**Course on Structural Analysis 1**
**By Professor Amit Shaw**
**Department of Civil Engineering**
**Indian Institute of Technology, Kharagpur**
**Lecture 38**
**Module 8**
**Analysis of Statically Indeterminate Structures: Method of Consistent Deformation**

Hello everyone, welcome to eight week of this course what we will be doing today of this week is we will apply consistent deformation that we have at least the concept of method of consistent deformation we learnt in the previous week we will apply that concept to various structures. This week we will be applying the method of consistent deformation for trusses and the next we will see how the method can be applied to beams and frames, okay.

So this so today we will see how the concept can be applied to method of application can be applied to indeterminate trusses finding support reactions and internal forces.

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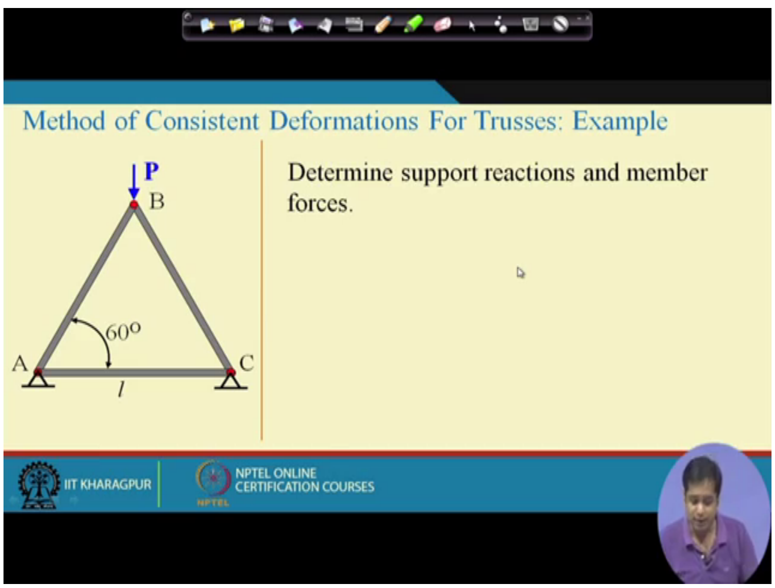


Let us demonstrate that through one example this is probably one of the very simplest example if you remember when we discussed analysis of statically determinate truss we also started with the same example, but in that case it is structure was statically determinate but in this case it is an indeterminate structure which we can easily identify by just looking at the structure the number of support reactions provided here four but for the stability of the structure we need actually three one hinge support and one roller support would have been enough for the stability point of view, but still instead of roller support one additional the hinge support is provided.

So it is a indeterminate structure, so what we need to find out? We need to find out what are the support reactions and member forces by method of consistent deformation, okay. So in the previous last week we discussed different steps for method of consistent deformation, now what we do is we just follow those steps.

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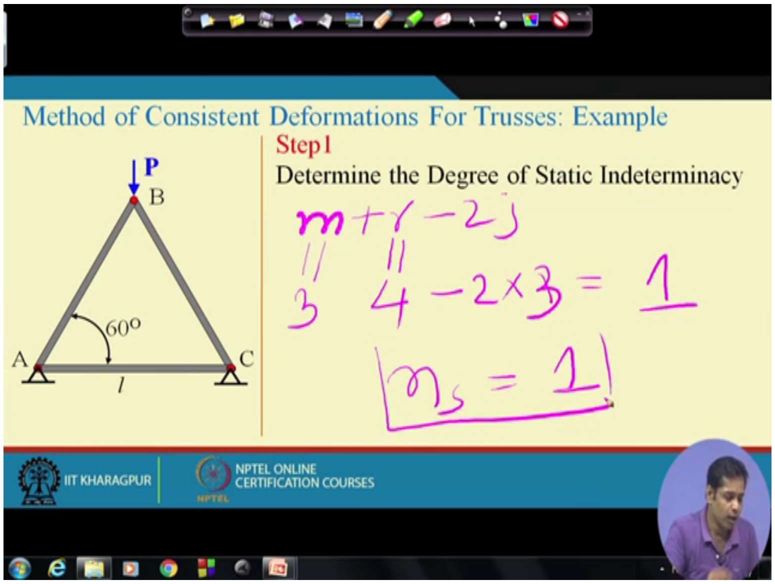


So step 1 was determine the degree of static indeterminacy let us find out what is the degree of static indeterminacy for this problem, we know for truss the static indeterminacy is m plus r minus 2 j where m is the number of total number of members, r is the total number of reactions and j is the number of joints.

