**Course on Structural Analysis 1**
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**Lecture 40**
**Module 8**
**Analysis of Statically Indeterminate Structures: Method of Consistent Deformation (Continued)**

Hello, welcome let us continue with what we have been doing since last two classes we have already demonstrated the concept of consistent method of consistent deformation for analysis of statically indeterminate truss through two examples but those examples was relatively easier in the sense the static indeterminacy was born whether in external or internal but the number of indeterminacy was born.

What we will do today is we will take a problem where the number of indeterminacy is just more than 1 and then we will see how to do this do the calculations, okay. Now so again today we will continue with the application of trusses application to trusses method of consistent deformation.

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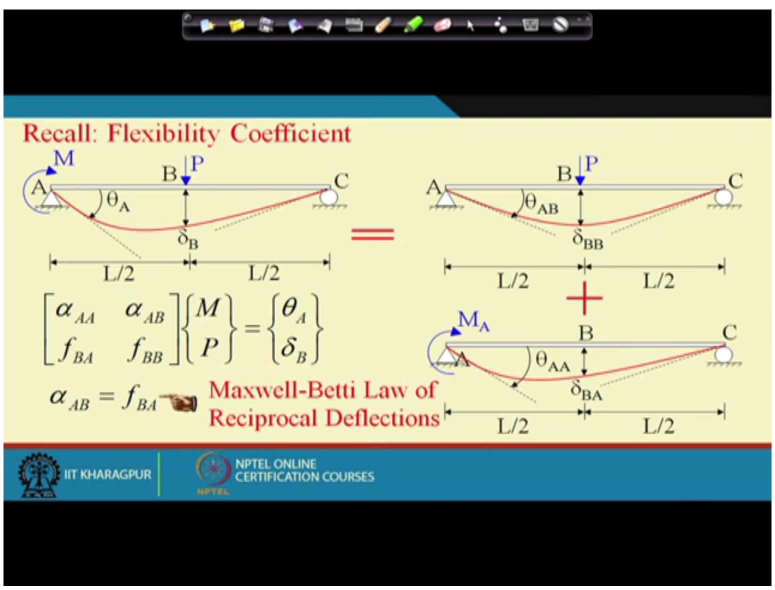


Now, before we go for the let us recall the flexibility coefficient the same slide I showed you some time back the essence of the slide is two things, one is suppose in this case we have two forces, force M which is moment acting at A and then force P at B vertical force.

Then deflection delta B is the in the vertical direction measure at B and theta A is the rotation at A and then we can we saw that how does theta A and delta B they are related to they are related to the corresponding forces external forces through this expression written in flexibility format. And this matrix is a flexibility matrix and all its coefficients are flexibility coefficients, okay. Now theta A theta in theta A M will contribute and P will contribute, right? So theta A is essentially the rotation due to M plus the rotation due to (externally) due to P.

So theta A the contribution from M is the coefficient associated with contribution from M is alpha A and coefficient associated with contribution from P is alpha AB. Similarly for delta B the coefficient for M is alpha fba and coefficient for B is fbb. So essentially alpha aa means it is the rotation at A due to the applied moment at A and alpha ab is the rotation at A due to the applied load at B, okay. So alpha a alpha ab these are the flexibility coefficients this is one thing and the second thing was this matrix is symmetric matrix and that directly comes from Maxwell-Betti Law of Reciprocal Deflections and since it is symmetric so these two values are alpha ab and fba they are same.

So we discussed this sometime back, okay. Now we will be using this while solving while analyzing some of the problems where we have more than 1 redundant forces, okay.

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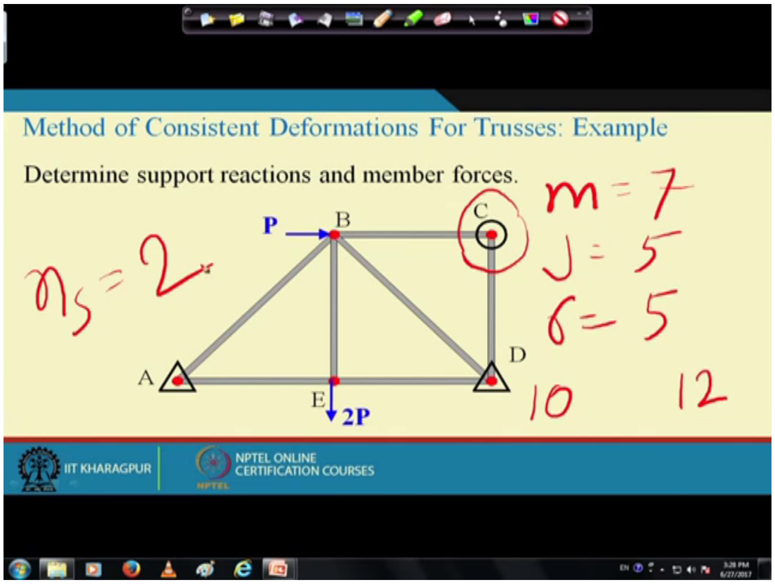

