Structural Analysis I
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Lecture 35
Analysis of Statically Intermediate Structures (Continued)

Hello Everyone! Welcome Back in the last class we introduce the force method. Let us continue with that today also. So today what we will do is this is a lecture number 35 we will still continue with the force method, ok.

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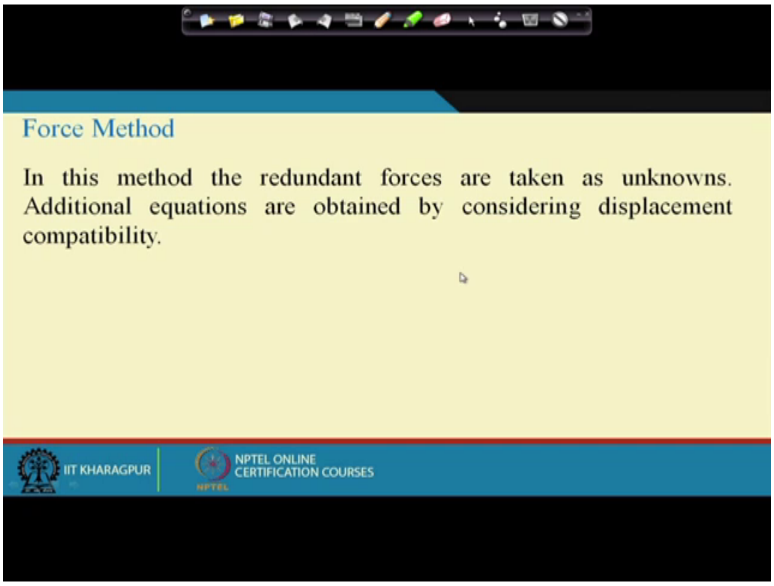

So what we have seen in the last class is force method is in force method the redundant forces are taken as unknowns, and additional equations are obtained by considering the displacement compatibility, ok.

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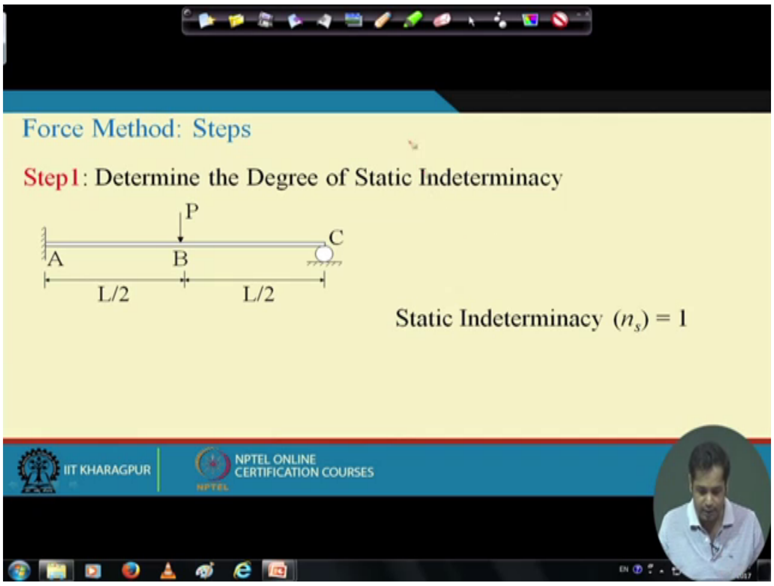

So let us summarize the different steps that involved in force method and the steps are let us take this example the same example that we that we analyze in the last class. The first step is to once we have any statically indeterminate structure and we need to solve it using force method.

The first thing we need to identify what is the static indeterminacy of this structure, right? We know how to determine static indeterminacy for beams for frames and for trusses as well. So for this problem the static indeterminacy is 1 because we have on four reactions three at support A and three at support A, three here and one here.

So we have total 4 and number of equations available is 3 so static indeterminacy for this problem is 1. So first step is to identify what is the static indeterminacy of the problem. So in this case it is 1.

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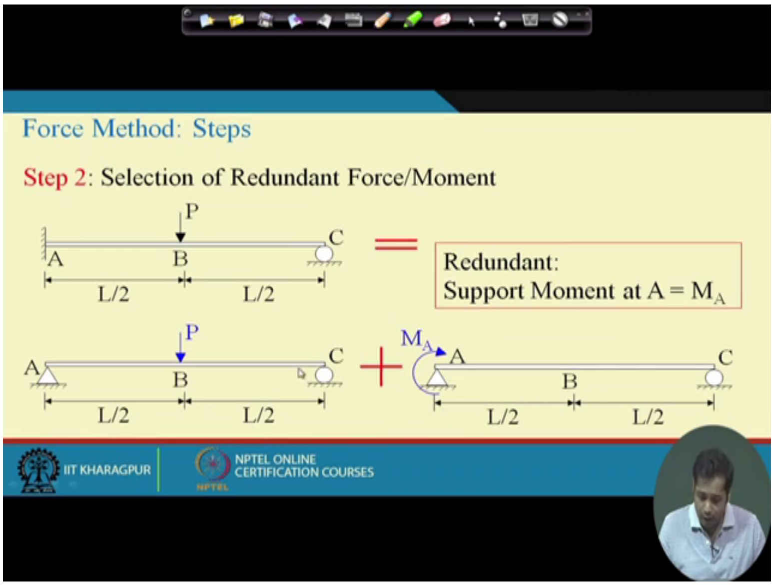

Then step 2 is now once we know the static indeterminacy then we need to select we need to choose the redundant force or redundant moment. For instance in this case static indeterminacy is 1 so we need to find out what is the redundant force or what is the redundant moment, ok.

For instance if we take redundant as support reaction at C C y is the redundant force then what we need to do is we need to we can decompose we can divide this structure into two parts one part is this one part is this where which is the remove the redundant force and then this is becomes the statically determinant structure. And then next part is apply on the same structure apply now the structure is subjected to the redundant force, right?

