Course on Structural Analysis 1
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Lecture 39
Module 8
Analysis of Statically Indeterminate Structures: Method of Consistent Deformation (Continued)

Hello everyone, welcome. If you remember in the last class we demonstrated the application of method of consistent deformation for analysis of statistically indeterminate trusses, let us continue with the same what we will do is today we will today and next two three lectures we will try to understand the method through some more examples, okay.

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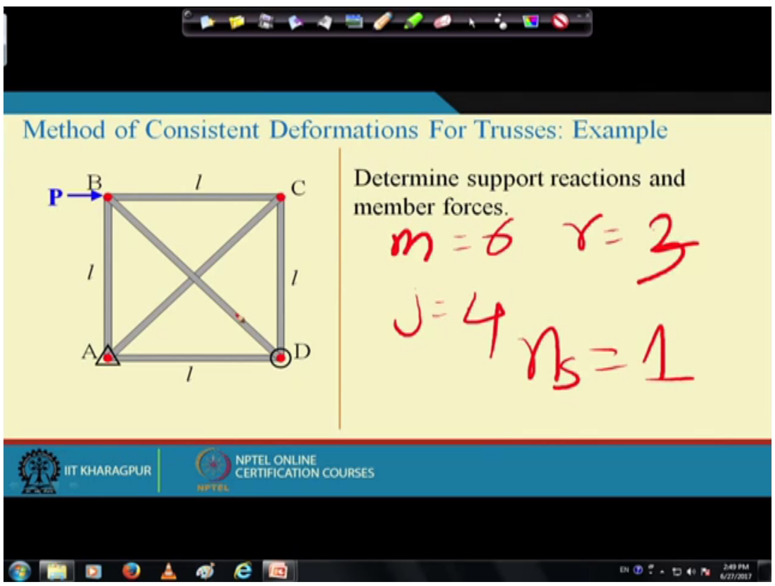

So let us take this is our second example this is again a statically indeterminate truss you number of total member is total member if you take in this case total member m is 1, 2, 3, 4, 5, 6 m is 6 and r is equal to 2 r is equal to 3 two reactions at A and one reaction at D and number of joints is equal to 4 so total number of equations available 4 into 2 8 and the number of unknowns are 6 plus 3 9.

So in this case static indeterminacy ns is equal to 1. Another important thing here is you see for this structure you can easily we can easily determine the support reactions, 3 we need 3 support reactions summation of fx summation of (())(1:42) and summation of moments at 0 these are the equilibrium equations available and through these equilibrium equations we can determine the reactions, but it is the problem what you get when you try to find out the member forces. So this is this is an indeterminate structure but the indeterminacy is internal indeterminacy unlike the previous example where the number of constraints provided in the structures in the structure was more.

But in this case the constraint are only three but it is the number of members providing the structure is more and that causes the indeterminacy in the structures, so these indeterminacies external indeterminacy. Now since in the previous case the indeterminacy caused by the support reactions supports so redundant force we took any support reaction was taken as the redundant force, but in this case since it is the member which causes the indeterminacy we need to consider we need to take one member, member force as redundant force, okay.

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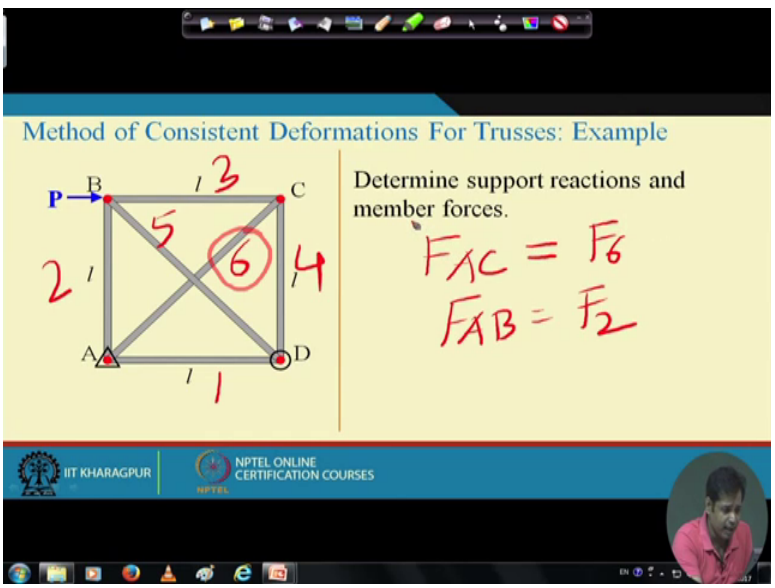

So the step 1 is the determination of static indeterminacy which is ns is equal to 1 here and then step 2 is identify the redundant force so one member will be redundant in this case one member force needs to be chosen as redundant force, let us first number these members let us say this is member number 1, this is member number 2, member number 3, 4 and then this is 5 and this member is 6, okay. So whenever we say that if FAC, FAC means force in member AC so instead of writing FAC we will say F6, F6 means force in member 6.