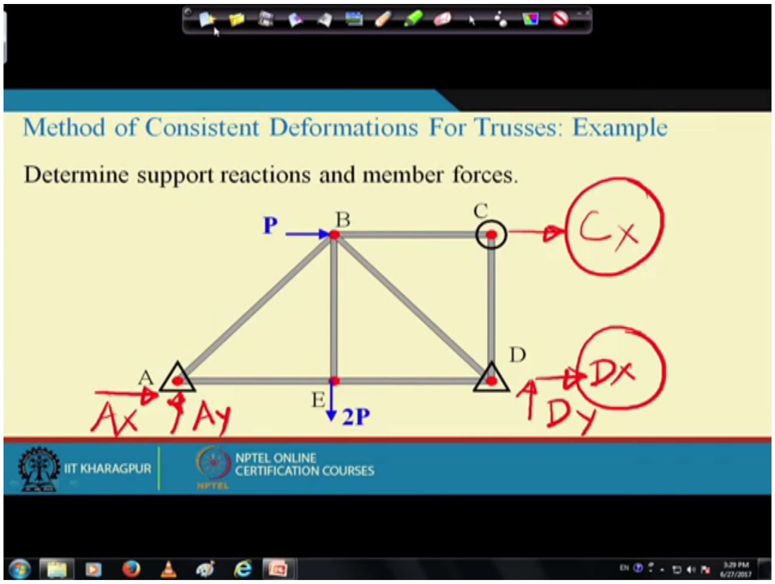


So let us take this problem this is again an indeterminate truss, okay so first thing is we need to find out what is the static indeterminacy you see here number of members are 1, 2, 3, 4, 5, 6, 7 so M is 7 and number of joints are 1, 2, 3, 4, 5 so J is equal to 5 and r is equal to here you have 2 reactions two reactions 4 reactions and then we have 1 reactions here so total 5 reactions.

Total number of equations are available 5 into 2 10 and number of unknowns are 7 plus 5 12, so 12 minus 10 is equal to so ns in this case ns is equal to 2. So first step was first step is determine the static indeterminacy in this case static indeterminacy is 2. And if you look at this in both static indeterminacy it is an external indeterminacy because you see these for stability of this truss this support is unnecessary this support is not required this is extra and instead of giving hinge at D if we give a roller at D then also it is stable it is a stable truss.

So this roller at C is additional is additional constraint and again constraint one constraint this constraint sorry the horizontal constraint at D is another additional constraint here. So we will take this 2 as redundant forces, okay. So first step is ns is equal to 2 and the second step is we need to identify what are the redundant forces, so redundant forces we will take see in this case support reaction will be, not this in this case yes support reaction is Cx and in this case support reaction is Dy and then Dx and in this case support reactions are Ay and then Ax total 5 support reactions.

What we will do is we will take this 1 redundant and this is another redundant, okay. So and then the structure become if we remove this redundant forces, then we get the primary structure which is determinate structure, okay. So let us do okay this is the structure first of all let us number the members so that we can identify then very easily.

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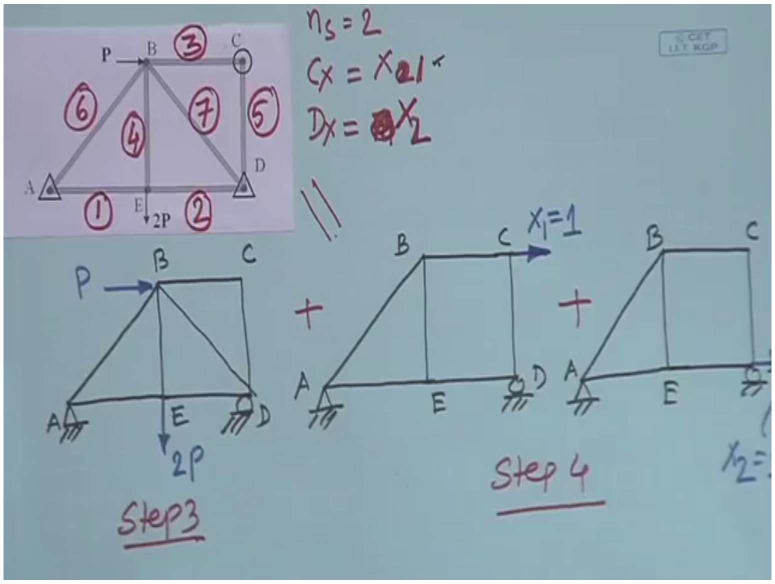


So let us this is member number 1 and then this is member number 2, this is member number 3 and then member number 4, member number 5 and then member number 6 and then member number 7, okay.