So this plus this will give us this structure that is linear that Principle of Super position. Now in the same problem instead of taking support reaction at C as redundant we could take the moment at A as redundant for instance if we take moment at A as redundant then what happen then the basic structures are the first structure is we need to release that release the associated constraint with the redundant now if we take redundant as M a moment at a then associated constraint is associated is the rotation so we need to release that at point A so fixed support become hinge support.

So this becomes the basic structure basic determinant structure primary structure very often it is called and then plus the same structure is subjected to only the redundant force in this case the redundant in this case it is moment redundant moment is M A. So again the principle of super position says this plus this is equal to this. So this is step 2.

And when any problem we can have say two redundant forces static indeterminacy is 2 or 3 or any number depending on the number of static indeterminacy we need to choose the redundant forces and correspondingly you need to divide the structure and then add them to get the final response, ok.

Now next is once we have divided the structure into two parts and both the parts you see this as determinant structure. Now put a star here towards the end of this lecture we see there as certain guidelines that need to be followed when we choose the redundant we cannot its just not any force cannot be taken as redundant.

Any force or any moment cannot be taken as redundant, ok. We will see that towards the end of this lecture. Now so the step 2 is selection of redundant forces, right?

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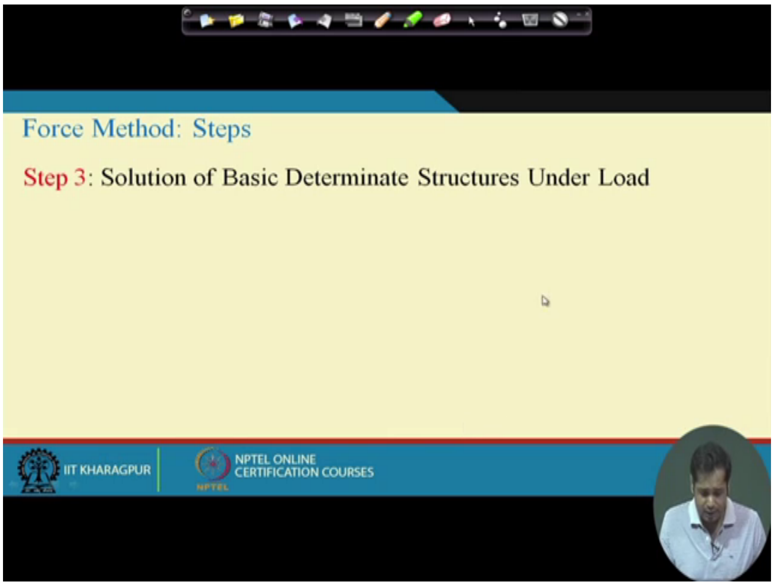

Now then step 3 is solution of the basic determinant,

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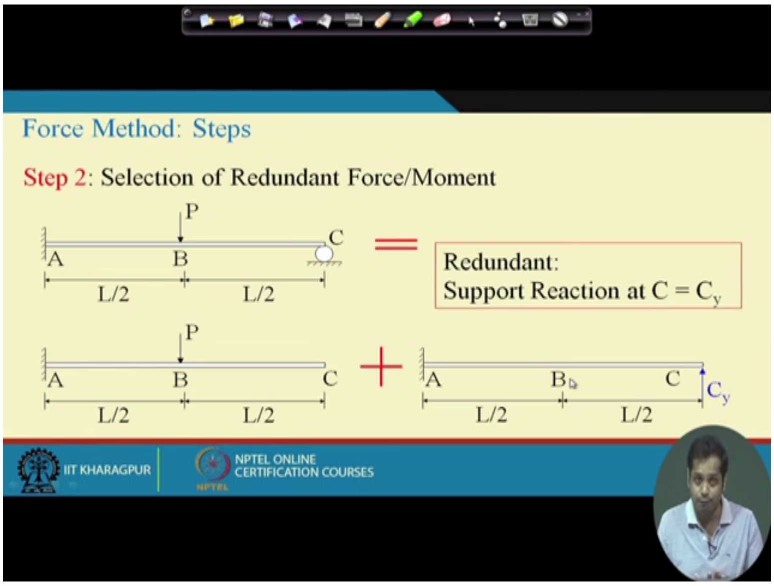

Now in this case we have two determinant structure because it is static indeterminacy is 1 had it been 2 had it been multiple more than 1 then we could have several other small sub structures as well.

Now in this case we have 2 basic structures 1 is subjected to the load externally applied load which is on the real structure and the next part is the same structure but now it is there is no externally applied load instead the structure is subjected to the redundant force or redundant moment.

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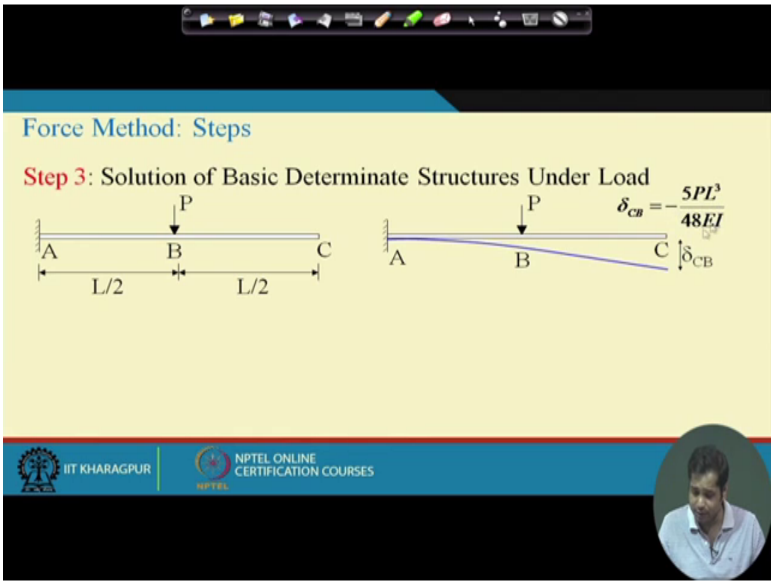

Now the step 3 is solution of the basic determinate structure under load. So this is the basic determinate structure under load and this is the basic determinate structure under redundant force, redundant moment. Now and similarly if we take redundant force as this and this is the basic determinate structure under load, and this is the basic determinate structure under redundant force, ok.

