**Structural Analysis I.**
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**Lecture-45.**
**Analysis of Statically Intermediate Structures by Force Method (Continued).**

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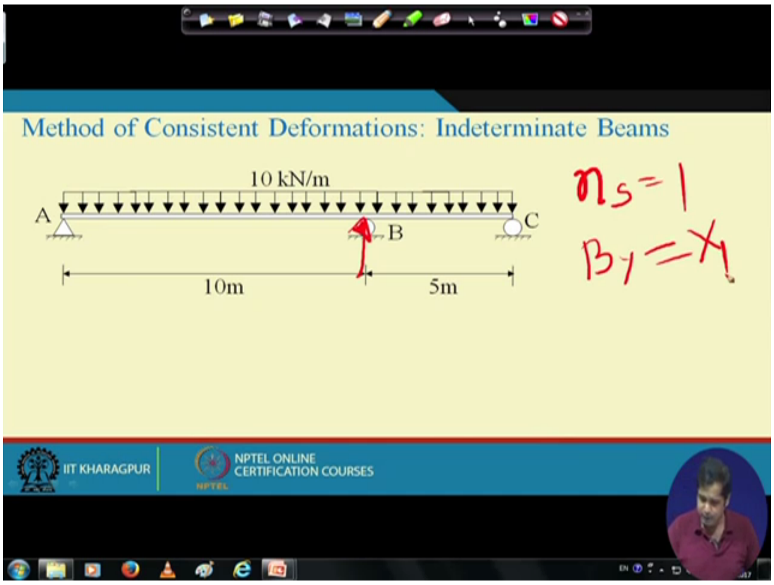


Hello everyone, welcome, what we will do today is we will apply, what we have been doing this week is we are applying force methods for analysis of statically indeterminate beams and frames. Today we will see how to use method of consistent deformation for indeterminate beams, okay. Now let us take the 1st example, the 1st example is, A is a 2 span, it is a continuous beam, 2 span continuous beam which is subjected to a uniformly distributed load, okay. Okay, now if you remember, if you remember the steps of method of consistent deformation, those steps remain same, in the last, last week we applied method of consistent deformation for indeterminate trusses.

The steps that we discussed, exactly the same steps but since the internal forces are different, there will be slight modification in the equation that we use in different steps. Okay, now the 1st step was, if you recall, the 1st step was determine the static indeterminacy. Now look at this structure, this is indeterminate structure and if we remove support B, then it becomes is simply supported beam subjected to UDL and because of the presence of support beams, it is it is now indeterminate. So in this case static indeterminacy is 1. And what is the kind of intimacy, it is external indeterminacy because indeterminacy is due to the support reaction. Okay.

So once this, once we understand, once we calculate the static indeterminacy, the next step is to identify the, identify the redundant force or redundant moments. Now let us take BY as redundant force, let us take BY as redundant force, okay, this BY, the vertical reaction at Y as redundant force. Okay. Suppose this is, since, suppose this is equal to X1, okay, the written, the reason I am writing it as X1 because if you remember in the class problem, when we had Multiple, the static indeterminacy is more than one, then this is how we identify each redundant forces.

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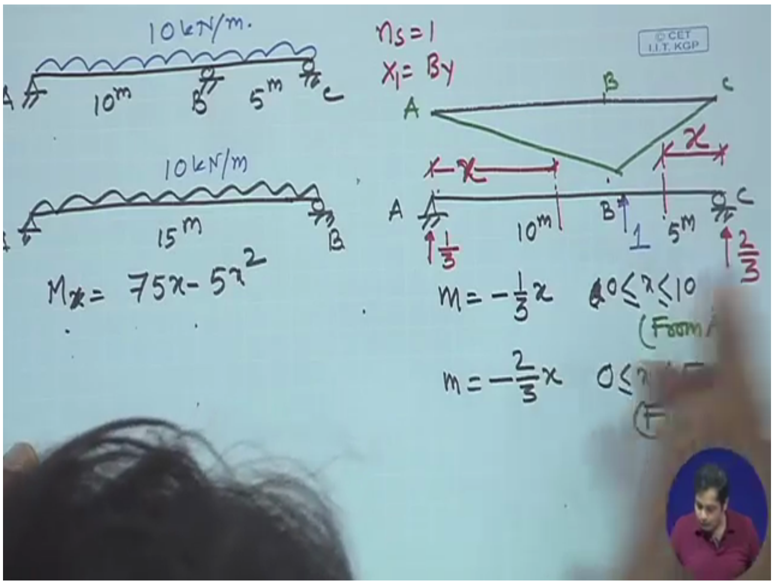

We denoted each redundant force as X1, X2, X 3 and so on, okay. In this case we have just one redundant force, let us write it as X1. Okay. Now let us see we can proceed with the calculation. So the structure, the problem what we have in hand is this. This is which is subjected to a uniformly distributed load 10 kilos Newton per metre. Okay. Now this is A, B, C, this is 10 metre and this is 5 meter, okay. Now we have already done step 2, we have identified the redundant force, NS is equal to 1, NS is equal to1 and X1 is BY, that is the redundant force, we have just one redundant force.

