Structural Analysis I
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Lecture 37
Analysis of Statically Indeterminate Structures (Continued)

Hello Everyone! Today is the last lecture of this week, what we were discussing in the last class is we just introduced what is kinematic indeterminacy, we demonstrated kinematic indeterminacy through some examples on truss and also one example on simply supported beam, now what we'll be doing today is we'll show you some more example to determine kinematic indeterminacy and then see what are the steps of force method.

And compare them with a displacement method and through that comparison we introduce what is displacement based method ok, now let us see some example, now first example is you see this is a fixed beam,

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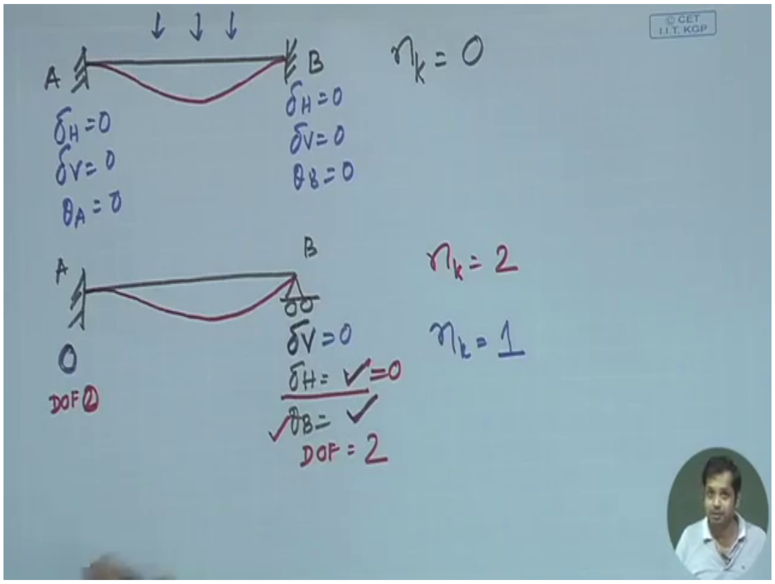

Consider a fix beam, a fixed beam means both ends are fixed and if the beam is fixed and it may undergo deformation like this for any arbitrary loading like this ok, now at this point delta is, suppose this is A, this is B so and this point horizontal delta H is equal to zero.

Delta V is equal to zero and theta A is equal to zero, similarly delta H is equal to zero here, delta V vertical displacement zero and theta B is equal to zero, so at this joint all the degrees of freedom is zero and the kinematic indeterminacy is the total degrees of freedom at various joints so in this case the total kinematic indeterminacy in k will be zero, so it is kinematic ally indeterminate structure, kinematic indeterminacy of the structure is zero.

Now take one more example same one end is fixed and suppose another end is roller ok, it is propt cantilever beam and if we deflects then it may deflects like this, as I said it is good to sense the deflection of a structure by looking at the boundary conditions and the applied load ok, so this is A B, what is the degrees of freedom here it is fix support, so degrees of freedom is zero here, all horizontal motion vertical motion and rotation they all are restricted.

Now in this case delta V is zero there is no vertical movement but delta H is allowed and theta B is allowed ok, so here degrees of freedom is zero, here degrees of freedom is two ok, so total degrees of freedom is N K is equal to two, now here is important point now if we assume that the axial rigidity of this beam is very high there is no axial deformation, then delta H is also zero ok, so because delta H will cause axial change in length of A B.

But if we assume that there is no axial deformation so delta H is equal to zero, if delta H is equal to zero then how many degrees of freedom at joint B has, joint B has just one degrees of freedom which is theta B, so in that case the N K for this beam will be one ok, so if we allow axial deformation then kinematic indeterminacy of this beam is two and if we do not allow axial deformation assume axial rigidity is very high then kinematic indeterminacy is one. We'll see in the problem that many problems we assume at least for the time being we'll neglect the axial deformation ok.