Let us demonstrate step 4, let us demonstrate step 3 with this example this choice of redundant force,ok. So now solution of basic determinate structure now this is the basic determinate structure now we need to solve it and this is the solution. Suppose the under this load beam deflects like this and at point C suppose it is delta C B.

Please note the notation that we are using here delta C B means it is deflection at point C due to the load applied at point B, ok. Now then step 4 is so this we already have seen in the last class that this delta C B is equal to this value, ok. Now this is again depends on your sign convention whether you take negative or positive ok.

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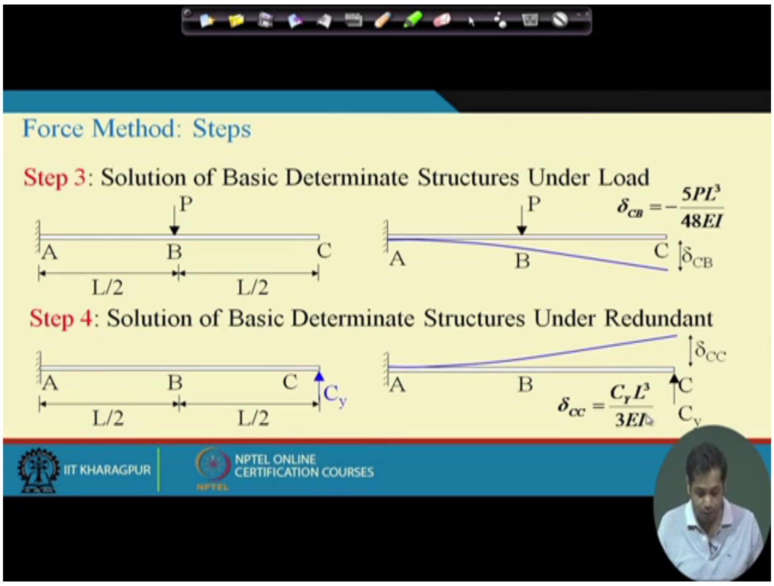

Now next is solution of basic determinate structure under redundant, so under redundant basic determinate structure under redundant force is this we if we consider that C y as redundant force is a cantilever beam subjected to tip load.

And each deflect it may deflect like this and this is delta C C again please note the notation it was delta C B was the deflection at C due to load at B delta C C is deflection at C due to load at C. And we have seen that this delta C C value is C y L cube by 3EI, ok. So this is step 4.

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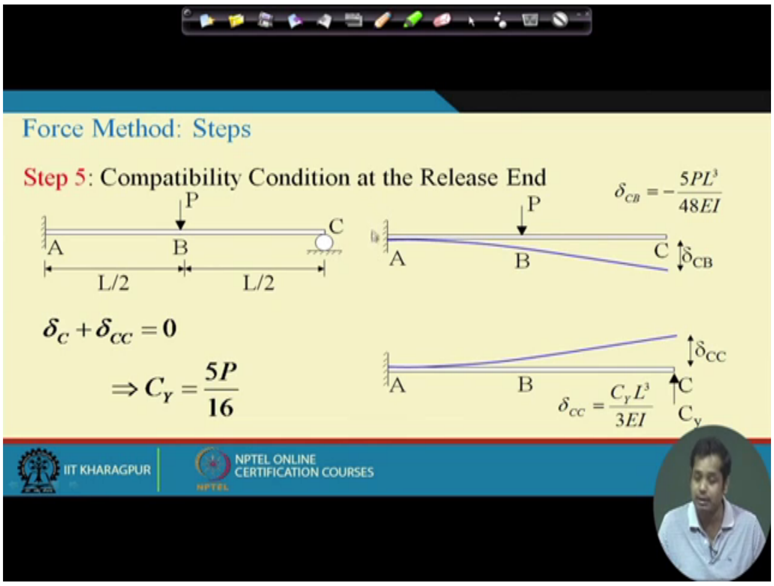

Now the next step is using the compatibility condition at the release end. Now in this case our release end is at C. So what is the compatibility condition at C, compatibility so what is the constraint at C we have. The deflection at C is equal to 0.

Now the compatibility condition says that then delta C B plus delta C C the total deflection at C will be this plus this that is the principle of super position. And that should be equal to 0 that is the compatibility condition. Now we know what is delta C B we know what is delta C C if we substitute that we get what is the value of C y.The value of C y is this.

So we had four unknowns here 3 reactions at A and 1 reaction at C and we could not solve using just equilibrium equation. Now and then compatibility equation is used to have one more equation and from that equation we could compute what is the value of C y, what is the reaction at C.

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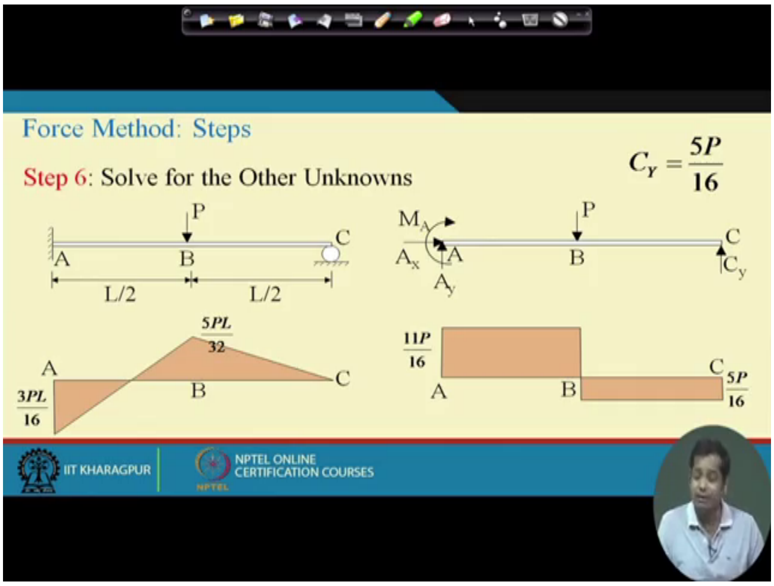

Now once we know C then how many it is a free body diagram of the entire structure this is the last step, you see here 3 reactions at A and 1 reaction C y. There are 4 reactions we could not solve using equilibrium equation. Now we have already determined what is the value of c y using the compatibility at compatibility condition at C.

