**Structural Analysis I**
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**Lecture 36**
**Analysis of Statically Indeterminate Structures (Continued)**

Hello Everyone! Now what we have done so far is we discussed what is indeterminate structures and there are majorly two different kinds of methods for solving them, one is force method and one is displacement method, we introduced what is force method just introduction, what we'll be doing today is we'll introduce what is displacement method ok, we won't go in details of the displacement method as we did not go in detail about force method. Just the concept we'll introduce today and the next week onwards when we'll apply those methods to different problems the concept will be probably more clear ok,

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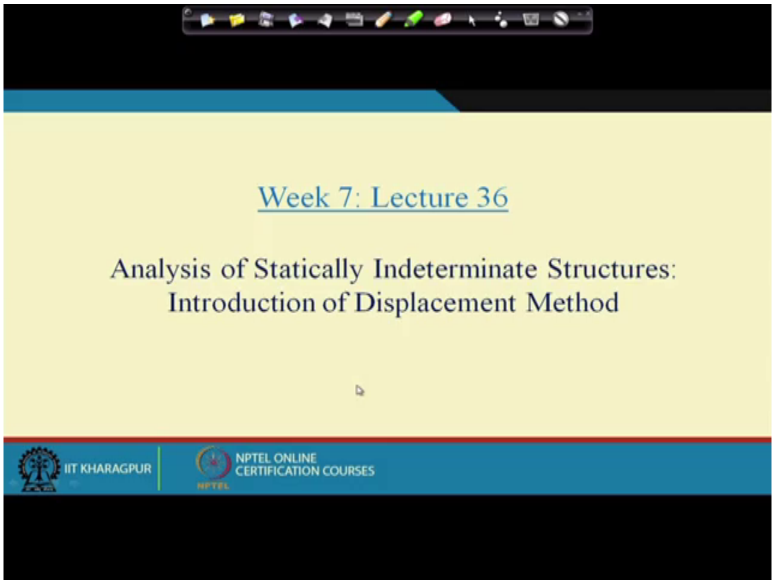


So today's topic is Analysis of statically indeterminate structures same we have been continuing and introduction to displacement method ok.

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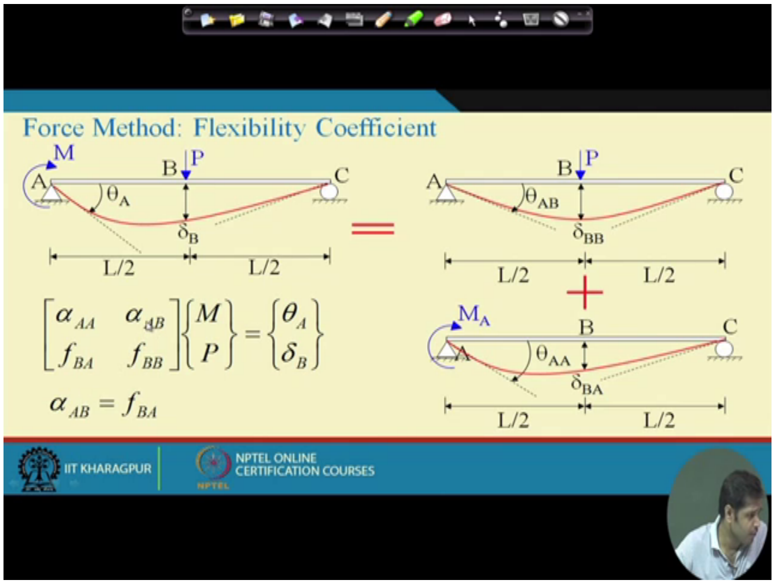


If you remember in the last class we stopped at this slide ok, when we introduce what is force method. And finally we arrived at the force displacement relation as this ok, in this case forces are one is moment applied at A and one is concentrated load applied at B and the displacements at theta A,

Rotation at A and delta B is the displacement at point B and this is the relation ok, and we also saw that this is flexibility matrix right and all this coefficients Alpha and f we can see, these are all flexibility coefficients, we also saw that this matrix is a symmetric matrix.