So in this case m is equal to 3, 3 members, r is equal to 4, 2 here and 2 here Ax, Ay and Cx, Cy and then minus 2 into number of joints are 3 so this is 3 so this become 1. So static indeterminacy for this problem ns is equal to 1, okay. So this was the first step, now next step is once we know what is the static indeterminacy, next step to identify the selection of the redundant force or redundant moment. So in this case the moment is not relevant because truss is all the members we can have at every joint we can have two translations so either force in x and force in y direction and if you take any member the force in member is always along the axis of the member.

So in this case we need to identify what is the redundant force, you see this is the step the most important in the entire process because the step before this and all the steps after this they were in somewhere mechanical they are you need to just follow the procedure once you know the procedure then you can so that, but this is the step second step where you need to identify the redundant force their some sense of structure analysis, some sense of stability, some sense of configuration or overall behavior of the structure is equal.

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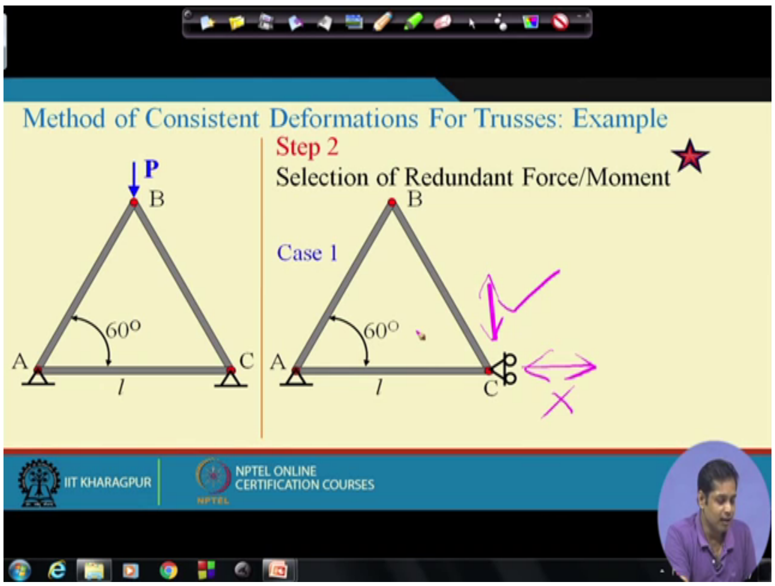


For example, in this case you see there are two possibilities this is a symmetric structure suppose we want to take one of the reaction as redundant force, okay. Now suppose consider joint C, so at joint C we have two reaction, one is Cx and one is Cy, right? So either we can take Cx as redundant or we can take Cy as redundant. We can take Ax or Ay as a redundant as well but this will be same because it is symmetric structure. So we can take either Cx as redundant or Cy as redundant, so let us see what happens if you take Cx and Cy as redundant forces, okay. So first case is first possibility is this, okay first possibility is this, in this case what is the release we have given in this structure? Suppose C is now allowed to move in this direction, C is now allowed to move in vertical direction C is now allowed to move in this direction, in this direction, okay.

The point C is allowed to move in this direction, but the movement in this direction movement in this direction is restricted, so this is allowed to move in this direction, okay. So in this case we have taken Cy as redundant, okay.

