Course on Structural Analysis 1
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Lecture 41
Module 8
Analysis of Statically Indeterminate Structures: Method of Consistent Deformations (Continued)

Hello everyone again let us continue with the analysis of statically indeterminate truss using method of consistent deformation. Now this is the last class of this week I believe by now you have at least understood the concept and its application to indeterminate trusses just when reinforce your concept reinforce the understanding of the method let us take the final example of this week.

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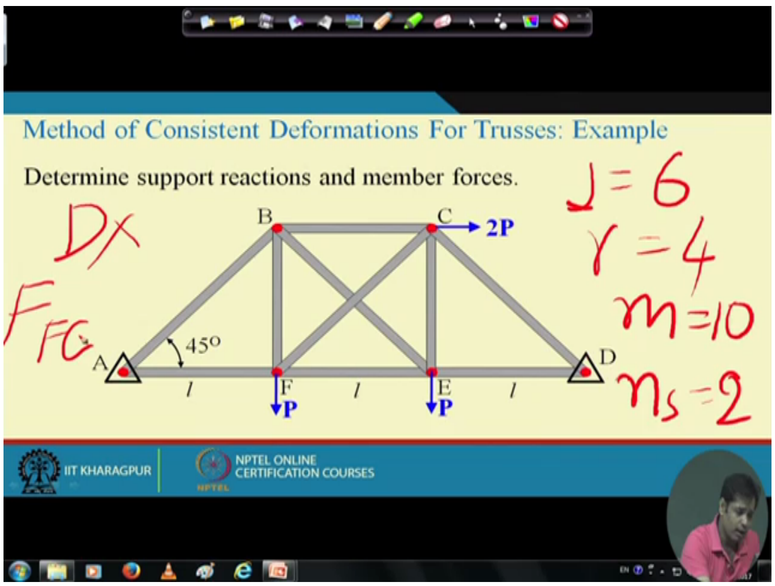

Now in this example, okay so this example is this, it is an again an indeterminate structure indeterminate truss let us find out what is the static indeterminacy in this in this case. So the number of joints we have 1, 2, 3, 4, 5, 6 number of joints J is equal to 6 J is equal to 6, so number of equations available 2J which is 12 and then number of reactions we have 2 reactions here and then another 2 reactions here total reactions r is equal to 4 and then number of members 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 member is member is 10 so number of unknown is m plus r which is 14 and number of equations available 6 into 2, 2 per joints 12. So ns is equal to ns become 2, okay ns is equal to 2, so static indeterminacy for this problem is 2.

Now see whether it is external indeterminacy or internal indeterminacy you see here the number of reactions are 4, but in this truss if we if we instead of hinge support here if we provide roller support at D, then also this structure is stable. So the additional horizontal constraint at D that makes the structure indeterminate, so 1 indeterminacy is due to the additional horizontal constraint at D.

And then another indeterminacy so this is external indeterminacy so the structure has external indeterminacy 1, similarly structure has external internal indeterminacy 1 because you see whether 1 of the diagonal members this member or this member they are additionally even if you do not give them one of them, then also this structure seems still stable. So the presence of this diagonal or this one of this member make the structure indeterminate. So that indeterminacy is internal indeterminacy, so the structure has 1 external indeterminacy and 1 internal indeterminacy so total indeterminacy is 2.

So this is the first step, and the second step is identify the redundant force so we need to since indeterminacy is 2 so we need to identify 2 forces, one is associated with the reaction and one is associated with the member force. So what redundant force we take? We take Dx as redundant so and then member BC say member FC so the force in member FC we take as redundant, okay. So this is external redundancy external, this is the reaction at D and this is the force in member FC, okay.