So alpha A B this term and this term they same ok, so alpha A B is equal to alpha B A, now I ask you please check why it is not accident if you even for different structural problem if you see and if you compute the flexibility matrix, you we'll see those matrixes are symmetric matrix and what exactly it says you know this theory this is essentially.

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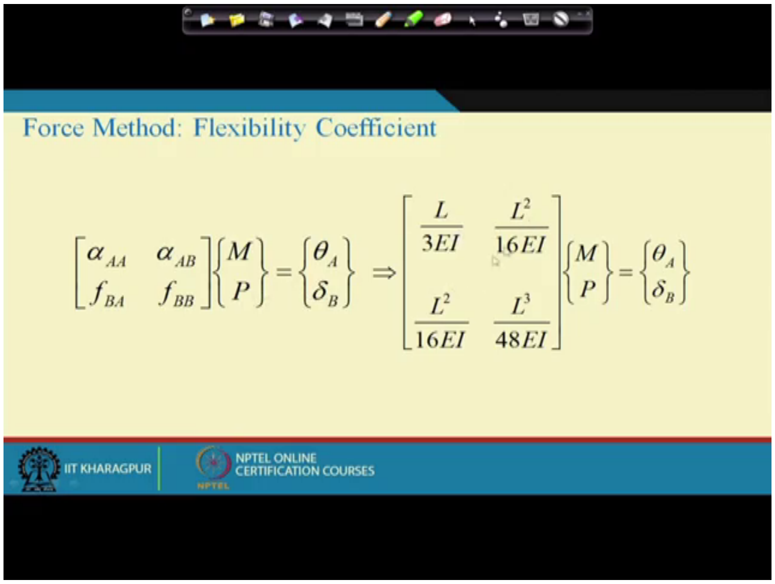


So these are the matrix you can see that this alpha A B is equal to F B A this is symmetric matrix ok.

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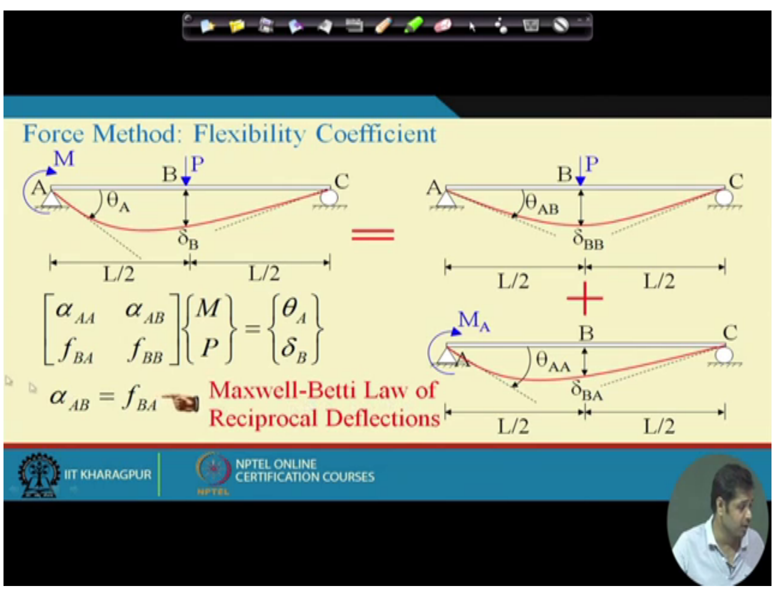


Now what exactly this is says, this is the theory of reciprocal deflection Maxwell Betti law of reciprocal deflection, what it says is we already discussed this reciprocal theorem when we were discussing virtual work principle what it says that if you put M is equal to one and P is equal to one, what is alpha A B, alpha A B is essentially is the rotation at theta A due to P and what is F B A, F B A gives you the displacement at B due to applied load M.

