Structural Analysis I.
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Lecture-48.
Analysis of Statically Intermediate Structures by Displacement Methods (Continued).

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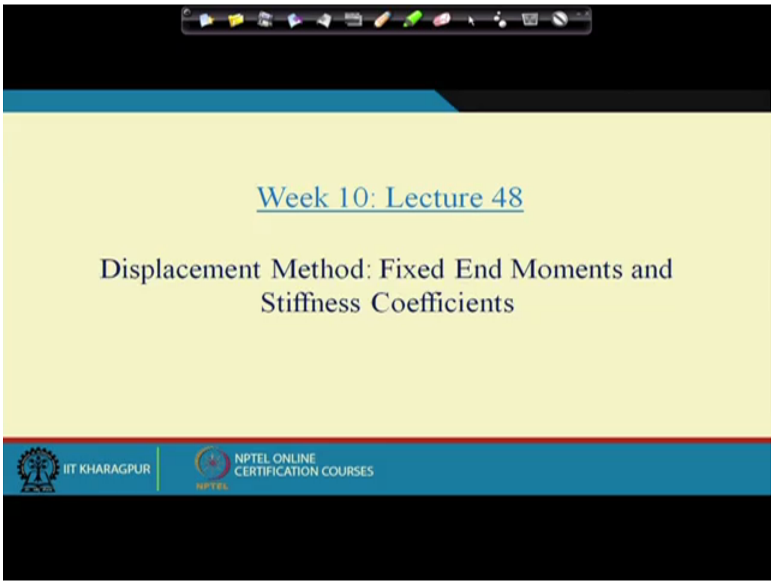

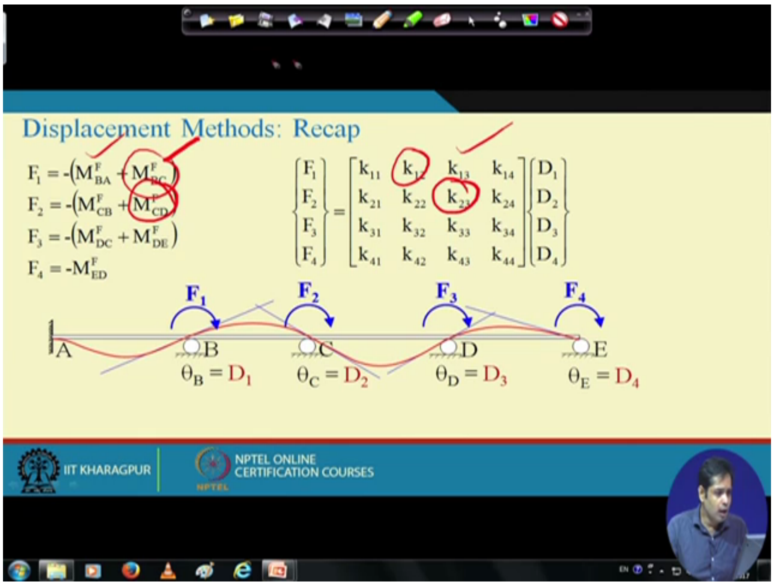

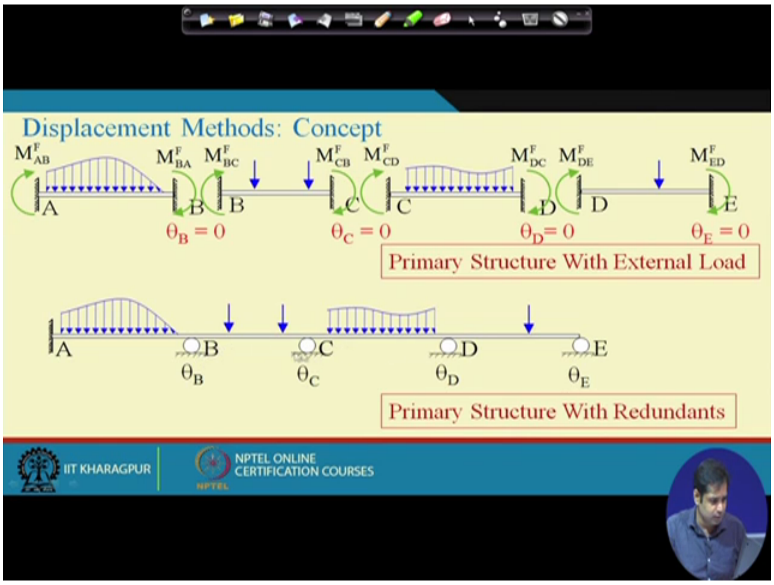

Hello everyone, so today, if you remember last class we just tried to understand the underlying philosophy of the displacement method and then finally we saw there are 2 important ingredients, one is the fixed end moments and another one is stiffness coefficients. What we will do today is we will see how to determine those fixed end moments and stiffness coefficients, okay. So today is the lecture 48 and the topic is fixed end moments and stiffness coefficients. Okay. This was our last slide in the in the previous class and these are the fixed end moments and these are the fixed end moments and these are the stiffness coefficients. Okay.

