Structural Analysis I.
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Lecture-49.
Analysis of Statically Intermediate Structures by Displacement Methods (Continued).

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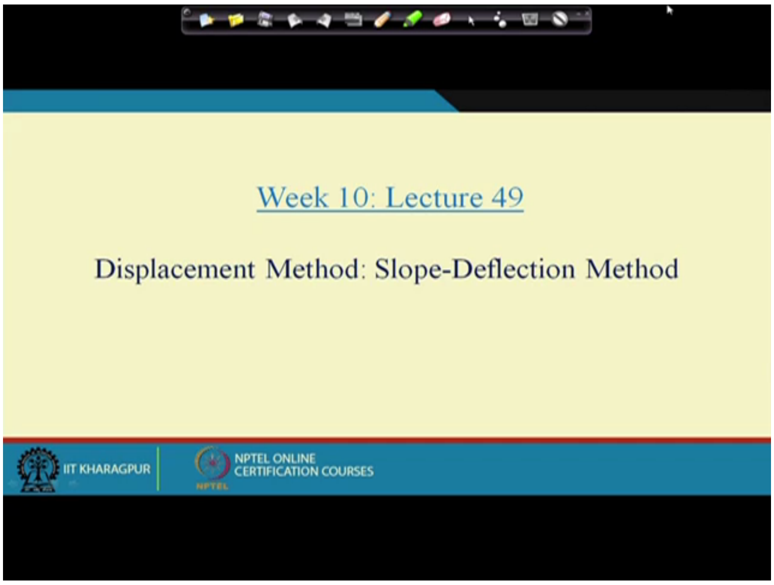

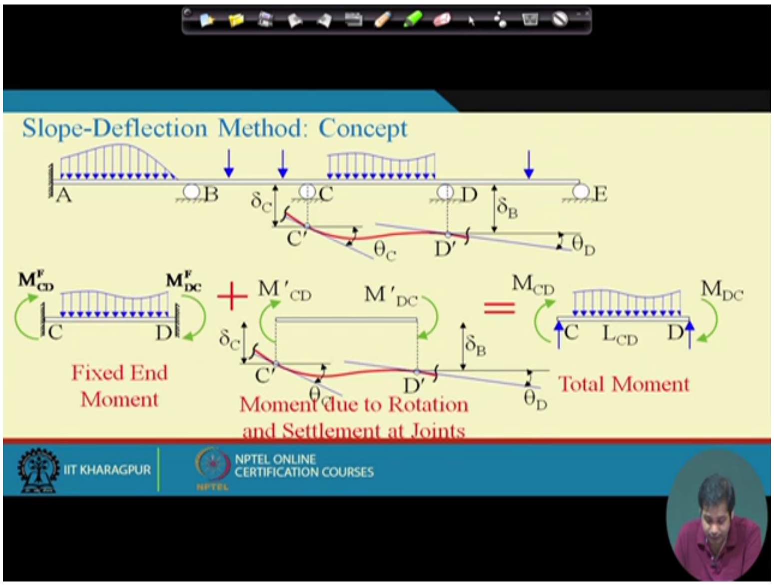

Hello everyone, welcome we have, what we have done so far in this week is we tried to understand the basic philosophy in displacement base methods. Now onwards, the similar philosophy, now different, we have different methods in a way how those that concept is being translated into methods in a different way. The 1st methods that we discussed is slope deflection method, what we will do today is we will see what is the method itself and then in the subsequent lectures we will see the application of this method in various indeterminate structures.

Okay, now let us start understand the method through the same example that I used to demonstrate the concept of displacement method as a whole. Now this is a continuous beam which is subjected to some external load, this is an indeterminate structure and degree of indeterminacy, static indeterminacy is 4, right. Now suppose consider only span CD, this bay may undergo a deformation in a different way depending on the, depending on the external load. But for some external load, suppose the segment CD, they undergo deformation like this, okay. So the point C goes to C dash and point D goes to D dash.

So we will demonstrate the concept only through segment CD and the final equation what we have, the same equation can be applied to all the segments separately and then combine them to get the final expression. Okay C dash and D dash is essentially the deflected shape of CD. Okay there are 3 things can happen, okay, suppose point C goes to C dash, if there is slope at C dash and this angle is Theta C. So the rotation at C is equal to Theta C and similarly if we draw the slope at D dash and the rotation at D is equal to Theta D. So Theta C and Theta D at the joint rotation C and D respectively.

In addition to joint rotation, we also consider the support settlement, the settlement of support C and settlement of support D. Suppose the settlement of support C is Delta C and settlement of support D is Delta D. So if we concentrate only on segment CD, 4 things can happen, one is the rotation at C and rotation at D and then settlement of C and settlement of D. So we have, now similarly if we take the segment BC or segment DE or any other segments, similar 4 things can happen, the rotation that the joint and the settlement of the joints, okay.

Now if you remember the concept, if you remember the last but when we demonstrated, when we discussed the concept behind the displacement based methods, what we did is, since the displacements are unknown here, so in the primary structure we need to we need to we need to remove those degrees of freedom, right, we need to make a structure, we need to kinematically admissible, kinematically determinate, okay, each segment of the structure. Now when you do that, the 1st step we get the primary structure which is a fixed, which is which is all the joints which are roller supports here, those joints become fixed joints.

Then we separated, then we took each segment separately, okay. Now similarly to get the primary structure we constrained, we constrained the degrees of freedom at C and degrees of freedom at D, so the primary structure becomes this, is not it which is a fixed been subjected to externally applied load. So this beam is CD. Now because of this loading the moment generated at fixed end is this. Now please look at the sign I am using for this moment the sign what we have been using, we will still continue with that when we draw the bending moment I shear force the exam, we see whether the moment is sagging moment or hogging moment.

But when we do the algebraic calculations, clockwise moment is positive, okay. Now that is why I took here the moments are shown in the clockwise direction because at the end when we do the algebraic calculations we do not need to see whether the moment is sagging moment and hogging moment, Straightaway we can add this moment, okay. Now so this is the fixed end moment generated at C and D, so MF, MF CD is the fixed end moment at C in span CD and MF DC is the fixed end moment at D in span DC or CD. Okay. So this is fixed end moment, okay.