So if you take both are unit M is equal to one and P is equal to one then alpha A B will be the rotation at A due to unit load applied at B and then alpha B will be the deflection at B due to the unit moment applied at A and as per the law of reciprocal deflection they should be same so that’s why this is same ok, so we already have seen it in the form of law but now it is a demonstration of that law. If you take any structure problem it is a consequence that the matrix that will get that is symmetric ok

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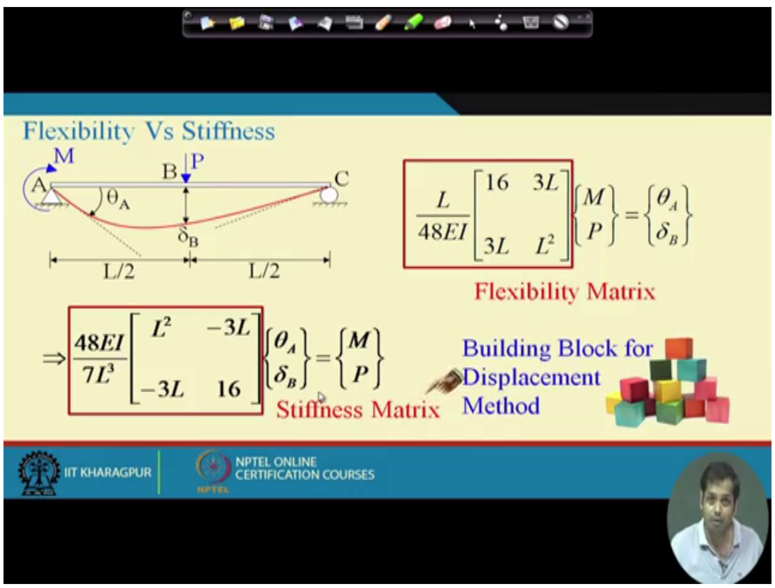


Now so this is the matrix what we have just the same matrix but only L by 48 EI term taken out. Now if we inward this matrix, if we inward this relation means now this relation give you theta, force displacement is equal to this flexibility matrix into force right, if we inward this relation what we get is this is called flexibility matrix right.

We discussed it now if we inward it then what we get is that inverse of this matrix is this so now we’ll get the force is equal to something into deflection and this something is called it is the stiffness matrix, if you remember how the force, how the displacement is related to force we discussed that if F is the flexibility then flexibility into force is equal to give displacement right, And if K is stiffness then what we get is stiffness into delta displacement is equal to give you the force we have demonstrated this through a spin problem ok in the last class so this matrix is called stiffness matrix so this is inverse of the flexibility matrix,

Now one important point here you can note that, M E I is the, E I is called flexural rigidity of a beam because if young's modulus is more, then the beam undergoes less deformation, similarly if the second moment of area is more, then the beam undergoes less deformation, so together their affect are similar, so together E I is called flexural rigidity ok.

So if E I is more for a given problem, for a given beam or a given structure if E I is more means the flexural rigidity of the member is more so the deflection of the member will be less, so the structure becomes more stiffer, so that's why this is called stiffness, now in this case if E I is less then what happen the member undergoes more deformation so the member or the structure becomes more flexible, it undergoes more deformation, so this is flexibility matrix.

Now why I am showing this line because you see in force method this flexibility coefficients plays a very important role, they are essential in the building blocks of force method and if you do it in a matrix form, force matrix method then the flexibility matrix is the building blocks, now similarly in displacement method it is the stiffness matrix is the building blocks for displacement based method ok.

So what we are going to introduce today. So essentially it is the same load deflection relation your writing but in the force method that relation is retained in flexibility format and in displacement method that relation will be retained in stiffness format ok, now before we introduce the displacement method there are few things which is important to reach the few concepts which is very important in displacement method ok.