What we will do today we will see how to determine these moments and these corresponding stiffness coefficients, okay. Now stiffness moments are, fixed end moments are essentially, if you see this, this is our actual structure which is subjected to external load, then we can get the primary structure like this. So all these correspondingly, all these joints which are, which we take as unknown, which the corresponding displacements are taken as unknown, those joints are now replaced by fixed joints and then we can divide this entire structure into small small statically, kinematically indeterminate structure.

All the beams are, all the segments are fixed beams. Now this is a fixed beam which is subjected to the external load and because of this external load, whatever moments generated at these ends, they are called fixed end moments. Okay. Now let us see how to, how to find out these fixed end moments. It is it is the, you already know it how to do it, just for the complete is we are discussing this today.

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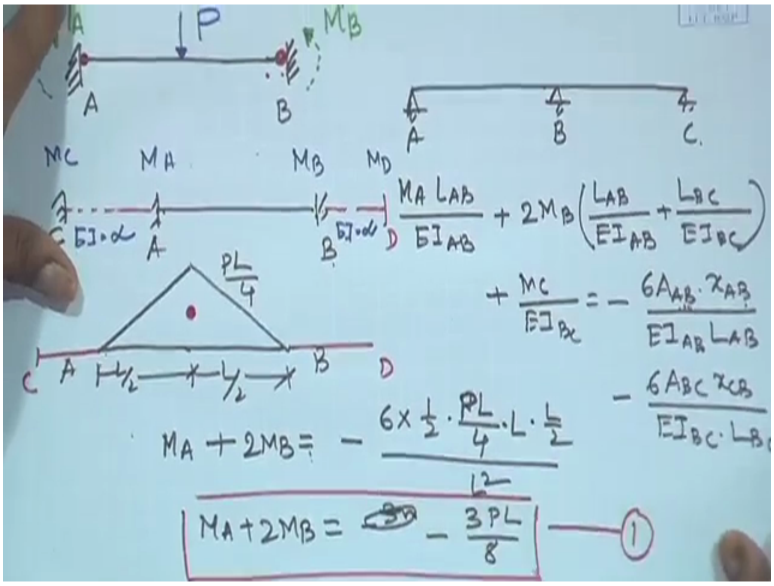

Okay, we will take 2 examples, 1st consider, a fixed beam is fixed beam which is updated with concentrated load, at the midspan, say that load is at P, concentrated load at P, okay. And corresponding moment will be and corresponding moment, this will be, this is A, this is B and say this is MB and this is MA, okay, or you can use M superscript F to say it is fixed end moment. Okay. Now this is a statically indeterminate problem, you see here, our objective is just to get the support reactions, reactions means not all the reactions, only the moments at A and B, okay.

And now there are 3 support directions here, there are 3 support reactions here, so total 6 unknowns, it is a statically, number of equations available is 3, so it is a statically indeterminate problem, statically indeterminate structure. So we need to find out what is , now what we can do is let us 1st, we can apply we can apply for instance we have already seen how to determine statically indeterminate structure using force method, right. So we can apply force method to find out the MA and MF, okay. Again we have learned different force methods, mainly method of consistent deformation but in a different form.

They can apply Three-moment theorem, that we discussed in the last week to get these to get these get these moments. And what is Three-moment theorem, Three-moment theorem says that if we take a beam like this, if we take a if we, any, suppose if we take a 3 span, if we take a beam like this, this is, this is A, this is this is and this is A, B, C, then the moments or moments at A, B, C, the relations are MA into LAB by EI of AB + 2MB LA B by EI of AB + L BC by EI of BC and then + MC by EI of BC, that is equal to - 6AAB into XAB divided by

EI of AB, when I write EI of, it is entire EI of AB, LA B - - 6A BC XCB by EI of BC into L BC. This was the Three-moment equation if you remember.

Now, another thing also be discussed, suppose in this case, since it is, since it is a fixed support, what we can do is, we can have assume, this is A maybe, this is B may be, this is B may be and then we can assume a fictitious beam, a fictitious, say this is this is C, this is C here and which is again fictitious beam for which EI is equal to infinity, EI is equal to infinity, is not it, because it is fixed support. Now similarly we can we can we can assume, it is another, say D and then here also EI is equal to, EI is equal to infinity, okay.

So it has become 3 span 3 span beams, span CA, AB and BD, now we apply, then we have moment, moment here MC, moment here MA, moment here MB and moment here MD, okay. Now we can apply Three-moment equation, this equation at A and we can apply Three-moment equation at B, get 2 equations for MA and MB and then solve these 2 equations to get the MA and MB. Since for this span and this span, EI is infinity and we have 1 by EI term in this equation, the corresponding term for this and for this segment will be 0. And finally, the equation that we get is is this.

And what is AB and A B, AB is the AB is the AB is the bending moment you take, you take a primary structure by inserting 2 hinges here at A and B. Then this becomes a simply supported beam subjected to a concentrated load and the bending moment diagram of that beam, area of the bending moment diagram of that beam is AB. And the distance of the centroid of that bending moment diagram from A will be XA B. Now for this, what would be the bending moment diagram, if we insert 2 hinges here, then the bending moment diagram will be, it becomes a simply supported beam subjected to concentrated load at the midspan.

