Structural Analysis I.
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Lecture-44.
Analysis of Statically Intermediate Structures by Force Method (Continued).

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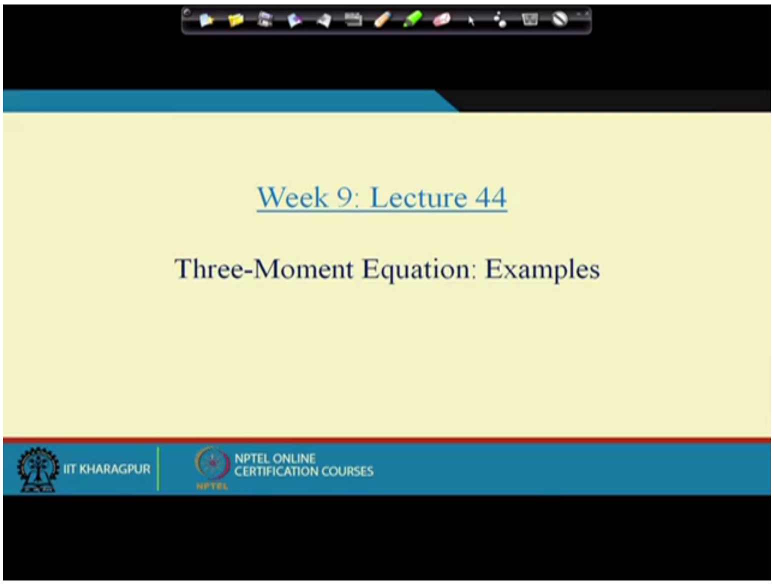

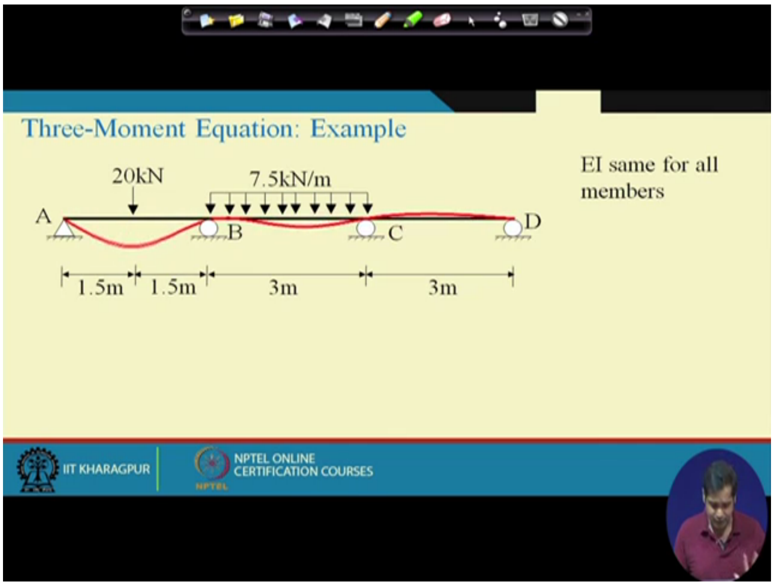

Hello everyone, let us continue with what we have been doing since last, since last class. Last class we had one demonstration of Three-moment equation that demonstration was through a continuous beam, 3 span continues Bhim, okay and they were subjected to some external loading and we determined what was the bending moment, what was the bending moment at different supports and then what would be the corresponding bending moment diagram.

Okay, and this was the problem we considered, we considered yesterday and this is the deflected shape of the beam.

Now, in this example, the fractural rigidity of all the span AB, BC and CD, all the fractural rigidities were considered as constant, okay. Now those fractural rigidity but in real life structure, that fractural rigidity for different members may have different values. Now we will see, if they have different values, then for the same problem, same beam, same length and subjected to same loading conditions, how they behave differently, different spans behave differently. Okay, now this is, this was the deflected shape of the beam where all these, all the fractural rigidity were EI.