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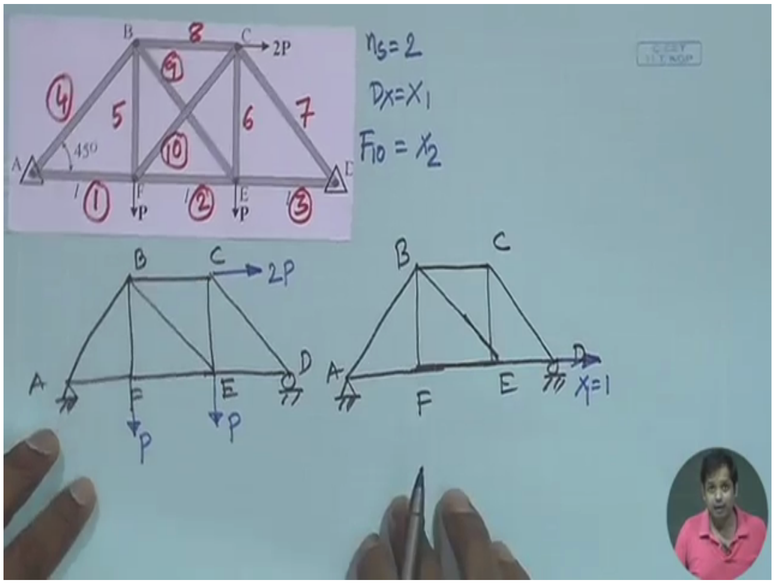

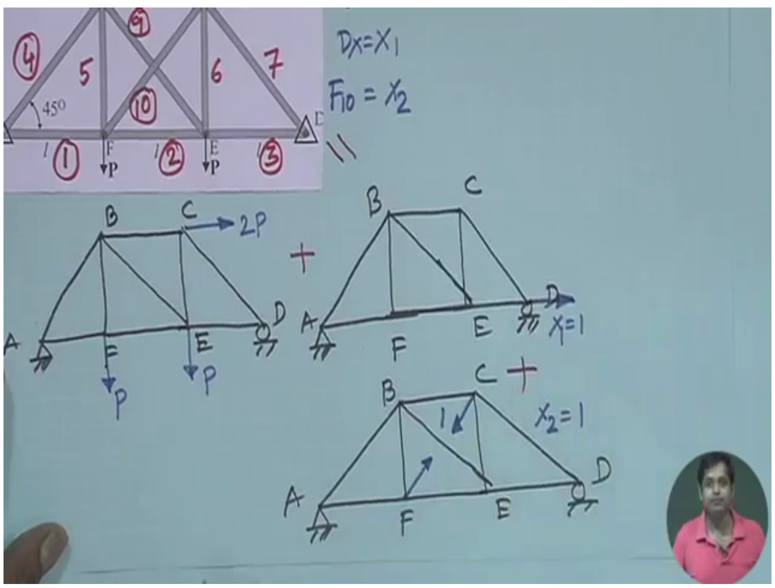

So let us see this is the example we have so just in order to identify the members let us give some numbers to them, this is 1, and then this is 2, this is 3, then finally this is 4, okay this is 5, this is 6, 7, 8 and then this is 9 and then this is 10, okay. So in this case ns is equal to 2, one is external, one is internal and then we take our what is this reaction Dx say Dx is equal to X1 so it is the first redundant and then we take then we take F member force F10 as X2, okay.

So these are our 2 redundant forces, okay. Now so X1 is our first redundant and X2 is our second redundant, okay. Now let us find out we need to decompose we need to divide this structure into now primary structures we have 3 such primary structures determinate structure, one is we need to remove the redundant, one is the primary structure subjected to the external load and then primary structure subjected to redundant force since we have 2 redundant forces so there will be 2 primary structures subjected to in two cases we have like the previous example.

So these structures will be the first will be this and then it is roller (support) hinge support here, roller support here so this becomes A, B, C, D, E and F it is subjected to horizontal this is the external load which is 2P and then P and then P here and so this is first primary structure and then the second is the same structure then this is A, B, C, D, E, F and then this is so member 10 is removed here and the horizontal constraint at D is removed here, okay.

And this is subjected to first is this unit load which is X1 is equal to 1, okay so this is first and the second will be roller support A, B, C, D, E, F and then this is the member and second primary second redundant force is this is and this is 1. So in this case X2 is equal to 1, so this is X1 and this is X2 here. So this will be this plus this plus this, okay. So next step is we need to we have already the first step and second step that both the first two steps are done, then the third step is to solve it find out the forces member forces here and then the fourth step is find out the member forces in this and in this.