So we have total 7 members and okay. Now great, now what we do next is next is we need to here it is ns is equal to 2 we have already seen, okay and the redundant force is r we have taken Cx as the redundant force and Dx as the redundant force, okay. So let us say that Dx is equal to F1 and Dx is equal to not F1 let us F we have already used for something else, so let us take Dx is equal to X1 and Cx is equal to X2, okay. So X1 is essentially the reason why we took X1 and X2 because it will help us to write the equation in a proper format, okay.

So the X is the redundant force in this case and the first redundant force is Dx and the second redundant force is Cx that how we I have given then numbering, okay, great. Now or say this is 1, this is 2 that is what we have written so this is 1, this is 2, okay. Now next step is we need to divide the structure into in such a way into primary structures and how many primary structures we can have here the one is the primary structure subjected to the external load, then another is primary structure is subjected to X1 the first redundant force and the third is primary structure subjected to second redundant force that is Dx which is X2, okay.

So the we have three primary structures and these three structures are first is this A, B, C, D, so what are the supports? Support at A is hinge support, okay support at D becomes roller support because we have already taken Dx as constraint as redundant and then there will be no support at C because it was only roller support at C and that reaction for due to that roller support we have taken as redundant, so this is the primary structure which is now subjected to load 2p here and then another load p here, okay.

So this is the primary structure subjected to external load, okay. And then these are the corresponding numbering also we can write here. Then the second part is the same structure but now it is subjected to first redundant force, what is the first redundant force? Okay write the name A then B, C, D then E here it is E so first redundant force is X1, X1 is Cx so it is subjected to X1 but if you remember the previous example we apply X1 is equal to 1 an unit load in this direction, okay.

So this is the second redundant structure and the third redundant structure will be A, B, C, D, E and the same primary same boundary conditions, but now it is subjected to second redundant force and second redundant force is X2 which is Dx, so it is subjected to force like this and which is in this case X2 we take X2 is equal to 1, okay. Now the next step so the total this is equal to this plus this, okay. So then this is our step 2, now the step 3 is step 3 is we need to solve this the primary structure subjected to the external load this is step 3.

So this is step 3 if you remember and then we need to solve this structure primary structure subjected to the redundant forces. So this is step 4 this and this, okay. Now again we know how to solve any statically determinate truss so I am not solving this again in this class so I have this solution what I am going to do is I am going to write the solution in a tabular form. So let us see this is the tabular form, okay.

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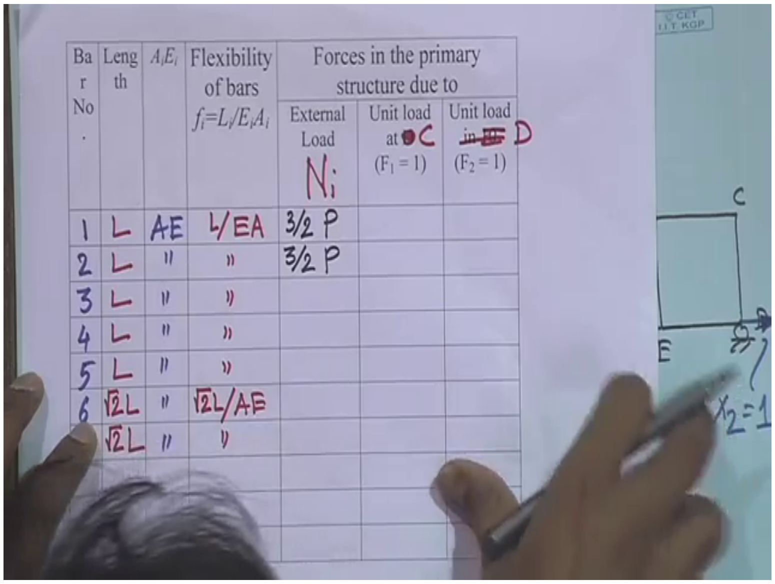


Now if this is unit load at C and this is unit load at D, okay. Now, how many members we have 7 members so these are the 7 members 1, 2, 4, 5, 6, 7 and the lengths are again all lengths are L except for members 6 and 7 so all are L 6 and 7 becomes root 2 L, okay.

