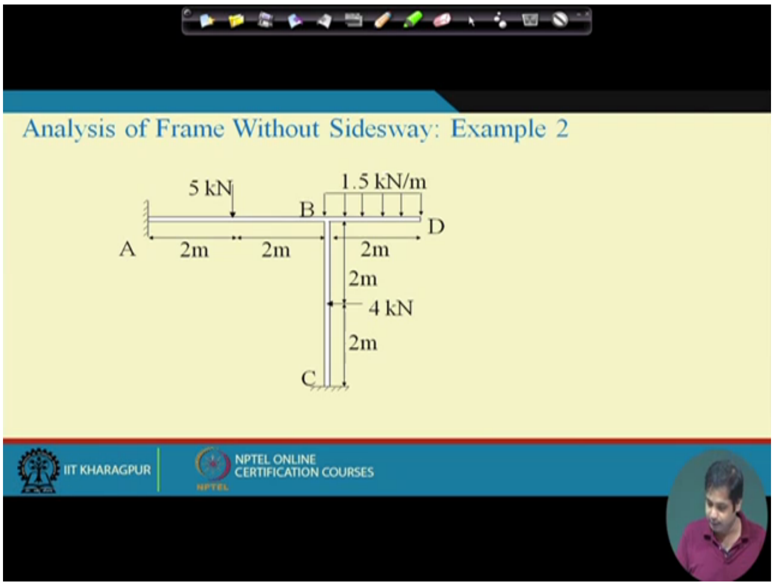

Now okay, so this is the first example for frame which does not have any sway, so let see very quickly the second example. The second example is this, which is again so quickly let us see what is the example?

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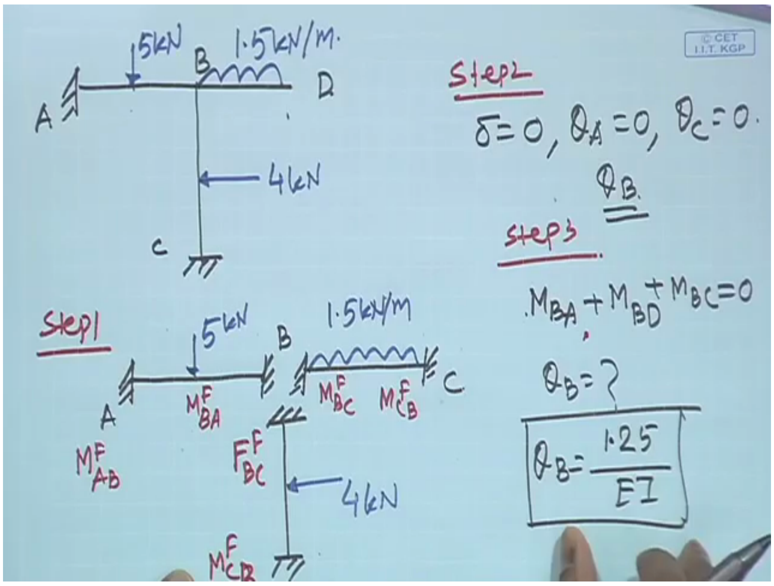

Example is this, it has 3 members, this is so this is A then B then C, D and the loads are this is subjected to 1.5 kilo newton per meter there is a concentrated load 5 kilo newton and then there is another load here which is 4 kilo newton, dimensions are given okay. Now the first step as as we know the first step is to step 1 is to draw the is to find out the fixed end moments, now what is the fixed end of AB so the primary structure becomes like this, so AB becomes this a then BD becomes this and then CD becomes and what are the loads on it?

This is 5 kilo newton, this is uniformly distributed load 1.5 kilo newton per meter and then this is 4 kilo newton.

Now you see so this is the primary structure subjected to external load, this is again a fixed beam subjected to a concentrated load at the main span, fixed beam subjected to uniformly distributed load and again this is the fixed beam subjected to concentrated load at the main span, so we can find out all this, this is A, B, D and then C. So we can find out what is M AB F and then M BA F, is the fixed end moment for AB then M BC F and M CB F this is the fixed end moment for BC and then similarly M CB F which is fixed end moment at C and then similarly M BC at F okay, so we know how to determine all these fixed in moments okay. Once we have these fixed end moments, what information then step 2, step 2 was what?

Step 2 was writing the writing the slope depression equation, now what information we have you see here, now fixed slope at Delta equals to 0 anyway there is no sway in this, another information we have that Theta A equals to 0 and then Theta C equals to 0 okay, so only unknown we have is theta B okay. Now you can say that the Displacement Delta is also unknown, in order to find out the member force say, without actually completing the Displacement at point D just by knowing the rotation at B you can compute the member forces okay.

Now that is a very important thing because when you take any structure, any structure can have any (())(26:31) now you need to reduce your problem, you need to reduce the computation right. You need to identify many kinematic indeterminate many degrees of freedom if you want to identify what are the degrees of freedom if you know then the internal forces can we determine okay. Now you take only those you compute only those those rotations and displacements, this makes your computation slightly or reduce your computations. So this is theta B now if we apply all these equilibrium conditions for slope deflection for AB, BD and BC and finally so this is step 2 and then step 3 was then step 3 is applying the equilibrium condition.

Now what equilibrium condition we have here is, equilibrium condition at B is you have only one unknown theta B so you need 1 equilibrium condition here and that equilibrium condition at theta B is M BA means at moment at B in segment BA + M BD moment at B in segment BD + M BC that moment of moment that B in segment BC that should be equal to 0. Now there is no external moment that will be an external moment at B, this should not have been 0 we we might have that values you need to write here. So in this case there is no externally applied moment at B so total internal forces they should balance each other. Now we know already expression for M BA, we know already exhibition for M BD and we know the expression for BC substitute that, get an equation and compute what is theta B okay.

Now once you calculate theta B and then I will tell you her final values theta B in this case will be 1.25 by EI okay. Now once you know theta B substitute theta B in all these slope deflection equations to get the corresponding moments at different joint okay. Now final results I leave it to you, once you know but this is the value of theta B, please check whether you are getting these values or not.

Now we will stop here today so now you see what we have done here is we just apply the slope detection concept into in frame, again we took relatively easier frame just to demonstrate the concept, now next class which will be the last class of this week we will take one more example of frame problem but in that case the frame will undergo some sway and we see how that sway to be considered in your analysis okay, thank you see you in the next class