Structural Analysis
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Module 10
Lecture No 50
Analysis of Indeterminate Structures by Displacement Methods (Contd)

Hello everyone welcome, what we discussed in the last class is we rewrite the slope deflection equation and then also discussed how that slope deflection equation for we need to derive we need to we need to obtain that equation for each segment separately and then combine them to get the final system of equation and solve that system of equation to get the unknown Displacement. Once we know the unknown Displacement, again we can apply them to slope deflection equation to get the internal forces. What we will be doing now today and then next 2 classes is we will just demonstrate the concept with several examples. The example that we consider today is the example of the continuous beam so this is the example that we will be considering today.

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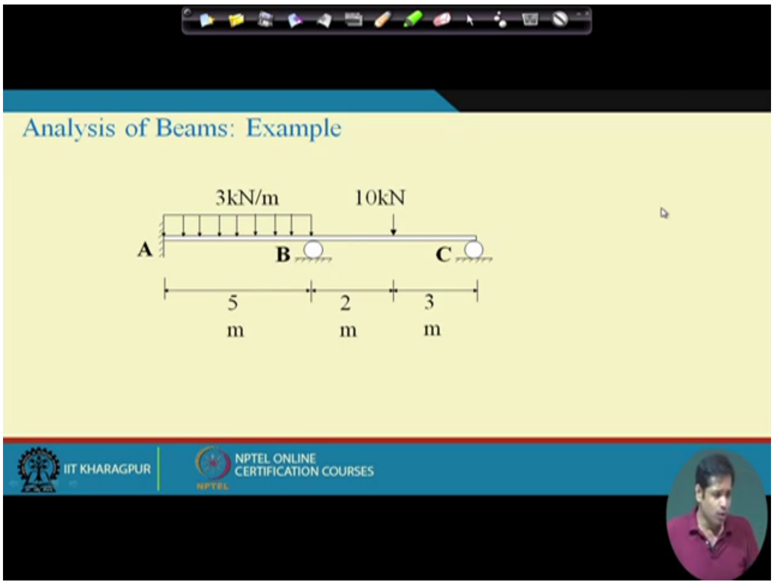

Okay so it is again the idea is to demonstrate the concept is the numerical demonstration of the concept, we could have taken more complicated structure with the number of spans and more, but the essence of the concept will remain same irrespective of the problem right. So the idea is as a before doing any example as I always say the idea is not to particularly solve an example in this in this class, the idea is through that example we demonstrate the application of the concept okay great. So we need to find out what is the what is the internal forces in this structure is an statically indeterminate structure so now let us draw the structure.

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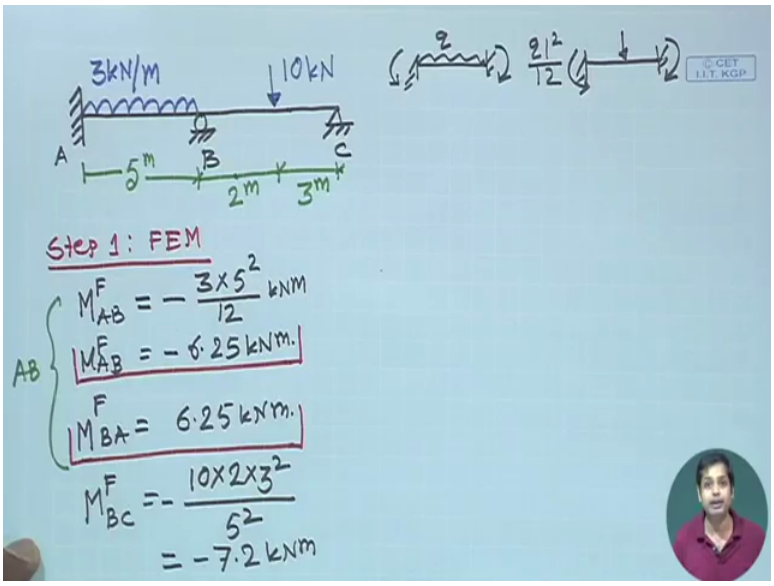

The problem is like this so this is the so this is a roller support and this is hinge support, this is its support, this is A, this is B, this is C and it is subjected to an internal uniformly distributed load which is 3 kilo Newton per meter then at the main span is the concentrated node which is 10 kilo Newton okay. This length is this length is 5 meter; this is 2 meters and 3 meter okay this is the problem great. Now the step 1, first that is again we take the member we consider the member A-B and B-C separately and then and apply this slope deflection equation for member A-B and member B-C separately and then combine them through equilibrium equations okay.