So this will be PL by 4, okay, right. And this is A, this is B, this is A, this is B and this is the centroid which is at a distance L by 2, L by 2, from here also it is L by 2. And for this segment BD, for AC, we do not need anything because anyway this corresponding these values will be 0, okay. So we now apply be, the Three-moment equation at A and again this Three-moment equation and B, if we do that, then the corresponding equations that we get is MA is equal to MA + 2MB is equal to - 6 into AB of this diagram, AB of this diagram is half into PL by 4, PL by 4 into L, then into, distance from A, it is L by 2, divided by, divided by L square.

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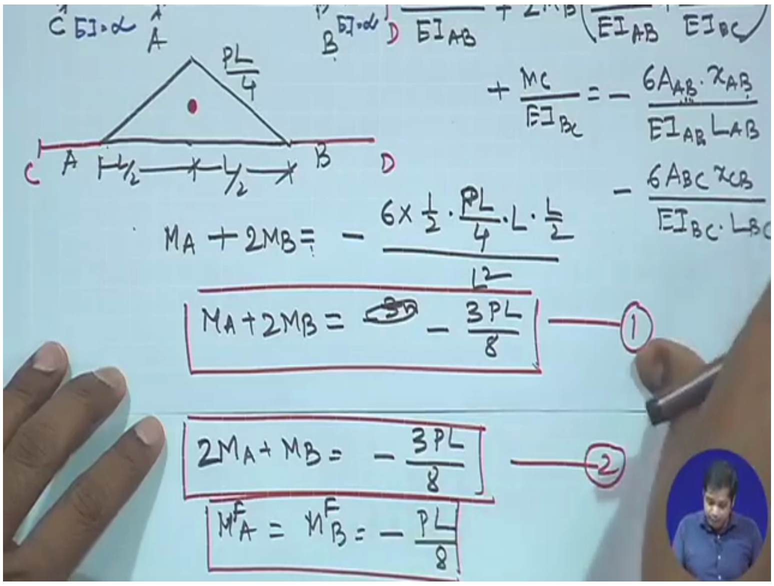

So this is this is one equation, this is equation we get, now this becomes, this becomes -3 PL by 8. So MA + 2MB becomes this. Okay, this is equation number-one, okay. Now similarly, similar to this we can have one more equation and that is and that is MA +2 MA, 2 MA + MB equal to -3 PL by 8. Now if we solve these equations, equation 1 and equation 2, then we get MA is equal to MB is equal to - PL by 8. So this is the fixed end moment, this is the fixed end moment. Okay. Now this you can write, this is MAF, so this is the fixed end moment. So if we if any structure which is subjected to, which is subjected to a load like this which is subjected to a load like this, fixed beam subjected to a concentrated load, corresponding fixed end moment will be this, okay.

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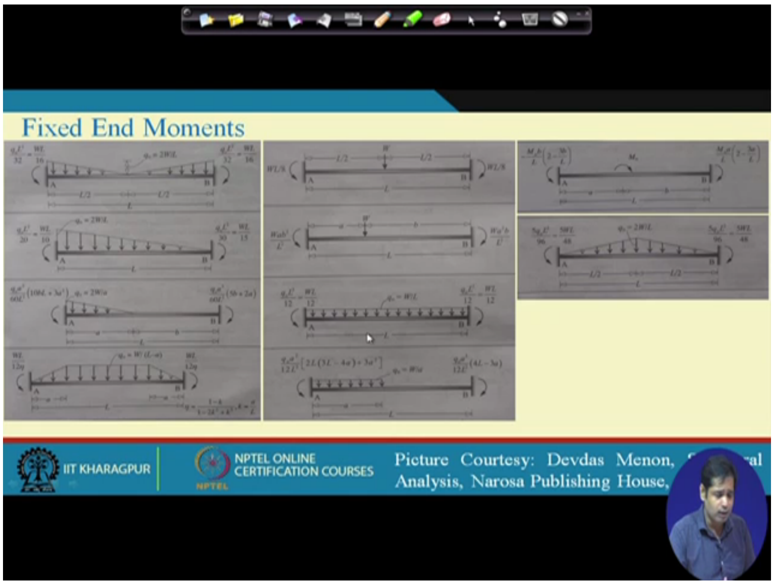