Similarly, instead of writing FAB which is member 2 here we will write F2, means F2 is force in member force in to in this case it is AB, okay. So let us take member 6 as redundant let us take this member as redundant member. So force in member AC F6 will be the redundant force, okay.

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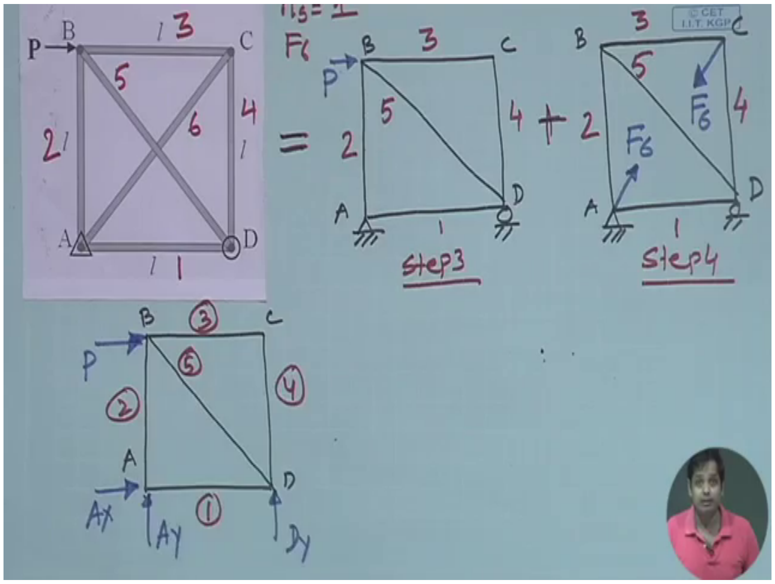

Now, so let us try to this is the structure so we let us write these numbering, so this is member number 1, member number 2, member number 3, member 4, member number 5 and member number 6.

For this we have seen that ns is equal to 1 and the redundant is F6, F6 is the redundant which is the force in member AC, okay. Now so once the second step is done, we already identified what is the redundant force next step step 3 is we need to release that force we need to release the corresponding constraint and then divide the entire structure into substructures the primary structures and then which are determinate structure and then solve those primary structures.

Now since in this case we have only one redundant the primary structures will be two. So what will be those structures? The first one is this just remove the redundant force remove the member corresponding member and then this is hinge support and this is roller support first structure will be it is subjected to the actual load this is P here, okay.

So this is A, B, C, D and the members are this is my member 1, member number 2, 3, 4 and 5 since member 6 is redundant so we need to remove that member. So then this is one primary structure and then the next one is the structure is subjected to only the redundant force, okay. So this is hinge support again roller support, we have a member diagonal member here and now and these members are A, B, C and D and then the redundant force is this, okay this is F6, this is F6, okay.

And as per our sign convention away from the joint is taken as tension and tension is positive, okay. So if we get F6 is a negative value it means that the force in this member is compression, okay. And then this is member number 1, member number 2, 3, 4, 5 and then member 6 is this force, okay. So this structure this is equal to this plus this, right? What we need to do is now step 4 if you remember, step 4 is solution of this problem, okay. Solution solve the primary structure which is subjected to the actual load this is step 4 find out the member forces and then step 5 is (solu) this is step 3 and then step 4 is solution of the primary structure which is subjected to the redundant force.

So let us find the member forces for these and member forces for these, let us first consider this, okay. So this is our step first this is step 4, step 3 if you recall the step that this is step 4, okay. So let us find out the forces in this structure, okay so draw the free body diagram, this will be the free body diagram, this is the free body diagram of this it is subjected we have (())(8:00) this is Ay and then Ax support reaction and then Dy is the reaction at D and then it is subjected to load P like this, okay and the members are this is A, B, C, D and the members are 1, 2, 3, 4 and then 5, okay.

