Structural Analysis I.
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Lecture-46.
Analysis of Statically Intermediate Structures by Force Method (Continued).

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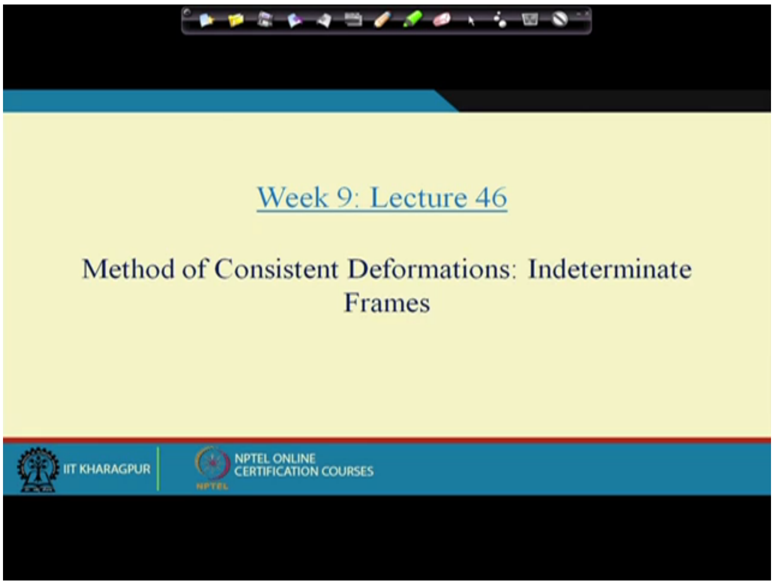

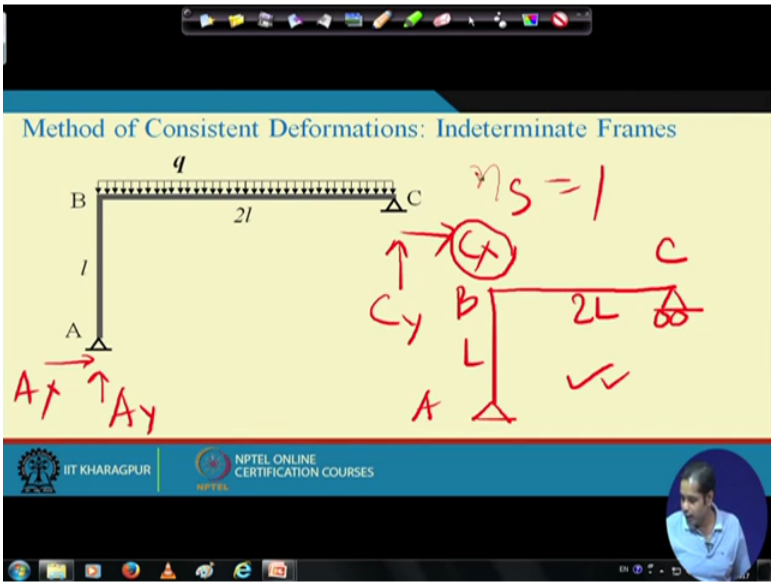

Hello everyone, welcome, this is our last lecture of week 9. What we have done so far is we have applied method of consistent deformation for trusses and then for beams. What will do today is we will apply the method of consistent deformation for indeterminate frames. The concept again remains same, only the applications are different. Okay, so to start with commercial today is method of consistent deformation apply to indeterminate frames. Let us start our 1st example, now you see, this is an indeterminate frame, if you remember the 1st part method of consistent deformation is to find out what is the static indeterminacy.

Just by looking at this frame we can see that for this it is static indeterminacy NS is equal to 1, okay. Now the 2nd step is, we have one horizontal correction here, one vertical and then one horizontal and one vertical, so total 4 reactions, 3 unknowns, so NS is equal to 1, external indeterminacy. Now the next step is to identify the redundant force. What we can do is we can say , keep the end A as it is and take CY, this is CY and this is CX, this is A Y and AX, AX, let us take CX as redundant force.