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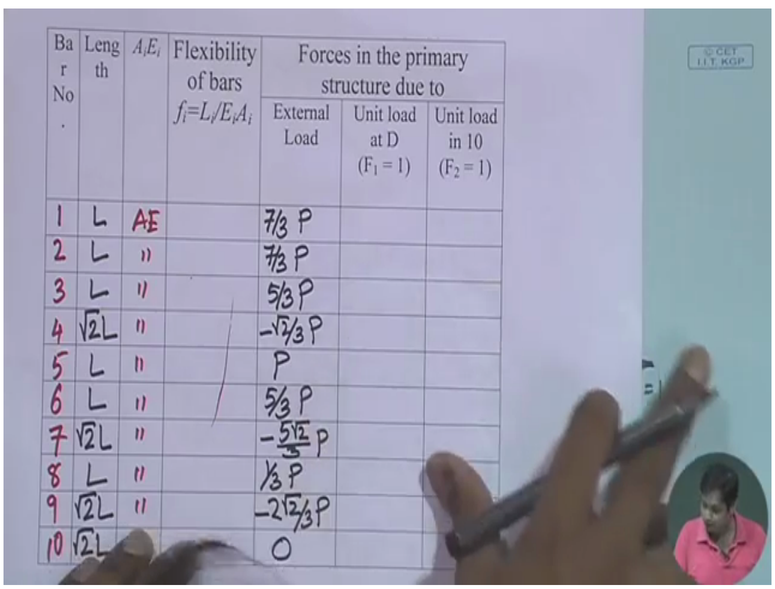

Bar No.	Length	A_iE_i	Flexibility of bars $f_i=L_i/E_iA_i$	Forces in the primary structure due to External Load	Unit load at D ($F_1 = 1$)	Unit load in 10 ($F_2 = 1$)
1	L	AE		7/3 P		
2	L	"		7/3 P		
3	L	"		5/3 P		
4	√2L	"		−√2/3 P		
5	L	"		P		
6	L	"		5/3 P		
7	√2L	"		−5√2/3 P		
8	L	"		1/3 P		
9	√2L	"		−2√2/3 P		
10	√2L			0		

So again these are all these structures are statically determinate structure so we can find out the member forces, so I have the member forces with me what I will do is I will just give you the what are the forces, okay. Now we have 2 we have 10 members here again it is very convenient to write this in a tabular form, but because of this space I could only do up to this part but you can have some more columns here where you can actually determine those displacement and flexibility coefficient in the table itself, okay so that will be convenient.

So we have total 10 members 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and then the lengths are these are all 45 degree intentionally I took 45 degree because then the computation is slightly easier, okay calculation becomes slightly easier. Now all these vertical and horizontal Ln are diagonals are root 2 L so this become L member 3 L then member 4 is diagonal so this becomes root 2 L, L, L member 7 is again member 7 is diagonal root 2 L then L, member 9 member and for all it is Ai so AE so they are same but again you can have different values for AE for differ members.

So we can compute the flexibility for each member separately in this column, okay. Now you can see there are 3 columns here first is external load the forces due to external load, forces due to the first redundant and forces due to the second redundant and these values are this is 7 by 3P, this is again 7 by 3P, this is 5 by 3P, this is minus root 2 by 3P please you do not take these values please compute these values and proceed and then proceed with the calculation, okay here the idea is to demonstrate the concept there is, okay let me see number 5 member 6 is 5 by 3 P member 7 and then member 8 is P by 3, member 9 is minus 2 root 2 by 3P I do not know whether it is properly visible or not but that is not important, what is important is to get the to understand the concept.

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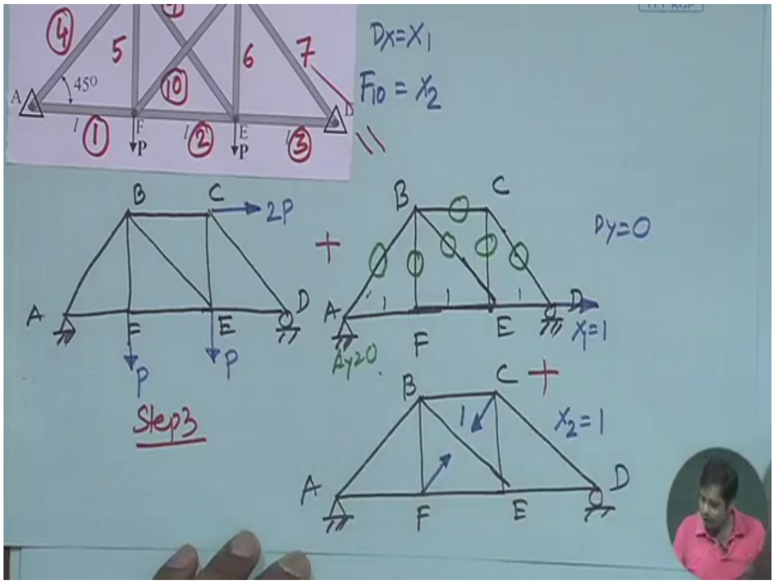

So these are the external load for this case this is our step 3, okay. Now step 4 is find out the forces in this structure and this structure you can again do that but again let us see whether we can slightly reduce the problem by identifying the member identify the 0 member force or not. Now you see this is look at this joint this is a 0 force member, okay and then if this is a 0 force member then also you see since this force is being applied here your reactions are if you take moment about A this will not contribute to this moment so reaction at point D, Dy will be 0.