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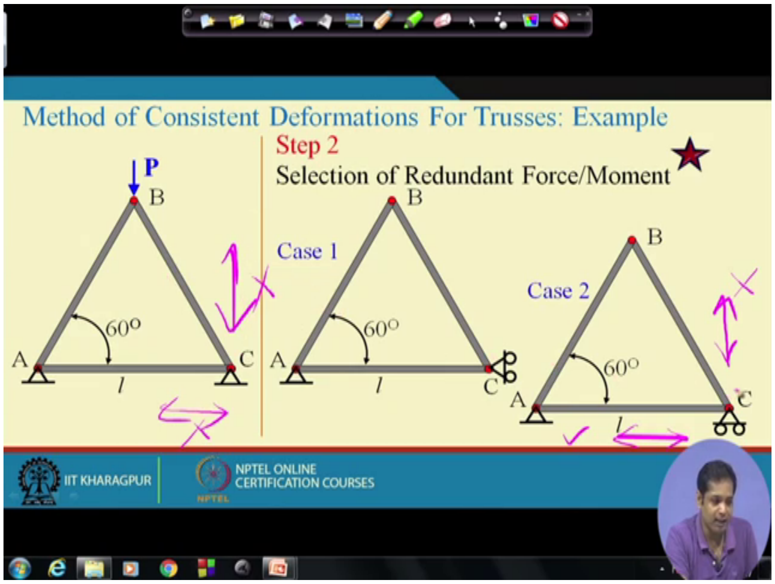


Now, then next is next possibility is this, next possibility is this, okay in this case what happens? In this case you see the structure is structure can move in this direction, the structure can this structure this point C can move in this direction this is allowed, but movement in this direction is restricted.

In original structure the moment in both the direction is restricted for C, this is also restricted and this is also restricted, but in case one we allow this and case two we allow this and restrict the other one. Now let us see, so this is case one and case two and both the cases we really do not know what case will be the possible case but as per as theory, both the case is at least as far as if we consider if the just the choice of if the point is select any redundant select the any reaction as redundant force the both two cases case 1 and case 2 both the cases seem to be fine, but actually they are not, let us see why they are not.

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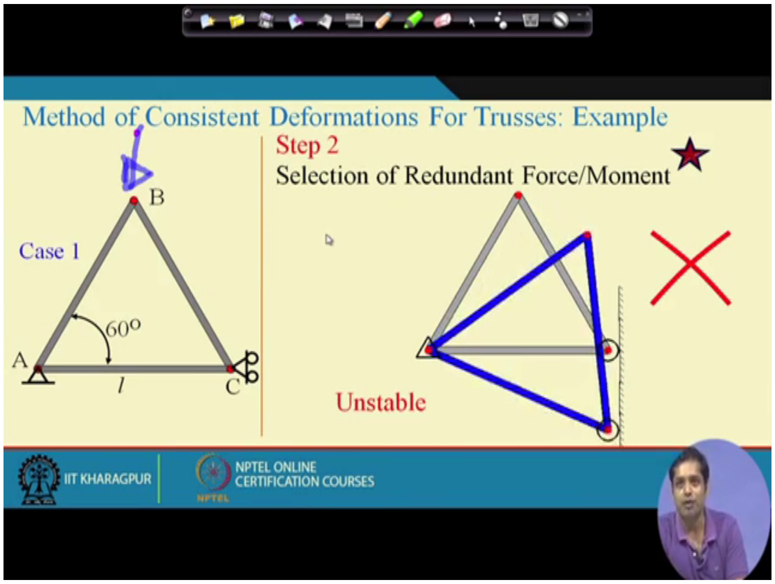


Now if we take first consider case 1, okay so this is case 1 where this movement is allowed and this is restricted and suppose this if we apply any load like this if we apply say any load, okay any load like this, then what happens the structure and let us see how the structure may deform this structure may deform like this you see. So this joint C can move in this direction but the movement in this direction is restricted. Now this is unstable, okay. So this is unstable and therefore this configuration the case one cannot be possible so this is not a possible choice, okay.