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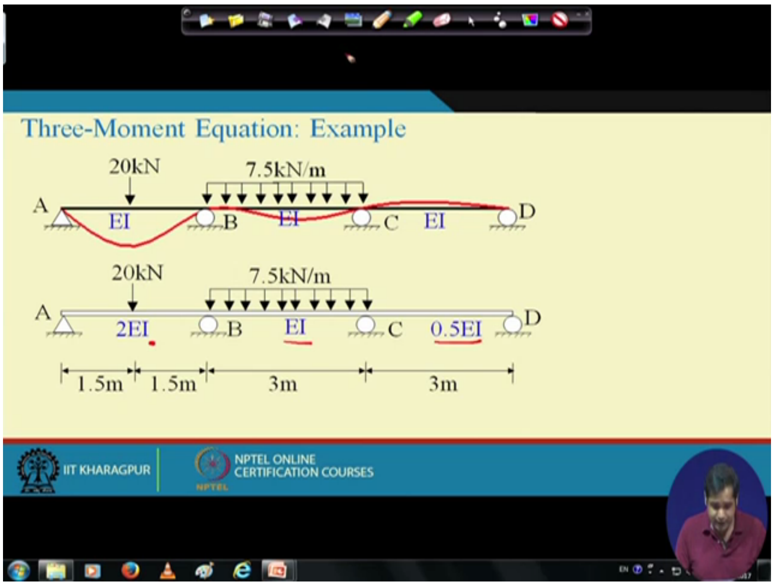

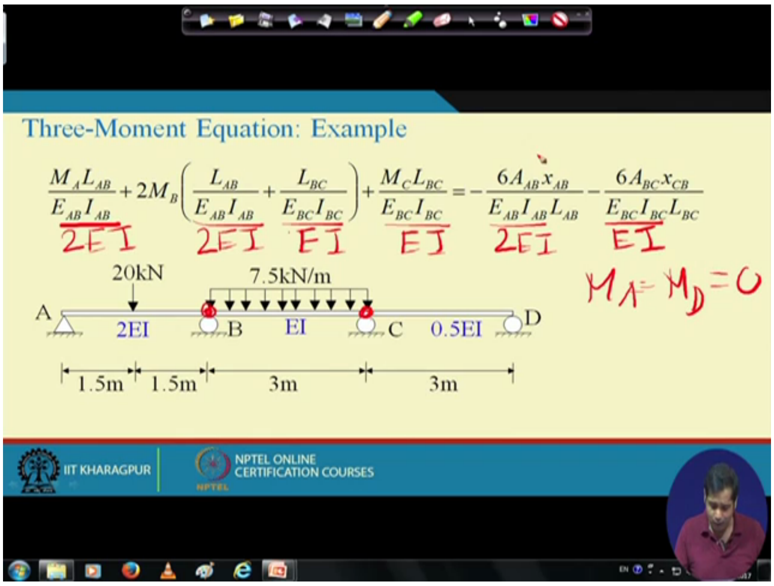

Now suppose, now the fractural rigidity is different for different spans. So for member, for member AB it is 2 EI and member BC it is EI and for member CD it is for member CD it is 0.5. Means what exactly we are doing, we are making member AB more stiff, member BC as it is and the member CD less stiff, more flexible. Okay. So we reduce the stiffness of member CD by 2, half, half of the previous stiffness and we increase the stiffness of member A B by by 2, okay. Now let us see what happens for this case now.

As far as the solution is concerned, it is exactly the same, the equations are concerned, it is exactly the same, only thing is now is, now since these EIs, these values are not same for different members, now these values are not same, these values are not same, these are different for different members, these are different for different members and we, in the same equation, the procedure is exactly same, we need to consider one hinge here, and another hinge here because for this structure it is, the static indeterminacy means 2.