So for this case Dy will be 0 since Dy is equal to 0 Ay will also be 0 and Ax will be 1. So since Dy is equal to 0, then this is also 0 force because that this it is vertical component there is no force to balance the vertical component here. Now if this is equal to 0, then what happens if you take joints C here it has horizontal force, vertical force then horizontal and vertical force there is no other forces to balance them.

So these two forces will again be 0, now again if you take point joint E here this force this member will have a vertical component but there is no other vertical force to balance them so this will also be 0. Now member the force the Ay we have just obtained that Ay is equal to 0, now if Ay is equal to 0 then this member has vertical component we shown be balance by anyone so this will be 0. So you can see that more all the member forces are 0 here except the horizontal member by intuition we can say that because you are applying the force along this line so all these forces you get all these forces you will get 1 these values are all these values are 1, okay.

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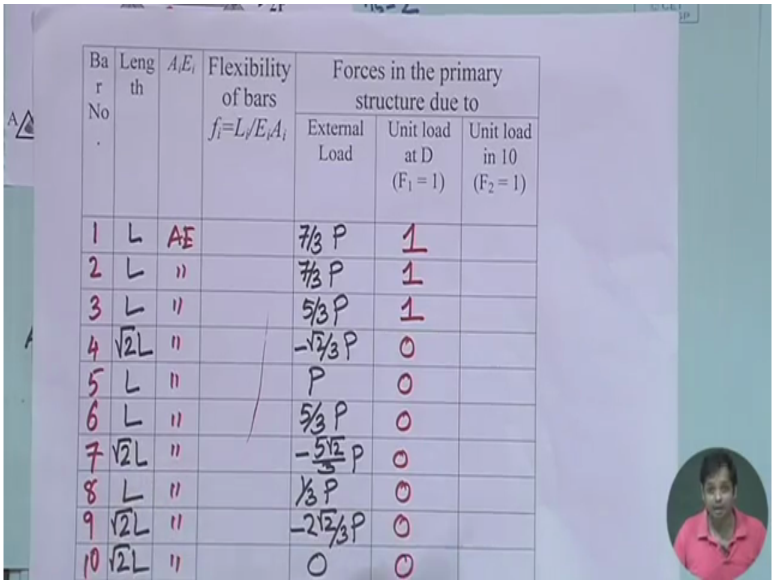

Bar No.	Length	A_iE_i	Flexibility of bars $f_i = L_i/E_iA_i$	Forces in the primary structure due to: External Load	Unit load at D ($F_1 = 1$)	Unit load in 10 ($F_2 = 1$)
1	L	AE		7/3 P	1	
2	L	"		7/3 P	1	
3	L	"		5/3 P	1	
4	√2L	"		−√2/3 P	0	
5	L	"		P	0	
6	L	"		5/3 P	0	
7	√2L	"		−5√2/3 P	0	
8	L	"		1/3 P	0	
9	√2L	"		−2√2/3 P	0	
10	√2L	"		0	0	

So then n1 you will get the first, second and third member you have the forces which is non-zero and the values are 1 and all other members the force are 0. So write down these values this values are 1 first member, second member, third member and all other member these values are 0, okay.