And for this problem Ai is same for all so for all members A and E are same. So flexibility again we can determine L by Ei L Ei this divided by so for this it is L by Ei which is L by EA which is same for all and for this case root 2 L by AE and again it is same, okay. Now external load due to subjected to when the structure is subjected to the external load, so this is ni, okay that is what we are using and these values the values are let us write it, so N1 becomes 3 by 2p please verify these values, 3 by 2P and but before that before I write the solution just look at the structure, can we reduce the problem slightly can we identify what are the 0 force members?

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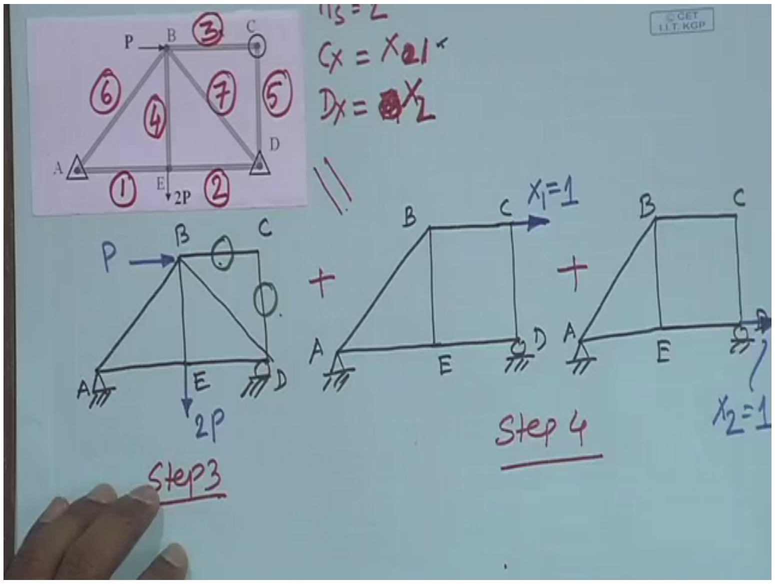


Yes, we can you see if you see look at join C then joint C the force in member BC is horizontal and force in member CD is vertical but there are no other forces to balance force in member BC and CD so naturally this is a 0 force member, this is a 0 force member. So member 3 is a 0 force member and then member 5 is a 0 force member, okay.

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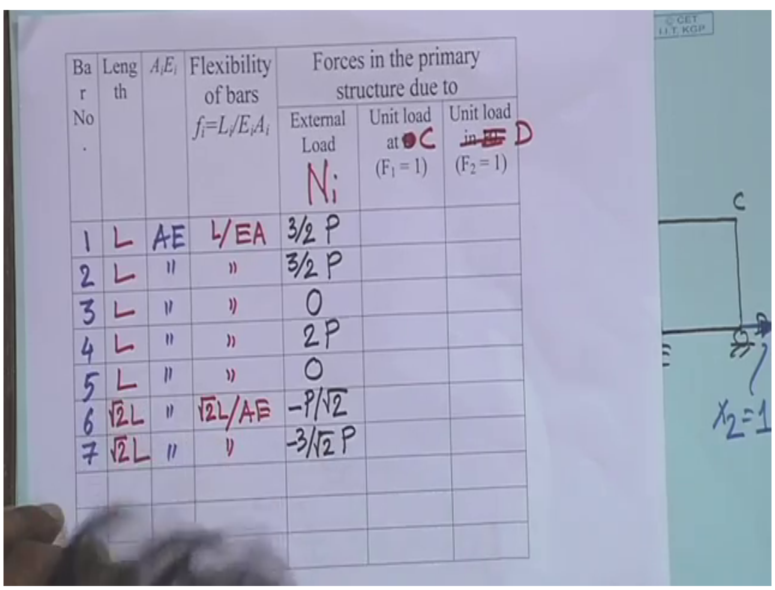

| Bar No. | Length | $A_iE_i$ | Flexibility of bars $f_i = L_i/E_iA_i$ | Forces in the primary structure due to External Load $N_i$ | Unit load at C ($F_1 = 1$) | Unit load in D ($F_2 = 1$) |
|---|---|---|---|---|---|---|
| 1 | L | AE | L/EA | 3/2 P | | |
| 2 | L | " | " | 3/2 P | | |
| 3 | L | " | " | 0 | | |
| 4 | L | " | " | 2P | | |
| 5 | L | " | " | 0 | | |
| 6 | √2L | " | √2L/AE | −P/√2 | | |
| 7 | √2L | " | " | −3/√2 P | | |

So member 3 is 0 force member and then member 5 is also 0 force member, okay. And N4 will be 2 P, N5 is 0, N6 will be minus P by root 2 and N7 will be minus 3 by root 2 P, okay please verify this values, okay.