Now so step one, so step one is that one is fixed end moment end moment, we need to determine the fix end moment for all the segments. So first consider for member A-B, member A-B is M AB fixed end moment at A in segment A-B is equal to, you see this is this becomes fixed this entire structure is subjected to uniformly distributed load and recall for this kind of for a fixed beam subjected to uniformly distributed load, we obtain this fix end moment becomes, if it is q, q l square by 12 okay and this the sign of this is q l square by 12 okay this is and this is this okay, this is how this moment this is the (())(4:27) moment okay.

Now in this direction it is anticlockwise and here it is clockwise and similarly that we have done and if you take any book so this is already given. So this is this fixed end become at A it is it is anticlockwise so this is – q l q l square by 12, q = 3 into l square 5 square 12, this kilo Newton meter which is equal to – 6.25 kilo Newton meter okay. Now similarly we can calculate M f BA, this will also be W l square by 12 but the positive sign because it is clockwise so this becomes 6.25 kilo Newton meter okay so this is M AB F. So we have this is the fixed end moment and this is the fixed end moment right. Now we need to find out the fixed end moment for this is for span A-B so similarly we can do it for span B-A, so span B-A will be span B-C will be fixed end moment at BC in BC

Fixed end moment that is fixed end moment at B due to externally applied load externally applied load at any arbitrary point, this p into b square divided by l square that you can find out from any book so this becomes 10 into 2 into 3 square divided by l is 5 square, l is 5 square, again this is negative. So if this is the structure and this is arbitrary load, we have some moment in this direction and moment in this direction okay. And so it is at this point it is anticlockwise, at this point it is clockwise so that is way it is negative and this becomes – 7.2 kilo Newton meter okay.

And similarly we have M CB F = 7.2 kilo Newton meters, this will be not 7. 2, if you apply the same formula this will get 4.8 kilo Newton meter okay. Now I live it to you to check the formula, you can you can get this so this is for so this is for span BC okay so we have done we have the expression for span A-B and span B-A, this is the fixed end moment, this was our first step okay now this was the first step okay. Now step 2, step 2 is step 2, step 2 is you need to apply slope deflection equation okay, so what is step 2?

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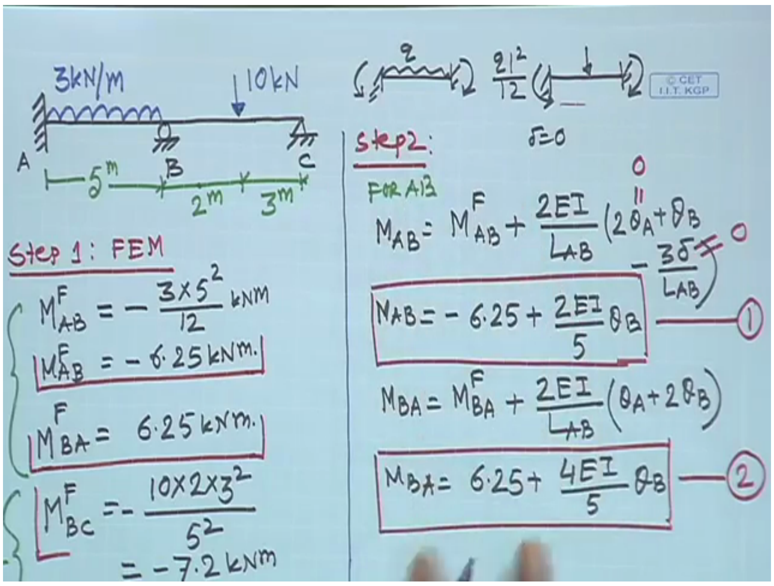

Now first we apply slope deflection equation for member AB and then slope detection equation for member BC. Now let us first for AB for AB, now what is if you recall the slope deflection equation final slope deflection equation was M AB = M AB F that is fixed end moment + the stiffness into corresponding rotation, this becomes 2 E I by L AB into 2 Theta

A + Theta B – 3 Delta by L AB okay, this is the slope deflection equation okay. If you the equation that you rewrite in the previous class, if you simplify this give you this. Now for this problem Delta = 0 there is no settlement so this will go okay.