And then once U i is known then we have 3 unknown we have 3 equation sation of f x is equal to 0, sation of f y is equal to 0, sation of moment is equal to 0. And if we apply the equilibrium condition on this free body diagram we can obtain in the final solutions. So for last let me solve for the other unknowns. So this will be the bending moment diagram and this will be the shear force diagram.

You see and if you apply the equilibrium condition these are the values that you may get please check these values, ok. So these are the steps that we follow in force method. Now (()) (09:34) of this structure it could be beam it could be truss it could be frame it could be difference it can have different support conditions.

The essence of this steps will remain same. Only difference will be the which we depending on the static indeterminacy you may have to choose the number of redundant forces and moments will be different and then depending on the choice of redundant forces and moment the basic determinant structures will be different.

But the essence of the steps they remain the same for all the problems, ok. Now before we go further in the next week we will be demonstrating this method we will be applying this method to different problems, next we will apply these to plain trusses and then next to next week we will apply concept of force method to beams and frames. But before we do that this week, this week the objective is to save the platform, right?

You understand this some of the to prepare the recipe to understand the recipe of the this method, ok or each ingredient of the recipe. One of the very important concept in force method is flexibility coefficient. Now let us see what is flexibility coefficient? We already we are all familiar with the term stiffness.

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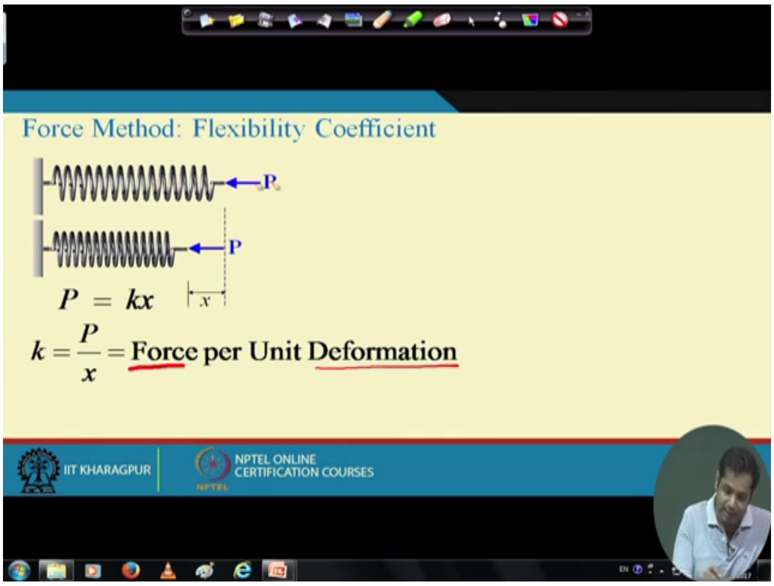

Now let us see if we have a spring which is subjected to actual load P and then because of this load P the this spring undergoes the formation and suppose this the change in length is x, ok. And then we know that P is equal to kx where k is the stiffness of this spring or sometimes it is also called spring constant.

But the more general term is the stiffness spring stiffness. Now what is the definition of stiffness? Definition of stiffness can be written as P by x and what it exactly? It is the force per unit deformation. So it is the amount of force required to deform this structure. Now you see here one important point is which force we are talking about and which deformation we are talking about?

In this case it is really straight forward because you have just it is in one dimensional it is just one dimensional where you have force in one dimension and in that dimension itself you have the deformation. So in this at least for this spring force is this direction and the deformation is in this direction.

But you can have we will see shortly some of the problem where the you are applying force at a point but your associated deformation you are majoring at different points. So in that case what could be the definition of stiffness what could be the expression of stiffness? But the as well as definition goes it is force per unit deformation.

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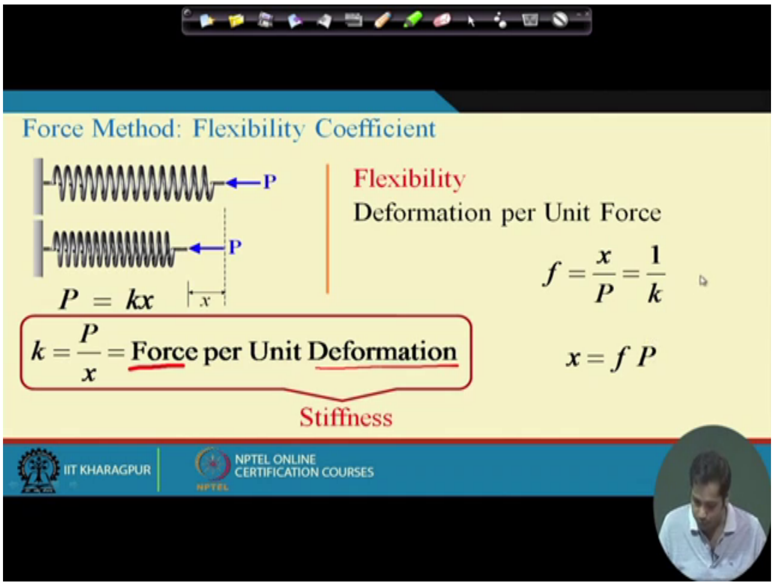

And we will see what kind of force and what is the associated deformation shortly. Now if it is then this is called stiffness, right? We are familiar with the term and flexibility is essentially the inverse of stiffness, ok. At least by intuition you can say without actually understanding what is stiffness?