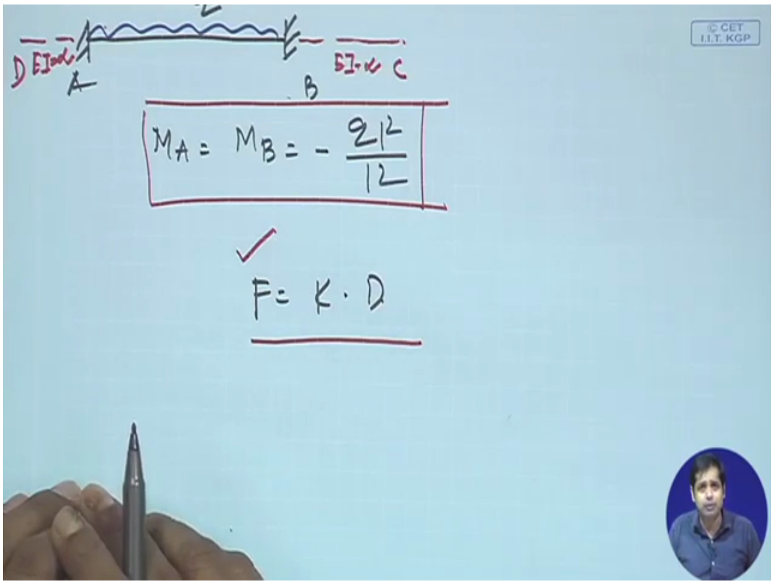

Now let us do the similar exercise for another case. Now this time we take another structure like same structure but fixed structure, fixed end moments, your beam is always fixed beam, now it is subjected to uniformly distributed load, okay. So this is A, this is B and this is Q and again we can apply the Three-moment equation and if we do the Three-moment equation, then for this case we will get, we is that there is a beam here which is C and there is another beam here which is D and for this it is EI is equal to infinity and for this also it is EI is equal to infinity and then apply Three-moment equation at A, Three-moment equation at B, get the 2 equations and solve them and if we do that, then we get for this problem MA is equal to MB is equal to - QL square by 12. Okay.

Now similarly for any loading condition, any loading condition when you determine the fixed end moments, essentially then you are solving is statically indeterminate structure. We know how to solve the statically indeterminate structure using force 1, so you can apply those force method to get the corresponding fixed end moment. Now if you if you if you see this , that is already available in the book but do not take it, these values for granted, at least, at least once you should compute the fixed end moments for different loading conditions and then see whatever moments you are getting, that are, that are, you are getting these values, okay.

If you take any structural analysis book, these fixed end moments moments for different kinds of loading are already given there. Okay. So fixed end moments, we know, right, once we know the fixed end moments, we need to, if remember the equations were, equations, equations were F is equal to K into D and F in order to get F, we need fixed end moments, now we have seen how to determine fixed end moments. So this is now known, this is, this is now known, now we need to find, we need to find out now what is stiffness matrix. Now stiffness matrix means what is stiffness coefficients.

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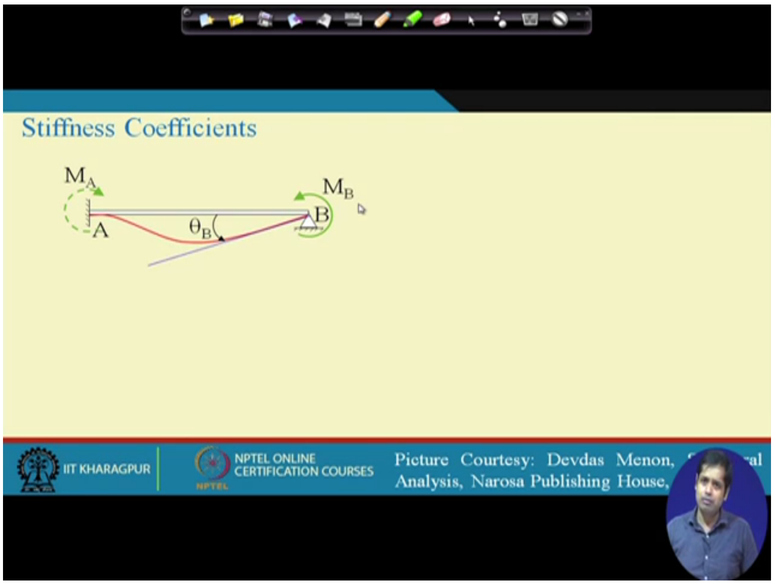

Once you know the stiffness coefficient, then you can apply this equation. Now again for stiffness coefficients, 1st of this coefficients , for stiffness coefficients we take 3 cases. A different boundary conditions for different loading, you can have, you can similar approach you can apply to get the stiffness coefficient for instance. Okay, take a propped cantilever beam, okay, which is subjected to a moment B here and because of that it undergoes rotation like this, okay. So this angle Theta B, okay. Now what is and then corresponding moment at this end is generated as MA, okay. Now what we need to find out here is, 1st we will see, forget about stiffness coefficients and flexibility coefficients, let us read this as a statically indeterminate problem, a propped cantilever beam which is subjected to a moment at the propped end and then what we need to find out, we need to find out what is the corresponding rotation at the propped end.

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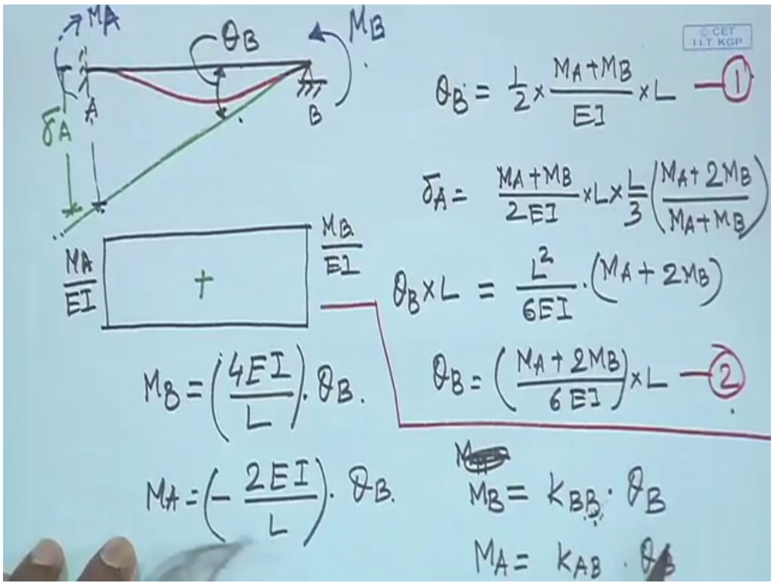

Okay. And the problem is this, okay, now, so this end is fixed end and this is a propped end, okay. Now one moment is applied here and this moment is say M, shades MB because it is applied at B, this is end A, this is end B. Now corresponding rotation is, since it is fixed and, this slope is 0 and at B, it is hinged, so there will be nonzero slope. And this slope is suppose, this slope is, this angle is Theta B, okay. So it is slope at Theta B. Now suppose this distance is Delta A, Delta A, okay. So or you can write Delta AB as well.