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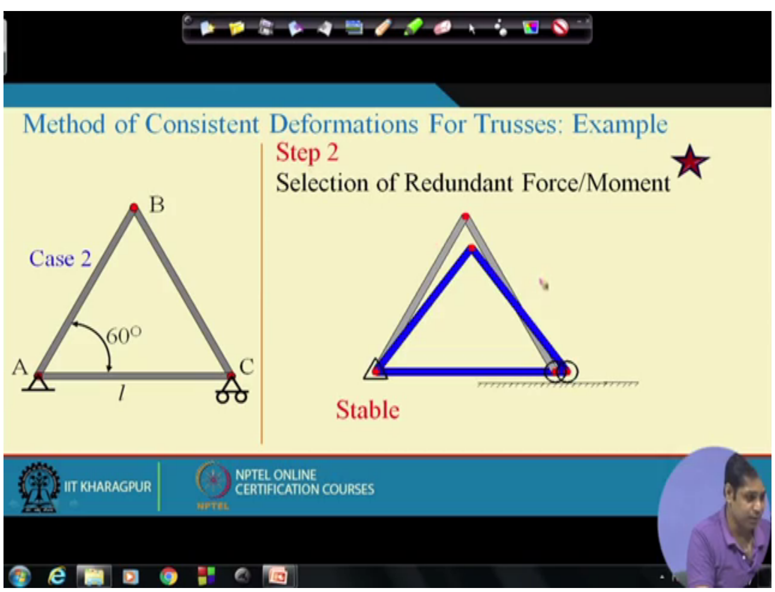

Now if we see second case, second case is case 2, case 2 is where this motion is restricted but C can move in horizontal direction, then what is the movement if it is applied to some load like this if it is applied to some load like this, then it can move in this direction and this configuration is stable configuration. So which configuration is stable configuration you there are two ways, one is you take any configuration and write the equation and then you find that the structure is not stable, but instead of finding or understanding the structure is not stable through that exercise it is always better to develop the sense that engineering sense so that just by looking at the structure and looking at the force and the configuration of the supports you can identify the which configuration will be stable configuration and the stable configuration when you take any force as redundant force, but the primary structure after taking the redundant force removing the redundant force allowing the deflection in corresponding direction the primary structures that you get that should be statically determinate and should be stable, okay.

Now, so case two is the possible case I suggest you take both the cases case one and case two write the equations of equilibrium and try to find out whether which steps are which step is stable and which is unstable, okay. Now so we will consider case two, okay so the step one was the calculation of static indeterminacy we have done it is 1, step two was identifying the support reaction identifying the redundant forces so we have considered these identified the redundant forces and then get the configuration of the primary structures so this is the our primary structure and the next step the step three is if you remember solve the primary structure subjected to the actual load and then step four was stove the primary structures subjected to the redundant force.

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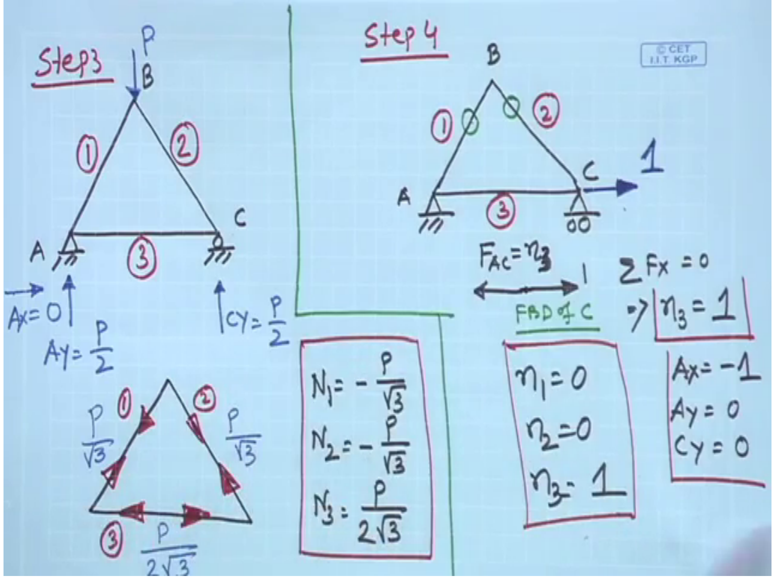


Let us do that, okay now so this was the first is step 3 first we will do step 3, step 3 is solution of the structure and analysis of the structure subjected to actual load. So this was the structure, this is A, this is B and this is C and it is subjected to a load P at B, okay. Let us give some name for this all these members, okay let us this member is 1, this member is member number 2, this member is member number 3, okay one way of identifying member is member AB, member BC, member AC but just to make it convenient to write these identify identification of the member it is better to give some number to the member.

So member 1 means AB, member 2 means BC and member 3 means AC, okay it would be easier if in this case it is only three members but if you any structure you can have many members many joints so for those cases it is easier to identify members through numbers, okay. So these are these are the and then we have a support here this is hinge support and this is roller support, right? We can we have already solve this structure if you remember when we discuss about how to analyze statically determinate (())(11:54) and the solution I just we are not doing that exercise once again you go back to that lecture and see what is the solution of this problem.