We need to introduce 2 redundant forces and the redundant forces in this case is moment at B and moment at C, that means the primary structure will be the same structure but with 2 internal hinges at B and C. So we are, we actually release rotational constraint. Then apply the Three-moment, Three-moment equation at B and then apply Three-moment equation at C. When we apply Three-moment equation at B, consider span ABC and when you apply Three-moment equation at C, we again consider span BCD. Moment at A and moment at, we know that moment at A is equal to moment at D and that is equal to 0.

So only unknown will be moment at B and moment at C. The entire process is exactly the same, that we have done in the previous class, but the only difference here is in the previous class, all EIs were constant, same value in the equation there was no, we did not, we did not write the equation in the terms, in terms of the EI. Now since it is now it is different, so when we consider span, say we consider span ABC, then this case, it becomes, it becomes 2 EI, it becomes 2 EI, then in this case it becomes 2 EI and then it becomes EI, EI and it becomes EI.

And then again it becomes 2 EI and it becomes EI. All other values LA B, M, LAB, AB XB BC XB, they are all same because they are the moment due to the primary, moment due to the primary structure due to the external load. So now then we get 2 equations in terms of MB and MC, solve them and then get the value of MB and MC, once we know the redundant forces, then rest is just to draw the free body diagram of different parts and apply the equilibrium conditions. Okay.

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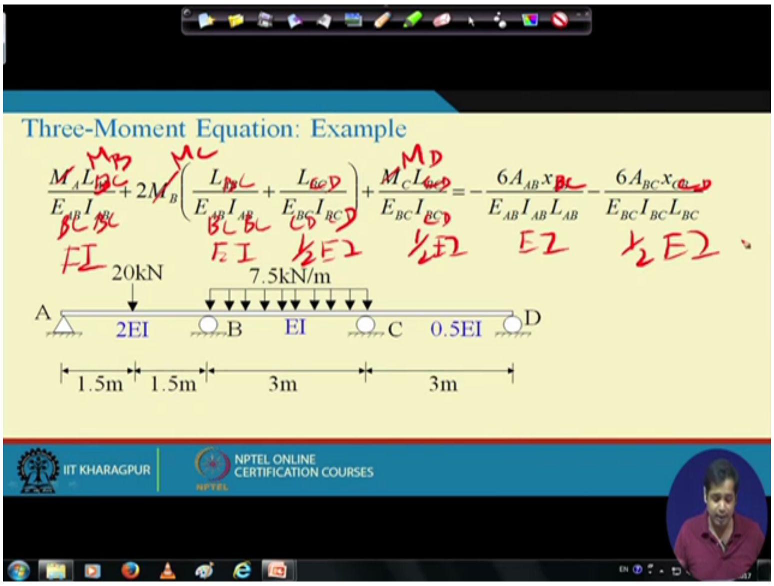