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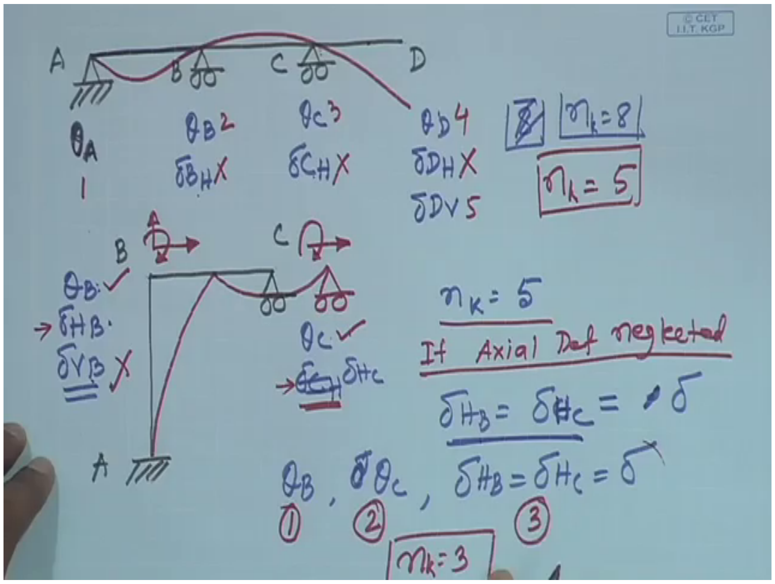

Now take one more example this is a continuous beam, this is simply support here then we have another roller support then another roller support and suppose this is A, this is B, this is C and this is D ok, now under axial of let see try to understand what could be the deflected shape of the beam under axial of some load deflected shape maybe like this ok.

It depends on the load what kind of load you have and what is the value of the load but when you draw the deflected, I am just drawing for some arbitrary loading case, suppose this is the deflected shape but when you draw the deflected shape few things you need to understand that displacement, the continuity of the slope, slope and deflection is continuous at this joints that we need to keep in mind ok, suppose this is the deflected shape.

So end A is a hinge support then what is the degrees of freedom here, what is allowed, there is no translation allowed only allowed is theta A ok, similarly if you at C B, B translation is allowed and rotation is allowed, so what is allowed here is theta B is allowed and delta B horizontal movement is allowed and at C then theta C is allowed and then horizontal is allowed ok, then what is at D, at D theta D is allowed then horizontal displacement of D is allowed.

And also vertical displacement of V is allowed because at D there is no support at all, so in planer structure at any point we have three degrees of freedom so all three degrees of freedom the point D has, so total how many degrees of freedom it is one, two, three, four, five, six, seven, eight, so here total degrees of freedom is N K is equal to eight, but in this when we say that the kinematic indeterminacy is eight.

We have not made any assumption whether the axial deformation is allowed or not but if we do not allow axial deformation then what happen, this is not allowed, this horizontal motion is not allowed because in this planer structure the point B moves here that means there is a change in length in A B, if C moves here then there is change in length in C B or if D moves here there is a change in length in A D, but the change in length the axial deformation is not allowed.

So delta C is also be zero an delta horizontal this is also zero, then what is the degrees of freedom now what is the kinematic indeterminacy now, kinematic indeterminacy become one, two, three, four, five so total kinematic indeterminacy is five for this problem ok, so it is very important whether when you applied the kinematic indeterminacy it is very important to know or state whether you are actually allowing axial deformation or not ok.

Now take one example of a frame problem, suppose this is a frame and this end is fixed end, this is A, this is B, and this is C ok, suppose at C we have a roller support ok then what happens, let's draw the probable deformation of the structure it may become something like this ok, for some loading condition ok, now at this point it is fixed support, the degrees of freedom is zero, rotation is not allowed in any translation ok.

And what at point B we have theta B and then we have delta H be horizontal motion of delta H and then delta V B at point B, so point B can rotate, point B can move in this direction and point B can move in vertical direction, if we just draw this, this point B can translate in this direction, point B can translate in this direction and point B can rotate as well ok, now what happens to C, the point C is, at point C we have theta C and we have delta C H ok.