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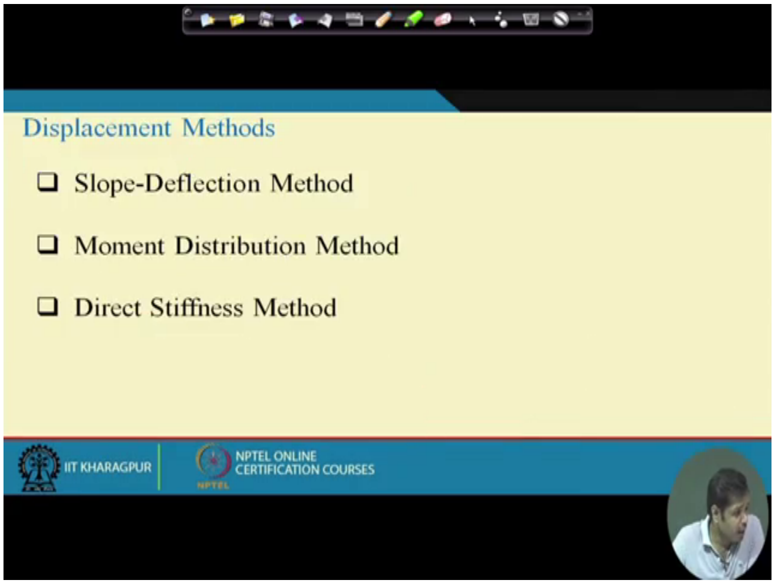


Before that there are three major displacement base methods. We'll be studying here in this course, one is slope deflection method, moment distribution method and direct stiffness method, details of this method we'll see as we derived those methods and apply to different problems, you see we know what is static indeterminacy right, we know when a structure becomes statically indeterminate or statically determinate structure and when we introduce that in indeterminacy.

I also mention there is a another term called kina mite in indeterminacy and we just mention the name but we did not discussed anything about kinematic indeterminacy and I told you whenever it is required, when we come across this kinematic indeterminacy we'll discuss that, now this is a time when we need kinematic indeterminacy ok, now before we introduce or define what is kinematic indeterminacy just recall what was degrees of freedom. Probably in the first week we introduced what is degrees of freedom.

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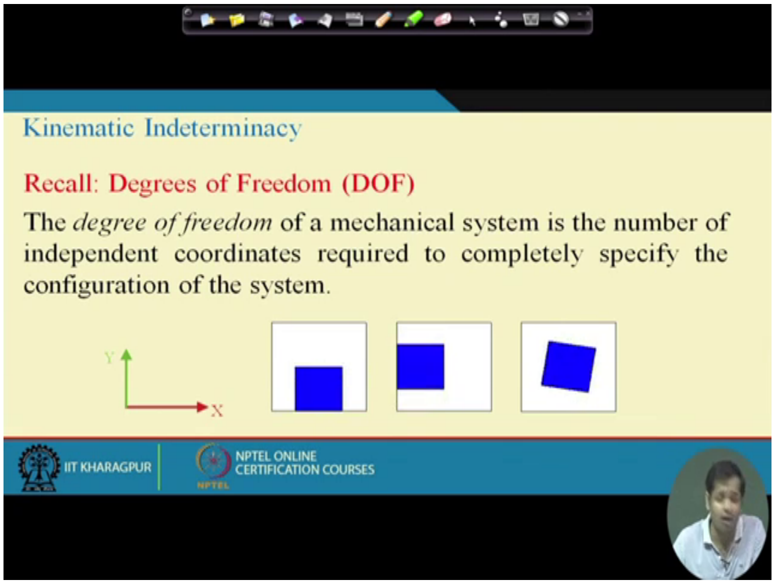


Degrees of freedom of a mechanical system if the number of independent coordinates required to completely specify the configuration of the system, in two dimensional there are three degrees of freedom at a point which is to translation, translation in x and translation in vibration and rotation on xy plane, since we are all the concept that we'll be starting here.

That we have been applying that to plane structures so for us at a given point total increase of freedom a point can have is 3, two translation and one rotation but when you extend this concept to space structures say space frame or space truss then at any given point the total degrees of freedom will be 6, three translation and three rotations ok, now this is degrees of freedom we know this ok, now when a skeletal structure deforms then what happens.