Now so this fixed end moment is calculated due to the external load on the primary structure. Now this segment CD now undergoes rotation and join settlement, joint displacement because of this rotation and the joint and settlement at the joint, it will induce some moment in the structure. And suppose these moments are M dash CD and M dash DC. So M dash CD is equal to the and due to Theta C, Theta D, Delta C and Delta B and similarly M dash DC is equal to moment at D in segment CD due to the joint rotation at C and D and support settlement at C and D, okay.

Now this is the moment to to rotation and settlement, rotation and settlement of joints, okay. Now then what we, then what we discussed, now this is the primary, now this is a primary, these 2 are primary structure, this is a primary structure subjected to the external loading and these are the corresponding moments at the ends and this is again the primary structure and the corresponding constants are removed. Okay. Sorry corresponding degrees of freedom so removed and then this structure undergoes some rotation and support settlement, these are the corresponding associated moment.

Now total moment at C and D will be this + this, right. So if we draw the free body diagram of member CD, forget about this thing, just the free body diagram of member CD, there will be shear force and the moment, okay. Now again recall when we when we actually, in previous lecture when we used free body diagram, always we showed the moment in the direction where it is sagging or hogging, sagging or hogging, we will still continue the same conversation while drawing the bending moment diagram.

Here the bending moments are shown, binding moments are shown in a clockwise direction because next, in the next, all the procedure to follow, we will do many algebraic sum algebraic calculations and it will help us to be to be consistent, okay. Now superposition said that, linear superposition said that if this is the total moment at C and D, then this should be equal to this + this. So primary structure subjected to the response of the primary structure subjected to the external load + the response of the primary structure with corresponding degrees of freedom constraint is equal to the total response of segment CD.

And this is true not only for CD, this is true for all the segments, right. Now so we have 2 parts, one is the fixed end moment and another is the moment due to joint rotation and settlement. Okay great. Now if we can find out these 2 values, the fixed end moment and these values, then we can find out what is the corresponding moments in the actual structure. The rest of the things what we see is the basic method is how to find out these values, okay, great.

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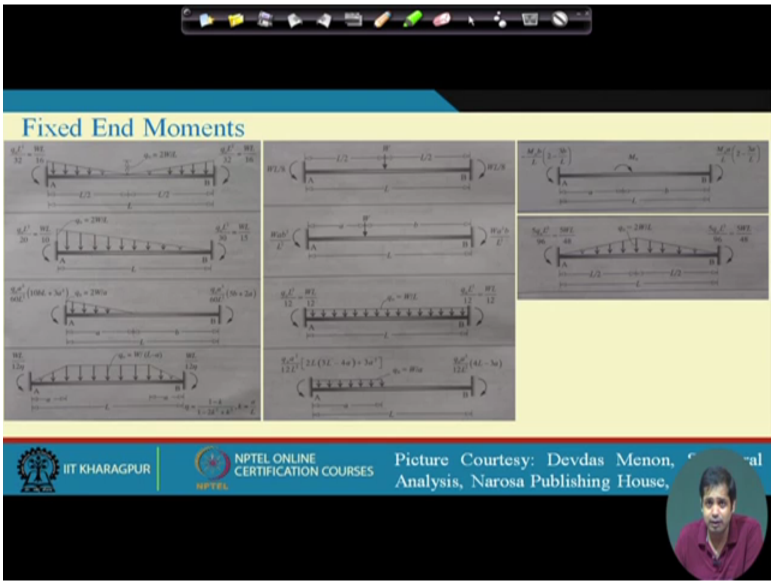

Let us see the fixed end moment, if you recall in the last class we discuss how to find out fixed end moment, fixed end momentum essentially is nothing but it is a fixed beam which are subjected to some loading and what would be the corresponding bending moment at the fixed and, that is the fixed end moment. Now we demonstrate, so this is, this is essentially few solve if you solve statically, this is statically indeterminate structure of degree 3, if you are you can use any force based method to solve statically indeterminate structure to get the fixed end moments, okay. If you take any analysis book, these are the, for some typical the fixed end moments are tabulated.

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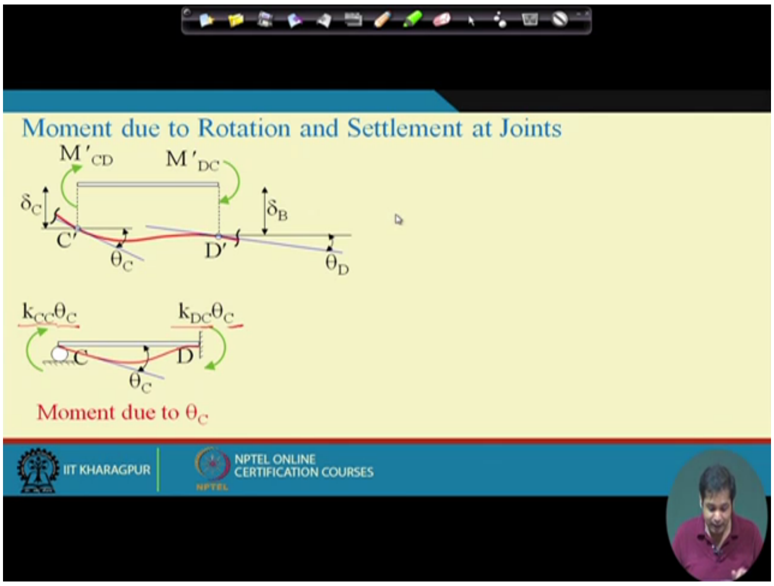