Because point C can move in this direction and point C can rotate in this direction there is no vertical displacement at point C because it is roller support here, then what is N K, then N K becomes one, two, three, four, five N K become five ok, now when we say N K is five for this case we have not made any assumption that the axial deformation is allowed or axial deformation is not allowed, now if axial deformation is not allowed.

If axial deformation neglected then what happens, now let see if the axial deformation is neglected what happens to this degrees of freedom ok, now what is the consequence of theta B, theta B tells you how the point B rotates ok and what is delta V B, delta V B ,means how this point B moves in this direction ok, now we’ll see if the point B moves in this direction then what happens then the length of A B changes the effect of this alone.

When we are looking at each degrees of freedom please see the effect of each degrees of freedom alone and then see whether this degrees of freedom is allowed or not, now if we allow point B only to move in this direction and this will cause change in length in A B, this will cause axial deformation in A B, now if they axial deformation is neglected then this cannot be possible isn’t it ok, now so delta H B is possible now look at point C, what is theta C.

Theta is the rotation at point C, rotation at point C is allowed, now see what is the effect of delta C H, effect of delta C H is point C moves in this direction and what is the effect of H B is, H B point move with this direction, now if delta, this value and this value they are different means the amount by which point B moves in this direction and the amount by which point C moves in this direction if they are different then what happens then the length B C.

Either they increase or they decrease, if point B moves more than point C then the length of B C decreases and if point C moves more than point B the length of B C increases isn’t it, but increase and decrease in length is not allowed here because we are neglecting the axial deformation, so the member B C will not experienced any axial deformation only when delta H B is equal to delta H C, so horizontal displacement at B is equal to horizontal displacement at C.

If this is ensured then only there will be no axial deformation at BC ok, and suppose this is delta or this is delta ok, so we can combine this and this then how many degrees of freedom we have, we have theta B, rotation at B we have then rotation at C we have then we have delta H B which is equal to delta H C is equal to delta, so we have degrees of freedom this is one, this is two, and this can be treated as one degrees of freedom because if we know delta then we know delta H B.

Or delta H C, so this is three so in this case total N K is equal to three ok, so if we do not neglect the axial deformation for this same problem the kinematic indeterminacy is five but if we neglect axial deformation then kinematic indeterminacy for this problem will be three, so you can find out kinematic indeterminacy if you see any book then there are different kinds of example given, different example on trusses, different examples of beams with different support condition.

And different example on frames with different support condition with internal hinge and so you can apply this same concept to find out the kinematic indeterminacy but when you find out the kinematic indeterminacy you please keep in mind whether you are allowing the axial deformation or not if you do not allow axial deformation whatever possible movements at various joints we have all the possible movements need to consider.

But if you do not allow axial deformation then the movements, the degrees of freedom which may cause the axial deformation in various member those degrees of freedom needs to be neglected ok and then kinematic indeterminacy needs to be determined so in order to be more comfortable with determination of kinematic indeterminacy of various structure I suggest you please look go through the books and see some of the examples given in the book ok.

Now what we do quickly is let us introduce what is displacement based methods that was our objective you see the inter concept of structure analysis that at least for this course what we are studying that is based on three pillars ok and what are those three pillars.

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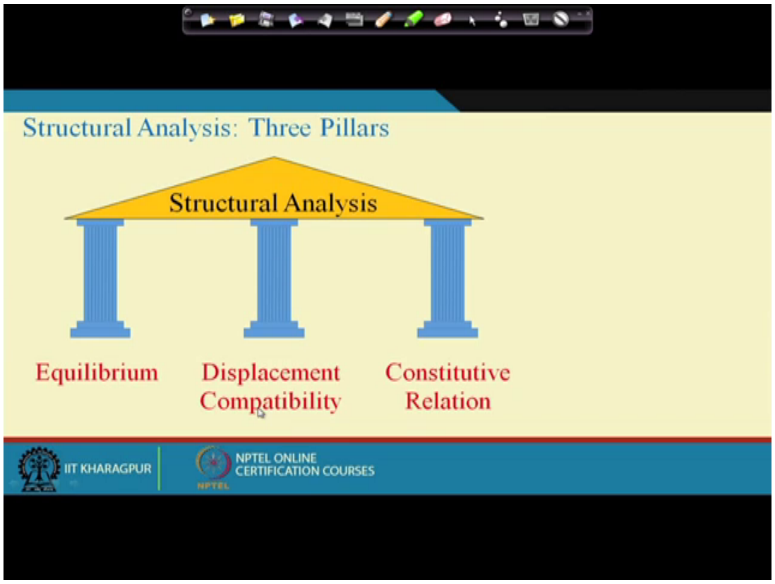

These three pillars are this the entire structure analysis is based on three pillars and this three pillars are one is equilibrium, what it says the internal forces is equal to external forces.