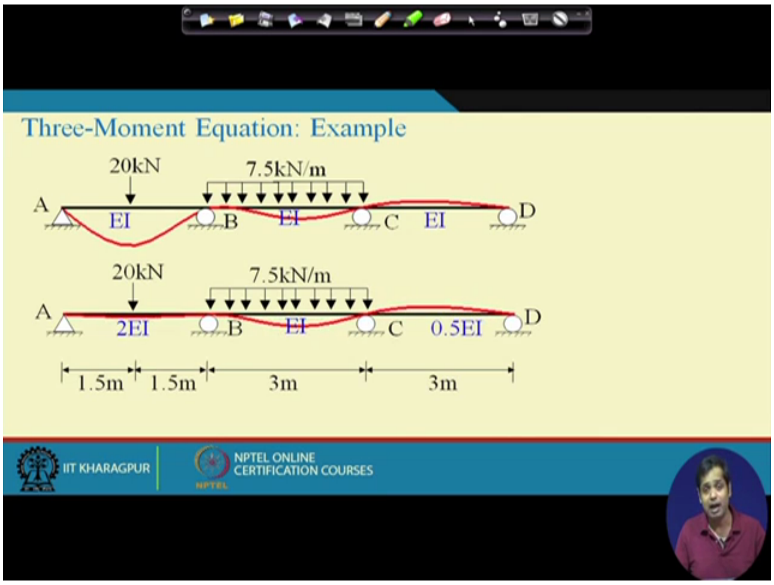

Now similarly if we have to do it for, this was for Three-moment equation at P. Now if we have to do it for BC, then, if we have to do it for BC, then suppose this is BC, this is BC, this is for BC, this is for BC and BC, this becomes CD, this becomes CD and this becomes CD, this becomes for BC, BC and this becomes for CD, okay, CD. And then this becomes MC becomes MD and this becomes M, MB, okay and this becomes MC. Then the difference will be, this is EI and this is EI, this is half EI and again this is this is half EI, this is half EI, half EI and this is EI and this is half EI and again rest of the process is exactly the same, you apply the equations, solve equations and then get the unknown and then finally draw the free body diagram of different parts and apply the equilibrium condition to get the unknown. Okay.

So this is the procedure when your the fractural rigidity of different members are different. Okay. Now let us take another example, so this is, this was the example for, this is interesting, this was the deformed shape for all the rigidity, fractural rigidity of all the members were same. Now if they are different, then this is the reflected shape. Now the, both the deflected shapes are scaled to the same value, their scaled by same factor. Now then you can, we can easily see here, when we increase the fractural rigidity of member AB, means it is not deformation of the member, the stiffness is a more, so natural the deformation in the member will be less.

On the other hand the member CD , the stiffness of the member CD was reduced and then therefore the deformation will be, we can expect more deformation in member CD and which is evident from this deformed profile that this deformation is more and this deformation is less compared to this part. Okay.

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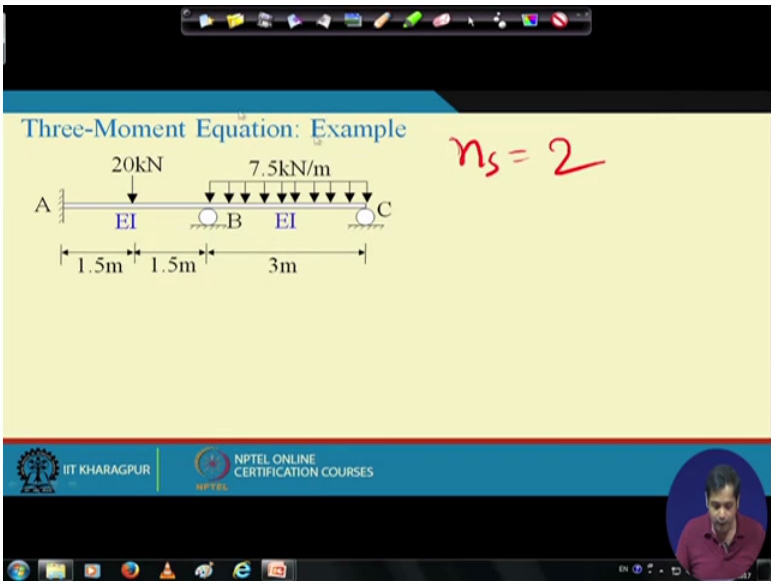