Now the next step, the step 3 is, we need to now 1st before we proceed 1st of 3, now the, we need to release, we need to remove this redundant force and get the primary structures. So this is the primary structure which is subjected to external load A, B, and this becomes 15 meter, okay. And then another primary structure which is now subjected to a unit load, so X1 is equal to 1, okay. Now this is 10 metre and this is 5 meter. This is A, then B, then C. Okay. Now step 3 is, we need, step 3 is find out the find out the internal, find out the associated deflection in the primary structure due to the external load.

And step 4 is find out the associated deflection in the primary structure in the subjected to the redundant force. Okay. Now this is simply supported beam which is subjected to the UDL, so we know that if we take any section at a distance X, then this becomes, M X becomes in this case it becomes 75X - 5X square, that you can easily check, WL by 2X - WX square by 2, okay, this is X, W is equal to 10 and L is equal to 15 meter, okay. So this is equal to, this is equal to capital M, okay, this slide, this is capital M.

Similarly what is the small m here, what is the bending moment diagram for this if we if we recall, this is simply supported beam subjected to unit, subjected to concentrated load at some distance, then the bending moment diagram of this will be like this, bending moment diagram will be like this. This is A, this is B and this is W, WAB by L, bending moment written down, bending moment is shown here, the downward because for this case this load is acting upwards and therefore the deflected shape, the deflected shape of the beam will be like this, so the the bending moment that would be generated, it is hogging bending moment because this load will cause hogging in this beam.

So hogging bending moment is negative as per our sign convention, that is why it is drawn like this, okay. Now what is small m, small m means the bending moment due to the unit load in the direction of redundant force. So small m is equal to, in this case your, vertical reactions will be if you see the vertical reactions, this vertical reaction will be, this section will be 5 by 15, means one $3^{rd}$ and this reaction will be 2/3, that you can easily verify, okay. So M will be, if we take a section between, if we take a section, this is X, this is X, then m will be, this value is X, then m will be -1/3 when X is between 0, X is between 0 to 10, okay.

And moreover X is measured from A, X is measured from A. Nycil take, this is the bending moment between this, this is equation for this line, bending moment between A and B, so bending moment between B and C will be M, which is m, in order to get the bending moment, take a section here which is distance X like this. Okay. We could have taken distance X from this support as well but we will see that if you take distance X from C, this equation becomes simple. At this expression will be -2/3 of X, okay, and in this case X is between 0 to 5 but this X is measured from C.

Okay, this is, this please be very careful, okay. So this is essentially equation of this line but this X is measured from C, okay. Now, great. So once we once we have this, once we have this, next if we need to solve this. Right, we need to find out the displacement at point B, displacement at point B, vertical displacement at point B due to this external applied load.

And suppose that displacement is equal to DL1, if you recall, when we, in the last week this is how we denoted the displacement or the associated displacement in the primary structure subjected to externally applied loading. Okay.

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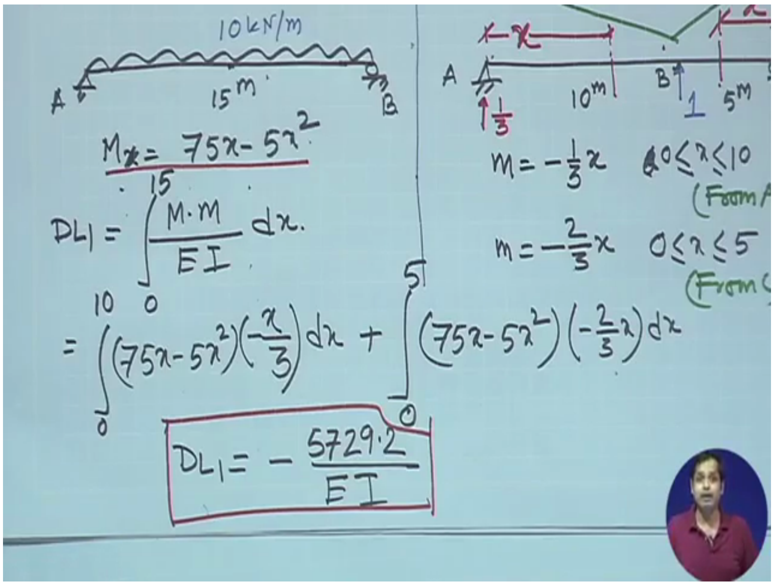