Now before we solve it we know how to solve it, right? We have spent entire week to discuss how to solve statically determinate trusses. Now using method of section, method of joints or using virtual (())(8:48). Now but before we apply any methods to solve any truss first thing is to see whether we can slightly simplify the problem, simplify the problem means not the simplify, simplify may not be the right word, we can we see whether the problem can be reduced, okay our computation can be reduced.

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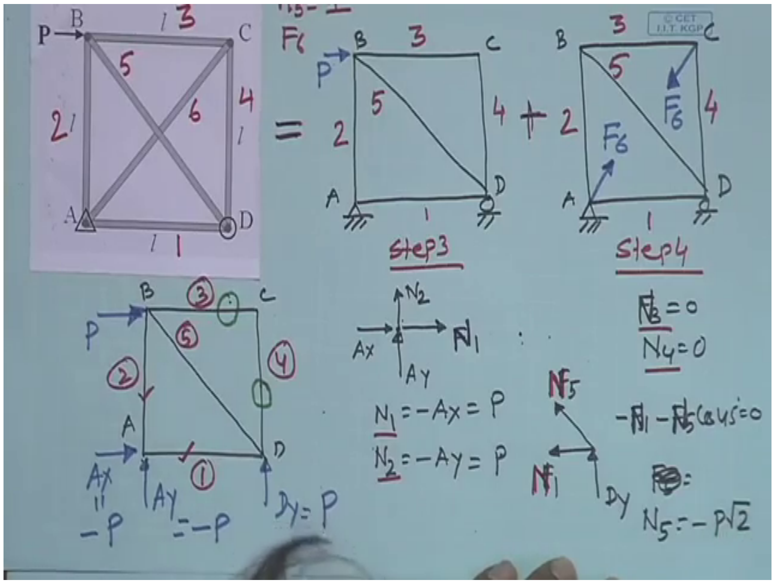

Now you look at this member look at this truss, just by looking at this truss we can easily say that this member, member BC means member 3 and member CD they are 0 force members, so this is 0 force member and this is 0 force member, right? So force in member so F3 will be F3 will be 0 and similarly F4 will be not F3 we use N3, N3 is 0 and N4 is equal to 0. So N3 is the if we reserve for the member force in the final structure the actual structure.

Now in the primary structure when it is subjected to load if you remember in the last class also we use symbol n so capital N, capital N3 is the force in member 3 and capital N4 is force in member 4 these are all 0, now let we need to find out the forces in other members as well. So we take moment about this point and then find out this moment these two will not contribute, P will contribute and then Dy will contribute if we do that, then we get this is equal to P we can calculate reaction is equal P.

Now summation of if we apply summation of Fy is equal to 0, if Dy is equal to P then Ay has to be minus P, right? That is why equilibrium will be satisfied. Now the horizontal forces are P here and only horizontal force is reaction at Ax so P is positive in this direction so Ax has to be minus P. So you really if you find that without any calculation just by looking at the (())(10:57) by simple calculation we can determine the forces then really you do not have to do all these free body diagram of the entire joint of different joints, different sections, okay.

So these are the reactions, now once we have the reaction let us see what are the forces in other member. Now you look at this joint, okay look at this joint, now at this joint what are the forces we have? We have force Ax, we have force Ay and then we have force in member AB means member 2 and then we have force in member 1, okay. Now okay draw the free body diagram of point A so point A this is Ay, this is Ay and this is Ax and then (mem) this is F1 this is N1 and then this is N2, so summation of Fx is equal to 0 summation of Fy is equal to 0 this is N we get N1 is equal to minus Ax, Ax is equal to minus P which is equal to P and then N2 is equal to minus Ay, Ay is equal to minus P this is equal to P.

So we have N1, N2, N3 and N4, right? So this is done, this is done, this is done, this is done and this is done only member left is member 5. Now if we draw the free body diagram of joint D, what are the forces we have here, Dy which is the reaction and then this is F1 and then F4 is 0, F4 is 0 here so there is no point in writing, then this is F5, F5 this angle is 45 degree. Now if we take summation of Fx is equal to 0, then which gives us that F1 minus F1 because F1 is this direction minus F5 cos 45 is equal to 0 and from there F5 we will get F1 already we have not F1 it is N1 again please correct it this is N1 N5 again this is N5 these are all N, okay these are all N, N, N, okay.

And then N5 will be N1 we already obtained 5 P, so N5 will be minus P root 2 so this is N5, okay. So for primary structure subjected to the load we have found all the member forces.