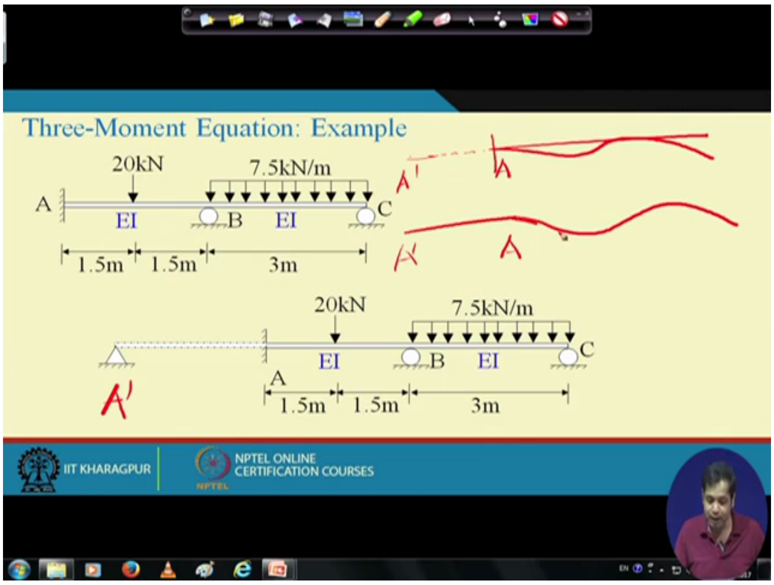

Now take the another example, okay. Take another example, this is again almost similar example but instead of what we did is, we removed the span AB and provided one fixed support here, one fixed support here at A. Okay. Now what is the kinematic, what is the static indeterminacy of this problem, again the static indeterminacy is, if we remove, if we remove B and C, then, then also it becomes the cantilever beam which is determinate and stable. The presence of support at B and C make the structure indeterminate, so in this case the static indeterminacy NS, NS is equal to 2.

So we expect 2 redundant forces, okay. Now next is, next is, now this is, you see this is the beam, now how to consider, how to treat, when your support is fixed support. Now what it means, suppose, what it means fixed support means the displays, it has all degrees of freedom of constraints, right. Now all degrees of freedom of constraints means and this means, what happens the, if we extend, suppose this is the actual length of the beam, AC is the actual length of the beam and if we assume a fictitious beam, say fictitious length of the, imagine a length of the beam, which is, the imaginary length of the beam which is say it is, say it is A dash, say it is A dash, A dash could be of any length, okay.

Now what happens, since at point A, the displacement and the rotation, all are, all constraints, the all degrees of freedom are restricted okay are constrained. Now we assume, we can assume that, that the, if we if we extend the beam up to an imaginary length A dash, that could be any value, then the rigidity of this beam, there is no deformation in the beam, there is no deformation in the beam. Now if we consider the beam ABC, then the deformed shape probably the deformed shape of the beam, if this is ABC, then the deformed shape of the beam probably this is something like this and then something like this, okay, probably for some loading condition.

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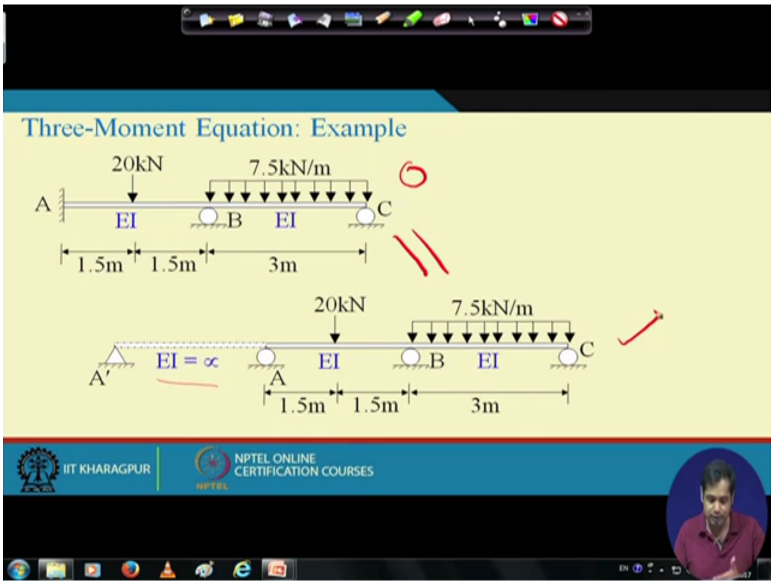