So DL 1 is essentially the displacement due to the displacement in the primary structure, due to the externally applied load but that displacement in the direction of X1 and this X1 is the value of X1, direction is, okay fine. Now this is equal to, if you recall unit load method, that is M into small m divided by EI, EI is constant in this case into dX. Right. Now these this expression can be divided into 2 parts, this is over the entire length 0 to 15 and this expression, this integration can be divided in 2 parts, one is between A to B and then between B to C because expression for bending moment are different for different parts.

So this becomes 0 to 10 and then then M is this value, 75X - 5X square, and then between 0 to 10, this is the value of small m, so this is X by 3 - X by 3 dx. Okay, so this value, it is from 0 to 10 + then 75X - 5X square, this is between B and C, and then again this is -2/3 of X dx and this is 0 to 5, okay. Please note that this expression, this expression remains same whether you calculate, whether you take X from this side or X from this side because it is a, it is a symmetric it is a symmetric loading.

So this is D L1 and if you do this integration, the value what you get is D L1 is equal to, I am not, I am not doing this integration here - 5729.2 by EI. So this is the displacement in the primary structure due to the external loading and this displacement is in the direction of redundant force. So this is the L1, okay. Now, if this is D L1, next step is to find out, find out the deflection in this structure due to this unit load. And that deflection is essentially the flexibility flexibility coefficient. So next we need to find out what is F11, what is F 11 for this structure, for this, F 11 for this.

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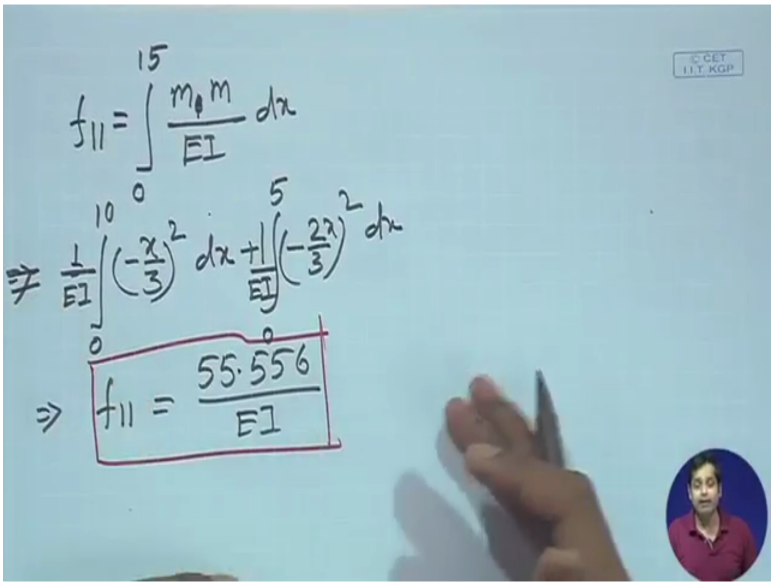


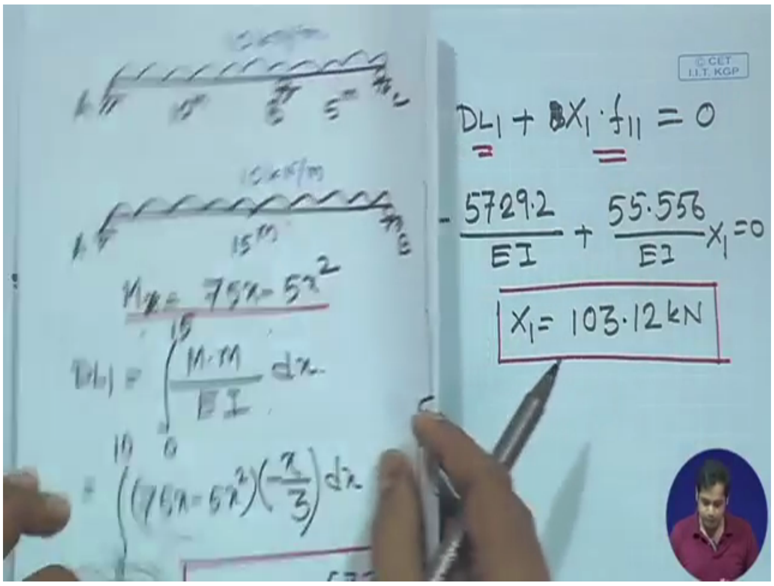


Now what is the value of F 11, S 11 will be 0 to 15 because this is the total length, then m1 into, in this case there is no m1 because there is just only one redundant force, so it is m into m by EI dx. Okay. And the expression for m, we know that this becomes 0 to 10, X is - X by 3 whole square dx 1 by EI and is equal to + 0 to 5, it is again -2 by DX 1 by EI, 1 by EI, okay. And this becomes, F11, if you do this integration and this becomes 55.556 divided by EI, okay.