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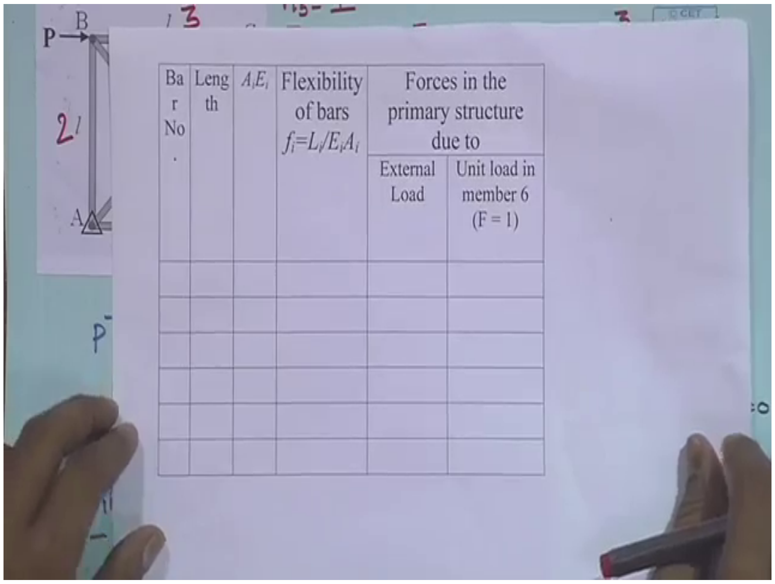

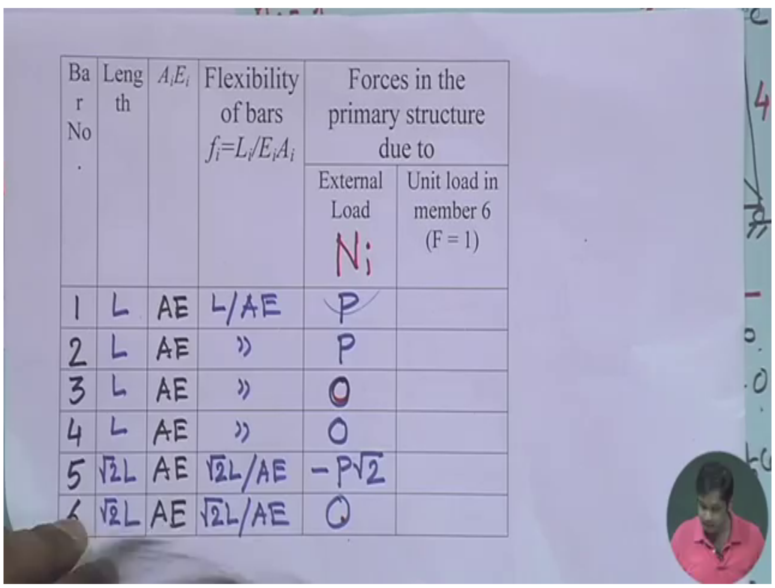

Now it is very convenient to write it instead of writing in a (())(14:08) way we will see it is very convenient to write the all these things in the tabular form, this is the tabular form you can see properly, okay the table is the first column is the bar number, okay and then the length of each bar, then Ai not Ai A into E if you find that all the cross sectional area young's modulus is same, then they will be all same but for different for some problem you may find that there is some values are different for different members.

Then we know the flexibility of bar is defined as L by AE axial in axial deformation so it is flexibility of individual bar and then external load now forces in the primary structure for due to external load and then due to the redundant forces, okay. Now in this case we have only 6 members so let us write 1 member 1, member 2, member 3, member 4, member 5 and member 6, okay and then what are the forces in different member due to external load, this is Ni just now we have determine that N3 is equal to 0, N4 is equal to 0 so write N3 is equal to 0 and N4 is equal to 0 let us use N3 is equal to 0 N4 is equal to 0 and then we have determined that N1 is equal to P, N2 is equal to P.

So N1 is equal to P and N2 is equal to P, N6 will be 0 here because there is no N6 here so N6 will be 0 when you are talking about the member forces due to external load and then finally N5 we obtained minus P minus P root 2 you can see here, okay so N5 is minus P root 2, okay. So these are all member forces written in a tabular form, now what are the length of different members? You see all these member 1, 2, 3, 4 all members they have length l and member 5 and 6 there length is root 2 L, so length is this is L this is L, L, L and then root 2 L and then root 2 L, okay.