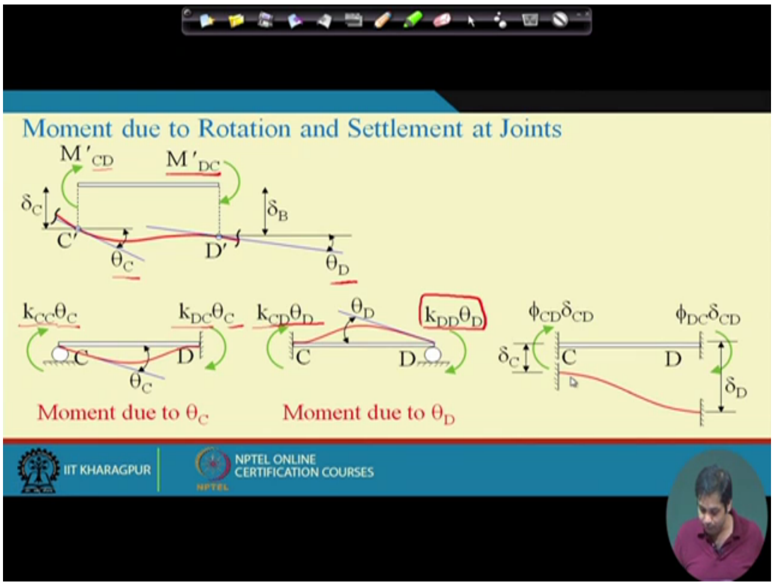

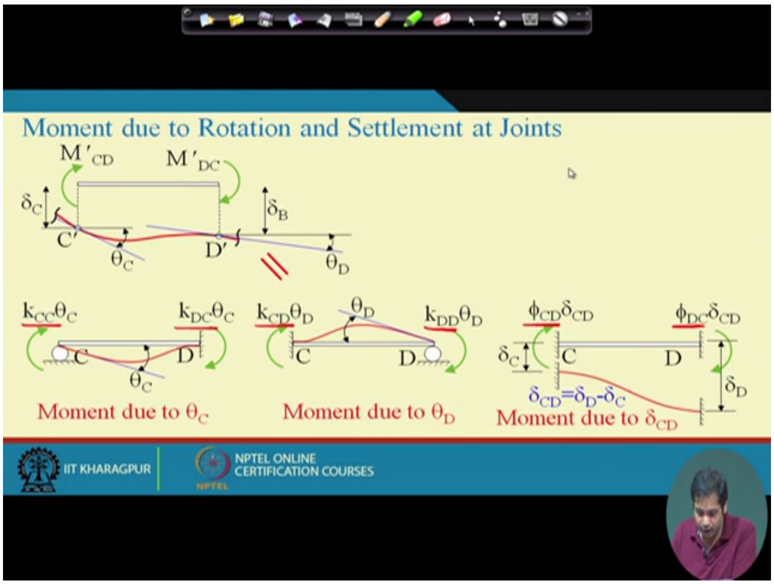

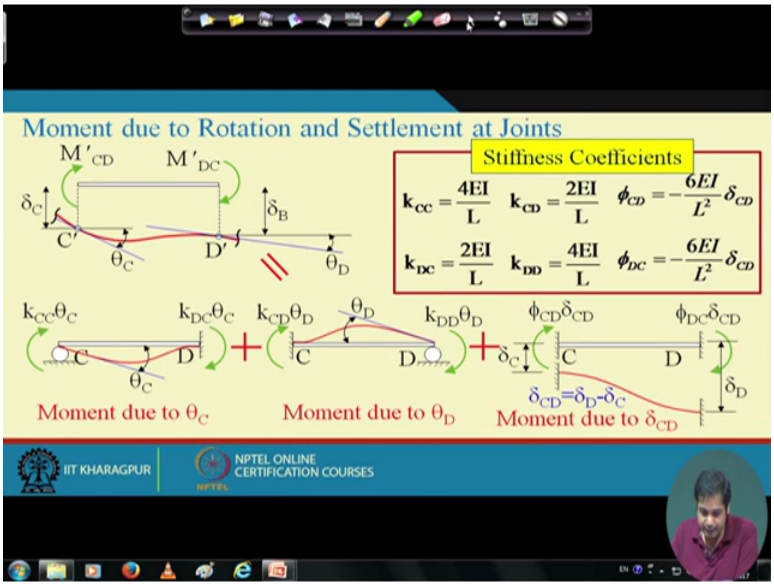

So you can directly use those values given in the table but before you do that, at least once in the entire course you find out these values, okay. Great. So fixed end moments, these fixed end moments are known for a given loading case. Now let us see how to determine this moment due to rotation and settlement. Okay. So next we discuss how to determine moment into rotation and settlement. Now this is the moment due to rotation and this is the moment, this is the this is the moment due to rotation and settlement. Now here we have contribution from Theta C, then we have contribution from Delta D and then we have contribution from the support settlement Delta C and Delta B. Okay.

What we do next is, we will, now again since it is the linear superposition is, the method of superposition is applicable here, what we can do is, this moment, since it has contribution from all these Theta C, Theta D and Delta C and delta D, we can say that this is equal to moment due to Theta C keeping all the values 0 and then moment due to Theta D keeping all other corresponding rotation and displacement 0. And keeping delta C and delta D nonzero and for delta C and delta D, and keeping Theta C and Theta D nonzero. Okay.

So in M dash CD and M dash DC, we have contribution mainly from 3 parts, one is Delta Theta C, Theta D and the relative support settlement which is delta D - Delta C. Okay. Now let us see 1st take the contribution for, contribution from Theta C. Now to take the contribution from Theta C, what we have to do is, we take the same beam, suppose same beam, which is, now this is the deflected shape which is which has, deflected shape is open by giving a rotation at C with equal to C. Now and keeping all these, all these other degrees of, other degrees of freedom or other rotation and translation 0. Okay.

So if you remember again in the last class we discussed, if you take a propped cantilever, this is essentially a propped cantilever beam, right, subjected to some moment at the, subjected to some rotation at C and because of this rotation, the moment generated at the free end and this rotation is caused by some moment, okay. Now if you recall in the last class we also discussed how to determine the stiffness coefficients and for and for some typical idealisation we see, we computed the stiffness coefficients, okay. Now suppose the stiffness coefficient in this case is K CC and K DC.

And what is K CC, K CC is the moment at C due to unit rotation at C. And then K DC is equal to moment at D due to unit rotation at C. Okay then if you multiply that stiffness coefficient with corresponding rotation Theta C, we get the moment, okay. So so that the moment at C will be the stiffness coefficient K CC multiplied by Theta C and similarly moment D will be K DC and Theta C. We will see what is the value of stiffness coefficient shortly. Now similarly this is, this is a moment generate, so K CC at C, this is the contribution of Theta C towards this and this is the contribution of Theta C towards this.