So then this was step 4, then if you recall, step 5 was applying the compatibility condition. What is the compatibility condition we have, compatibility condition is at this point, displacement is equal to 0 because it is supported. So compatibility condition is that delta DL 1 which is the displacement in the primary structure due to the external applied load + X1 which is the redundant force into F 11 which is the flexibility associated with this redundant force is equal to 0. So this gives you the displacement in the primary structure due to the externally applied load and this gives you the displacement in the primary structure due to the redundant force, okay.

And F 11 is the associated flexibility coefficient, so this is equal to 0. Then if we substitute the value of D L1, DL 1 is equal to -5729.2 by EI + it is 55.556 by EI into X1 is equal to 0 and finally we will get X1 is equal to 103.12 kilos Newton, okay. Now once we get X1, the value of X1, in this in this problem , value of X1 means then BY is equal to, this is equal to BY, so we know what is the value of BY. So with the knowledge of value of BY, another structure becomes determinate, we can find out, we can draw the free body diagram of the entire structure, apply the equilibrium condition and then find out the other support reactions and member forces.

Okay, now this is the, this is the 1st problem, okay, now let us let us take, the step 6 is that apply the once we once we know the redundant force, apply the equilibrium, draw the free body diagram and draw the, determine the other unknown forces, that is the step 6, step 6 we are not doing here because we have done it several times in several problems, okay.

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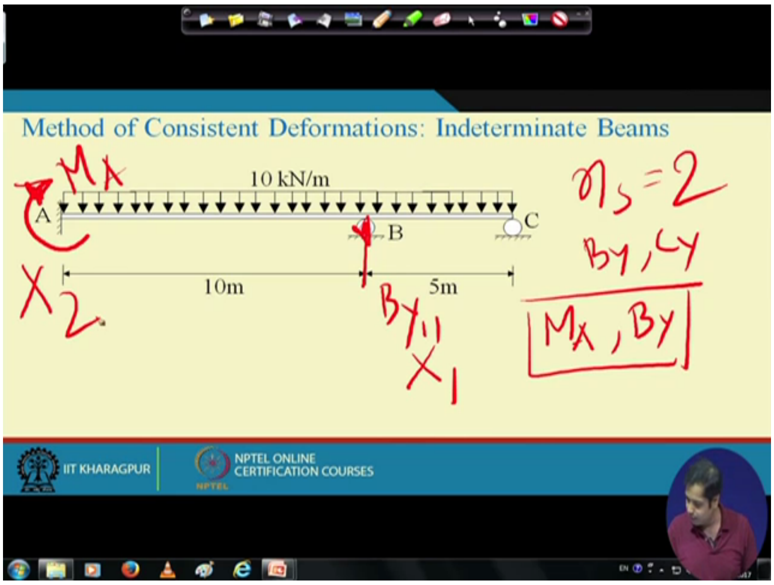

Let us take another example, this example is similar to the previous example, similar in the sense, the length and the loading condition but only difference is the pin support which was that A, that is now replaced by, that is now replaced by one fixed support here, if you see that is now replaced by one fixed support. Okay, now then what happens, what is the indeterminacy we have, it is, what is the indeterminacy we have with this problem, we , once we make, once we make, once we replace in support by fixed support, means we are adding one additional constraint, it is rotational constant, right.

So we are , for that the indeterminacy of the structure is increased by 1. So in this case, now what is the indeterminacy, indeterminacy of this problem will be NS is equal to 2 because if you remove B and C, support B and C, then it becomes cantilever beams which is stable and determinate and because of this additional constraint, the structure is indeterminate, the degree of indeterminacy is 2. Now the next step is the choice of redundant forces. Now there you have, when you have, when you have multiple redundant forces, more than 1 redundant forces, then your choices also become more.

Okay, but you have to choose those primary structures which are determinate as well as stable, okay. Now in this case, you can have you can have some choices like, we can take BY as we redundant, BY and CY as redundant, this is one choice. Another choice is we can take, we can take MA, MA and BY as redundant. Okay. And this in both the cases, the structures are determinate and stable. What we do is we go with this choice, okay, we will take the rotation at the moment at A and that support reaction and this moment, this moment and the support reaction at, so this is BY and this is MA, we will take these 2 as redundant, okay.