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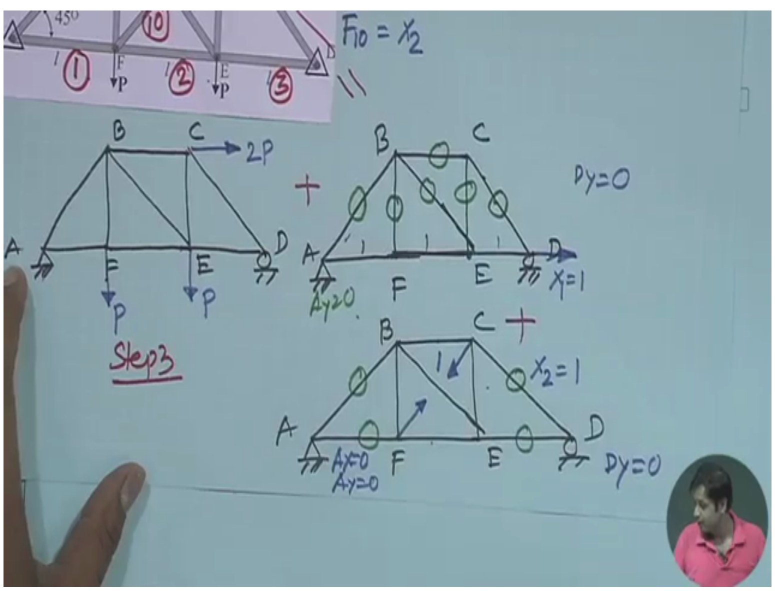

Now similarly let us do it for this structure, now again like previous one if you take these 2 forces are collinear they whatever components you take whatever point about which you take the moment these two forces will cancel each other balance each other.

So what you get is all these reactions, reactions at D and reaction at A all these reactions will get 0. So in this case you get Dy is equal to Dy is equal to 0 in this case we will get Ax is equal to 0 and then Ay is equal to 0 all the reaction forces are 0 and if you now if you apply this similar logic then what we get is, we get these forces these members are 0 force member, okay. See it is very important step to identify a 0 force member because once you identify them your computation become very easier or reduced rather.

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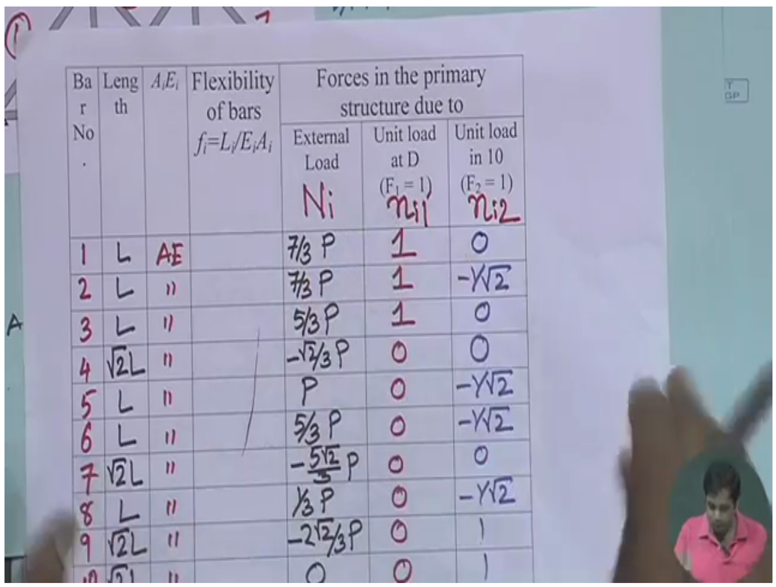

Bar No.	Length	A_iE_i	Flexibility of bars $f_i=L_i/E_iA_i$	Forces in the primary structure due to External Load N_i	Unit load at D ($F_1 = 1$) n_{i1}	Unit load in 10 ($F_2 = 1$) n_{i2}
1	L	AE		$7/3\ P$	1	0
2	L	"		$7/3\ P$	1	$-1/\sqrt{2}$
3	L	"		$5/3\ P$	1	0
4	$\sqrt{2}L$	"		$-\sqrt{2}/3\ P$	0	0
5	L	"		P	0	$-1/\sqrt{2}$
6	L	"		$5/3\ P$	0	$-1/\sqrt{2}$
7	$\sqrt{2}L$	"		$-\frac{5\sqrt{2}}{3}P$	0	0
8	L	"		$1/3\ P$	0	$-1/\sqrt{2}$
9	$\sqrt{2}L$	"		$-2\sqrt{2}/3\ P$	0	1
[illegible]	[illegible]	"		0	0	1

Now so then rest we need to find out the forces in these 5 members and these forces will be let us write the forces and these values will be, okay. The first member it is 0 and then it is minus 1 by root 2 and then 0, 0, then minus 1 by root 2 again, minus 1 by root 2 again, 0, minus 1 by root 2 and then 1 and 1. So this is N capital Ni and this is small ni 1 and this is small ni 2 this is associated with the first redundant and this is associated with the second redundant and it is due to the externally applied load, okay.