Stiffness without really having conceptual structure analysis and solid mechanics, if you look at the stiffness, the term stiff means which if we apply some kind of agitation the structure is or the object is very stiff. The object the deformation it undergoes is very less, whereas we say the structure is very flexible means the deformation of the structure will be more if the structure is more flexible, ok.

So flexibility is the inverse of stiffness and what is the definition of flexibility? The definition of flexibility is deformation per unit force, it is just opposite to this, ok. So for this problem what is the flexibility? The flexibility f we will be using f to denote the flexibility, f is equal to 1 by k which is x by p means it is the deformation per unit load, ok.

Now let us see once you have defined flexibility, let us see what implication it has in the case of structural problem, ok. So once we so then similar to this expression p is equal to kx we can express the same thing in terms of flexibility x is equal to f into p, ok x is equal to deformation and f is equal to flexibility and p is equal to the applied load, ok. Let us see what happens what is the interpretation of flexibility in the case of normal this different structure problem, ok.

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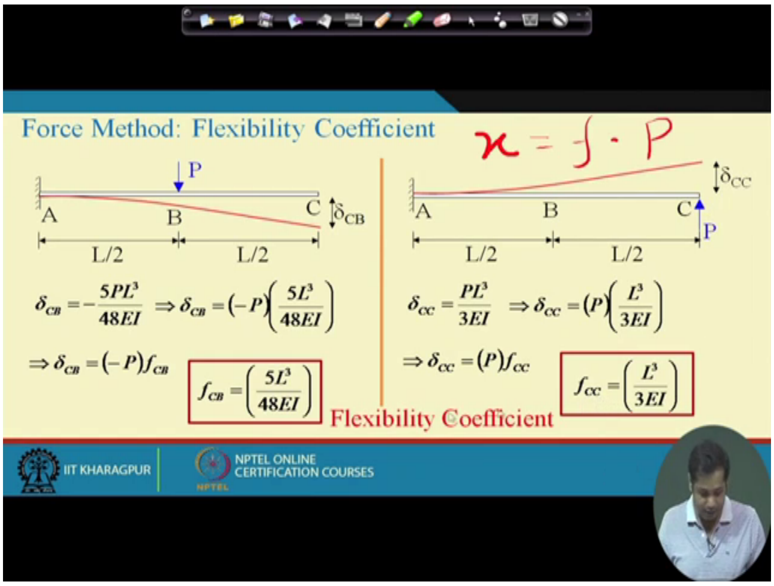

Suppose let us take 2 cases here the first case we have just now seen these two cases these are the two basic determinant structure this is the determinate structure under applied load and this is the determinate structure under redundant forces ok. And suppose we are concentrating as I just now said that what is the force and what is the deformation.

Suppose in this case the deformation we are talking about at C in displacement at C that is the displacement that is the deformation we are focusing on, ok. Now so delta C B in this case it is delta C B means it is deform at displacement as C D to applied load P and in this case delta C C displacement at C due to the applied load at C.

Now what we know is just now we have seen that delta C B is equal to the expression for delta C B is equal to this now the same expression can be written as delta C B P is taken out and then this ok. Now suppose this expression the same expression can be written as minus P into f C B where f C b is equal to 5L cube by 48 EI, ok.

Now see in this case delta CC is equal to PL cube by 3E I just now we have seen and then delta CC is equal to write the similar way take P out and L cube by 3EI and this delta CC can be written as P into f CC where f CC is equal to L cube by 3EI.

Now if we compare just now we have seen that the relation between the deformation force and flexibility is x is equal to f into p right? F into P where x is the deformation f is the flexibility and P is the force, ok. Now this term is this term are called flexibility coefficients, ok.

Now y you again please note the the notations that we are using here. F CB is the flexibility coefficient and this is associated this coefficient is associated with displacement at C and load applied at B whereas F CC is the flexibility coefficient where the load displacements are displacement at point C and the load is also acting at P that is why it is F CC and it is F CB.

So whenever we talk about flexibility coefficient similar as stiffness then flexibility and stiffness what they exactly do they relate the displacement and the force deformation and the load, right? Now which deformation and which load we are talking about depending on that this flexibility coefficient will also change. For instance in the same problem in this we are measuring deflection at point C here also here also we are measuring deflection at point C.

But in this case this is caused by load acting at B and this is caused by load acting at C therefore there flexibility coefficients are different. So this flexibility coefficient associated with this load displacement and this flexibility coefficient is associated with this load displacement, ok.

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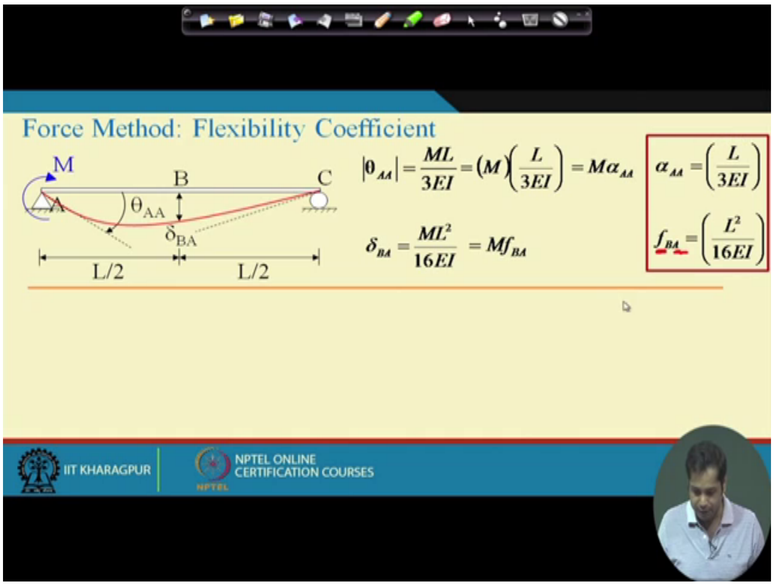

Now let us see let us see a more detail let us express this flexibility coefficient through another example. Suppose a simply supported beam which is subjected to concentrated moment at point A ok and because of the moment this is the deflected shape of the beam. The red line that you can see that is the deflected shape of the beam.