Now then this is, say this is X1, this is X1 and this is X2. Okay. Now, okay, let us let us find out, let us see how we can proceed with this method of consistent deformation. Now the 1$^{st}$ is, the step 1 and step 2 are done, then the step 3we need to solve the primary structures for different loading conditions. So since, now we need to solve the determinate structures, one is primary structure subjected to the external load and 2 primary structures which are subjected to 2 redundant forces, okay.

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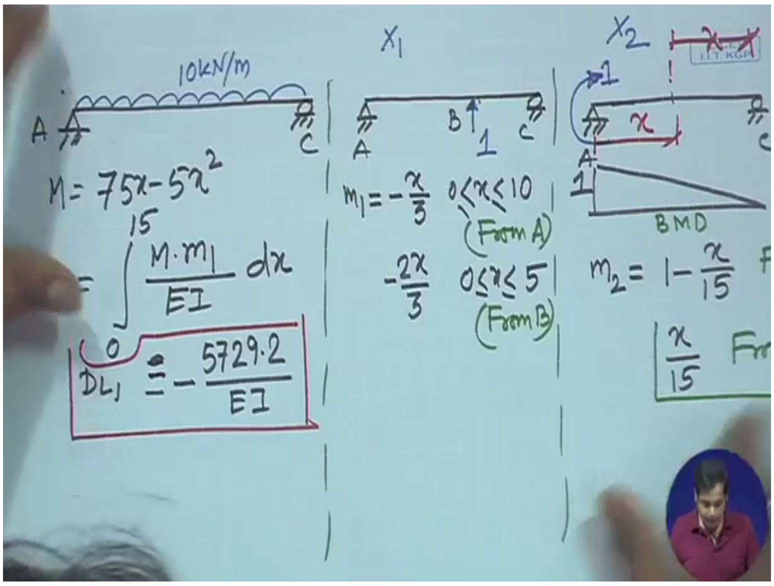


Now this is the 1st structure, the 1st structure, primary structure which is which is subjected to the subjected to the external load, okay. Then 2nd primary structure which is subjected to unit load and then 3rd primary structure which is subjected to unit moment. Okay. This is 10 meter, this is A, C, A, B, C, A, C, okay. Now, let us just divide this into 3 parts. Okay, now this is already we have done, right. Now this is X1, in this case it is X1 and in this case it is X2, the redundant forces is X1 and X2.

Now what is the, what is the expression for M, expression for M, just now we have seen that 75X - 5X square. Okay, now let us find out what is, what is M1 here, what is M1, if you if you remember, just now we wrote what is M1 and the M1 is - X by 3 and when between A and B, when X is between 0 to 10 and this is from A, measured from A and then M1 is equal to - 2X by 3 between 0 to 5 it is measured from B. That we have done it just now.

Now for this case , for this case what will be the pending moment I am, the bending over diagram for this problem will be something like this, okay, this value is 1, this will be the BMD, bending moment diagram for this. And what is the expression for m2, expression for m2 will be it is 1 - X by 15, if - X is measured from A, if X is measured from A, otherwise its value will be axed by 15 if it is measured from C, okay. This is important from C.

Okay, so if you take a section, if you take any section if we take any section and if you take X is this value, then this is the bending moment expression for bending moment but if we take X is this value, then this is the expression for bending moment which is obvious from this figure, okay. Now next step is we need to, next is what we need to find out, we need to find out the displacement in the associated displacement in the primary structure and then corresponding flexibility coefficient for this. So we need to 1st find out what is DL 1, DL 1, what is DL 1, DL 1 is the displacement in the primary structure due to the externally applied load and that displacement is in the direction of 1st redundant force, okay.

So DL 1 is essentially displacement at B and then we need to find out what is D L2, DL 2 will be the displacement in the primary structure due to the external load and that displacement is in the direction of X2. So DL 2 will be essentially the rotation at A. So and we know from unit load method, DL 1 will be, this is 0 to 15, 0 to 15, this is M into m1 into by EI. Now M, expression of M we know, expression of small m1 is this and then if we do that, it is exactly same, we have already determined it that DL 1 will be DL 1 will be in the previous example, that 5729.2 by EI, this DL 1, this is DL 1.