If CX is a redundant force, then the primary structure becomes we need to release this corresponding, corresponding constraint. So this N becomes roller support, with this, this is the primary structure, this is A, B and C, this is 2 L and this is L. So this is our primary structure, okay. Now this was step 2, another step 3 is we need to analyse the primary structure subjected to the external load and then this primary sector, analyse the primary structure subjected to the redundant force, okay.

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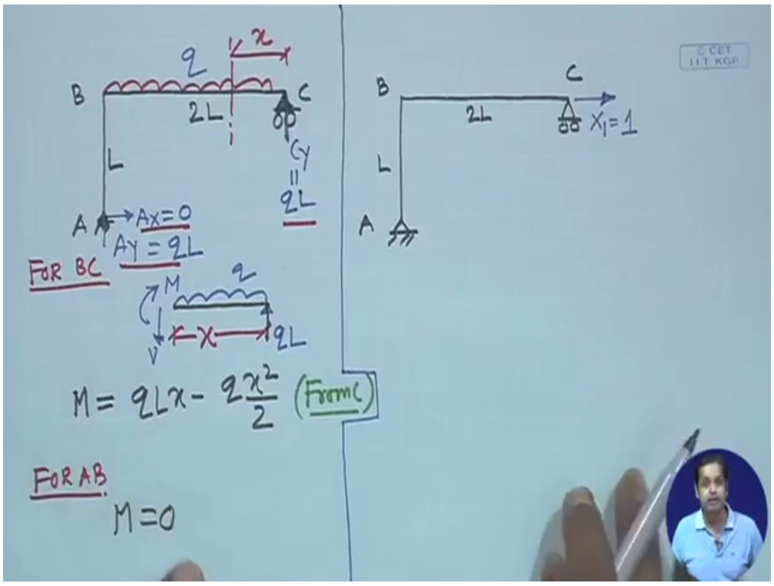

So let us do this here. So the primary structure we have seen the primary structure is this, right. So this is, this point was A which is hinge, this is roller, this is B and this is C which is subjected to uniformly distributed load, uniformly distributed load, okay, fine. This is Q, okay. Okay, now this is N, this is L and this is 2 L. Now this is the primary structure subjected to external load. Similarly we have the primary structure subjected to the redundant force. So this is again roller, hinge, this is roller, A, B, C, this is L, this is 2 L and the redundant force is, this is redundant force which is X1 and we have to apply the unit load here.

So this is the primary structure subjected to external load, primary structure subjected to the unit load, okay, now we need to analyse it, okay. Now let us find out what is the, what is the expression for bending moment for this. So we have 2 members in this frame, member AB and BC. So bending moment expression we have to find out for both the members. Let us 1st find for BC. Okay, before that, what will be the free body diagram of the entire structure, this is an indeterminate structure, free body diagram will be, this force will be CY, this is CY then we have AY and AX.

Okay, since there is no horizontal force, AX will be 0 and then if we take summation, if we take moment about A, then we get, we get CY and the value of CY will be value of CY will be Q into L, this will be Q into L, similarly A Y will be Q into L. So this is the support reactions for the primary structure. Okay, it is just the application of equilibrium equation. Now let us take one section at say any section at a distance X from C, distance X from C and draw the free body diagram of that section.

So this is, this is CY which is Q into L and then it is subjected to Q and this distance is X, this distance is X and then we have, this is V here and this is M here. Now if we take moment about this point, we get summation of, then we get moment is equal to expression for moment is equal to QLX - QX square by 2. So this is the expression for moment between B and C but what is important here, the X is measured from C, X is measured from C. So when we integrate it, then we have to take the limit from C to B. So this is the expression for bending moment for BC.

And let us take the expression, similar expression for AB. So for AB, we can see that if we take any section between A and B and draw the free body diagram, we will, you can say, you can find that M is equal to, M will be, the expression for M will be 0. So for this case, for this primary structure subjected to external load, bending moment is quadratic as per this equation between B and C and between A and B bending moment M is equal to 0. Okay. So this was for this was for primary structure subjected to external load. Okay.