And the B is the point at the mid span so this L by 2, this L by 2 ok. Now theta AA is the slope at point A and delta BA is the deflection at point B. Again please note the please see the notation why it is theta A because it is theta A it is the slope at A due to moment at A, ok.

At delta BA why it is delta BA it is deflection at B due to moment at B at A, ok. So that is why it is BA. Now this is a determinate structure so if you can solve this determinate structure you will see that theta A is equal to ML by 3EI ok. That you can we know how to solve it, you can use conjugate B method or any other method to solve it and get what is the expression for theta A it is ML by 3EI.

And now if it is ML by 3EI then again it can be written as M into force into some coefficient and this is equal to M into alpha AA where alpha A is equal to L by 3EI. So what is alpha AA, alpha AA is the flexibility coefficient and this flexibility coefficient is associated with the moment at A and rotation at A that is why it is AA.

Now we will be using two symbol for flexibility coefficient one is F and one is alpha F for displacement and alpha for rotation. Now similarly for the same problem what will be delta

BA again we can find out the deflection at the mid span and if you do that then delta BA will be ML square by 16 EI.

Again it can be written as M into F BA where F BA is equal to L square by 16 EI. And what is F BA? F BA is the flexibility coefficient associated with the moment at A and displacement at B. So displacement is this flexibility coefficient relates displacement at B due to the moment at A, ok. Now again take another problem so these are flexibility coefficient, ok.

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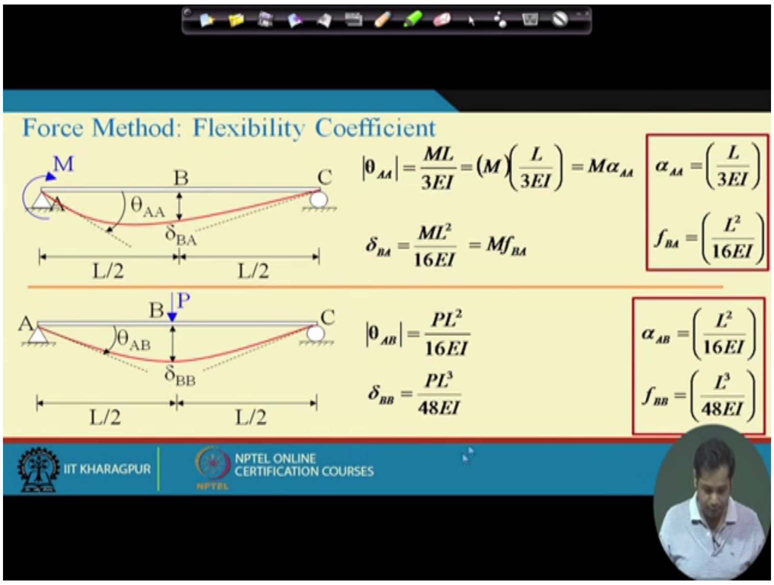

Now take one more problem take another case where the same simply supported structure, but now it is subjected to a concentrated load at the mixed span. And the red line that you can see that is the deflected shape of the beam. And these slope at theta is theta AB, and deflection at B is equal to delta BB. Slope at theta AB means it is slope at A due to force applied at B.

Similarly it is delta BB is the displacement at B due to force applied at B. So again this problem we can solve using any method that we learned in indeterminate structure and if you do that F AA will be PL square by 16 EI. And then what will be the flexibility coefficient? flexibility coefficient will be L square by 16 EI, ok.

Similarly delta BB will be PL cube by 48 EI and associated flexibility coefficient at F BB will be L cube by 48 EI. So in this case we can write that theta AB is equal to alpha AB into P, ok. And similarly we can write delta BB is equal to F BB into P. So deflection is equal to flexibility coefficient multiplied by the load, ok.

Now so this is again flexibility coefficient right? This is also flexibility coefficient for this problems, ok. We are concentrating only on two points one is point A and point B in this case load applied at A and in this case load is applied at B and displacement are measured at point A and B displacement means rotation at A and displacement at B and rotation at A and displacement at B, ok.

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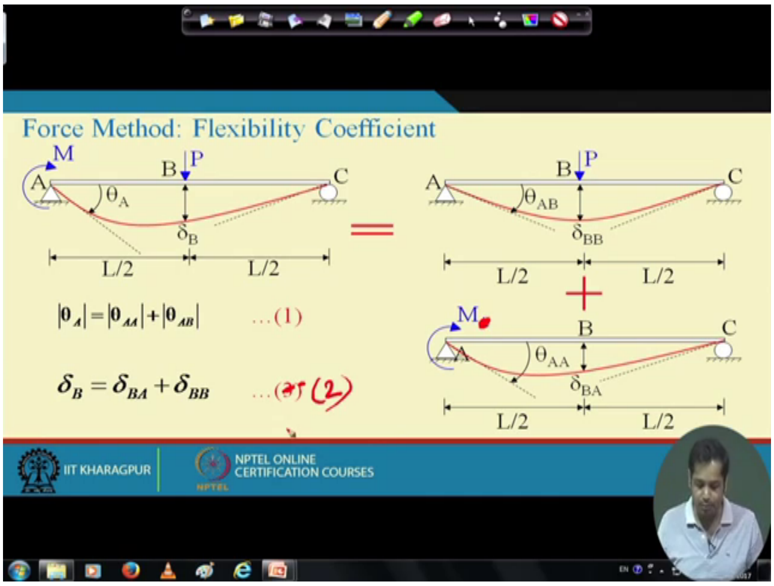

So now what we have is now suppose consider now in this problem the loads are acting separately it is only concentrated load it is only the moment at A. Now suppose we consider a problem like this where the same simply supported beam but it is subjected to both a concentrated load at the mid span and moment M at point A, ok.