Now next we take contribution from Theta D, so idea is, we take the same beam and give a rotation other rotations and settlement as 0. So again do the same thing, so this is CD and the moments are shown in clockwise fashion and because of this clockwise moment, this beam will deflect like this, the rotation at D will take place like this. So again so what would be the moment, moment at C do to the rotation at D, this will be K CD into Theta D. K CD is the stiffness coefficient which is essentially the unit moment generated at, the moment generated at C due to unit rotation at T.

And similarly the moment corresponding moment at D will be KDD into Theta D and K DD is the stiffness coefficient and what is this, this is, the moment generated at T due to the unit rotation at D. Now this stiffness coefficient when multiplied by the corresponding, corresponding displacement, we get the corresponding, the associated force or moment, okay. So again this is, this is the contribution to this because of due to Theta D. And similarly this is the contribution to this due to Theta D. Now we have obtained the contribution of Theta C and Theta D, next we need to see what would be the contribution of the support settlement, okay.

Now suppose suppose this beam, now when we when we take the contribution for support settlement , then all other degrees of freedom, Theta C and Theta D have to be 0, okay. So we allow only the segment CD to , we allow the deformation such that only the only the settlement of the joint takes place, there is no rotation of the joint is allowed. Okay. Now so, this is the settlement delta C at joint C and corresponding and the settlement at Delta T is equal to at D. Okay, now suppose corresponding moment generated at C is Phi CD and delta CD.

Delta CD is, delta CD is Delta D - C. So it is essentially the relative, relative settlement, okay. Now what is Phi CD, Phi CD is the moment generated at C due to relative settlement, due to unit relative settlement or settlement of D with respect C. And similarly Phi DC is equal to the moment generated at D due to the due to the unit relative settlement at T. Okay, please note the delta C is computed, delta C is defined like this, okay.

Because if you define delta CD as delta C - Delta D then the corresponding stiffness, sign of the stiffness coefficient damn will also change. So whatever stiffness coefficient term that I am going to show you, that is consistent with this, this diagram. So if you are using different sign convention and if you are defining the delta CD in a different way, please make sure that you take the take the associated coefficient accordingly. Okay so these are all stiffness coefficients, right. These are all stiffness coefficients, these are all stiffness coefficients. Now we will see what are the values of these stiffness coefficients.

Now if you recall, the last class, last class this is moment due to CD, last class, so now before we give you, before we show you what are the expressions for stiffness coefficient, then what we will, we know is this is equal to this + this + this + this. So total, the moment generated due to the entire, considering all the deformation is equal to the moment generated due to Theta C + moment generated due to Theta D and moment generated due to support settlement, okay. There is, we are not we are not discussing anything new here, we had been doing the same thing right from the beginning of this course, right.

Now then what is the expression for this the stiffness coefficient, the stiffness coefficient expression for this, we have already done it in the last class, right. These values we have already obtained in the last class. Please check please note that Phi CD is negative here, it is negative because this is how we defined delta CD. If we define delta CD as Delta C - Delta D, then this becomes positive, okay. Now so this depends on the, the way you the way you define delta CD, okay. Now these are the stiffness coefficients and we know how to how to get the stiffness coefficients, right. Great.

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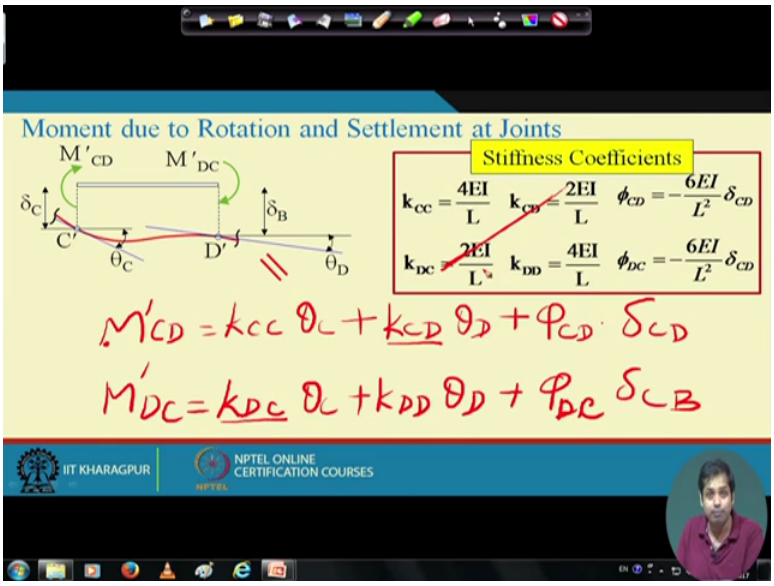

Now next is, once we know the stiffness coefficient, then what will be, what will be, then what will be M dash CD, M dash CD will be, say M dash, M dash CD will be what, this is the moment at C. Moment at C will be the moment at C due to Theta C, moment at C due to Theta D and moment at C due to due to delta CD. And what would be the moment at C due to Theta C, moment at C due to Theta C will be KCC into Theta C where KCC is the associated stiffness coefficient. Similarly +, what would be the moment at C due to Theta D, this is K CD into Theta D, okay.

And then again +, the moment you to, moment at C due to settlement support settlement, this will be Phi CD into Delta CD. So this will be the total moment at C. Similarly we can write M dash DC is equal to, then moment at C due to Theta C which becomes K DC into Theta C and then + moment due to D at because of Theta D, this becomes K DD into Theta D and then + Phi CD or Phi DC, Phi DC into delta CB, right, delta CB. So this is the contribution from this. And we also know that this K DC and K CD should be same. And which is evident from this figure, you see that K CD and K DC will be same, okay.