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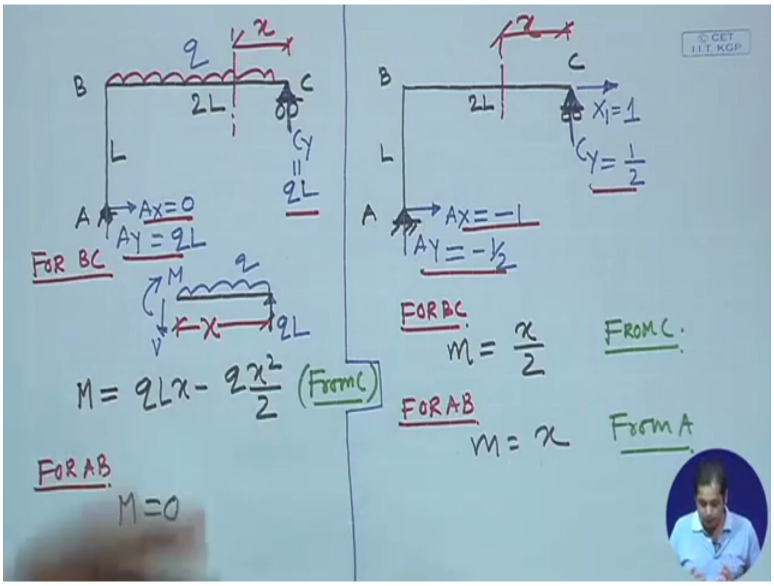

Now let us do the similar exercise for the primary structure subjected to unit load. Again if we draw the free body diagram, this will be AX, AY and then AX, AX and then you have CY here, CY here. I have already told you many times that in this case I am writing the, I am drawing the free body diagram on the actual structure itself but actually when you draw the free body diagram, unit to remove the support and represent the support by the characteristic forces. Okay. So support and the reaction cannot be shown together. Okay. So CY you will get if you draw the equilibrium, if you apply the equilibrium conditions, CY will be half and then similarly AX will be 1, AX will be -1 and A Y will be - half, these are the support reactions.

These are all support reactions, okay. Now draw for BC, for BC, the expression for M, for primary structure subjected to, subjected to unit load, we use symbols small m. If you take any section X, we take any section X from C, any section X from C, then draw the free body diagram, you get m is equal to X by 2. But in this case X is measured from C. Okay. Now similarly draw the free body, find the expression for bending moment for AB and again you can take any section here and apply the equilibrium condition and if you do that, then you get m is equal to X.

And in this case X is measured from A, from A, okay. So we have primary structure, expression subjected to external load, expression for moment, then the primary structure subjected to external load , subjected to redundant force we have the moment, this is step 3 and this is step 4. Okay, now what we need to find out, we need to find out what is the displacement at point C due to this external load and what is the displacement at point C due to the redundant force and then apply the compatibility condition to get the unknown redundant force.

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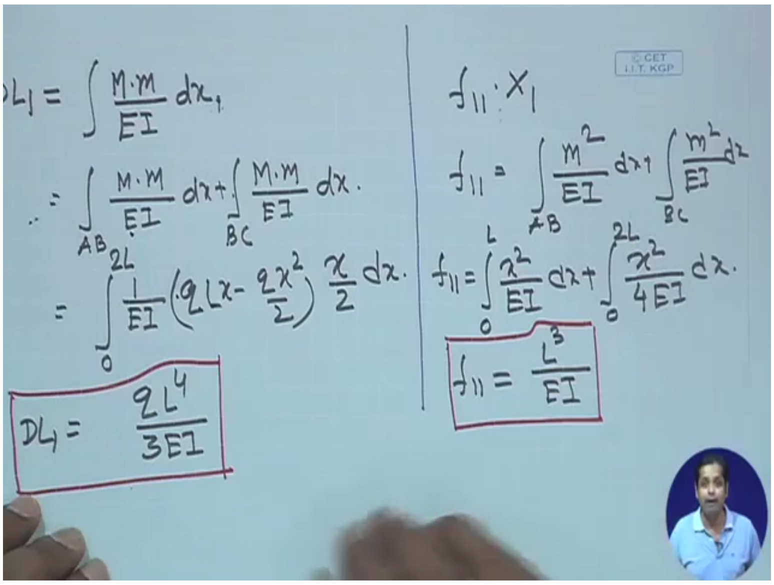