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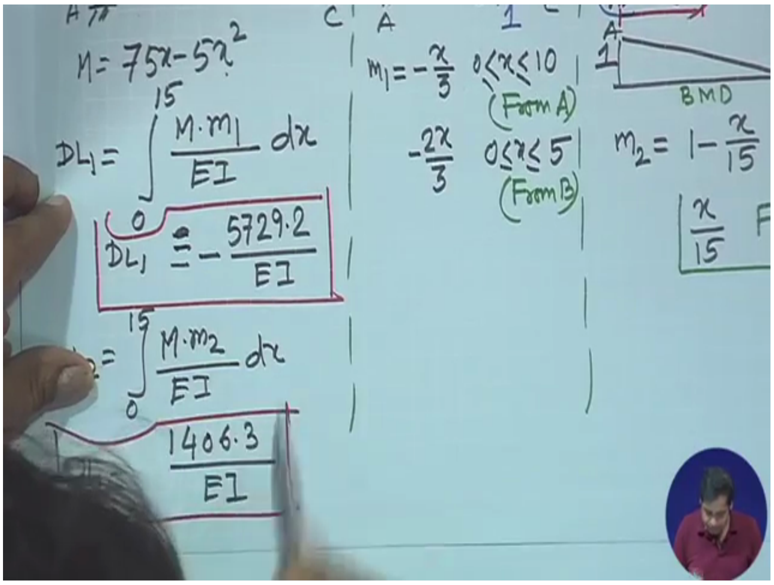

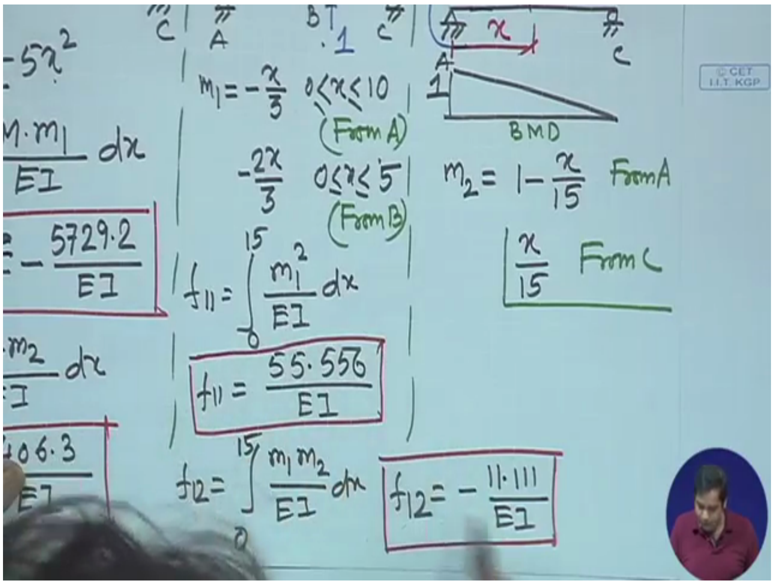


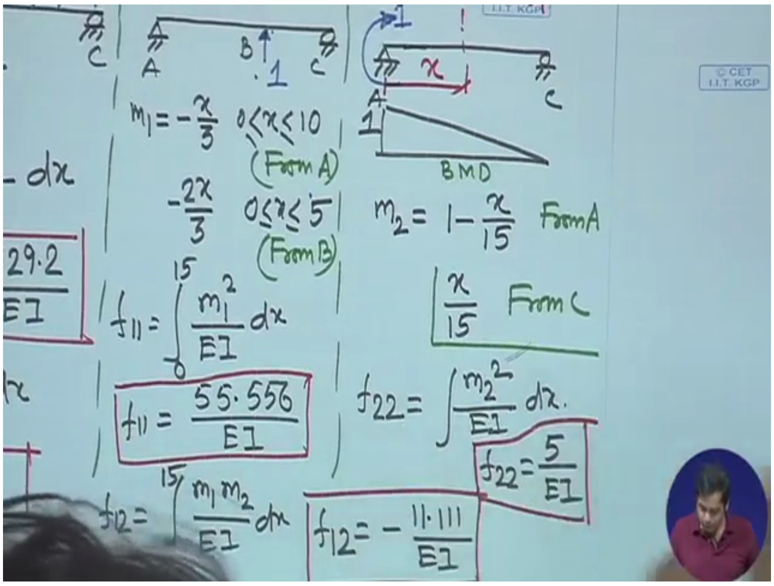


Similarly we need to find out what is D L2, DL 2 will be, this is just straightforward implementation of my unit load method M to by EI, okay. So DL 2 will be essentially the rotation at A to do the externally applied load. And what is the value, you can substitute M from, you can substitute M from this and small m2 from, m2 from this and if you do that, this expression becomes1406.3 by EI, DL 2. Please do these integrations and check these values. So we have obtained DL 1 and DL 2, the displacement in the primary structure due to the externally applied load.