So delta A is essentially the deviation of A on slope drawn at B, is not it. So at B you draw a slope and on this slope, what is the deviation of A, so that is equal to delta A or you can write Delta AB as well. Now next is suppose we do not know what is this value, corresponding moments here is, say it is MB, okay, what we need to find out, MA, sorry, it is MA, we need to find out how is this Theta B is equal to MB and MA, okay. Now let us let us 1st find out, let us 1st draw the bending moment diagram.

If we know it is the moment at B is MB and moment at the moment at A is MA, there is no other forces in between, so bending moment at B will be MB, bending moment at A is MA and in between they vary linearly, okay. So the bending moment diagram will be, so this is MB and this is MA, we do not know whether MA is more in what value, so this is MB. This is the bending moment diagram, this bending moment diagram is positive, this is positive because when we draw the bending moment diagram, then the sign convention we use, whether it is sagging moment or hogging moments.

We have taken it is sagging moment, so the bending moment diagram is positive, okay. Now remember, remember moment area method, what moment area methods as is that the angle at B will be, angle B, Theta B will be the area of this diagram. Suppose this is not, this is MB by EI, moment, MI, M by EI diagram. So moment area method says that this slope will be area of this diagram, right. So Theta B will be area of this, area of this means it will be half of MA + MB divided by EI into L, okay. So this is Theta B, suppose this is equation number-one. Okay.

Now we also know that the 2nd moment, 2nd theorem moment area method that the deviation of A on slope drawn at B is equal to moment of M I by M by EI diagram from A. So moment of this diagram from A will give us this deviation. So delta A will be, Delta A will be the moment of this about this. And moment of this about this will be the area multiplied by the distance. So area will be MA + MB divided by 2 EI into L, that is the area of this and that multiplied by the centroidal distance, this is a trapezium, etc. centroidal distance of the trapezium is L by 3 and then this is MA + 2MB by MA + MB, okay, this is the Delta A.

So delta A then becomes, this MA and this MA, they get cancelled and this becomes L square by 6 EI into MA +2 MB. Now what is delta A, Delta A if we , that was our assumption right from beginning that the displacement and slopes are small, then linear superposition is valid. So I have delta small, that they can write Delta A is equal to Theta B multiplied by, multiplied by L. So this delta is essentially Theta be multiplied by L. So what we get from this Theta B is equal to, Theta B is equal to MA + 2M B by 6 EI into L. Suppose this is equation number 2.

Now let us solve the equation number-one and equation number 2, find out MA and MB in terms of Theta B, okay. Now if I do that, then what we get is, MB is equal to, MB is equal to 4 EI by L into Theta B and MA is equal to -2 EI by L into Theta B. Okay. Now let us write this in a different form. This, let us write it as MB is equal to, MB, MB is equal to K BB into Theta B. And MA is equal to KAB into Theta B, okay. What is K BB, K BB is this and KAB is this. Okay. So K BB is the, K BB and KAB are the stiffness coefficients, okay.

Now why this K BB is essentially what, K BB is, K BB is the relation between at B and rotation at B, okay, so this is K BB. And similarly KAB is equal to the relation between the moment, relation between moment at A and Theta at B or in a different way we can say that KAB is the, if we apply unit rotation at, if we apply a unit rotation at A, then corresponding momentum generated unit rotation at B, then corresponding moment they attend at A will be K AB. Okay, this is the stiffness coefficient.

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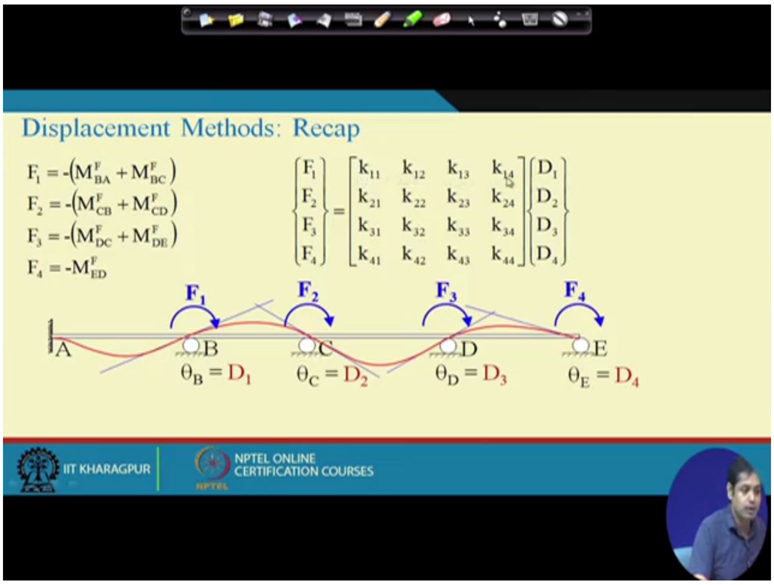