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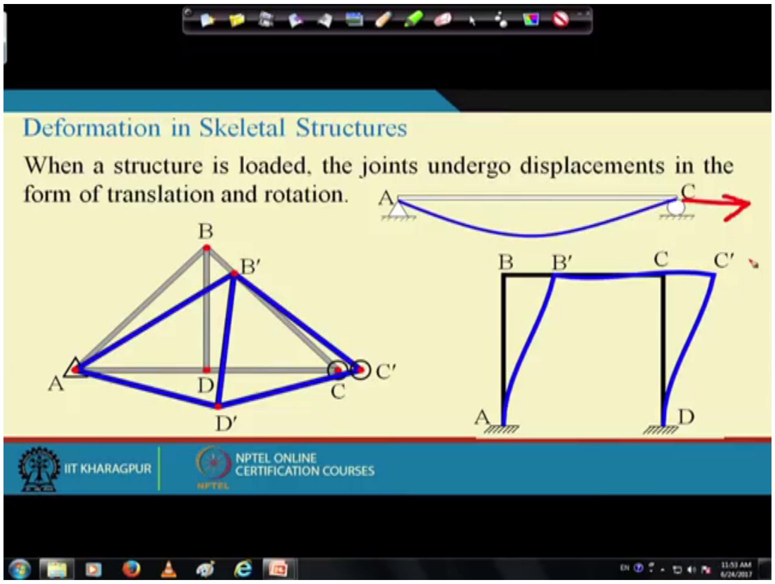


When a skeletal structure is loaded then the joints undergo displacements either in the form of translation or in the form of rotation because that is the degrees of freedom a joint can have, just to make it more clear suppose there is a truss here it is a statically determinate truss let's see because it has four joints so number of equations are available eight and then hoe many members it has one, two, three, four, five members and then three reactions.

So five plus three total eight are known and eight equations available it is statically determinate structure, now when it is applied some load say if it is applied to a load something like this a load here and at point you apply a load here then it undergoes deformation like this, so point B comes to B dash, point C goes to C dash and point D goes to D dash ok, so what happens, now take another example of a beam, beam A C which is subjected to maybe any random load on this.

And it is transversely deform like this so in this case what happens the joint A undergoes rotation and joint C undergoes rotation ok, similarly if we take a frame like this and it is subjected to again a horizontal load like this then the blue line that you can see that is the deflected shape, so point B use to B dash point C goes to C dash the point A and D they are fixed support that they neither translate nor rotate.

So what happens when a skeletal structure is subjected to any kinds of load their joints, in this case joints are A B C, these are the joints, those joints they undergo deformation either in the form of translation or rotation or both, for instance in this case it is only translation but many other cases it could be only, in this case it is joint A and C, joint A is only rotation, joint C it is a roller support, it can rotate and it can translate in this direction as well. So it is rotation plus translation and for this case joint B goes here, joint C goes to C dash, so this is also combination of translation and rotation depending on the type of joints we have ok,

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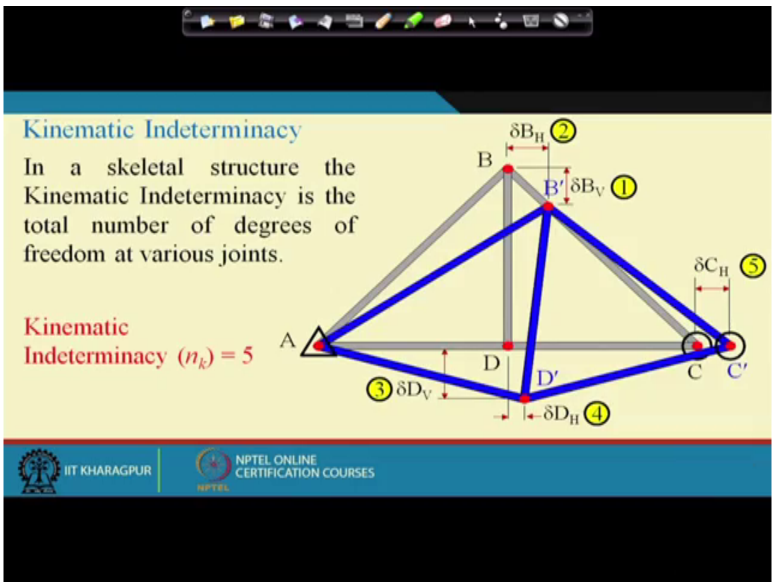