Now what we need to find out, we need to find out the flexibility coefficient. F 11, F 11 will be the integration m1 square by EI dx 0 to 15 and m1 is this. And if we do that, that is equal to 55.556 by EI, F 11, this is F 11. Similarly we need to find out F 12, F 12 is the flexibility coefficient associated with displacement in the direction of X1 due to unit load in the direction of X2. Or we can say it is displacement in the direction of X2 due to unit load in the direction of X1, both are same because the matrix is symmetric matrix. Okay.

So this becomes m1 m2 divided by EI dx 0 to 15, okay. And if you do that, then F 12 comes, F 12 is -11.111 by EI, F 12 is this, okay. So we have already determined F 11, F 12, F 12 and F21 will be same, next we will determine what is F 22. F 22, F 22 will be m2 square by EI dx and m2 is this value, and that becomes F 22 becomes 5 by EI, 5 by EI, okay you can check it. Okay. So we have almost all the information available. We know the displacement in the primary structure and also we know the flexibility coefficient, next step is apply the compatibility condition.

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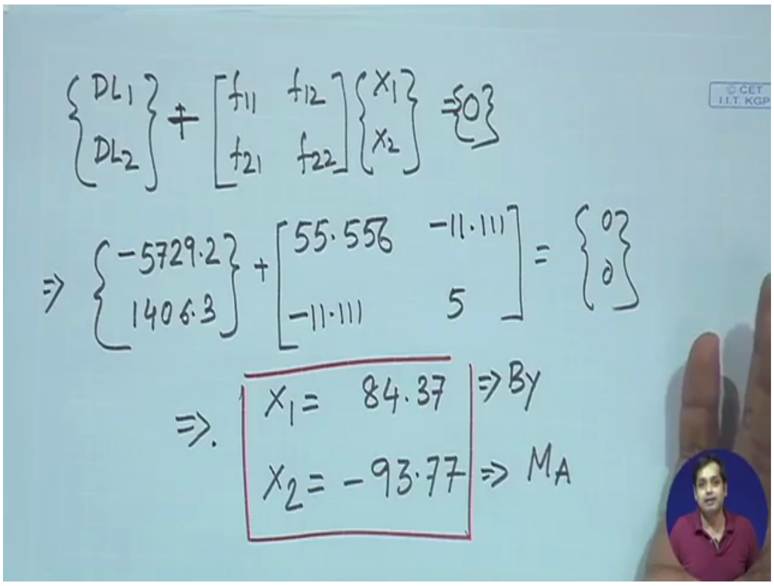


And what is the compatibility condition, compatibility condition is DL 1, DL 2, similar compatibility condition we have used for class problem as well. This is F 11, F 12, F 21, F 22 and then X1, X2, this is equal to 0. What it means, it says that, it says that displacement, if we see here, it says that displacement at point B, displacement at point B is equal to 0 and this rotation at point A is equal to 0, okay. Now if we substitute all these values, DL 1 is equal to -5729.2 and then 1406.3+ F 11 is 55.556, F 12 -11.111, F 21 will be same as F12 and then F 22 is 5 is equal to 0.

Now if you solve it, then we get X1 is equal to 84.37 and to X2 is equal to -93.77 -. Okay. So what was our X1, X1 is equal to support reaction BY, support reaction at BY and X2 is MA. So for this problem we have already, now these, with the information of BY and MA, now the structure is determinate, we can apply the, we can draw the free body diagram and apply the equilibrium condition to get the other support elections and internal forces.

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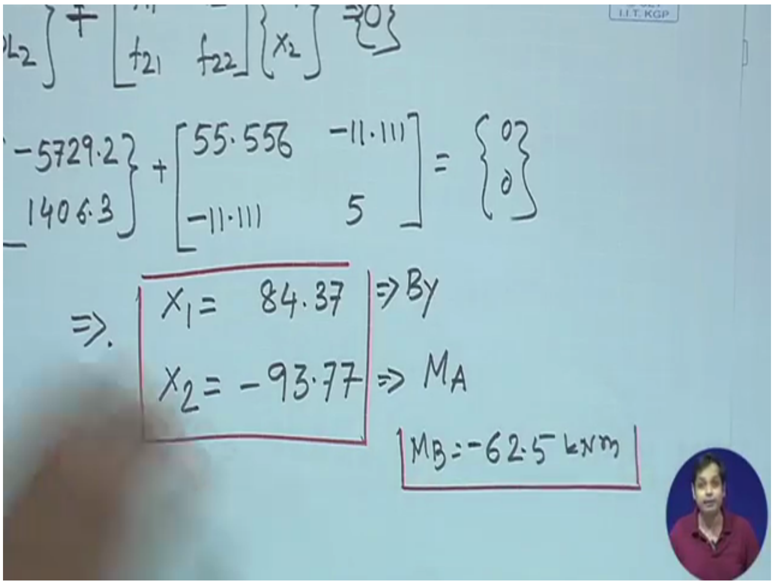