So the solution of this is your support reactions are like this, Ay is equal to P by 2 and Cy is equal to P by 2, Ax is equal to 0, okay Ax equal to 0 and the forces in each members are like this member 1 is equal to, okay let us write the let us draw the force diagram, the force diagram is like this, this is the force in member 1, then member 2 and then member 3 we have already done this in second week please see that lecture and these value is P by root 3 this is also P by root 3 and this is P by 2 root 3, okay.

So this means this member, this is member 1, member 2, member 1, member 2 and member 3, right? So member 1 is subjected to compression and the value is P by root 3 it is as a compression P by root 3 and member 3 is subjected to tension and the value is P by 2 root 3. Now if you remember unit load method so this is the this is the structure subjected to actual load so we write Ni N1 which is the member force in member 1 it is minus P by root 3 then N2 is equal to same minus P by root 3 and then N3 is equal to P by 2 root 3 this is the forces members when the structure is subjected to actual load, okay.

And the when the we use the convention of the unit load method when we discussed that method if you remember we mentioned the method is very powerful method and it has huge application in most of the subsequent methods that we learnt in this course. So let us and the usual notation is when the structure is subjected to the actual load, then it is written as N1, N2, N3 and then it is or capital letter M if it is moment then M1, M2, M3 and if it is when it is subjected to unit load dummy load then it is written in small letter.

So this is step 3 then take step 4, step 4 is the same structure is now subjected to redundant force, okay so draw the same structure let us this was step 1, okay. So the draw the same structure it is hinge support it is roller support and then this is A, B, C this is member 1, member 2 and then member 3 and this is subjected to a load like this, okay because that is the redundant force, okay. Now instead of applying a force Cx which is the redundant force support reaction at C in the direction x let us apply an unit load, okay.

The reason is if we know the value at the unit load what is the member forces support reaction those values if you multiply by Cx will get the corresponding member forces and reactions due to load Cx, okay because load displacement those relations are linear we can apply the method of super position. So this is the second this is another this is again determinate structure, but in this case it is subjected to redundant force of value 1, so let us analyze it and if we analyze it then you see by just looking at this structure we can say consider this joint if you remember we again when you analyze its truss first thing we see if we have some information about the truss or not.

If you can reduce our problem, our computation in this case one way is identify the if there are members which have 0 force, okay 0 force members. Now if you look at this structure, then this member and this member this member AB and member BC they are 0 force member because if you take joint B, then you can apply the free body you draw the free body diagram of joint B and apply the equilibrium condition you get this member this member as 0 force member but again instead of doing that just by looking at the truss you should be able to identify what are the 0 force members.

So these two members are 0 force member, so our problem becomes easier let us now draw the free body diagram of supports of C if you draw the free body diagram of C, then free body diagram of C will be this is point C and this force is 1 and this force is Fac which is also is equal to N3, N3 means actual force in member 3 or Fac. So apply so this is Fbd of Fbd of C, okay. Now apply summation of Fx is equal to 0 and this gives us N3 is equal to 1, right? N3 is equal to 1.

Now then if we apply if we draw the entire free body diagram of the entire structure and apply the summation of Fx is equal to 0 and we will get Ax is equal to we will get Ax is equal to again minus 1, okay and then Ay is equal to 0 and Cy is equal to 0 we can get it please check this, okay. Now then what we have here? We have N1 is equal to 0, N2 is equal to 0 and N3 is equal to 1, okay.

Now, instead of 1 if the structure is subjected to Cx, then the corresponding forces in member will be N1 0, N2 0 and N3 will be 1 into Cx is equal to Cx, okay. Now step 4 is done, then what was the next step? Next step was to apply the compatibility equation, okay. Now what is the compatibility condition in this case? You see in this in the primary structure because of this load the C will the movement of C in this direction is allowed so because of this load the C will move in this direction slightly.