Or the total forces in a particular direction or total moment about any point this should be equal to zero, so that is the length equilibrium equation, then the next pillar is the displacement compatibility means if any structure and it's force system satisfy the equilibrium condition we say that this statically admissible force system, so equilibrium condition gives you the statically admissible forces ok.

Now displacement compatibility means displacement at various joints should be compatible to each other, if it for instant the slopes it talks about the continuity of the slope, it talk about the whether the displacement is allowed, whether rotation is allowed, all the problems, all the constraint that in a structure we may have, all these constraints should be satisfied properly, so this is displacement compatibility and if the displacements are compatible.

Then we say that the system that we have that is kinematic ally admissible, so a solution which satisfy equilibrium condition and which satisfy compatibility condition that solution is the unique solution that is the statically admissible as well as kinematic ally admissible so in order to get the unique solution for indeterminate structure then we need both equilibrium and displacement compatibility and then finally we have the constitutive relation.

Which tells you how the forces are related to displacement, stresses are related to strain so we have the forces, we need to find out displacement, get the force displacement apply, substitute those forces in the force displacement relation, get the displacement or if we know the displacement in order to get the force you substitute the relation and then get the forces, so whether the forces displacement relation.

Whether we know force and determine the displacement or we know displacement, determine the force depending on that we either use the flexibility format or use the stiffness format, the flexibility and stiffness format we discuss in the last class ok, now if the structure is statically determinate, the equilibrium condition is alone sufficient to give you the solution but if it is not then we need along with equilibrium conditions.

We need displacement compatibility and in order to get the relation between force and displacement we need constitutive relation as well, now how this three pillars are used ok, in what order this three pillars are used depending on that we have two methods, one is force based method and one is displacement method bit in all the methods either force or displacement methods they are premise of both the methods are these three pillars ok.

But how these three pillars how this three ingredients are used, in which order they are used depending on that we have two methods force method and displacement method, now what I will do is we discussed the different steps of force method, we introduced the force method through an example but that we are not going to do it for displacement method, we know the different steps for force method and we'll see what are the corresponding steps in displacement method ok.

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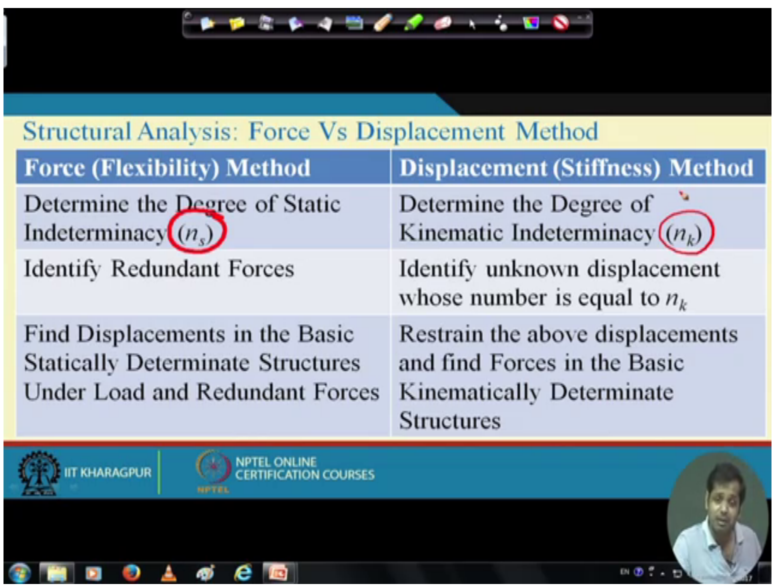

So these are force method which are also called flexibility method, displacement method which are also called stiffness method that we discussed in the last class, if you remember in the force method the first thing what we do is, we calculate the what is the static indeterminacy of the problem ok, but in the stiffness method what you do is to find out what is the kinematic indeterminacy.