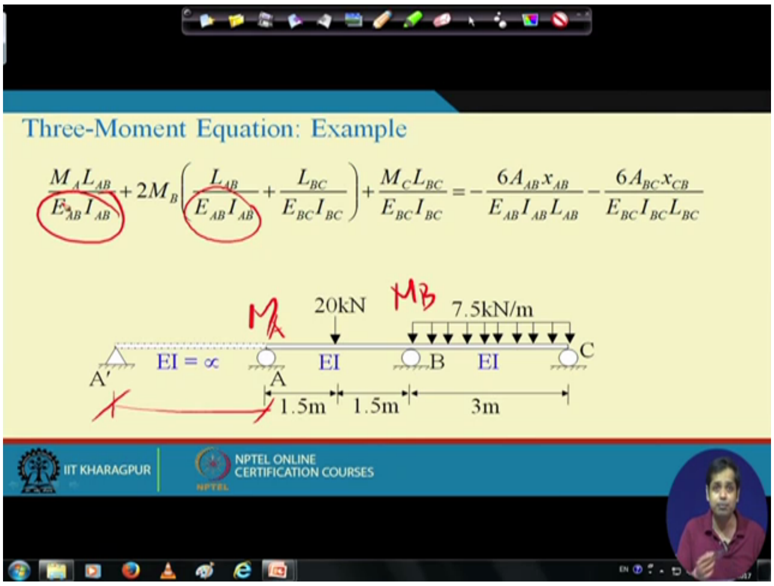

Now if it is extended, if it is extended to some length, some length say it is A dash, A dash, then it means that up to it is A, up to A dash to A there is no deformation and then from there the deformed shape becomes like this. So means A dash A is the beam length, an imaginary beam length but which has infinite structural rigidity, infinite rigidity. Therefore since it has infinite rigidity, so there is no deformation in the member in the in this fictitious length A A dash.

Okay now if it is common then take this beam remember, the beam ABC, actual beam is ABC, beyond this fixed support, assume there is a fictitious length A A dash which is A A dash and then this fixed support, if you remove, if you remove by a roller support like this and make this is A dash, and make this rigidity fractural rigidity of the member A dash is infinite, infinite, then this is an, this idealisation of this beam, they are similar, okay, behaviour will be similar. So what we do is instead of considering this beam, we consider these beams because they are similar, okay.

Now if we do that, then what happens , so the equation, this is equation, the same equation but important part is, now we need to apply the Three-moment equation, 1st at point A and then apply Three-moment equation at point B, okay. Now when we apply Three-moment equation at point A, then we need to consider this plan A dash A and AB. Irrespective of the length of A A dash, there is no length written here, what is important here is now the fractural rigidity is infinite, so if the fractural rigidity is infinite, then the corresponding contribution, this part becomes infinite, this part becomes infinite, so therefore their corresponding contribution, since it is 1 by EI, the corresponding contribution becomes 0.

So what you get is, if you apply Three-moment equation at point A, so contribution from this part becomes 0 and then what you have, you have a relation between member, between bending moment at B and bending moment at B. Then you consider the Three-moment equation at B and then MC is equal to 0 here because it is it is roller support, then again from by considering Three-moment equation at B you can get a relation between moment at A and moment at B. So you have 2 equations which give relation between moment at A and moment at B and then if you solve those 2 equations, we can get what is the value of moment, what is moment MA, what is MA and what is MB, we can get this.

Now this step is, this length, this length is immaterial because you see here, irrespective of the length, if this is A dash A, always we have 1 by EI term, 1 by EI term, so irrespective of the value of this length, this, since it is 1, this EI term is infinite, this contribution for the support A dash, for A dash A, this span, this contribution will become 0. Okay, so this is how you can idealise a fixed a fixed support when you apply Three-moment equation, okay.