And similarly because of this unit load here C will move in this direction slightly. So total deflection total deformation at C will be total movement of support displacement that C will be this plus this and the compatibility condition says that because in the original structure this point was hinge support there should not be any deflection at C at deformation at C so compatibility equation say that deformation obtained from this and deformation obtain from this there summation should be equal to 0. So suppose this is delta C1 this is suppose this is delta Ch this is delta Ch1 means the horizontal displacement at C in this structure is equal to delta Ch1.

And the suppose this is delta Ch2 horizontal displacement at C in this structure should be equal it is delta Ch2, then step 4 compatibility equation say that delta Ch1 plus delta Ch2 they should be that should be equal to 0, okay.

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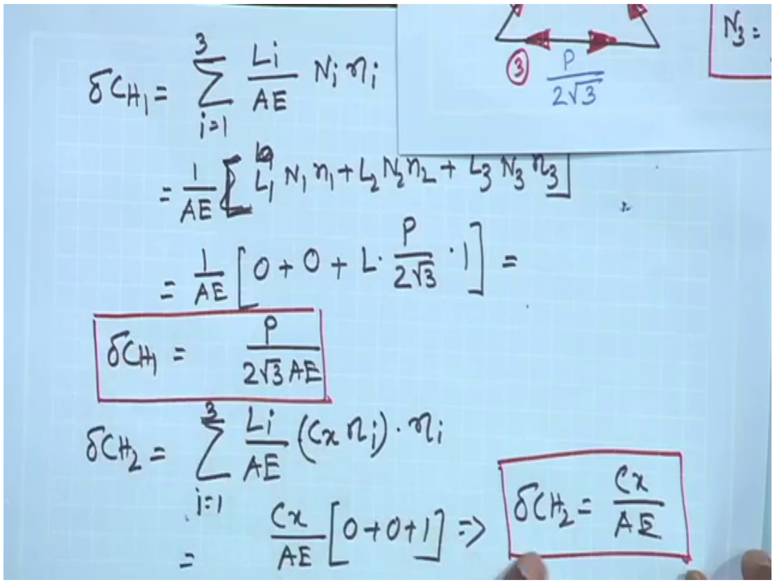


Let us before but before we apply compatibility condition we need to find out what is delta Ch1 and what is delta Ch2 we simply apply the unit load method and if you remember the unit load method, unit load method says that delta Ch1 will be summation over i, i is the 1 in this case 1 to 3 because there are only 3 there are 3 members, okay and then Li divided by Aei in this case A and the cross sectional area and young's modulus they are constants so we can write AE here length of Li is different for different members.

Then Ni into small ni that is unit load method, right? We discussed in fact we applied this method to truss problem in one of the previous weeks, okay. Now if we apply that, then what we get? That is equal to, okay so 1 by Aei we can take it out 1 by E please remember if the A is not constant it is different for different members then that you need to write this is for different member you need to write here, but in this case it is constant that is why we can take it out.

So this would be then L1 into N1 n1 and then you can take it like this plus L2 N2 n2 and then plus L3 N3 small n3, okay. Now you see here we have already determined N1, N2, N3 and small n1, small n2, small n3, so small n1 and n2 are 0 so this cross this 0 this into this 0 so what we are left with n3, small n3 and that is equal to so 1 by AE this part will be 0 because n1 is 0 this part is 0 because n2 small n2 is 0 and this part is L3, L3 means length of this member which is capital which is L into N3 is equal to P by 2 root 3 and into 1 small n3 is equal to 1.

So this becomes P by 2 root 3 AE that is delta Ch1, okay delta Ch1 is this, okay. Now we need to find out what is delta Ch2, now delta Ch2 that will be equal to same way you see here unit load is here if it is applied to unit load, then this is the these are the deflection and now if it is not unit load if it is Cx, then the deflection then the corresponding member forces will be 0, 0 and Cx. So in this case again i is equal to 1 to 3 and then this is Li by AE which is Li is length at different members and then now unit load for dummy load this is the result and for actual load in this case actual load will be Cx applied here the result will be 0, 0 and then Cx.

So this will be say Cx into ni which is the member forces when actual load in this case actual load is the dummy force Cx is applied and then due to unit load which is ni, okay that is so this becomes if we take Cx Cx by AE out and then this becomes ni square now n1 and 2 are 0 and n3 is equal to 1 so this become 0 plus 0 plus 1 so this become so delta Ch2 become Cx by AE, okay Cx by AE, okay.