So this is the solution of the primary structure subjected to external load, so step 3 is done. Now let us do step 4, step 4 is solution of this and this, so before you do that again let us see whether we can identify this 0 force members or not, you see take join C, now join C has an horizontal load which is 1 and this horizontal load will be balanced by force in member BC. Now force in member CD is vertical there is no other vertical force at join C which can balance the force in member CD.

So naturally it has to be a 0 force member, okay. Now similarly if you see joint E here, now the AE and ED there forces are horizontal and they may balance each other, but there is no other force to balance member force the vertical force in B so this has to be a 0 force member, okay. Now, so at least we can say that these forces are 0 force member, okay member so what are those member number 4 and then member number 5 they are 0 force members, okay.

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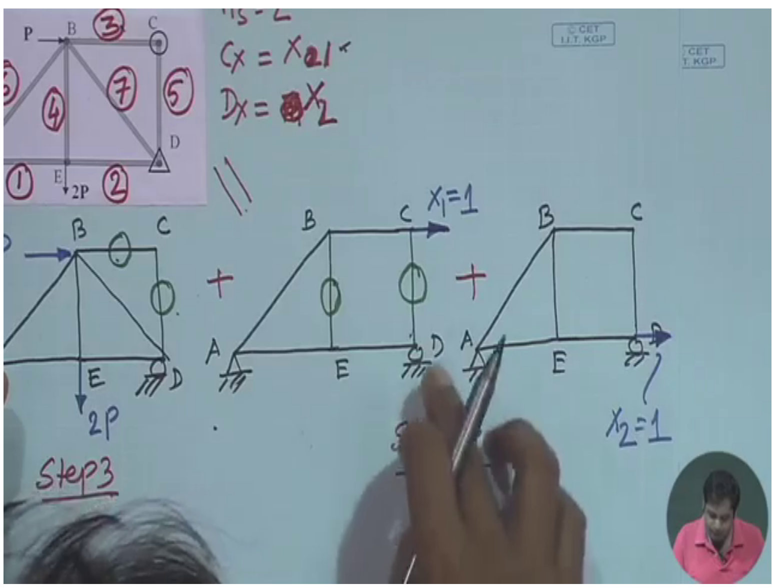

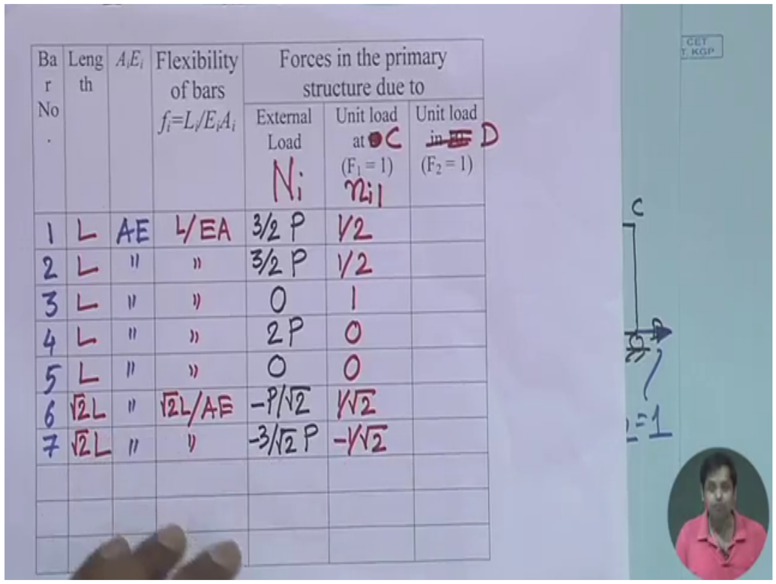


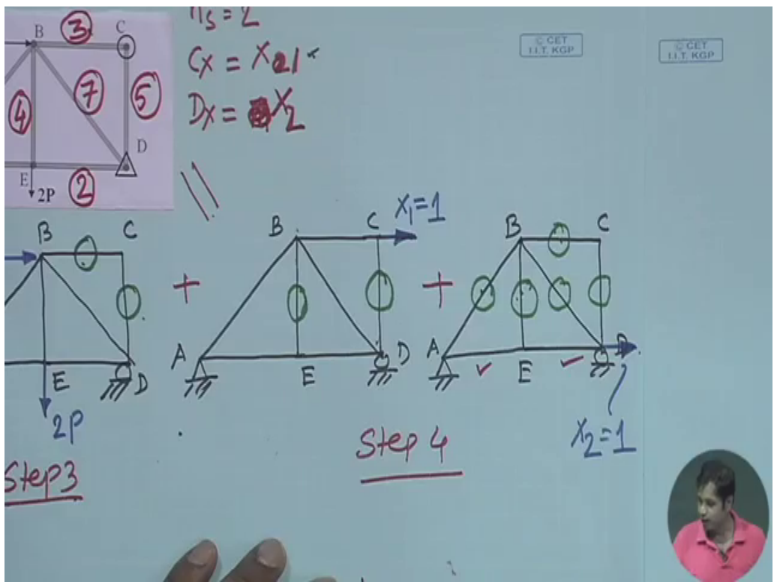


Now once we know these are 0 force member and it is a statically determinate truss we can compute the forces in other members and if we do that I have the force ready with me this is written as ni small ni and then write 1, 1 because it is due to the redundant force 1, okay whose value is X1, but it is obtained these values are obtained when the unit load is applied in the direction of X1, okay.