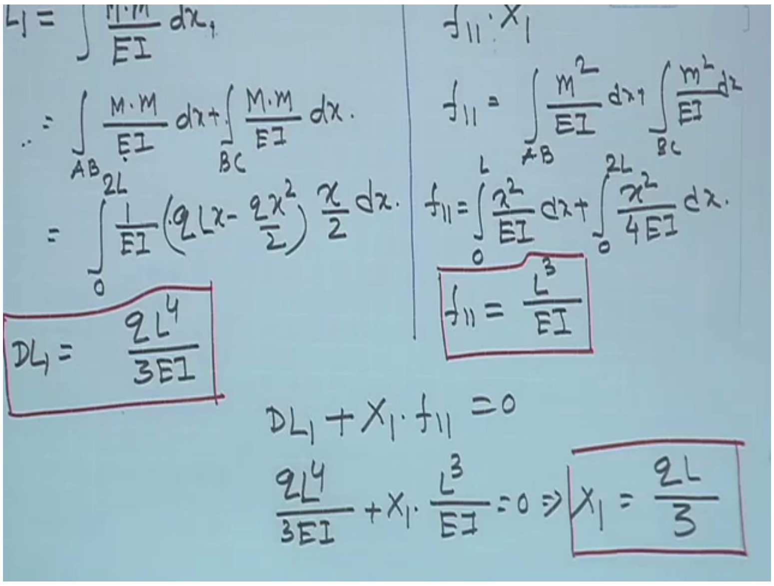

So let us find out what is the displacement at this point. Again we can, we can apply the unit load method and what unit load method says that DL 1, DL 1 is equal to, that is the expression used in the case of beam as well, DL 1 means the displacement, displacement of point where your redundant force is X1. And that displacement is measured in the direction of X1, this displacement due to the primary , external load. So that is integration, integration of M into small m by EI dx. Okay, now we have 2 parts, AB and BC, so that integration has to be done over AB and BC. So over AB M by EI dx + integration of BC M by EI over, that is dx.

Now, for AB, for AB, capital M is equal to 0, so this part will, this part will become 0 and for BC, your X varies from 0 to 2L, that is how we have taken X. And then M is this and small m is X by 2. Okay. So substitute that, so this becomes 0 to 2L, 1 by EI into capital M is QL X - QX square by 2 and then small m is X by 2 that is dx. Okay, now if we do this integration, the expression what we get is QL raised to 4, this is capital L used, divided by 3 EI. So this is DL 1, okay. So this is the displacement in the primary structure due to external load and this displacement is measured in the direction of in the direction of X1.

So similarly we can obtain the displacement in the in the primary structure due to the return in force. And that displacement will be F 11 into X1, X1 is the redundant force and F 11 is the associated flexibility coefficient. And what is the expression for F 11, expression for F 11 is equal to integration m1, in this case it is m1 and m is same because had it been many redundant forces, means more than one static indeterminacy, we use, we would have used m1, m2 and so on but since it is just one static indeterminacy 1, so we have used small m.

So this is m square by EI. Okay and this is this will be over AB + and then BC m square EI dx. So, yes now F 11 becomes, if you, if we substitute that, we know what is the expression for m between BC is X by 2 and AB is X. So this becomes 0 to L and it becomes X square by EI dx +0 to 2 L and m is X Y2, so it becomes X square by 4 EI dx. Okay, now if we do this integration and we get F 11 is equal to L cube by EI. So this is the coefficient for this is the associated coefficient. So this is flexibility coefficient, okay.