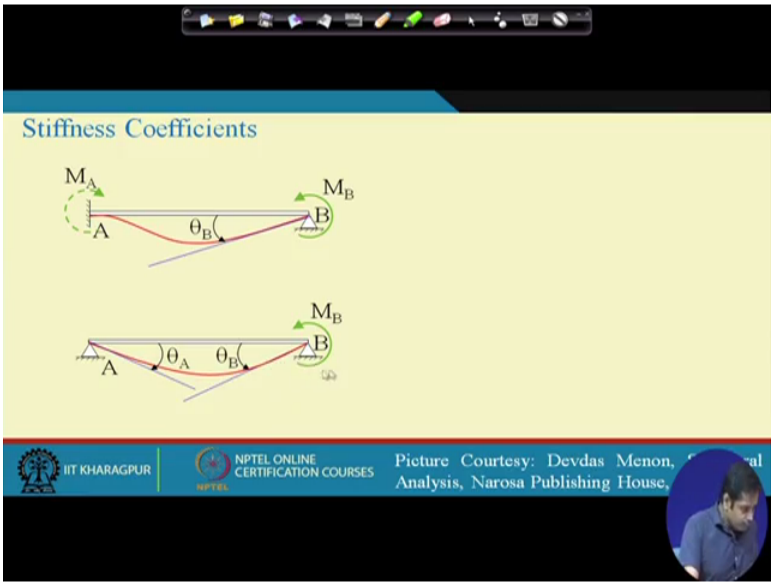

So when we construct, when we construct the, when we construct this stiffness coefficient, if K 11, K12 and so on now if we see the boundary conditions of the beam is such that it can be idealised, as the problem that now we have seen, then we know what would be the corresponding stiffness coefficients. Okay. Now let us take one more example where it is slightly different. Now the example is, the 2nd example, the same thing but now instead of fixed support at A, make it a hinge support. Now there is a moment at MB, it undergoes deformation like this, what we need to find out, how this MB is related to Theta B and Theta A, okay.

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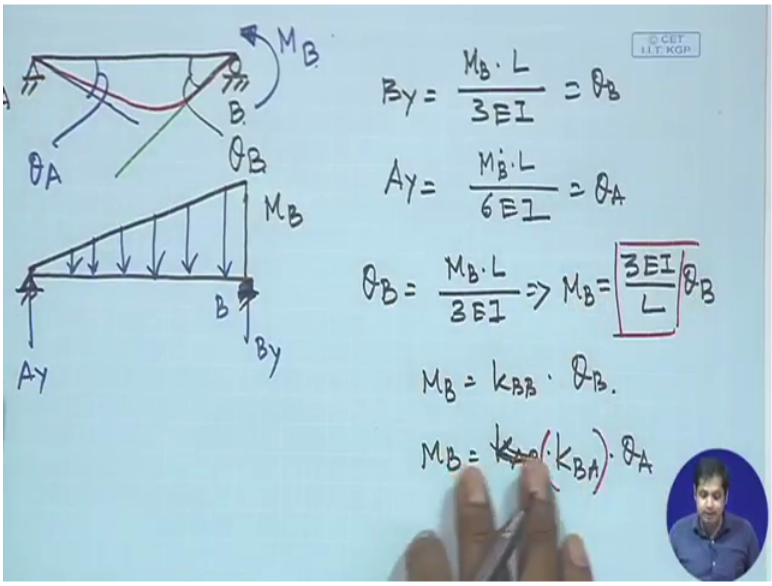

Now let us do that once again. So take this, this is, okay, now we can, this is subjected to, we have a moment which is MB here because this is A and this is B, okay. Now we could have taken hinge support here, both will give you the same thing because there is no horizontal horizontal load, so horizontal direction will be 0. So this will go deformation like this, deformation like this and this angle is, this angle is Theta B, this is Theta B and this angle is similarly you can have an angle like this and this angle is Theta A. Okay.

We need to find out what is relation between MB, Theta B and Theta A. It is very straightforward, this is a statically determinate problem, so we can apply the conjugate beam method. The conjugate beam method says that you 1st draw the bending moment diagram of this beam, the bending moment diagram will be, this is MB, positive here and then there is no bending moment at A, so this bending moment is linearly varying, okay. Now next the conjugate beam says that whatever bending moment diagram you take, you assume that as loading diagram on the conjugate beam.

And conjugate beam, how to form the conjugate beam, conjugate beam, it is the same length, same beam but depending on the support, the boundary condition, your conjugate beam, the boundary condition conjugate beam will be different but forcibly supported beam, the conjugate beam is also simply supported, so it is a conjugate beam, right. It is the conjugate beam which is subjected to a load like this, load like this. And then conjugate beam says that if you get the, if you get the reaction at any point, that reaction will give you the, give you the corresponding rotation, okay.