Now using the and this deflect like this suppose, and corresponding rotation at A is theta A and at B the displacement is delta BB. So theta A see note here in this problem we have not written theta AA or theta AB or delta in this case it is not delta AB or delta BA or delta BB.

When I write theta A it is now cumulative action of A and P. When it is delta B it is cumulative action of total action A and P so theta A is the rotation at A due to both the forces. And delta B is the displacement at B due to their total action, ok.

Now we know that using the principle of super position this problem can be divided into two parts. One is remove the M as simply supported beam subjected to P and as simply supported beam subjected to M at a, ok. Now and this is M ok. Now then this plus this should be equal to T that is in the principle of super position, right?

Now just now we are expressed we have seen the expression for theta AB delta BB and theta A and delta BA for this two cases ok and what are now what principle of super position says that if you are measuring displacement or rotation or moment at any point in this wing that will be the this plus this. So what will be theta A? Theta A will be theta AB plus theta A, ok.

Now in this case you it is written the absolute value is written here but as far as sign convention these theta is positive so whether you write absolute value or not, it does not matter as long as you are concentrating on this rotation. Similarly delta B will be delta BB plus delta BA, ok.

Now suppose this is equation number 1 and this is equation number 2, ok. Now then what we have these two equations, right? Then we have this equation we have, right? At delta BB that we have already obtained.

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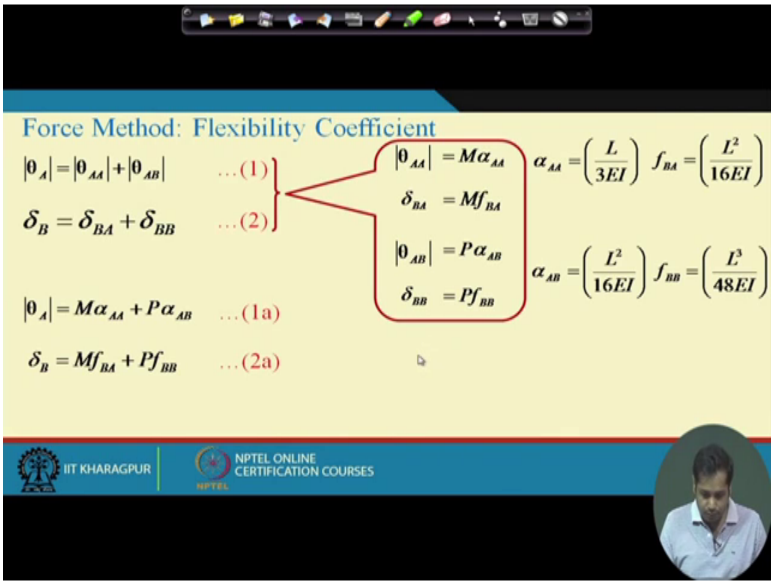

Now if we apply this into these two equations which we obtained from Principle of super position. Then what we have? We have this theta A is this theta AB is this and theta BB BA is this and theta BB is this. So we have these two equations suppose this is equation number 1 a and this is equation number 2 a, ok.

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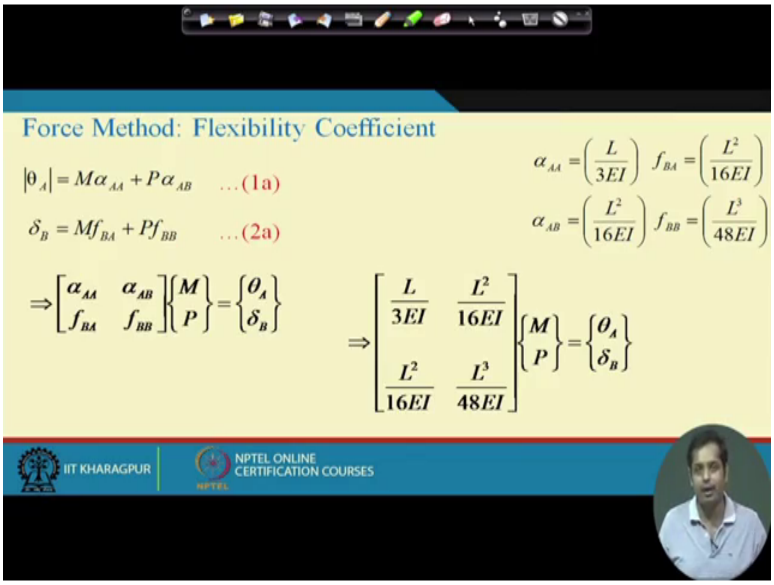

Now then what we have is equation number 1 a is this and equation number 2 a is this and in this equation alpha and F they are the flexibility coefficient associated flexibility coefficient that expressions are this, ok.

Now if we substitute this in this equation though what we had before that let us write this expression in a matrix form, so if we do that then this is the matrix where which are flexibility coefficients into M into P M by P is equal to theta and delta these are the displacement, ok. Now if you substitute these expression of flexibility coefficient here, then this becomes this, ok.