Then what is kinematic indeterminacy, kinematic indeterminacy is is in a skeletal structure the kinematic indeterminacy is the total number of degrees of freedom at various joints ok, just to understand in a better way considers the same truss here which undergoes deformation like this ok.

Now you see joint A cannot translate, in this case since it is a truss there is no rotation allowed in any joint so it is only translation, so mode of deformation is of any joint is only translation, now joint A doesn’t undergo any translation because it is hinge’s joint but joint C which is roller support it cannot move in Y direction but it can translate in this direction, but it cannot translate in Y direction and joint A in both the direction this translations are restricted.

But all other joints, joint B, joint D they can translate in both the directions isn’t it, now at every joint how many degrees of freedom we have in the structure, at every joint we have two degrees of freedom ok, what are those two degrees of freedom, translation in x direction and translation in Y direction, there is no rotational degrees of freedom because all the joint are hinge or rotation ok, so what happens then total how many displacement are known we have here.

You see joint B we have horizontal displacement and also we have vertical translation, joint C we have horizontal translation and joint D we have horizontal translation as well as vertical translation then joint A there is no translation and joint C there is no translation in vertical direction, so how many degrees of freedom at the joints, total degrees of freedom at the joints we have is it is one then two then three then four and then five.

So in the structure if you see only the joints then the total degrees of freedom at various joints are total five at this point B we have two degrees of freedom, at point D we have two degrees of freedom, at point C we’ve just one degrees of freedom, at point A there is no degrees of freedom because joint A cannot go anywhere ok, it is supported so in this case in this problem the kinematic indeterminacy is five ok.

So the kinematic indeterminacy is the total number of degrees of freedom at various joints so we know what are the total degrees of freedom we have at various joints and that would be the kinematic indeterminacy ok.

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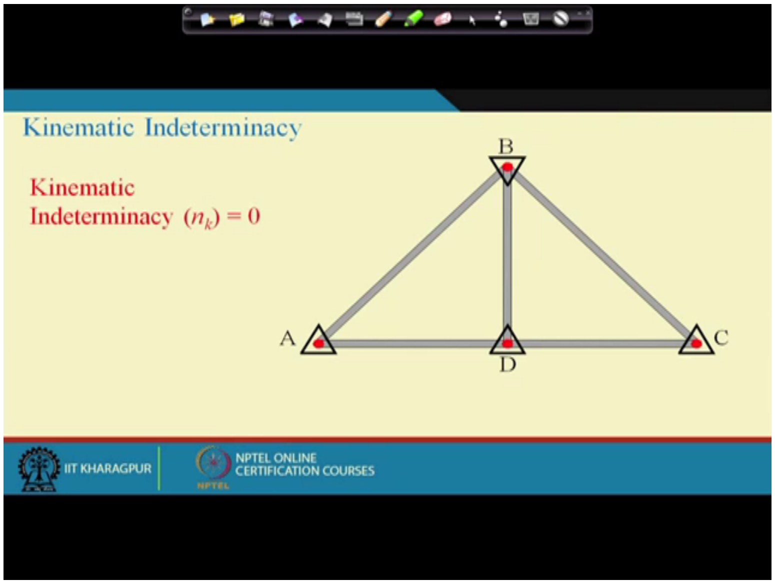


Now take one example suppose this is the structure the same structure. Same truss but we have simply support all joints, so what happens then what is the kinematic indeterminacy of the structure.