So let us find out what is the reaction, so at this point your reaction is AY and at this point your reaction is BY, this is A, this is B and what is BY, if you take a moment about A, if it take a moment about A, then BY will be ML by MB into L by 3 EI. Okay. And if you take, and AY will be ML by, MB into L by 6EI. Okay. So this is BY and A Y. Now since BY is equal to Theta B and AY is equal to Theta A, so Theta B will be MBA into L by 3 EI. So which can be written as, which can be written as MB is equal to, MB is equal to 3 EI by L into Theta B.

And similarly MA can be written as, similarly Theta, similar in terms of , in terms of Theta, in terms of Theta B. In terms of Theta A, MB also can be written. Now this is the flexibility coefficient. Now what we can write as, we can write MB is equal to K BB into Theta B where K BB is the moment at B due to the unit rotation at B. Similarly M BB, MB can be obtained as KAB into , KB A into Theta A, Theta A will be, this will be, this will be Theta, this will be Theta B and this will be Theta A, right, that is conjugate beam method.

So similarly MB can be obtained as Theta A, so K BB will be, K BB will be this corresponding stiffness coefficient. So it essentially says that it is the, it is the moment generated at B, moment at B due to the unit rotation at A. So K BB was the moment at B due to the unit rotation at B and KB A is the moment at B due to the unit rotation at A. So this is the, this is the flexibility coefficient, again the problem when you solve, if you see that can be idealised, the support system is such that it can be idealised as this, then you know how to get the flexibility coefficient.

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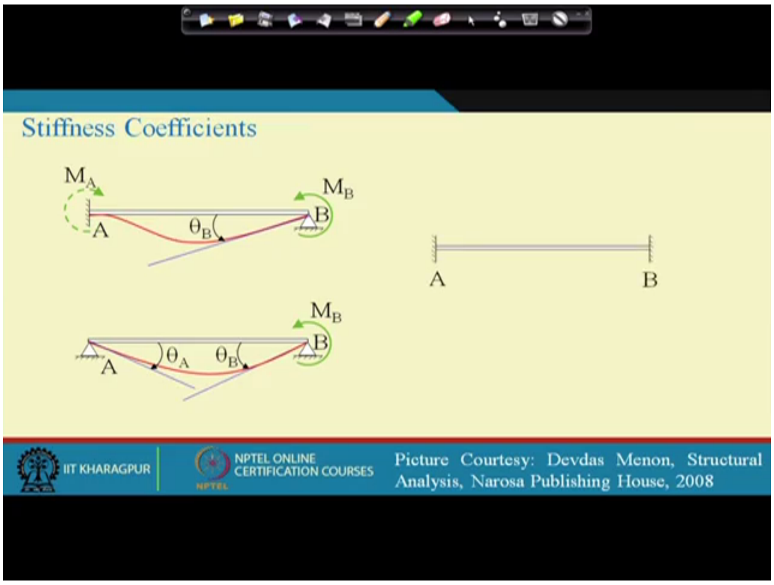