L AB is the length of AB length of AB and E I fractal rigidity of AB, in this example it is as you the fractal rigidity is the constant for both these span and he M AB F is the fixed end moment at A. So if we if we apply that what we get, we get M AB is equal to fixed end moment is – 6.25 + 2 EI by 5 into Theta B. Now another important thing is Theta A is also 0 here because it is fixed end so fixed end your rotation at A = 0 so straightaway we can write Theta A = 0 and Delta = 0, so only we are left with left with left with Theta okay, so this is equation 1 so this is equation 1 okay. See this is the moment slope deflection equation for member AB. Now similarly we can have M BA, M BA will be again same thing fixed end moment at B + okay let us write it, so M BA F, fixed end moment at B F + 2 EI by L AB into Theta A + 2 Theta B, Delta I am not writing here because Delta is 0 okay.

Now M BA is equal to M BA F is 6.25 + again Theta A = theta A = 0 and it is fixed end, so this becomes 4 EI by 5 this is Theta B so this is equation number this is equation number 2 okay. So this is the slope deflection equation for member AB and this is at A and this is at B okay. Now similar exercise we have to do it for other members and means members BC, let us let us do that now for so for BC and what is the expression?

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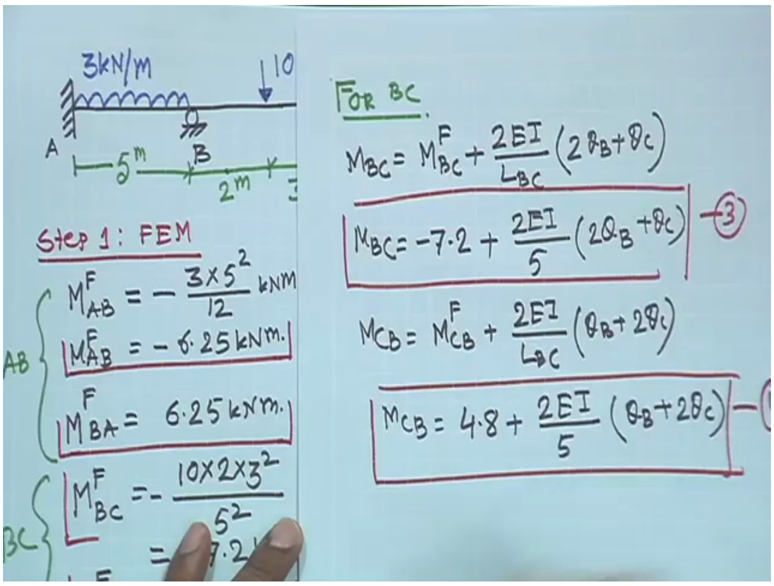

We have M BC, M BC will be then fixed end moment + 2 EI by L BC into 2 Theta B + Theta C you can write the general expression Delta, but Delta is 0 so not written explicitly. So now

M BC is equal to we have already obtained this is – 7.2 so M BC becomes – 7.2 + 2 EI by 5 2 Theta B + Theta C, so this become equation number 3 okay this become equation number 3 okay. Now let us do it for M CB, MCB is M CB F + 2 EI by L BC into Theta B + 2 Theta c, you can do it in matrix form as well, this becomes M CB is equal to this is 4.8 + 2 EI then this is 5 then theta B + 2 theta C okay, so this is equation 4 okay.

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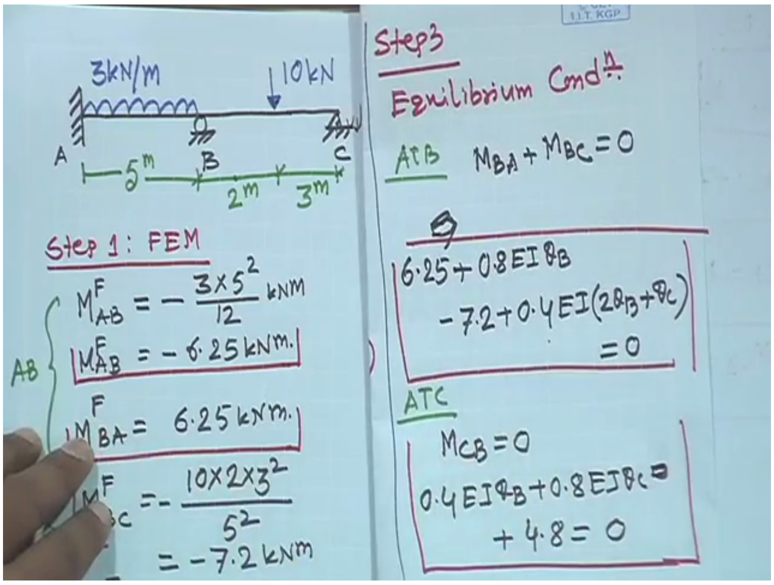