Now these values are this is half and then this is half, this is 1, then member 4 and 5 0 force member we have already checked member 4, 5 0 force member and then 6 is 1 by root 2 and 7 is minus 1 by root 2, okay. So these are member forces for this and then find out member forces for this again, can we identify some of the 0 force member? Look at joint C this member and this member they will be 0 force member, for the same logic like this this has to be 0 force member. Now look at joint B this is a 0 force member, this is a 0 force member, so there will be a component the force in member AB has component vertical direction and horizontal direction but no other forces at B which can balance those components so member force in AB will be a 0 force member, okay.

One thing I missed here please, there is a member here as well, okay. Now so and then look at these at these joints all these are 0 force member so this has to be a 0 force member. So in this case most of the members are 0 force member only members which are non-zero forces are this and this and their values are just without any solution just by looking at this we can say that if this 1 this has to be this will be 1 and this will be 1.

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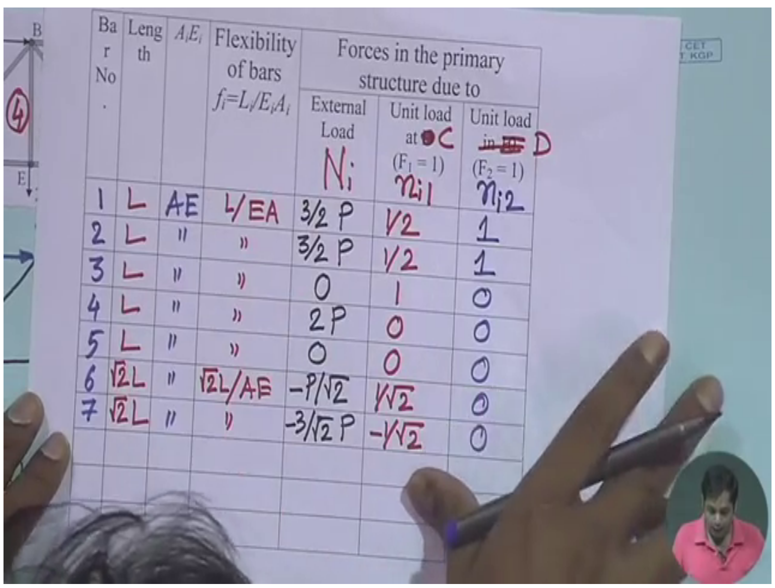

| Bar No. | Length | $A_iE_i$ | Flexibility of bars $f_i=L_i/E_iA_i$ | Forces in the primary structure due to External Load $N_i$ | Unit load at C ($F_1 = 1$) $n_{i1}$ | Unit load in D ($F_2 = 1$) $n_{i2}$ |
|---|---|---|---|---|---|---|
| 1 | L | AE | L/EA | 3/2 P | 1/2 | 1 |
| 2 | L | " | " | 3/2 P | 1/2 | 1 |
| 3 | L | " | " | 0 | 1 | 0 |
| 4 | L | " | " | 2P | 0 | 0 |
| 5 | L | " | " | 0 | 0 | 0 |
| 6 | √2L | " | √2L/AE | −P/√2 | 1/√2 | 0 |
| 7 | √2L | " | " | −3/√2 P | −1/√2 | 0 |

So write these values, so this is n this we can write ni this is 2 because it is due to unit load but that unit load acting in the direction of second redundant force which is X2. So this value is 1, this value is 1 and all these values are 0, 0, 0, 0, okay. Now so we have done step 3 and step 4, now next is step 5, step 5 is to apply the compatibility condition. Now in order to apply the compatibility condition we need to find out the deformation in those members. Now what is compatibility condition in this case, okay.