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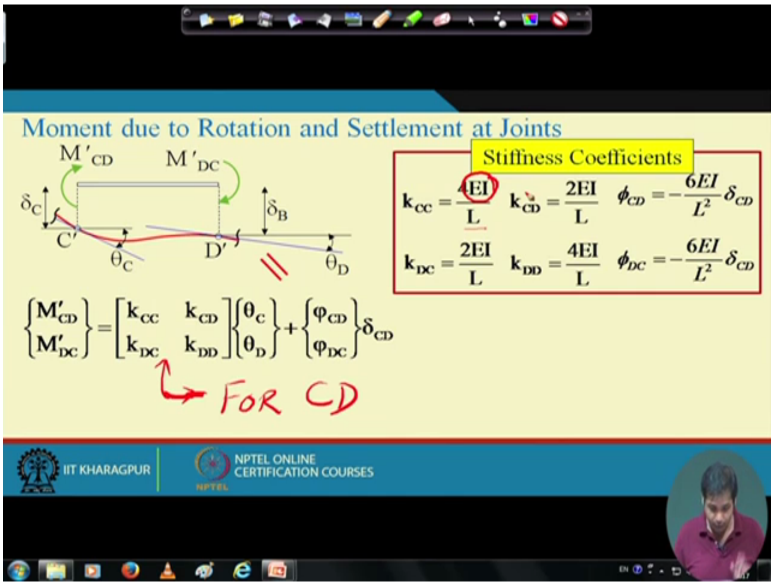

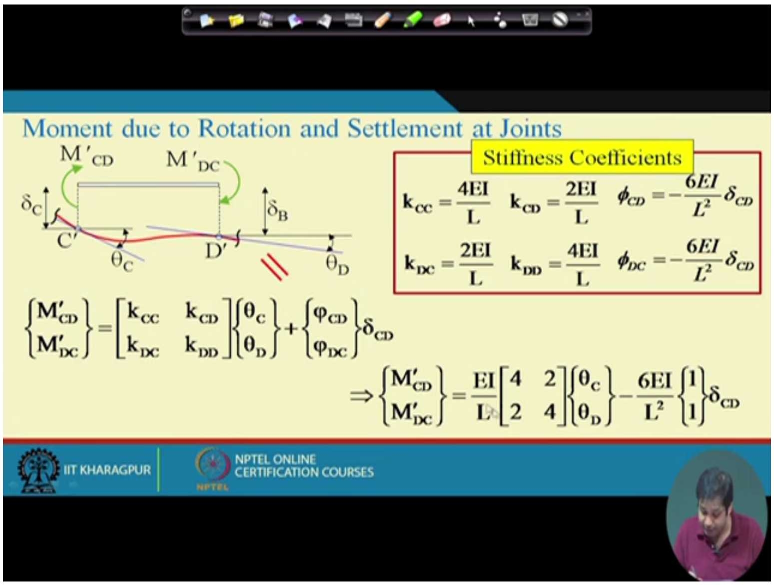

Now if we write that in a matrix form, than what wehave, we have a matrix like this, okay, the same expression but written in a matrix form. And so this is called, this is a stiffness matrix for segment CD. Now here one thing, please note this is only, this is for CD, this is, this is for, only for this is for CD. And similar stiffness matrix you can have for different members. Now when you are calculating the stiffness matrices for different members, the coefficients, for instance the coefficient is 4 EI by L, K CC is 4 EI by L. Implicitly it means that in this case EI of C and L of C, means length of segment CD and fractural rigidity of segment CD.

Now when you go to another member, say it is AB, then corresponding coefficient will be , in order to get the corresponding coefficient, we need fractural rigidity of that segment and length of that segment. Okay, it is not explicitly written here but implicitly it is assumed when we say L, this L means the length of member CD, when we say EI, it means the fractural rigidity of member CD and that may change for different segments, okay, great. Now next , so if I, if we substitute these values from this, then this becomes this expression.

This is a stiffness, this is a symmetric matrix, okay. Now so this is the expression for the bending moments, this M dash CD and DC for member for member for segment CD, okay.

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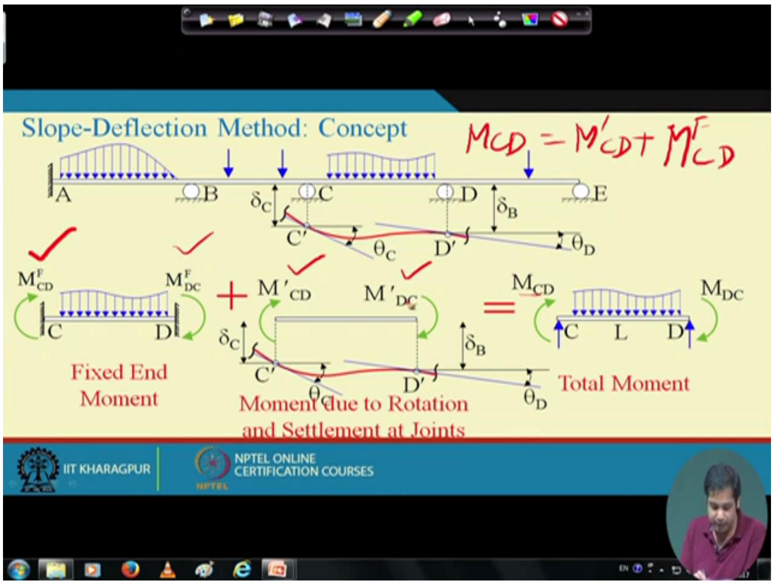