So we have octane what is M AB, M BA and M BC and M CB, this is what is the slope so we have applied slope differential equation for all segments. Now next part is ex part is application of equilibrium equation. What we do is next part, now step 3, step 3 is equilibrium condition, no equilibrium condition means we need to apply equilibrium at B and similarly we need to equilibrium at C, what is equilibrium at B? Equilibrium at B will be at B is M BA + M BC = 0 okay. Now we have already obtained the expression for M BA and M BC and if you substitute that here we will get M BA was 6.25 + 0.8 EI Theta B and M BC was – 7.2 + 0.4 EI 2 Theta B + Theta C that is equal to 0 okay, so this is the equilibrium condition at B.

Now similarly we can have equilibrium condition at C and what is the equilibrium condition at C? Equilibrium condition at C will be M CB = 0 because it is only moment you have is M CB, then M CB will be 0 because it is a simply supported entry okay. Now M CB = 0 we have obtained the expression for M CB and if we substitute that here then it becomes 0.4 EI Theta B + 0.8 EI theta C that is equal to + 4.8 = 0, So this is one equation and this is another equation. Now this how many unknowns we have here? We have 2 unknowns, rotation at B and rotation at C, so we have two equations, this equation and this equation okay.

Now this equation is applying equilibrium at joint B and this equation is applying equilibrium at joint C. So we have 2 equations and 2 unknowns theta B and theta C, if we solve them then we get the expression for Theta B and C as this.

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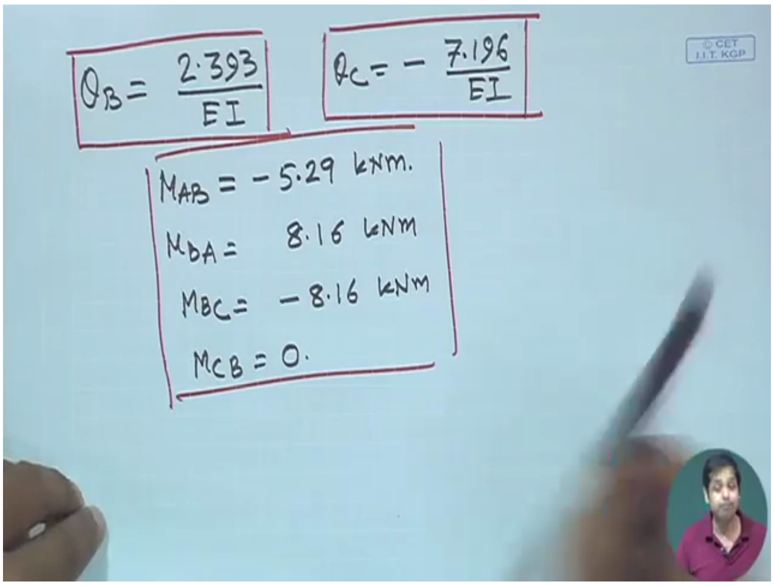

We get Theta B = 2.393 by EI and Theta = – 7.196 by EI, this is the Theta B and theta C okay. Now once we have Theta B and Theta C what we need to do next is we need to apply we need to just substitute these values of Theta B and Theta C in this, see this is the slope deflection equation for AB now in this expression if we substitute the value of Theta B, we will get what is the value of MB. And in this expression if we substitute the value of Theta B we will get expression for M BA and similarly this is the slope deflection equation for BC and if we substitute Theta B and Theta C here we get BC and if we substitute Theta B and Theta C here we get CB you should get CB = 0 because the moment M CB = 0 because moment at CB = 0 here okay.

So now if you do that then the final result that you have is M AB = – 5.29 kilo Newton meter and then M BA = 8.16 kilo newton meter and M BC is – 8.16 kilo newton meter and M CB = 0 so that is the final interval forces you have okay. Now if you have to draw the bending moment diagram, how we can draw the bending moment diagram? The actual structure was something like this, this end was fixed, this is roller and this is hinge support, here you have uniformly distributed load and a concentrated load like this okay. Now let us talk the bending moment diagram, bending moment diagram will be first is you draw the bending moment diagram for these extra loads individually for all segments and then the bending moment for this okay.