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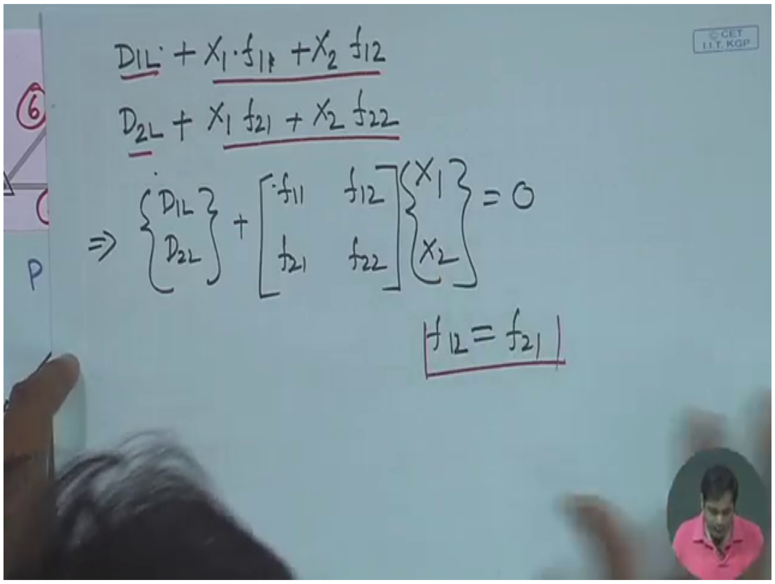

Now suppose okay, now we have so we have 2 joints 2 redundant forces here, X1 and X2, X1 is at point C, X2 at point D so we need to consider the displacement at point C and displacement at point D, okay horizontal displacement at point C and horizontal displacement at point D.

We can see from the actual structure that horizontal displacement at C and horizontal displacement at D will be 0, right? Now suppose D1 suppose D1 L is the horizontal displacement in the direction of redundant one due to the applied external load, okay. And D2 L is the horizontal displacement in the direction of redundant second redundant due to the externally applied loads. So D1 L and D2 L are essentially obtained from this structure, okay. So D1 L not this D1 L yes D1 L due to from this structure, okay. So D1 L is the horizontal displacement at C and D2 L is the horizontal displacement at D due to the externally applied load, okay.

Now, then what is the you see now when we apply a point load at C, this will cause displacement at C and this will also cause displacement at D. Similarly when we apply unit load at D this will cause displacement at D this will also cause displacement at point C. So unit load X1 or first redundant force first redundant will cause displacement at C and displacement at D and second redundant also will cause displacement at D and displacement at C. Then what is the total displacement at C? Total displacement at C total displacement due to the redundant forces will be displacement due to this and then displacement due to this, and what will be the displacement due to this? Displacement due to this will be the force X1 into f11 and force X1 into f11.

What is f11? f11 is flexibility coefficient, flexibility coefficient for what? Flexibility coefficient associated with displacement in 1 displacement into 1 means displacement in the direction of first redundant due to the force acting in the direction of first redundant, okay. So X1 if f11 essentially is the displacement at C due to this load, okay. Now plus then plus X2 into f12, what is f12? f12 is displacement at C due to this load or displacement at C due to this. So displacement in the direction of first redundant due to the second redundant, okay.

So that is why it is (())(22:57) f12. Similarly, what is the displacement for the redundant force at point D or at second in the direction of second redundant? This will be X1 into f21 and plus X2 into f22, okay. Then what would be the so this displacement this part is due to the redundant forces and this part is due to the primary structure subjected to the external load. So total displacement at point C will be this plus this and total displacement at point D will be this plus this, this is very important.

Now this we can write in a matrix form as D1 L, D2 L plus f11, f12, f21, f22 this is X1 and X2 the same way we wrote in the case of simply supported beam just before starting today's class I showed you that slide, okay this is the same expression similar expression. Now again this is a this matrix has to be a symmetric matrix and therefore f11, f12 should be equal to f21, okay this is important, okay.

This f11, f12, f21, f22 they are flexibility coefficients, so this is displacement so this is the compatibility condition, this compatibility condition is displacement due to this part which is this, displacement due to this part which is this the total displacement is this plus this should be equal to 0 that is the compatibility condition and that is written in a flexibility format, okay. Now what is left now is we need to find out what is D what is these values.

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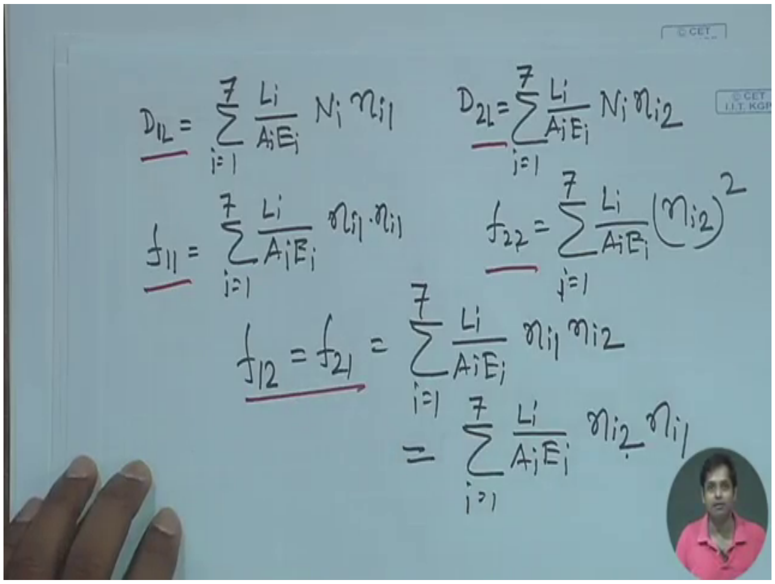