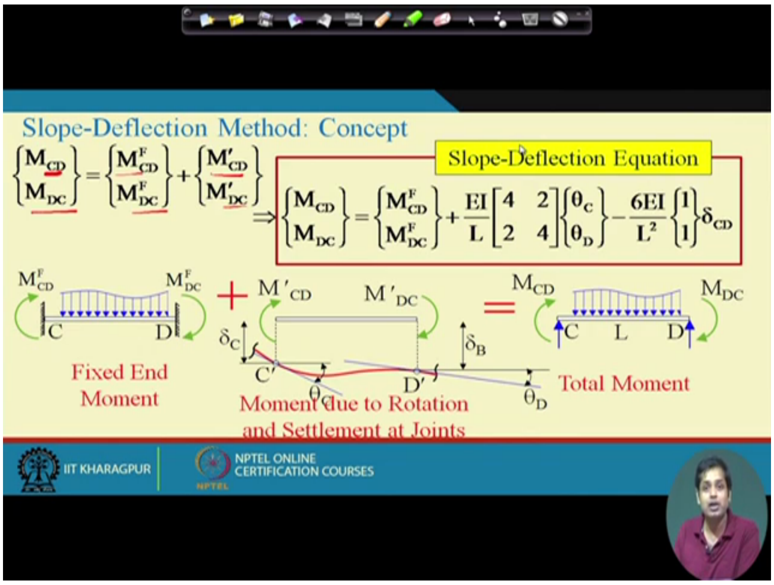

Now, so, now we have determined, we have seen how to determine the fixed end moments and just now we have seen how to determine this. So we have this with us and we have this with us. Then this + this is equal to this, though M CD will be, so M CD will be, M CD will be M dash CD + M fixed end moment CD. And similarly MDC will be this + this, right. So then what we have is, if we write that again in a matrix form, say M CD will be, M CD will be this + this and M DC will be this + this, okay.

Now let us express this, just now we have we have derived the expression for this and if we, if we write that and here we have M CD is equal to MCD, this is a fixed end moment + this is the stiffness term, stiffness for that segment, okay, into corresponding rotation - this is the contribution from the support settlement. If for a problem, if there is no support settlement, there is no, for frame problem we will see that the frame can sway or non-sway, if we find that there is no support settlement, then this part becomes 0 and only you are left with this.

But this is a very general expression where we, when we consider all the possible degrees of freedom, okay. Now this equation, this equation is called slope deflection equation. But to be to be specific, this equation is a slope deflection equation for member CD, okay. Now if we take another segment, similar slope deflection equation can be obtained. But in that case only difference will be, for instance it is for MB, so M, this becomes, so CD it becomes M, it becomes M AB, fixed end moment for MAB and fixed end moment for MBA, corresponding league, this becomes Theta C becomes Theta A, Theta B and then delta C becomes Delta CD becomes Delta AB. Okay.

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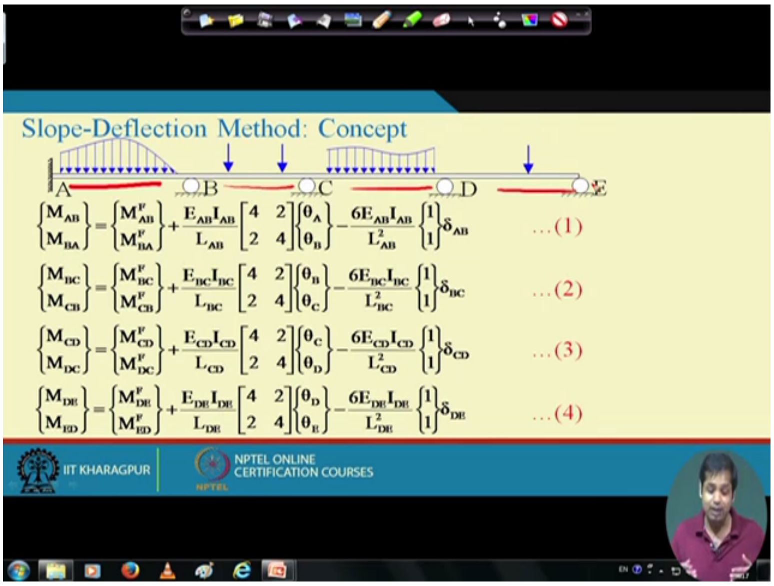

So similar expression, similar slope deflection equation can be derived, can be obtained for all the members, okay. Now, now once we have the equation, slope deflection equation, let us see how the slope deflection equation to be used. Okay, now as I said , so we can have, this is the slope deflection equation for member, for member AB and similarly we can have slope reflection equation for member BC, for member CD and member DE. For all the members they can have the slope deflection equation for this case. For a given problem, in this case we have 4 segments here but if your problems have, the problems can have any number of segments, so whatever may be the number of segments, you have to apply the slope deflection equation for those segments, okay.

Now once we have this, let, next is this leaves you, this gives you the, the slope deflection equation for each segment separately. This is for this segment, this is for the segment, this segment and this segment separately. Now but in actual circle, these segments are not separate it, the segments are connected together. When there connected together, they have to satisfy certain conditions, compatibility and equilibrium conditions. Compatibility conditions we already considered, okay but now the equilibrium needs to be satisfied.

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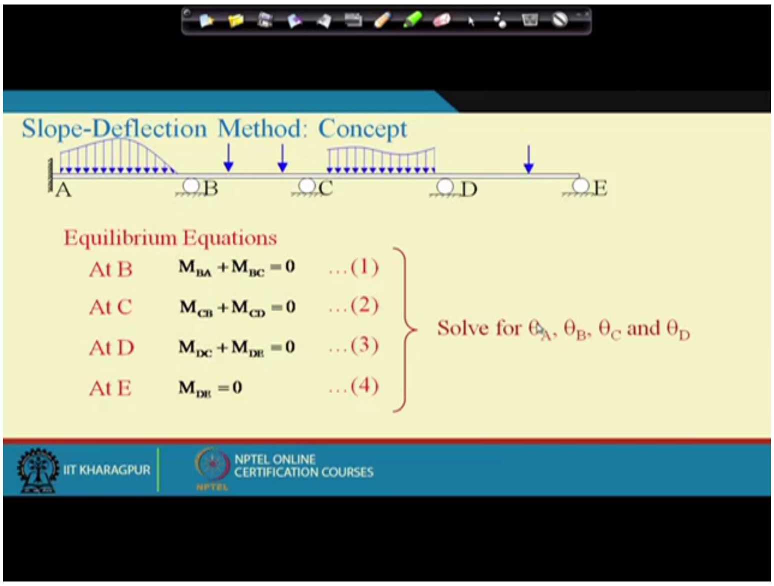

And what is equally blame condition, what is equally brain condition, equilibrium conditions says that, at point A, at point B say, it is M BA is the, MBA, the moment at B obtained from segment AB and moment at B obtained from segment BC, they should satisfy the equilibrium condition. Okay. And similarly for, similar at C, similar at D and similar at E. D is equal to 0 in this case because it is a roller support, so at this end T is equal to 0, that is equilibrium condition. Now suppose at BA, instead of, at B, if we have, if we have a moment like this, suppose there is no actual moment at joint B, suppose at joint B we have an external moment like this. Then what happens M BA + M BC, M BC will not be 0, this will be the corresponding, moment here.