What is step 5, step 5 is the compatibility condition and the compatibility condition at C is, since it was hinge support in actual structure, so that is, that should not be any horizontal displacement, so compatibility condition will be displacement from this class displacement from this, their submission should be equal to 0. So displacement from, displacement for external load applied to displacement for in the primary structure of light to action a load is D L1 and displacement in the primary structure due to redundant force is equal to X1 into flexibility coefficient F 11, that should be equal to 0.

So if we substitute that DL 1 is equal to QL to the power 4 by 3 EI, 3 EI and then + X1 into L cube by EI is equal to 0 and this gives us X1 is equal to Q X1 is equal to QL by 3. So horizontal reaction in this, so in this problem, it is an indeterminate problem, so horizontal reaction will be QL by 3. Now once we know the horizontal reaction, means in this case it was redundant force, than with the knowledge of that information about the horizontal direction, the structure now becomes determinate structure.

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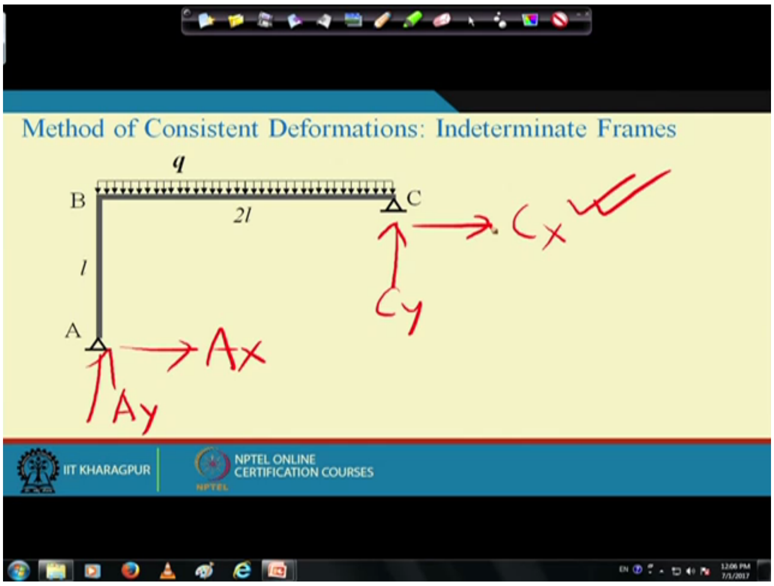

Now we can apply the apply the, we can apply the all these equilibrium conditions. As I said, here we have 2CY and CX, CX and here it was AY and AX. So initially it was 4 unknown, we could not determine, now CX is known, so with the knowledge of CX, only unknown we have is AX, AY and CY, we can apply equilibrium condition to get these unknowns and then internal forces at any, at any point, okay. Now this is the 1st, that finding the internal forces, applying the equilibrium condition is step 6, that we do not discuss your because we have done it many times indeterminate structures, okay.

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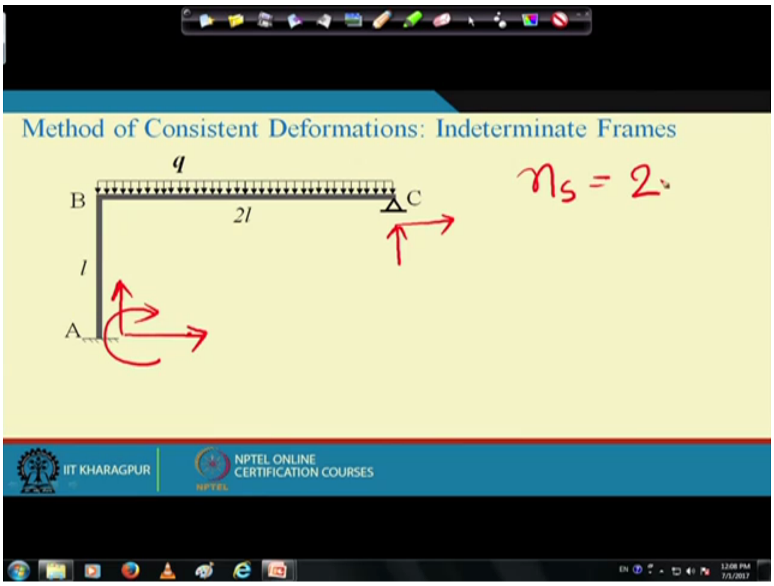

Next is, next example is, take the same example but the difference is this the end A, for the 1st example end A is hinge, pint, now make it fixed support. Now once we make it fixed support, means what exactly we are doing, we have we provided one more additional constraint and that constraint is protection at A. So what is the static indeterminacy in this problem, here we have 3 reactions AX, this and then this and then moment at A and here you have CY and then CY and CX. So total 5 unknowns, 3 equilibrium equations we have, so NS, static indeterminacy is 2.