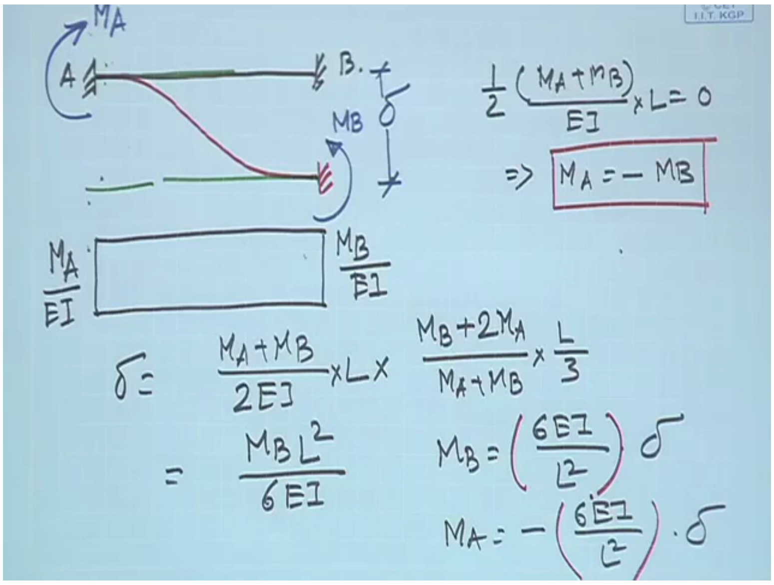

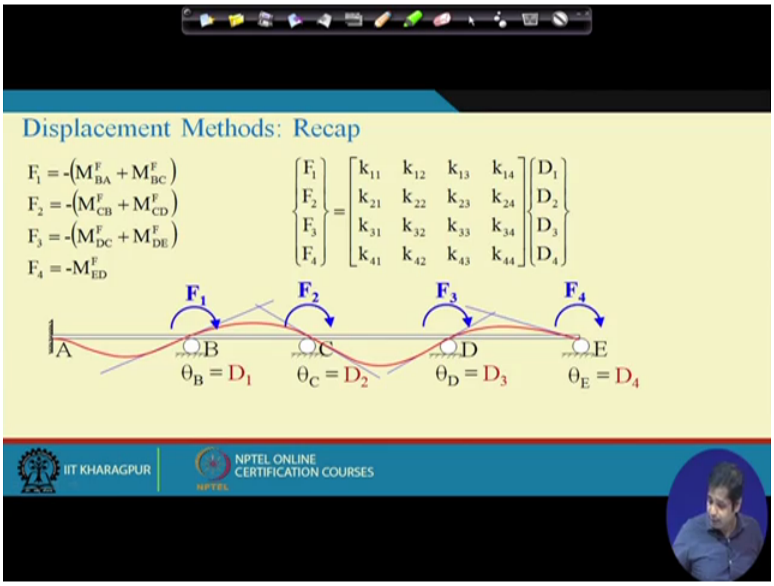

And one last case will consider is the same beam but now it is subjected to settlement of support. So if the support settles like this, if the support settles like this, then what would be the corresponding , so point B, there is a settlement at point B, or relative settlement at point AB and that will cause moments at A and B. So how these moments are related to this Delta B. So let us consider this again. So initially this is the beam, both ends are fixed, this is A, and this is B. And suppose the beam deflects like this, deflects like this, so this is the settlement Delta.

Okay. And Delta will cause some moment at A and moment at B, we need to find out those moments. Suppose this is MA, and corresponding moment at B is MB, okay, MA and MB. So then what would be the, what would be the bending moment, what would be the bending moment diagram, bending moment diagram for this will be again there is no other load in between, so bending moment will be like this. So this is MB, we know whether MA and MB is same or different, it is a trapezium MA and MB, okay. Now then apply moment area method, what moment area method says is that rotation at A the rotation at A or relative rotation, if we draw slope at A slope at B, their relative rotation between, the angle between those slopes will be the area of the bending moment M by, the bending moment by the EI diagram.

This by EI diagram, now we see this is a fixed support, so your slope is a horizontal line here, this is again a fixed support, slope is again a horizontal line here, there is no relative rotation, so your bending moment diagram area of this bending moment diagram should be equal to 0. Now the area of the bending moment diagram if we take, then the, it is half into MA + MB by EI into L, that is equal to 0, okay. So this gives us MA is equal to - MB. Okay. This is 0 because if you draw a slope like this, if you draw a slope like this and if you draw a slope like this, their relative rotation is 0. So area of the bending moment diagram by EI diagram will be 0. That is the moment area method, the 1st theorem and moment area method.

Let us now apply the 2nd theorem of moment area method, then what is this Delta, this is Delta is essentially, this Delta is this delta is this Delta is your rotation, your deviation of point B deviation of point B drawn on slope at point A or the same Delta, deviation of point A, if you look at from this, deviation of point A on the slope drawn at point B. So what we can do is, we can take the, that deviation will give us, that deviation can be obtained by taking the area, moment of this bending moment, bending M by EI diagram, okay.

Now Delta will be, moment of this diagram if we take, the area which is MA + MB divided by 2 EI into L, that is the area of this bending moment diagram into MB +2 MA divided by MA + MB into L by 3, L by 3, okay. That is Delta. So this you can , just by some algebraic manipulation we can get and substituting MA is equal to MB, we can this is equal to MB into L square by X EI. Okay so what we get, we get MB is equal to 6 EI by L square into Delta. So this again we can write that this is the flexibility, this is the stiffness coefficient, okay.

So stiffness coefficient corresponding to the settlement. Similarly MA will be what, MA will be, MA is equal to - MB, so it will be -6 EI by L square into Delta. So this is the, this is the corresponding stiffness coefficient, okay. So this can be written, so again if we see, if we see the, the problem is such that it can be idealised like this and stiffness coefficient can be obtained. Similarly for different support conditions, for different conditions you can get the stiffness coefficient, okay. Now for that please go through the books, any structural analysis book you take, there are different cases given and please see those cases and try to derive the corresponding stiffness coefficient.

And when you derive the stiffness coefficient please be careful about the sign, okay. All the, one important thing you get, suppose if this is the, this is the stiffness matrix, all the diagonal terms in the stiffness matrix should not be, all the diagonal terms in the stiffness matrix that you can see, this diagonal term should be positive. Okay, and this stiffness matrix is symmetric and again another important thing, it is positive definite. Now what is positive definite, that I will not, we will not discuss here now.

But this is a property of the stiffness matrix, I request you see any linear algebra matrix algebra book to see what is positive definite matrix, okay. Now so when you calculate stiffness coefficients, be careful about the sign, if you are getting wrong sign, if you get the wrong sign, then your results will be wrong, okay. So be careful about the sign convention. Okay, so what we do is, now once we have understood the concept behind displacement method and also we have understood that there are 2 ingredients in that in that in that general formulation, that is the fixed end moments and stiffness coefficients.

We have also understood how to get the fixed end moments for different loading conditions and we have also seen how we can calculate the stiffness coefficient for different boundary conditions. Next step is to derive the , to write the equations for displacement method. Now we have 3 different methods that we will be discussing in this course, one is slow deflection method, then moment distribution method and then the stiffness matrix. We will do this make slope deflation method but again the underlying principle of all this method is that just now what we have discussed in 1st 2 lectures of this week.

So next class what we will do is we will start slope deflection method , next class we will start slope deflection method, derive the corresponding equations and demonstrate that through some example. Okay, then see you next class, thank you.