The essence is, whatever internal forces, MBA, MBC, these are all internal forces, right and equilibrium conditions says that internal forces is equal external forces. Now since internal forces means here internal moments that should be summation of all the internal moment should be equal to the external moment. And external moment is 0 here, that is why the summation of all the internal moments are 0, this is equilibrium condition, right. Now then next, what we do is, so next now if we solve these 4 equations, we get Theta A, Theta B, Theta C and Theta D, okay.

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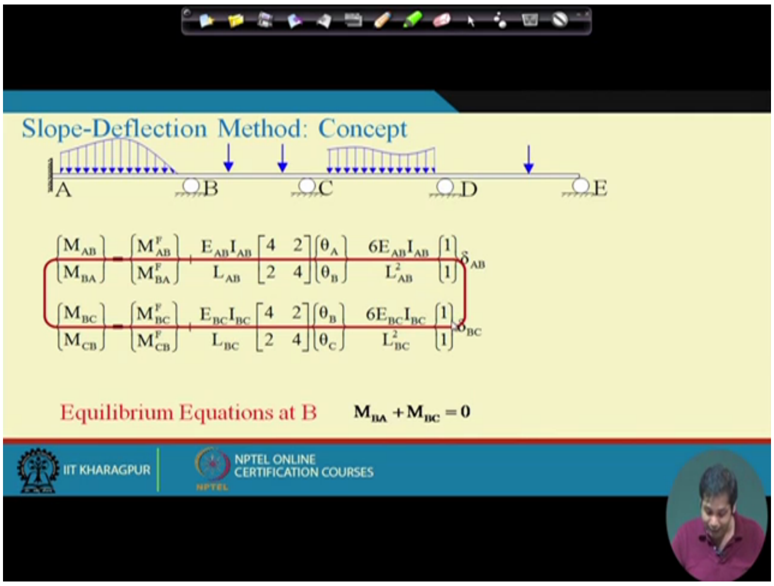

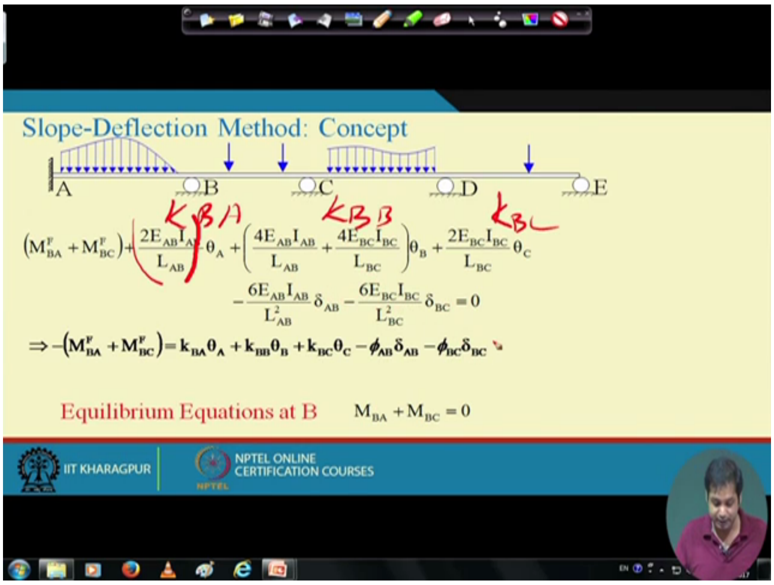

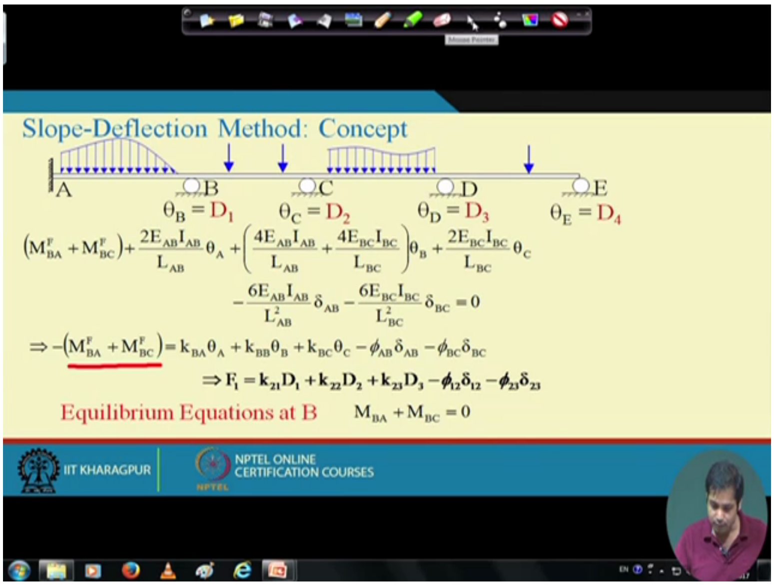

Now next is, suppose, so this is final approach, okay. So this is the slope deflection, this is the slope deflection method. Once we obtain these Theta A, Theta B, Theta C and Theta D, then we can apply, we can use these terms to get the internal forces in all these members. Now before you demonstrate that through some example, let us find out a relation between the expression that we showed in the previous, in the general discussion in displacement method and the slope deflection method. Consider member, consider these 2 members, member BA and member BC and member B is common and then satisfy the equilibrium condition at B.

What is the equilibrium condition B, equilibrium condition at B says that MBA + MBC is equal to 0. So MBA is this and MBC is this. So this + this should be equal to 0. Right. Now so this +, this + this should be equal to 0 for, if we if we add them, then we get an expression like this, okay. You can try this, so what you have to, it is determinant matrix form, so you have to write the MBA in algebraic form, MBA in algebraic form and then add them. If you add them, you will get an expression like this, okay.