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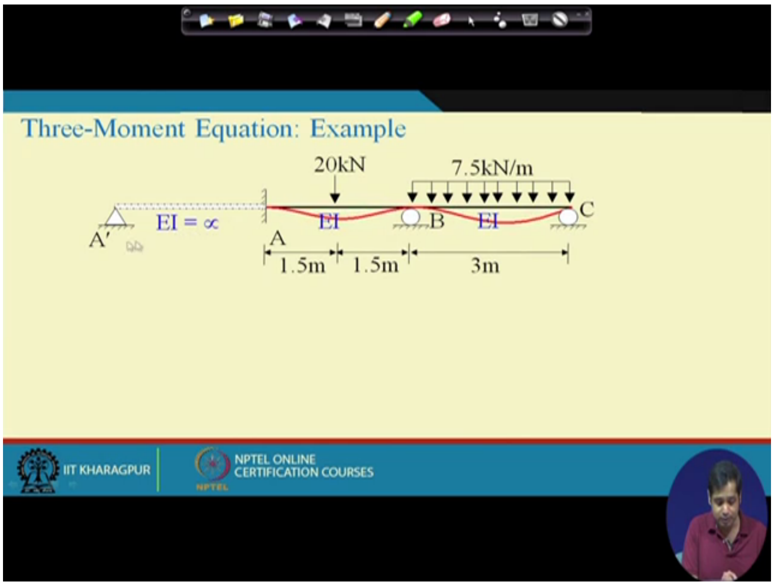

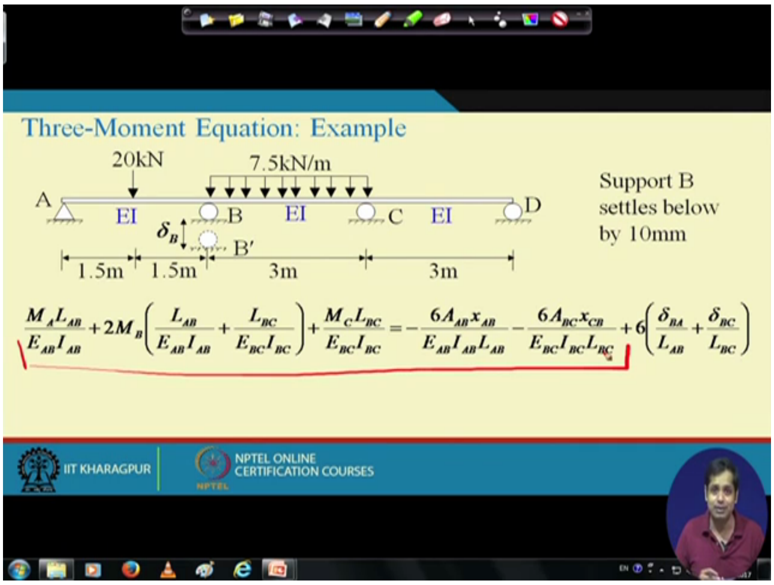

Now next is if we see the deformed shape of this, this is the deformed shape. Okay, the deformation between A dash and A because it has infinite rigidity, okay. Again the deformation is scaled here, so this is how the deflected shape of this bend will look like. You can solve this applying the Three-moment equation and get the solution, I am not giving the solution here, I am just discussing how you can solve it. Okay, there are several examples given in the book, you can go through those examples and practice those examples, okay.

But the concept behind applying Three-moment equation for this kind of problem is this, okay. Now, so so far what we have done is, we apply Three-moment equation to a problem where it is mainly subjected to the some external loading, either in the form of distributed forces or concentrated forces or moments, okay. Now as I said the external agitation may be given to the structure by allowing a support to move certain distance, so the support may settle for some reason. And because of the settlement of the support, the structures, instead of in spite of any presence of any external loading, the structure may, structure may be, some stresses may induce in the structure and correspondingly moments and stress results and moments and shear, also we can have in the structure.

Now this is one example, with the same example like the previous one, but in this case, in addition to the externally applied load, what we have is the support B settles below by 10 MM, the support B settles below by 10 MM. So if the support is, if the support B is settled below by 10 MM. And if the B point B goes to B dash and this distance is Delta B and this Delta B is 10 MM. So this delta B, this is equal to Delta B, the shabby is equal to 10 MM. Now if you recall, this is a general expression, in the previous example we did not consider this, we did not write this part because there was no support settlement in the previous example.