The kinematic indeterminacy of the structure is zero because joint A cannot translate anywhere because it is supported, similarly for joint B C D all the joints are supported so it cannot go anywhere so the degrees of freedom at all the joints are zero so total degrees of freedom is zero, so kinematic indeterminacy of the structure is zero, so when the kinematic indeterminacy of the structure is zero the structure is called kinematic ally determinate structure.

Now see the difference between static indeterminacy and kinematic indeterminacy for this truss it is not statically determinate truss because if we have to find out reactions, how many reactions we have one, two, three, four, total eight reactions we have and then one, two, three, four, five total five member forces we have, so total thirteen are known but how many equations are available, only three equations. So this structure is statically indeterminate structure but it is kinematic ally determinate structure ok, so kinematic indeterminacy of the structure is zero, ok?

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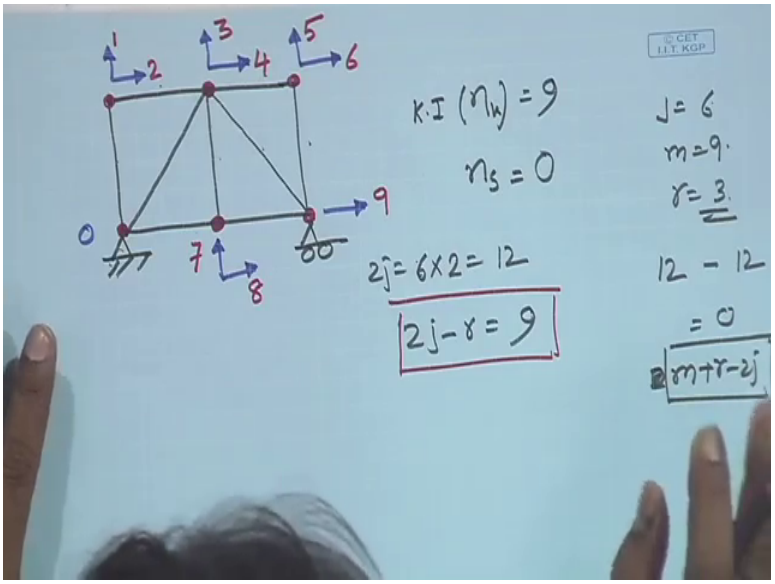


Now see just one more example for truss itself so you see take a truss here, we draw quickly a truss ok? And these all are hinges ok, now we have support here say we have a support here and we have a roller support here, so what is the kinematic indeterminacy of this structure you see at this point there is no.

Because this point neither translate in this direction nor translate in this direction, you see this point can translate in this direction as well as in this direction, this point also can translate in this direction as well as in this direction, this point can translate in this direction and this direction, this point can translate only in this direction there is no vertical translation and this point also can translate in this direction and also in this direction.

Then what is the kinematic indeterminacy it has, it is one then two, three, four, five, six, seven, eight, and nine so the kinematic indeterminacy is nine, kinematic indeterminacy or in K is nine ok, now what is static indeterminacy, static indeterminacy N S please check how many joints we have one, two, three, four, five, six; six joints so J is equal to six, how many members we have one, two three, four, five, six, seven, eight, nine so member is nine.

And the reactions R is equal to three, so we know that two J, so total equations available is two into six, twelve equations available, how many unknowns are nine plus three, twelve unknown so static indeterminacy is zero, so it is statically determinate structure ok, now for static indeterminacy we know the formula is M plus R minus two J, if you remember this was the formula M plus R minus two J.