So that is our 1st, the 2nd thing is identifying the redundant, identifying the redundant forces. Now when we have more than one static indeterminacy, we have, we have more than 1 choices of redundant forces or selection of redundant forces further for example one case is we can take case 1, case 1, case 1 we can take CX and CY as redundant force. And if we take CX and CY as redundant force, our primary structure becomes like this. It is, it remains as fixed and there is no support at CX because both the reactions, CX and C Y we have taken as redundant force. Now this is again a statically determinate structure, so this is, this could be one choice.

And another choice could be, take CX, take CX and MA as redundant force. Means horizontal reaction at C and the moment at point A is redundant force. And if we are taking CX and MA as redundant force in the primary structure, associated degrees of freedom needs to be released. Now CX, so the primary structure becomes in this case, if we release, if we allow horizontal movements, this seam support become roller support and then if we release the rotational constraint, means MA, then this fixed support becomes hinged support, okay.

So this is our primary structure. Now this primary sector is very similar to the research we have taken it for, we have taken in the previous example. Okay. In this case, these are the 2 redundant forces. Now what we do is, we go with this choice because some part of because for the 2 reasons one is already we have done some calculation in the previous example, that calculations we can use for this for this example and the 2nd thing is, if we take this, if we take this, if we take this case then what happens you are taking 1 force and one moment as as constraint. So let us go with it.

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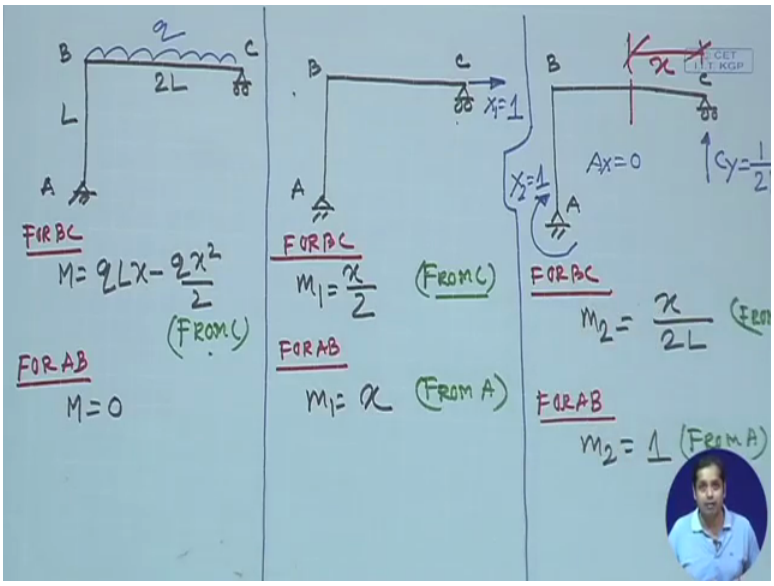

So our primary structure, now we have seen 2 degree of static indeterminacy, we need to solve 3 determinate problems, one is primary structure subjected to the external load and then 2 and 3 is primary structure subjected to 2 redundant 1 and the primary structure is subjected to redundant 2. So 1st is the primary structure, again this is the primary structure which is hinge here. And then again roller here which is subjected to uniformly distributed load of intensity Q. This length is 2L and this length is L. This is the primary structure subjected to external load.