So lengths are written all these members they have the same A and young’s modulus and cross sectional area, so for this all are same AE, but for some problem you may get that these values are different for different members, and then what is the flexibility for each member under axial under axial force? So these becomes L by AE all are L by AE all are L by AE and these become root 2 L by AE this also root 2 L by AE, okay. So this is the length this is AE and then flexibility of each bar and then the forces in member due to external load which is capital N, then we need to find out this is step 3 if you remember this is step 3 then we need to find out we need to do the step 4 which is the forces in different member due to redundant force.

Suppose this is written a small ni, okay. Now so this now this is done, right? So this is done, now we have to do the step 4. Now what we do instead of applying F6 what we do is we apply F6 is equal to here F6 is equal to 1, okay so we apply the an unit load in the direction of redundant force because since we know the super position linear the super position linear super position is valid, so what we can do is if we know the unit load forces due to unit load, then we can easily find out forces due to any load by just multiplying the corresponding member forces, okay.

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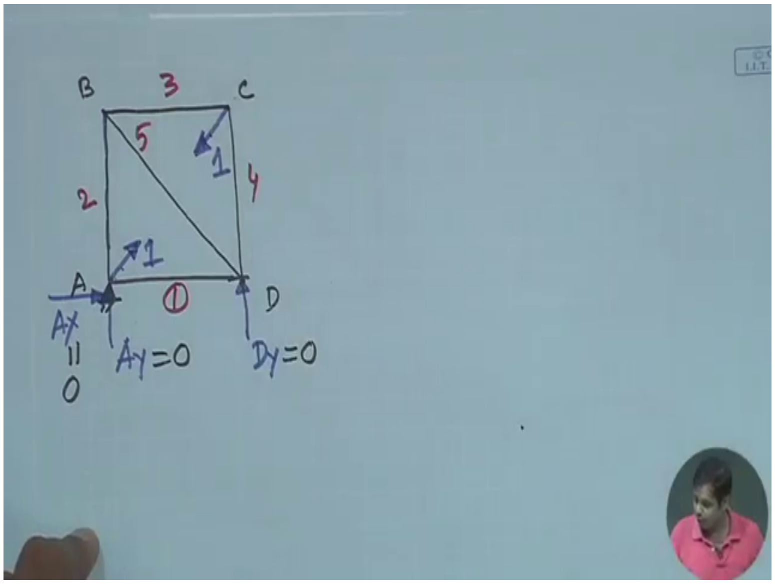

That is what unit load method we discussed some time back, so we apply an unit load here. So let us find out what is the corresponding member forces, so draw the structure once again free body diagram of the structure, so this is the truss and then this is A, B, C, D another member here member AC, then what are the forces we have? We have Ay, then Ax and then Dy and then unit load here unit load 1 here, okay and then member force member numbers are 1, 2, 3, 4, 5 and this one is 6, okay.

Now, let us find out what are the forces we have. Now you see we need to first find out say what is the support reactions without really any without doing any calculations we can say in this case all support reactions are 0, why we can say that? See this structure is subjected to only these two load 1, 1 this two load, right? And these two are collinear, right? So whatever orientation we whatever at any point we take moment or summation of forces these two force will cancel each other whatever moment or they component they will always cancel each other, okay. So what we get is we get Dy is equal to 0 we get Ay is equal to 0 and we get Ax is equal to 0 you can easily verify this, okay.

Now we need to find out forces in different members, we can find out again we can apply method of joints, method of sections or any any method that we are comfortable with and determine this forces.

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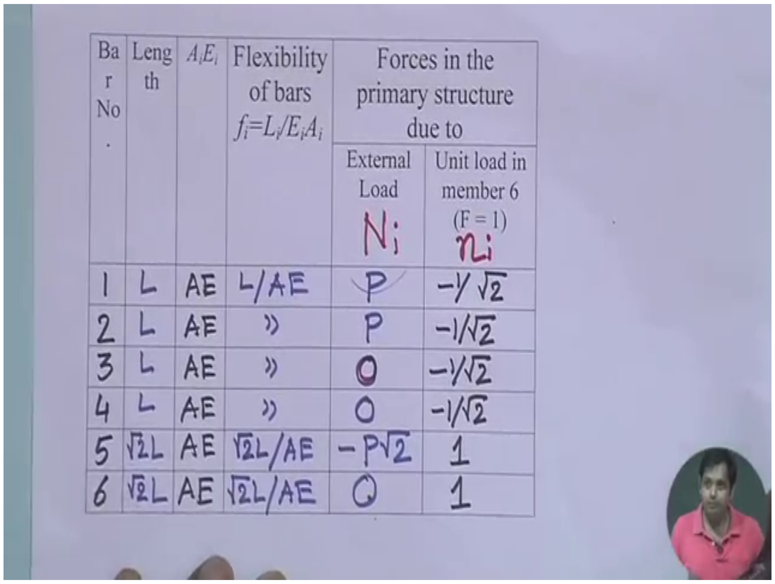