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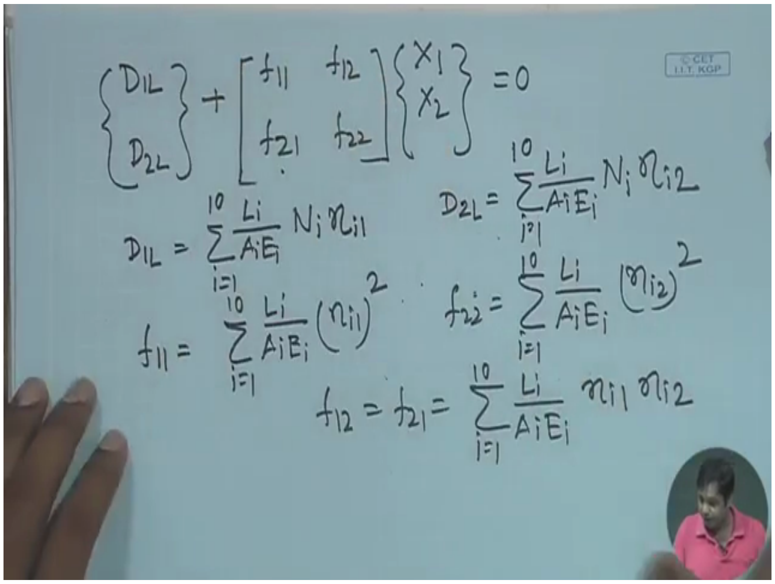

Now again we can write the flexibility format like this, D1 L, D2 L plus this is the compatibility condition f11, f12, f21, f22, X1, X2 that is equal to 0 that is the compatibility condition, and what is D1D1 L and D2 L? D1 L is equal to D1 L is this is our first redundant, this is our first redundant is horizontal force reaction at D so D1 L will be the horizontal reaction at D due to the externally applied load and D2 will be horizontal deformation in member FC due to the externally applied load, okay. So this is the deflection deformation due to externally applied load.

Now f11 is the flexibility associated with X1 but the force applied in the X1 and displacement is measured along X1 and f12 is equal to force applied in direction of X2 but displacement is measured in the direction of X1 and similarly f21 is equal to force in measure in the direction of X2 but the force is applied in direction of X1 and f22 is displacement measure in the direction of X2 force is also applied in the direction of X2 like previous example and then we can write that D1 L is equal to summation of i in this case i is equal to 1 to 10 then Li by AiEi and this is Ni small ni 1, okay.

And please this is very important and this is D2 L is equal to summation of i is equal to 1 to 10 and then Li AiEi and then capital Ni small ni 2, okay. So this is due to the this is in the direction of second redundant and this is in the direction of first redundant. Similarly, f11 will be i is equal to 1 to 10 same Li by AiEi and then ni 1 square and f22 is i is equal to 1 to 10 Li AiEi and then ni

2 square and finally f12 is equal to f21 you can write i is equal to 1 to 10 Li by AiEi and then ni 1 ni 2 or ni 1 ni 2, okay.

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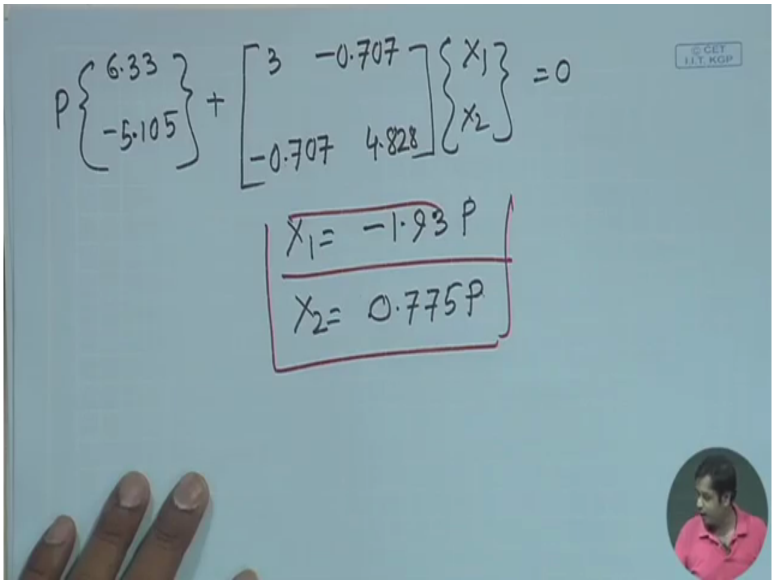