So these values of Delta were 0. But since in this case, these values are nonzero, we need to consider this part as well. Now this is the general expression, now up to this example, up to this part, this part is exactly same as before. Okay. Now this part is due to the settlement of the support, okay. Now this part will, this part if you consider only this part, this is the, this part will give you the moments generated in the structure due to the external load and then this part, the this additional part will gives you the additional momentum generated in the structure due to the support settlement. Okay.

Now what is Delta, what is Delta BA, Delta BA is equal to Delta B - delta A, it is the relative displacement between point B and point A and similarly Delta BC is equal to relative displacement between point B and point C. And we know that Delta B is equal to Delta, this is delta B, in this case we know delta B is equal to 10 MM but delta B is equal to 10 MM and delta A and Delta C, delta A and C is equal to 0. So when we apply Three-moment equation at B, delta A and Delta C is equal to 0, we can calculate correspondingly what is delta B and BC and use that in this equation, get the expression, get an equation for M B and MC and then apply Three-moment equation at point C.

But in this case again, delta B is 10 MM but delta C and Delta D is equal to 0, again we get an equation for MB and MC and then by solving those 2 equations we finally get what is the value of MB and MC. And once we have MB and MC, rest of the thing is very straightforward, it is same as an statically determinate structure. Okay. Now if you see the, if you have to see the deflected shape of this obvious beam, the deflected shape looks like this.

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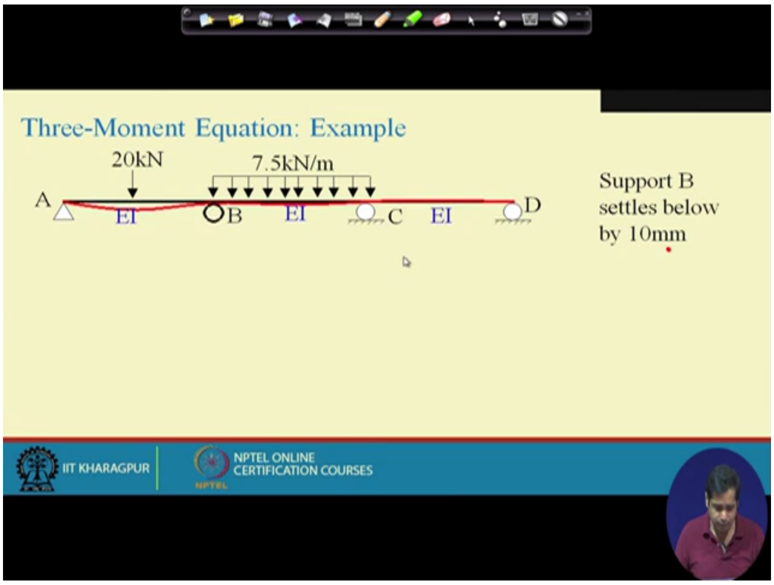

So you can see here in this example, how this, how the support, the support settled below, the moment of the support can be easily seen. The moment of the support can be seen here, this moment of the support can be seen here. Okay. Now this is the application of, okay, so this, these are some applications of Three-moment equations for continuous beam. As I said we actually in this class we actually analyse just one example and rest of the examples are in some way slight modification of the 1st example.

Again the idea has not been to show you how to solve, to show you the different steps, because that we have already discussed, the idea is if you have, if you come across different kinds of problems, then the same Three-moment equation, the general Three-moment equation, how that Three-moment equation can be used for different conditions, okay. So the concept will be clear when you, when you actually practice some examples that if you can take any book a structural analysis and please do some examples to make yourself comfortable with the concept of Three-moment equation.

Okay, so what we will do in the next class, next class we will use the method of consistent deformation, this is also a method of consistent deformation but in a different form as we discussed. The Three-moment equation is also the premise is consistent deformation. Now what we do next is we will, we will apply method of consistent deformation next class to indeterminate beams for solving beams which is indeterminate. Okay, see you in the next class, thank you.