Now for kinematic indeterminacy also similarly we can write per each joint the degrees of freedom are two, so total number of joints here it is six, so total number of is J is equal to six, so total degrees of freedom is six into two is equal to twelve means two J is equal to twelve and how many reactions we have, here two reactions and here one reaction so R is equal to three so it is two J minus R is equal to nine, so this is the kinematic indeterminacy of this structure ok. Now similarly for any structure you can find out the similar way for any truss you can find out the kinematic indeterminacy ok, now this was for trusses, now let's take for beam

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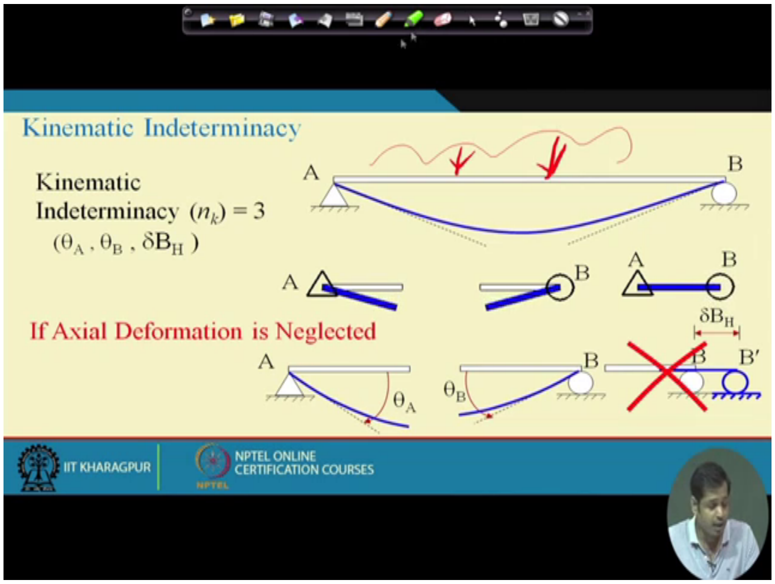


Suppose this is again a simply supported beam now two joints A and B and if it is subjected to some kind of load some kind of random load like this ok and it deforms like this the blue is the deformation ok, now if it deforms like this then what happens joint A it undergoes deformation.

Joint A is a hinge support so it can only rotate and joint B is the roller support so what are only constraint at joint B is the vertical displacement but the releases are it can allow rotation and it can also allow translation in horizontal direction so now if you see these are the degrees of freedom at various joints, joint A only it rotates, joint B it rotates and also due to the horizontal, since it is the roller support at B so it can also move in this direction ok.

So in the structure if we only concentrate at the joints then at joint A we have just one degrees of freedom which is rotation at theta A and joint B, we have rotation which is theta B and also at joint C translation so if point B goes to B dash because it is roller support and translation in this direction is allowed, there is no constraint in horizontal movement, so in this case total degrees of, then what is the kinematic indeterminacy, kinematic indeterminacy is three.

One degrees of freedom at A and two degrees of freedom at two total kinematic indeterminacy is three, now there is one important point here you see what does it mean when we say that joint A can rotate and joint B can rotate, what is the effect on the deformation of the structure ok, because joint A can allow rotation, joint B can allow rotation, the deformed shape will be like this ok, but still this even in the deformed shape if we don't consider right now this part.

If we only allow say joint A, joint A can rotate it can rotate about A but joint A still remains at the same position right, similarly if we only see the rotation at joint B, point still remains at the same position but it rotates only about that point B ok, but what does it mean that when we allow the translation at joint B as well, so when the translation is allowed at joint B, if the joint B is initially at this point, then after deformation the joint B takes the position B dash ok.

It means that initially the length of the beam is A B, it was A B now the length of the beam becomes A B dash ok, so the length of the beam is now different means this translation will cause change in length of the beam, means it causes axial deformation in the beam ok, now if we neglect the axial deformation, if we assume that axial rigidity of the member is very high ok, if we neglect that deformation then in this case if we allow axial deformation.

The degrees of freedom are rotation at A, rotation at B and delta H at B, now if we don't allow axial deformation means if we assume that it's axial rigidity is very high then this mode of deformation there will be no this mode of deformation ok,

So this is neglected and if you neglect that then the kinematic indeterminacy of this structure is two ok, now we'll see some example for kinematic indeterminacy here quickly some of the problem for instance you see. We'll stop here now and we'll see some more examples on kinematic indeterminacy and then once we know the kinematic indeterminacy then we'll introduce what is displacement method by comparing their steps with force method ok,

Thank you.