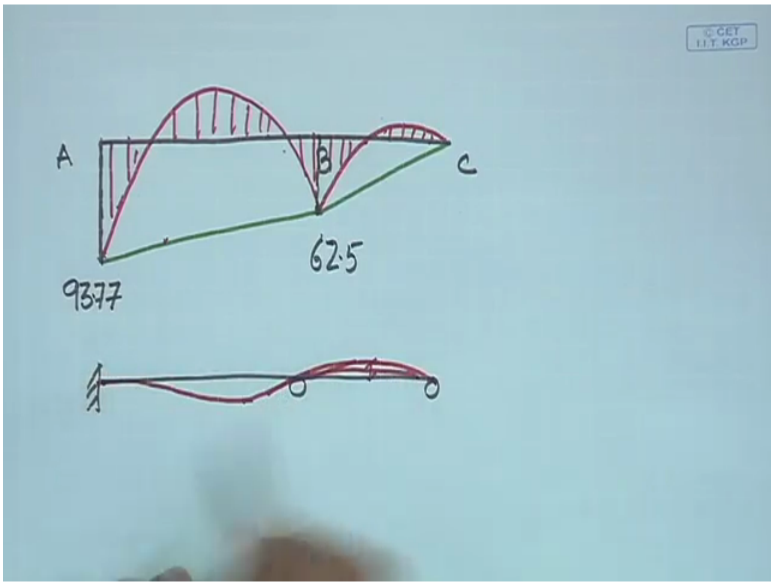


Now we can draw the, if you have to draw the bending moment diagram for this problem, let us draw the bending moment diagram. This was A, this was B and this was, this is C. Okay. Now one approach as I said, you see this bending moment is negative, this bending moment is negative, okay. Now one way is you draw the, if we apply, if we apply the equilibrium conditions, then we get MB is equal to, MB is equal to, I am not doing it, you can check it please 62.5, this is negative, kilo Newton meter. Okay, 62.5 kilos Newton meter.

Now then MA is 90, MA is, this value is 93.77 and then this value is 62.5 and then due to the redundant forces, the bending moment is and then constant because that C to 0. Now what is the bending moment due to the externally applied load, bending moment due to the external applied load is this. Okay. Now if you superimpose that on this, then your final bending moment diagram becomes like this. So this is bending moment diagram, okay, not the green one. So your bending moment diagram becomes this part, so this is bending moment diagram.

The advantage of writing, drawing the bending moment diagram like this, you do not really have to specify the sign. Now whatever bending moment is below this, it is hogging bending moment and above this, this is sagging bending moment. Now just try to guess what would be the deflected shape of this structure, the deflected shape of the structure, there is a support here, there support here and there support here. So naturally at this point, the slope will be 0, the deflected shape you may get, please verify this, the deflected shape something like this you may get you may get.

Okay, or if it is not something like this, you may get. Another, the reason I am drawing bending moment deflected shape here is you see by just looking at the bending moment diagram and the support condition we should be able to, we should be able to have some understanding about the deflected shape of the structure. Because you see naturally in this case between this part, it is hogging moment. So whatever way this beam deflects, the deflected position will be hogging position.

And between this point and this point, it is sagging moment, so between this part and this part, the beam will sag and again between this part and this part, the beam will, there will be hogging and again between this part and this part the beam will sag. But based on that you need to identify the deflected shape. Okay, this deflected shape is not drawn in scale, you may have this deflected shape or these values will be very small, if this value is very small compared to that deflected shape may look like differently.

But the point is it may deflect like this or you may have very small value, something like this, okay. So whenever you draw, whenever you, whenever you solve any problem whether determinate structure or indeterminate structure, whenever you draw bending moment shape diagram, parallely you try to draw the deflected shape, try to understand, try to, by intuition you try to find out, try to guess the what could be the deflected shape. And once you draw the deflected shape, then later we find out the deflectation and compare whether your intuition was right or not.

This exercise will help to develop the sense through which without actually solving the structure you can have some understanding of the behaviour of the structure. Okay, now what we do is, we stop here today and then next class we will see how the method of consistent deformation can be applied to frames, indeterminate frames, okay, see you in the next class, thank you.