Then we have primary structure, we have primary structure which is subjected to redundant force in the direction of, the 1st redundant force, this. Okay. In this case X1 is equal to 1, unit load in the direction, in the direction of 1st redundant redundant force. And then 3rd, we have primary structure, this is the primary structure which is subjected it to 2nd redundant force and the 2nd redundant is this moment here. So here X2 which is equal to 1. So X1 is CX and X2 is moment at A, in this case they are subjected to unit load in the direction of X1, here it is subjected to unit load in the direction of X2.

Now let us find out what is the next we need to find out the expression for bending moment for the entire structure A, B, C, so A, B, C, then A, B, C. Let us let us do that, we have already done some calculations, let us divide it into 3 parts. Okay. Now, so for this, for this we have just now done for AB, for BC, for BC, expression for M will be QL X - QX square by 2 and in this case your X is measured from C. And then for AB, M is equal to 0 right. So this we already have.

And then, and the 2nd case for this, this also we already have, for BC, just we have done it in the previous example, m1 is equal to X by 2 where it is from C, X is measured from C and then for AB, for AB, it is m1 is equal to X and it is from A. Now we need to do it for this. Okay. Now again if you apply the equilibrium condition, then what we get, we will get CY equal to, this is CY, we get CY is equal to 1 by 2L okay. And AX will be 0, AX will be 0 and at AY will be, AY will be -1 by 2 L. Okay, now then for BC, the expression for moment, if we take any section here, if we take any section here and this is at a distance X.

Please remember one thing, in the primary structure, for a given member, say for instance here BC, in the primary section, whatever moment expression we have obtained, in all the expression, X is measured from C. And then in the primary structure which are subjected to redundant forces, then also you make sure that you use the same same Convention because when you, if you do not use the same convention, then the integration would be problematic, because you have to essentially integrate product of this and this, in both the cases, definition of X is different, then integration will be different.

So in all the structure, the, how is the X is measured, how the X is defined, that should be uniform. So for BC, m2 will be X by 2L, this X distance, multiplied by 1 by 2 L is a reaction and then similarly for AB, for AB, we have m2 is equal to 1. Your X is from A, in this case it is from C and from C and in this case it is from A. So we have already obtained all the expressions for bending moment M, m1 for the 1st redundant and m2 for the 2nd redundant. What do we need to find out, we need to find out the corresponding displacements and the flexibility coefficients, okay.

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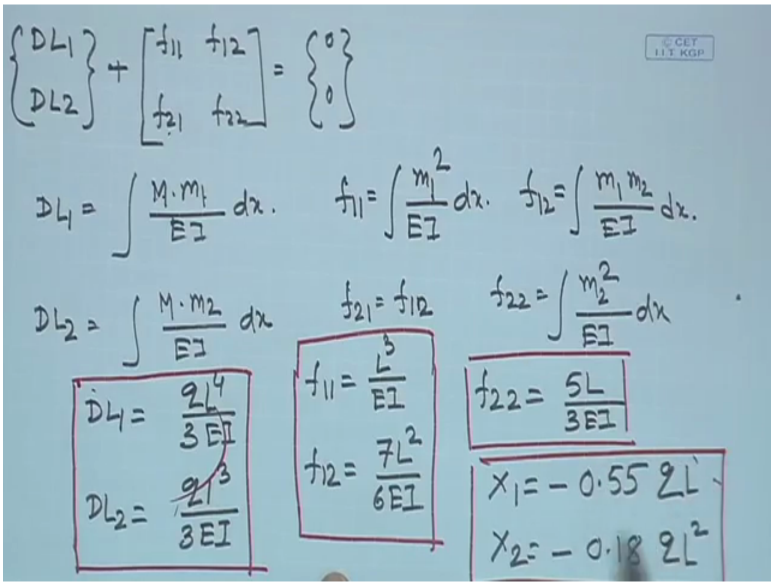

What are the corresponding displacement and flexibility coefficient? The 1st displacement is D L1, D L1, okay. The final expression will be DL 2, if you remember, +, this is final compatibility equation that we may have, F 21, F 22 is equal to 0. Now what is D L1, D L1 is the displacement in the or the deformation in the primary structure subjected to external load in the direction of X1. And D L2 is the primary structure, displacement in the primary structure to do the external load integration of X2. And F 11, F 21, F 12 and F22, they are flexibility coefficient.