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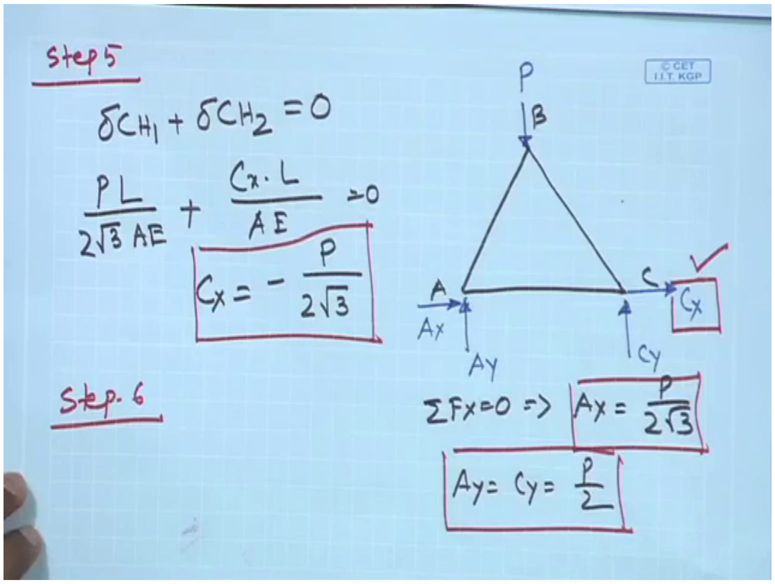


Now if it is now next compatibility equation is this plus this should be equal to 0, so step 5 compatibility equation, compatibility equation says that delta Ch1 plus Ch2 that is equal to 0. Now we know what is delta Ch1 here and delta Ch2 here so write it delta Ch1 was L by delta Ch1 there is a correction it will be P into L there should be L here, okay and there also there should be one L here because it is L, okay this is L here please correct it.

So this is PL by 2 root 3 AE and then delta Ch2 is Cx into L by AE that is equal to 0 and from that we get Cx is equal to minus 1 by 2 root 3 or P by this, okay Cx is equal to this, okay. Now if you draw the actual this total structure and this is A, this is B, this is C and the free body diagrams are this is subjected to a load P, this is Ay, this is Ax, this is Cy and this is Cx, okay. Now there where 4 unknown, but now Cx is known so once you know Cx then rest is you apply other three equilibrium condition take summation fx is equal to 0 summation of fy is equal to 0 and summation of fz is equal to summation of moment is equal to 0 we get what is the value of other support reactions.

So if we do that if we take summation fx is equal to 0 this gives us Ax is equal to minus Cx so Ax will be P by 2 root 3, okay Ax is equal this if you take summation of fy and forces summation of force in y direction and summation of moment is equal to 0 we get Ay is equal to Cy is equal P by 2, okay and rest the thing is you take free body diagram of every joint and calculate the member forces, okay now that was step 6 so step 6 was once we have this reaction once we have

Cx, then apply the free body apply the equilibrium draw the free body diagram and apply the equilibrium condition on this structure to get the other unknowns in this case other support reaction that the member forces, okay.

So this was this were different steps for demonstrated demonstrated for this problem, okay. Now demonstrated for this problem this problem, okay. Now you see it was a very simple problem only three joint three member and supports are also degree or static indeterminacy was also once we needed only one redundant force that is why we could do it like this, now but in a structure when we have when we have many indeterminacy when many redundant forces many members many joints so the concept is same but the calculation in this way may not be it becomes tedious.

So in that case either we can write in a matrix form or a tabular form those thing we will see in the class we will take slightly complicated problem which has more number of static indeterminacies more than 1 so number of redundant forces are more than 1 we will see how that can be solve using method of consistent deformation, but the essence of the method is still the same, but in this case again the unit load method plays very crucial role we discussed that in one of the previous weeks please go through them and make yourself confortable with the method because very often all the problems that we will be doing next the unit load method plays a very important role there.

So please revise that so we stop here today and next class what we do is next class we will see some more examples on trusses analysis of trusses using method of consistent deformation, thank you.