Now we know that D1 L is equal to how to determine that, D1 L is equal to summation of i is equal to 1 to 7 here because we are 7 members that is then Li by AiEi into Ni and since it displacement in the direction of first redundant this will be ni 1, okay. And similarly D1 L, similarly D2 L will be summation of i is equal to 1 to 7 Li by AiEi and then Ni and it will be ni 2 due to second redundant, okay.

So this is just a unit load method, right? So this is the displacement in the first redundant in the direction of first redundant, displacement in the direction of second redundant due to the externally applied load. Now we need to find out flexibility coefficient f11, f11 will be y is equal to 1 to 7 Li by AiEi it is f11 is displacement in the direction of first redundant when the force applied in the direction of first redundant, so it will be ni 1 into ni 1 or ni 1 square, okay. Similarly f22 will be summation i is equal to 1 to 7 Li AiEi and then ni and then Ni 2 square, and what is f12? That has to be equal to f21 that is equal to summation of i is equal to 1 to 7 Li AiEi and it is ni 1 ni 2 means it is the displacement in the first redundant due to the force applied in the direction of second redundant you can write this 1 to 7 you can verify this, this ni 2 displacement in the direction of second redundant due to the force applied in the direction of first redundant.

So this is f so like we can find out all these values, okay.

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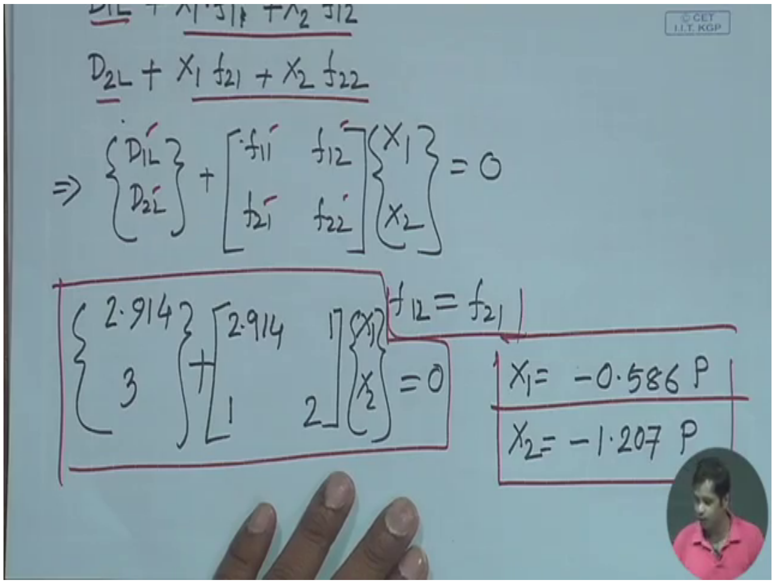


Now next step is we need to substitute this in this expression in the compatibility condition and if we do that I leave it to you if we do that these equations what we get is like this, 2.914, 3 this is plus 2.914, 1, 1, 2 this is X1, X2 and that is equal to 0 this is the we get and this is the flexibility format, okay. Now we can solve it and determine what is X1 and X2 and what we get is X1 is equal to minus 0.586 and X2 is equal to minus 1.207 this is P, this is P.

Now once we know X1 and X2 then rest is very straight forward rest is we need to once we know X1 is X1 is the horizontal reaction at C and X2 is horizontal reaction at D so out of 5 reactions two reactions are now known rest we have 3 reactions so with the knowledge of Cx and Dx the structure becomes statically determinate and we can find the reactions and the member forces and that I leave it to you. So please verify these results, okay and convince yourself. So that is again method of consistent deformation applied to truss now in this case we have seen the number of unknown number of redundant forces are 2 here, but in this case the both the redundant are external redundant, okay.

Redundant due to the support reactions, okay. Now next class we will take an example where we will see the structure which has both which as redundant too but the redundant is one is external redundant and one is internal redundant and we will see how to again the concept is same we will see how this method how the concept can be applied to that structures as well, okay see you in the next class, thank you.