And F 11 is actually the displacement in the direction of, in the direction of X1 due to unit load, again in the direction of X1. Similarly F 12 is the displacement in X1 due to the unit load in the direction of X2. And F 21, F 22 also can be defined as this and their compatibility equation is this. Now what we will do, you need to find out all these, these displacements and this compatibility and these flexibility coefficients. Now what is D L1, D L1 is equal to again, we can apply unit load method, that is M into small m1 by EI dx. And then DL 2 will be the integration M, small m2 by EI into dx.

Okay, now this integration can be done over AB and BC separately and then sum them, again it can be done over AB and BC and then sumthem. Similarly F11 will be integration of m1 square by EI dx and F 12 will be integration of m1 m2 by EI dx. And similarly F 21 will be same as F 12, that is this matrix is symmetric and then F 22 will be integration of m2 square by EI dx. Now all these, all these moment expressions, capital M, small m1 and small m2, that we have already to mind here.

Now what we need to do is, we need to substitute these values in those integrations and then put the limit and get the integration. And if we do that, then the expression that we will get, D L1, D L1 we will get, D L1 will be same as the previous example which is QL to the power 4 by 3 EI and DL 2 will be QL to the power L cube by 3 EI. One good check is, so this is D L1 and DL 2, okay. Now you see just by looking at this expression can at least say whether they are correct or not, that depends on the, on the integration that you have to do this integration.

But whether they are wrong or not, that you can say by looking at them because you see their coefficients, in both the cases Q and 3EI, they are same but in 1st case it is L to the power 4 and in 2nd case it is L cube. The reason is, the reason is obvious because in the 1st case I degrees of freedom is the displacement and in 2nd case degrees of freedom is the rotation, okay. And the so they are dimensionally consistent, okay. So whenever you get the expression of anything, 1st you check whether they are dimensionally consistent or not and if you find they are dimensionally not consistent, then there has been some mistake and then you check check your calculation.

Now similarly you can get F 11, F 11 is, again it will be same as the previous one, L cube by EI and then F 12, what you get is, F 12 you will get 7L square by 6 EI, 6 EI. Similar observation it can see from, again F 11 and F 12 as well. Okay, F 11 corresponds to rotation and F 12 corresponds to displacement. And are they dimensionally consistent, yes, they are dimensionally consistent because you have to multiply these expressions, you have to multiply these equations by force and if you do that, then you will see this dimension and this dimension will be same.

And again this dimension, this dimension will be same. And similarly F 22 you will get, F 22 will be 5L by 3 EI. Okay 5L by 3 EI. Now all these expressions we have, now this we need to substitute in this expression and solvate and if you solve it, then you get X1 is equal to -0.55 and X2 is equal to - 0.18, of course it has to be QL, this is the coefficient at this has to be multiplied by QL, QL and this is QL square, okay. So this so this is the force and this is the moment. And again you can see they are dimensionally consistent.

So this is the, this is the 2nd example which is very similar to the 1st example but we have just provided one more, one more constraint. Now the idea has been to demonstrate the concept but once you understand the concept, similar concept can be applied to any kind of structure, intentionally I took a problem where you bending moments are, the sum of the, the expression for bending moments are not very complicated. The reason is we have to we have to demonstrate the concept in a very in a very short time but again once you understand, once you have understood, if you have understood the concept, please go through the exercise given in the book and make yourself comfortable with the method.

Okay, so this week we have discussed, we started with previous week, we discussed what is the force method and how the force method can be used to find out internal forces and support reactions for different indeterminate structures for trusses, for beams, for frames. There is another method called displacement method, if you remember to, one or 2 weeks back we just briefly outlined what is displacement method Vis-a-vis force method. Now next, next week , this is the 9^{th} week, we have another 3 weeks for this course, so next 3 weeks what will do is we will discuss different kinds of displacement method. So next week what we will start is displacement method, okay. See you in the next week, thank you.