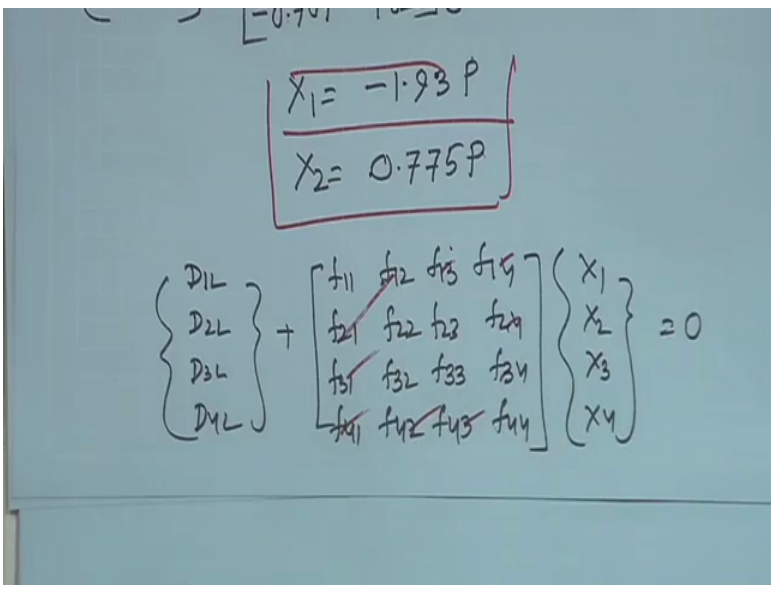

Now so this is the flexibility coefficients, now we can obtain calculate this flexibility coefficients and write here and then final expression what we get is like this I will give you the final expression for this and the final expression becomes D1 L is 6.33 and D2 L is minus 5.105 P plus f11 you get 3 and f12 is minus 0.707 it is symmetric 707 and f22 becomes 4.828 that is then X1, X2 is equal to 0 and finally you get X1 is equal to if you solve them minus 1.93 P and X2 is equal to 0.775 P, okay. So once we know X1 and X2, X1 and X2 are essentially now that was the redundant forces once we know the redundant forces rest of the thing is then with the knowledge of this redundant forces value of the redundant the structure become statically determinate.

Now we need to we can find out the supports reactions on forces in other members, okay. So this is how we can apply method of consistent deformation for trusses, now for demonstration purpose we chose some example which has 1 or 2 redundant forces but in actual structure you can have many redundant forces but if you have that then your equation becomes something like this if you have say 4 redundant forces, then it becomes D1 L, D2 L, D3 L, D4 L and then plus this is f11, 12, 13, 14, 21, 22, 23, 24 and then f31, 32, 33, 34 and finally a f41, f42, f43, f44 that into X1, X2, X3, X4 that is equal to 0 that your compatibility equation.

If you have n number of unknown then this flexibility matrix becomes n by n, but this remain always symmetric so this always symmetric, okay this always symmetric this is symmetric matrix, okay. So the concept is again same, only thing is you need to your computation is the calculation is required calculation is more when your problem your more number of redundant forces. As I said earlier that if you look at all this steps the first step is very straight forward then step number 3 to step number 6 is also very straight forward it is once you know the procedure you can apply the procedure to any problem, step 2 is very very important where you need to identify the redundant forces, okay.

You need to identify the redundant forces in such a way that primary structures are determinate structure, but you cannot afford to have an unstable primary structure, okay. So at step 2 you need to apply your whatever structural engineering sense you have is developed so far that you need to apply you need to understand how the behavior of the structure I have already told you in the first class but I am repeating it and probably I will be repeating many more times that calculation is fine but before actually you do the calculation you need to understand the problem you need to try to analyze the problem in your mind and you need to solve the problem in your mind and the calculation is required finally to get the numerical values, okay.

With this I will stop here today, so what we have done? We have seen the demonstration of consistent method of consistent deformation for trusses next thing we will see how the method of consistent deformation can be used to analyze statically indeterminate beams and frames, okay. With this thank you very much, see you in the next week.