We just discussed how to determine kinematic indeterminacy for different structures, now in force based method then once we know the static indeterminacy then we need to understand what are the redundant forces when you say the forces it includes force and moments both, so identify the redundant forces right, but in this case in the displacement method we need to identify what are the unknown displacements and the number of those unknown displacement.

Should be equal to the kinematic indeterminacy of the structure, so in the force based method we need to identify the forces and in displacement based method we need to identify the displacements as name suggests you know it is very obvious from the name in a force method, force is essentially the unknown and the displacement method it is a displacement which are unknown ok, then what you do in the force based method we find.

Then we find the basic determinate structure which are subjected to load and the redundant forces but what we do in displacement based method is restrain the above displacements, whatever displacements are unknown displacements we have those displacements need to be restrain and find the forces in the basic structure, now when we divide the structure in the force based method we need to ensure that all those primary structures.

The basic structures are statically determinate and in displacement method when we divide the structure into small small sub structure we need ensure that all this small sub structures are kinematically determinate and then we find the forces here ok.

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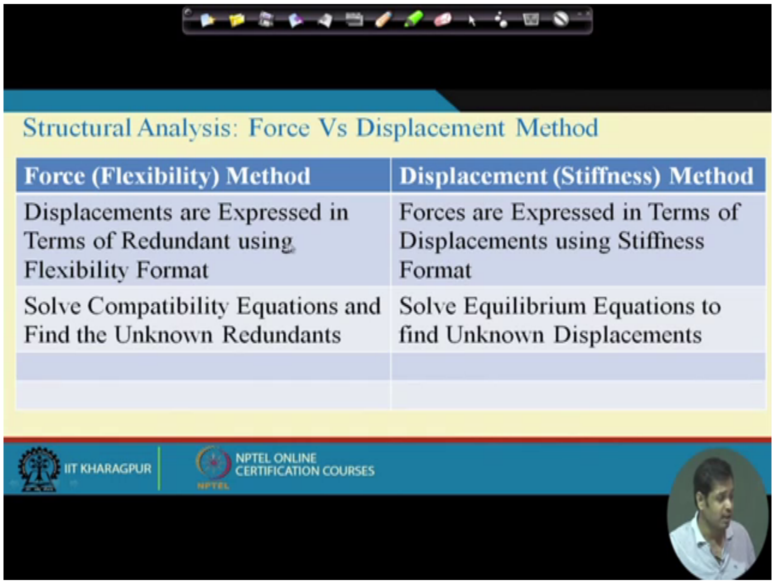

Now next step is once we know the displacements then the displacements are expressed in terms of redundant forces using the flexibility format. But in displacement method, we need to express the forces in terms of displacement in the stiffness format ok.

And then in flexibility method we write the compatibility equation and solve the compatibility equation and finally in displacement based method once we have this form we solve that form and solve the equilibrium equations for unknown displacements, now some of the steps will not be clear which is very obvious but the reason I did not introduce through a problem.

Because we already have done that for force method, here the objective is, we know the steps for force method, what is the corresponding steps for displacement method ok, but those steps will be explained in detail when we actually formulate the displacement methods and apply to different kinds of problem, here the objective has been to just we tell you that in what sense those two methods are different, what are the recipe for all these methods ok.

Of the entire week that has been the purpose of the entire week, now the recipe is ready what we need to do is we need to start the cooking and that we'll be doing from next week, next week what we do is we'll start with force method, next week we'll spend on force method and then after that another two three weeks on displacement bend method, the force based method it is, we'll discuss method of consistent deformation.

We’ll first apply the method of consistent deformation for trusses and then method of consistent deformation for beams and frames and then once we had done that, and after that we’ll derive the displacement methods and apply to different problems ok then thank you, see you in the next week.