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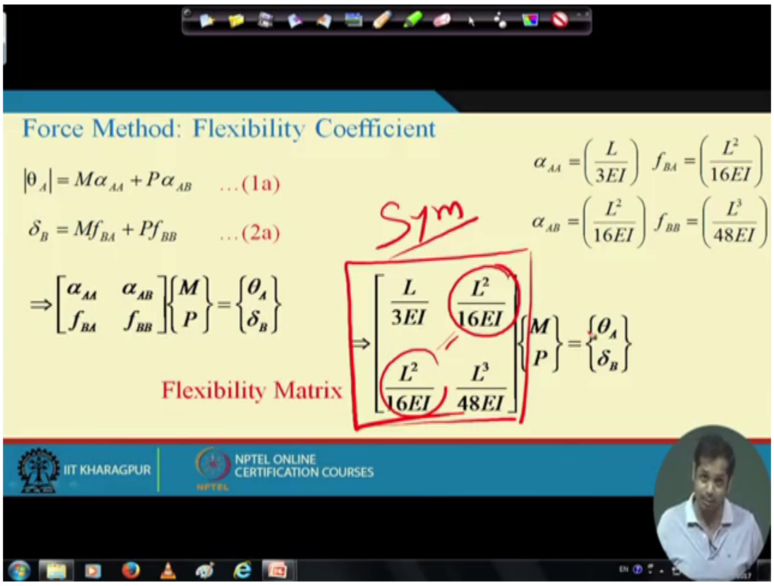

Now so this what we have done is now is if we consider only two points A and B and these loads are acting on A and B we are measuring displacement or rotation at A and B. Then that displacement force they can be related as this, right? And since we are concentrated only on two points A and B in this case your this matrix is 2 by 2 matrix, right?

Now this is called flexibility matrix. Now you see for a spring case if you remember when it was a spring we wrote that x is equal to f into P ok it was just only one force and in that direction only we computed the displacement and therefore it is a flexibility coefficient in a scalar value.

But now what we have is in this case it is just a 1 value we have displacement rotation and those are different points. Therefore the relation between force and displacement will not be a scalar relation, it would be a several system of equation and that system of equation written in this form linear system written in this form and this expression is called flexibility matrix, ok.

Now you may ok we will appreciate this way of writing the displacement and the relation between force and displacement once we actually start analyzing different kind of structure, ok. Now what is important here in this flexibility matrix you see this matrix is symmetric it is a symmetric matrix ok, and this value and this value they are same, ok.

Now why they are same? You know why they are same you have learned we have discussed the associated theorem which says that they are same they should be same. Now but we will not tell you in this class please think on that why they are same and if they are same then what exactly they are telling about the effect of forces or effect of moment or in general effect of load on structure.

We will discuss that when we actually solve this problem. Now this matrix is symmetric now your job is to check why this matrix is symmetric, ok Now before we stop today as I said choice of determinate structure is very important because you cannot take.

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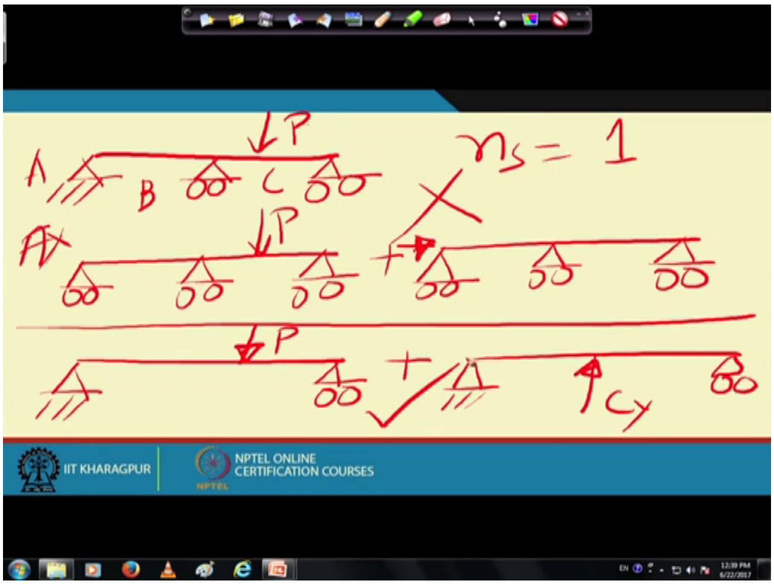

For instance suppose you have a beam like this you have a beam its a hinge here and then roller and then roller , ok. And this is an indeterminate structure and what is ns in this case ns will be 2 reactions here one reaction here one is the total 4 reactions number of equations available so ns will be 1, ok. Now what we need to do is we need to identify one redundant force and then divide the structure into some basic structure, ok.

Now one you can do is suppose identify the redundant force as suppose as this, ok. If we, we can this is roller this is roller and this is roller right? And then this plus we have taken redundant force as Fx, F x is taken as redundant force this is A, B, C.

So suppose I take x as redundant force. So as per steps these is the basic structure and suppose it is subjected to any load like this P. So this is subjected to any load P. So first step is the calculation of ns, ns is equal to 1 second stage is identification of choice of the redundant force suppose our redundant force is x and then divide the structure into parts depending on the value of ns where the basic structure basic is subjected to load and the basic structure is subjected to the redundant forces, ok.

So this basic structure subjected to load and in this case basic structure subjected to redundant force, ok. Now then what we will have to do? We have to solve this we have to solve this and apply the principle of super position and compatibility equation. But look at this, this structure is not though here we have see reactions here also we have three reactions this is not a stable structure because I am not the action of these horizontal load the structure is not stable, ok.

So it is not that you cannot take any force or any constraint as redundant we have to make sure that whatever constraint or whatever force you whatever reaction you take or internal force you take as redundant that will not lead to a system which are unstable, ok. So this is not the correct composition.

What could be the better theme here you take redundant force C y as redundant force so if you remove this C y so this is your basic structure which is subjected to a load P here and then plus again this is the basic structure and subjected to load C y, ok.

Now you see in this case it is simply supported beam it is determinate structure and stable it is determinate stable structure so you can open the solution and then apply principle of super position and compatibility. So this is correct way of doing it. So when you have any problem when you need to apply force method then please make sure that the redundant forces or moments that you can take that should not lead to a system which are unstable this is very important.

So we stop here next class what we do is we will introduce what is displacement method and then once the displacement method is introduced we go to the next week to apply displacement and force methods for various structure vitalizations, ok

Thank you!