Bar No.	Length	A_iE_i	Flexibility of bars $f_i=L_i/E_iA_i$	Forces in the primary structure due to: External Load N_i	Forces in the primary structure due to: Unit load in member 6 (F = 1) n_i
1	L	AE	L/AE	P	$-1/\sqrt{2}$
2	L	AE	"	P	$-1/\sqrt{2}$
3	L	AE	"	0	$-1/\sqrt{2}$
4	L	AE	"	0	$-1/\sqrt{2}$
5	$\sqrt{2}L$	AE	$\sqrt{2}L/AE$	$-P\sqrt{2}$	1
6	$\sqrt{2}L$	AE	$\sqrt{2}L/AE$	0	1

So I am not doing this exercise because I have already calculate the forces let us now do that, so let us write these forces here, okay. Now what N1, N1 we have minus 1 by root 2, N2 have minus 1 by root 2, N3 is again minus 1 by root 2, N4 is minus 1 by root 2 and N5 is 1 and N6 is anyway 1 because we are applying unit load, okay.

So this exercise I have not done because you know how to analyze determinate truss, okay. Now what we have? We have already done step 1, step 2, step 3 and step 4 so then step 5 is applying the compatibility condition, right? Now what is the compatibility condition we have? Now this is the redundant force, this is the redundant this member is you see the force in member AC we consider as redundant force. Now the compatibility condition here is we neglect if we neglect the axial deformation, then the deformation in AC will be 0, will be then whatever deformation we get axial deformation we get for this and whatever axial deformation we get for this there summation should be equal to 0 because we assume that there is no axial deformation in this member, okay great.

Now let us say that the axial deformation let us use let us take DL, D is the deformation DL, L for load so let us take when we say DL, DL is the deformation in bar AC due to actual load, right? Due to deformation that obtain in the primary structure when it is subjected to actual load, okay. And then we will see what then let us find out DL first, then what is DL?

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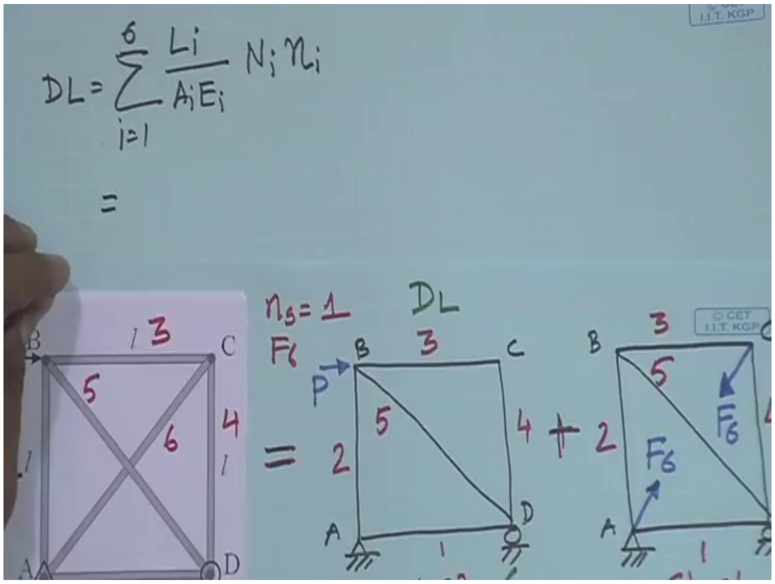

We can apply unit load method DL if we apply unit load method then DL will be the unit load method is in the direction wherever you want to determine the deflection we apply unit load in that direction and then analyze it and then once we get the results then DL can be obtained as for truss summation of i is equal to 1 to 6 in this case because there are 6 members and then Li divided by Ai Ei into Ni and small ni, okay.

Ni is the forces in member in this primary structure subjected to load and ni small ni is the forces in the primary structure subject to unit load, that is what unit load method we have we already discussed this unit load method, right? Now let us see what is these values, now advantage of writing this in a tabular form is once we know this table, then it is just multiplication of this, this into this into this and then we can write what is this, what is this, what is this, what is and what is this, okay.