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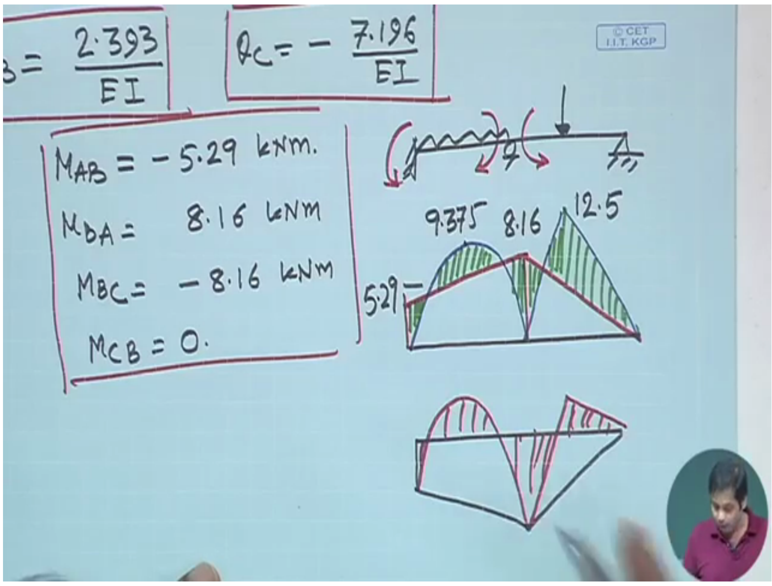

Now if you do that than your external load bending moment diagram would be something like this and then something like this, this is the bending moment for external load and then finally you have now one thing please make sure these values that what you get, the sign that you get that is that sign gives you whether the moment is clockwise or anticlockwise right, this sign will not tell you whether the moment is sagging moment and hogging moment but when we draw the bending moment diagram, we use the condition sagging moment and hogging moment. Let us see whether these moments are how these moments look like, see M AB is – 5.29, – 5.29 means it is or anticlockwise, anticlockwise means this moment is in this direction okay this is M AB.

And then M BA is 8.16 it is positive, it is positive means it is clockwise in this direction and M BC is – 8.16 which is again negative, negative means anticlockwise direction and M CB is 0 here. Another important thing to be notice you see M BA + M BC = 0 here and that is very obvious because at this point that is no external loading so the internal forces they should balance each other cancel each other right. Now when you draw the bending moment diagram, we check whether the moment is hogging moment or sagging moment. Because of this external load, the moments will be sagging moment because this external load will cause sagging in this segment that is why it is drawn on the upper side okay.

Always at least this is the convention that you have been following in this course that we draw the bending moment in the compression side okay. Now this is positive bending moment okay, now next is on top of this these are all negative bending moments so this value is 5.29 and this value is 8.16 and then become this okay. So this is positive and this is negative so some part will cancel each other soldier bending moment will be only this part, only this part will be your bending moment. And let us write the values, so this is 5.29 and this is 8.16 and this value is 12.5 and this is value is 9.375.

Another way of drawing bending moment that we also discuss that you draw this bending moment like this, first you draw the negative bending moment which is like this and from the negative bending moment we can draw the positive one okay something like this. And then from that we can get something like this okay, so then in that case this becomes your bending moment, we will check this bending moment diagram do not take it for granted okay, so this becomes bending moment diagram. Now you look at this is your positive, all dispositive and all this positive and this becomes negative, this is also negative okay. So let us see so this part is negative, we have a negative part which is below the this is our beam negative part then this part this part and this entire this part is this part and then this part is this part okay.

So this can be your bending moment diagram or this can be your bending moment diagram, that advantage and disadvantage. The advantage here is the drawing has become easier, but the advantage here is you really do not have to say which one is negative which one is positive, just by looking at the bending moment diagram you can say that part is sagging and which part is hogging in other words, which part is under tension, which part is under compression okay. Okay, so this is the demonstration of slope deflection method for a beam which is relatively simpler, as I said the idea was not to analyse any particular structure, the idea has been to demonstrate the concept of slope deflection method. What we will do next is we will again demonstrate the concept through some examples okay. We stop here today see you in the next class thank you.