Now the interesting thing is, now this is, this is say K BA, so K, and this is K BB this is K BA and this is K BC, this is K BC. If you do that, that this expression becomes K BA Theta A, K BB Theta B, K BC Theta C and this is Phi AB and this is Phi BC. Now if we define your degrees of freedom like this, that Theta B is equal to D1, Theta C D2 and so on, means point B is 1, 2, 3, 4, then this equation can be rewrite, this expression can be rewrite as this, okay.

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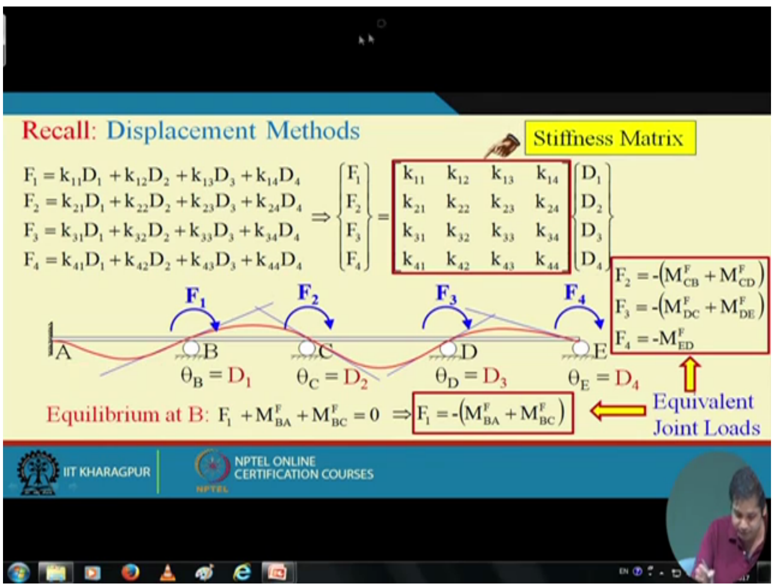

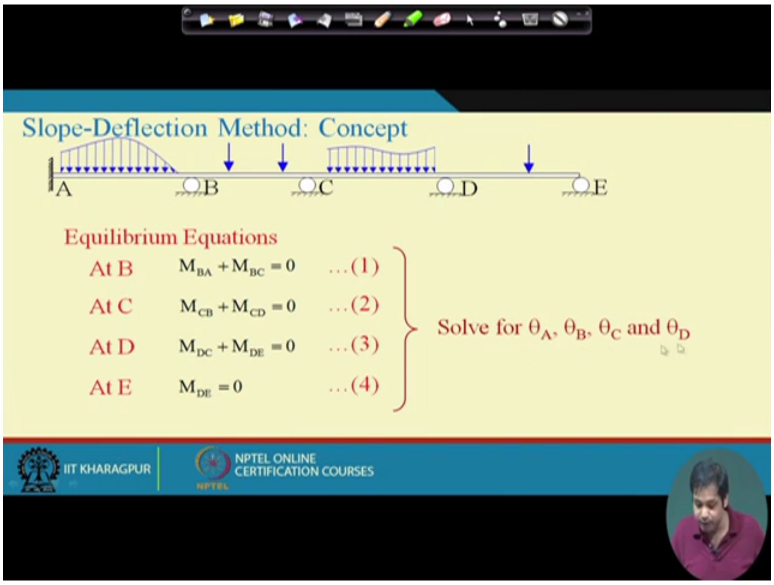

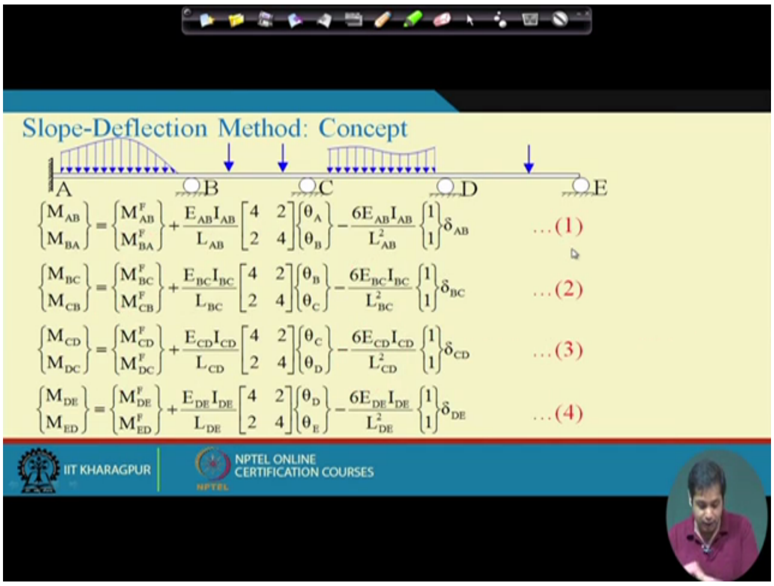

Okay, now what is this, what is MBA + MBC F, if you if you, if we go back, if we recall, this is the slide I showed you in the previous class, what is MBA, MBA + MBC, this is equivalent joint load F 1, right. So, this becomes equivalent joint load F1 and K BA becomes K 21 because the point the point B is now denoted as 2 and point A is one. So this is denoted as this and now you see this expression, this expression is very similar to this expression, the situation where K 14 is equal to 0 and there is no support settlement, okay.

So the expression just now, the expression, this expression already we discussed while discussing, while mentioning the general, while giving the general overview of the displacement method and this is the, this is a particular form in slope deflection, in a method which is termed as slope deflection method. Now this is, the purpose of this is to show the similarity between the final expression that we get at the discussion at the previous class. Okay, let us just summarise this.

So you get, use all these equilibrium equations to get the corresponding displacement, once we have the corresponding displacement, this corresponding displacement can be substituted in all these, all these segments and, the slope deflection equation for all the segments and get the member forces, okay, the MAB, MBA, that the member forces. So this is slope deflection method, probably when we demonstrate this through one example, the idea will be clear, that we will be doing in the next class, okay. See you in the next class, thank you.