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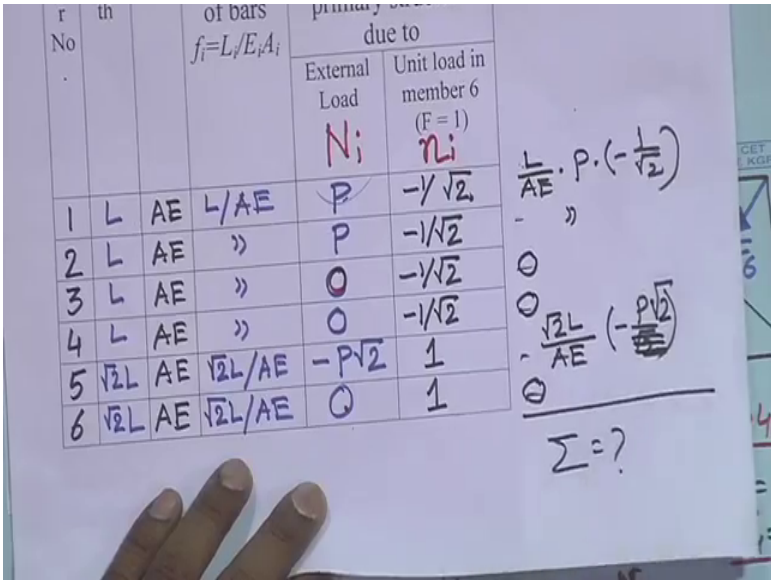

So let us do that, so if we do that then this becomes L by AE this is L by AE, right? The length is L by AE into this P into minus 1 by root 2, right? And then this also same this is also same this is 0, this is also 0, this is also 0 and this becomes root 2 L by AE into minus P by root 2, okay. Now so then what we have is P by root 2, P into root 2 sorry P into root 2, okay.

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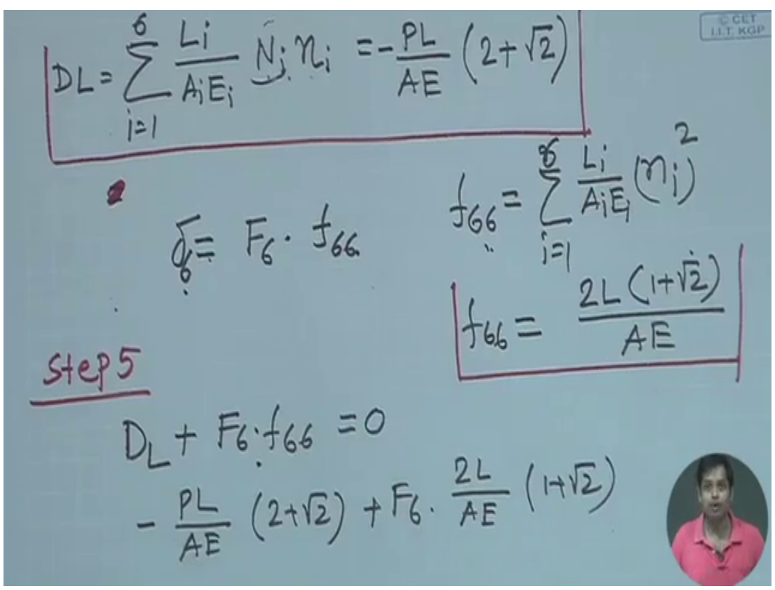

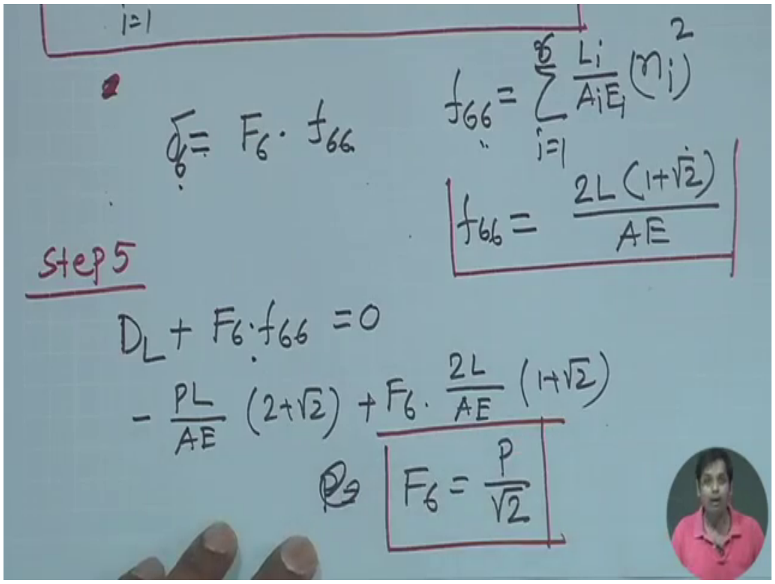

Now then what we have to do is we have to sum them, now if we do this sum then what we get is we get this is equal to PL by AE because AE is constant 2 plus root 2 and this is negative sign. So DL is equal to this, okay. So DL is the deflection change in length in member 6 due to when the primary structure is subjected to actual load, okay.

Then we need to find out what is the deflection in primary structure when the when it is subjected to unit load, and how to find out that deflection? Suppose that deflection will be say it is delta, that deflection delta will be actual force F6 into deflection into change in length this is subjected to unit so whatever deflection whatever change in length in member AC we get for this if we multiple that with F6 that will be the total deflection in member 6. So delta it is delta F6 delta 6 say, okay delta in member 6 is equal to F6 into deflection in this, okay.

Now you see you please if you recall the way we wrote the flexibility we wrote the load deflection in flexibility format that is deflection is equal to force multiplied by flexibility, right? So here delta 6 is equal to F6 into the deflection in deformation in member AC due to this unit load, so deformation in member AC due to the unit load is essentially the flexibility of this member, right?

So that flexibility is written as f66, why it is f66? Because this is associated with deformation in member 6 due to the load applied in the direction of member 6 that is the way if you recall when we wrote the flexibility Fab, Fbb alpha ab in one of the previous classes that is how we wrote the flexibility coefficient, right? So then deformation, deformation in member 6 will be the force F6 into the flexibility coefficient, and what is f66? f66 if we call the unit load method f66 will be summation over i in this case i is equal to 1 to 6, 6 members and then Li by Ai Ei into ni square, okay.

Why it is ni square? Because it is you see it has it was capital Ni small ni because in this case we are measuring deflection in this direction due to the load in this direction, okay that is why this is the forces due to the load in this direction and this is the forces due to the unit load in this direction. Now what is f66? f66 is the deformation in (mem) same deformation in member AC due to the force unit load acting in the direction of this member, okay that is why it is ni into ni which is ni square that was unit load method was, right?

Now we have already determined what these are the ni's so what we need to do is we just we have to substitute this ni into this expression, right? And if we substitute that, then what we get is f66 we get is this f66 we will get 2 L 1 plus root 2 divided by AE that is f66, okay. Now the compatibility equation step 5 which was compatibility equation says that DL which was the deformation in member AC due to external load plus F6 into f66 which is deformation in member AC or member 6 due to the member force F6 that should be equal to 0, right? That is compatibility condition.

Now delta L we have already obtained this so we can write this minus PL by AE 2 plus root 2 is equal to plus F6 into this 2L by AE into 1 plus root 2, okay. And if we solve it then what we get is this we get P is equal to sorry F6 is equal to P by root 2 F6 is equal to P by root 2. Now you see again we go back to the original structure that was our original structure and we found that it is it was externally indeterminate structure and degree of indeterminacy is static indeterminacy is 1 and we chose AC as redundant so member force in AC as redundant because we have just 1 redundant and the problem the entire problem was to find out this redundant forces redundant force in this case.

And we finally obtained this redundant force F6 is equal to P by root 2, now once we know the member force in the force in member AC is equal to P by root 2 so if we know it is P by root 2 this is P by root 2 then what happens then we can see that this structure become rest of the things become determinate and we can just apply equilibrium condition that was our last step, step 6 apply equilibrium condition once the redundant force is not computed apply the equilibrium condition to find out other unknown, in this case other unknowns are forces in other members and we know how to do that you can use any method, method of joint or method of sections, okay.

Now this problem again it was the if we compare this with the problem that we consider in the previous class, in the previous class the problem was it was indeterminate but the indeterminacy was external indeterminacy. In this case the it is indeterminacy but it is internal indeterminacy but in both the cases the indeterminacy is static indeterminacy was 1, what we will do in the next class is we will take a problem where the indeterminacy is more than 1 and we will see then how to the basic concept is same, but how to do the calculations or the demonstration of the same concept through a slightly complicated problem we will see in the next class, okay thank you.