Structural Analysis I
By
Prof Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 34
Analysis of Statically Intermediate Structures (contd)

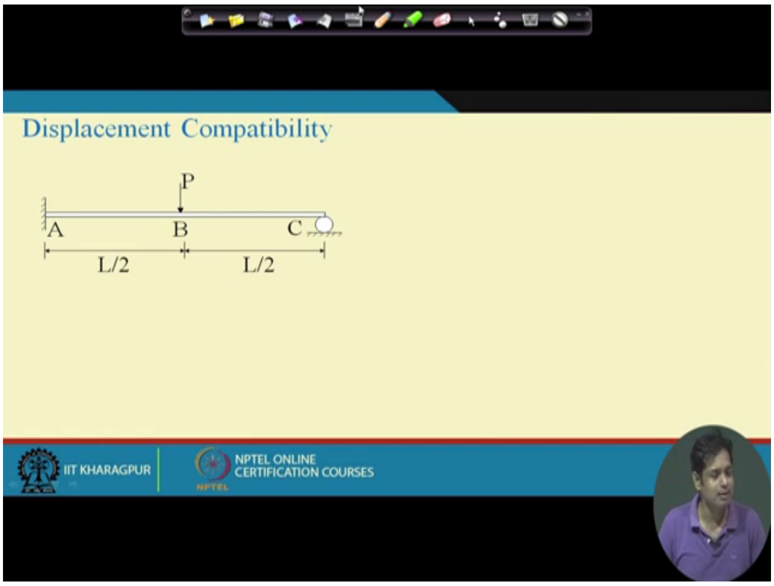
Hello Everyone! What we will be doing today is we will introduce let me first recall what we have done in the last class. We introduced indeterminate structure you know indeterminate structure even we discussed the indeterminate structure even during solving determinate structure but formally.

We introduced what is indeterminate structure and then how to determine the degree of static indeterminacy external and internal for trusses beams and frames that we discussed. And also we discussed some of the concept that we used earlier but is very important for indeterminate structure that is compatibility condition and another super position.

We also discussed that number of equations available equilibrium equations alone cannot provide sufficient information to analyze this structure. So we need some more conditions we need to append the equilibrium conditions by some other conditions and how those conditions are formed depending on that we have mainly two methods one is force method and one is displacement method.

What we are going to do today is we are going to introduce what is force method. Ok Let me before I formally introduce what force method what are the steps to be followed and let me demonstrate that through one example.

(Refer Slide Time: 01:49)



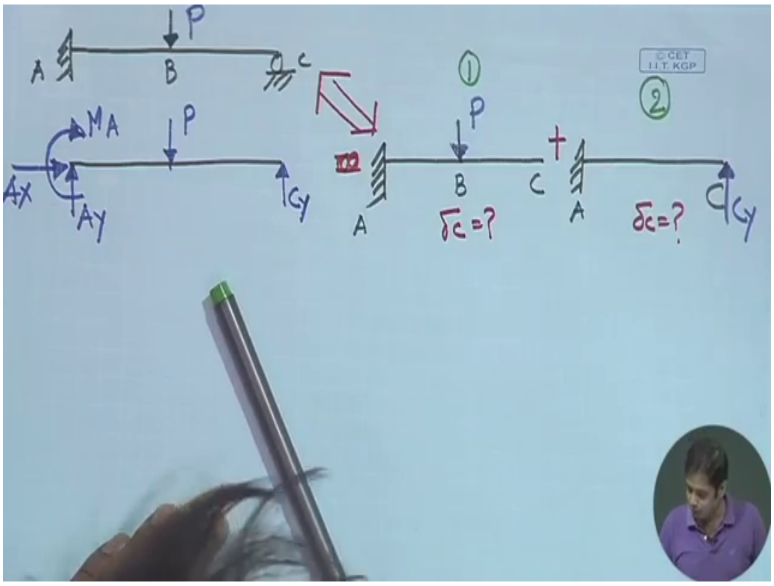
This is an example probably we have seen many times in this course there is a prop cantilever beam subjected to the concentrated load let us say in this case they may span.

Suppose we do not know what is indeterminate structure we do not know any methods to determine the forces and reactions in indeterminate structure only thing we know is the concept of determinate structures and several methods to determine the internal forces and reactions for determinant structures ok.

So now what we do let us do not bother about force method displacement method more than three and four kinds of technique and the steps. Let us whatever information we have whatever we have learned in last 6 weeks can we apply them. Can we use them to device a method or device a strategy to find out the member forces?

Or in this case atleast support reactions for this problem,ok. Using equilibrium equations alone we cannot do that that is we have seen it. We need some additional condition let us see what additional condition we can provide in this problem so that it can be support reactions can be obtained, ok.

(Refer Slide Time: 03:14)



Now let us see what we can do Now the problem is like this it is a the Cantilever beam and then it is a ok then we have on externally used blue ink externally load say P here. So this is A this B and this is C. We need to find out what are the support reactions.

Now let us draw the free body diagram of the entire structure free body diagram is this is applied load P then we have support reactions which c y and then A y A x and then moment M a this is the free body diagram ok. The number of unknowns are 4 one is A x A y M a and C y ok By applying equilibrium equation f x is equal to 0 we can get A x is equal to 0 but still we will be having unknown A y C y and M a and number of equations available for this unknown are 2 sation of force F y is equal to 0 and sation of moment at any point is equal to 0 ok.

Now we learnt Principle of Superposition in the last class and also know the compatibility condition, ok. Let us and we already know the equilibrium conditions and how to apply equilibrium conditions and all, ok. Let us now this is the problem right? Can we divide this problem into two part like this like this or ok fine.

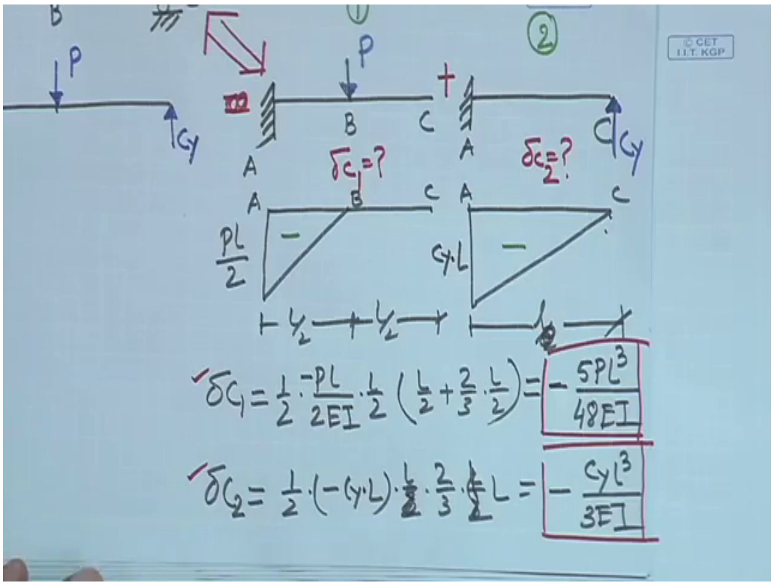
One is this where we have or where we have it is or rather this problem or this problem ok this problem where we have this is A then concentrated load P at B this is B and this is C plus same A C which is subjected to C y ok and this is this plus this, ok. So essentially this support is essentially this four system what we have done here is in the first case we removed the prop here at C and this is one case.

And the second case we remove this force because force because force is already applied here and only thing in this case was prop was not there and we applied this prop by C y ok Now you look at so this is principle of super position right So if this is equal to this plus this then anything we can obtain from this internal forces displacement the support reactions obtained for this and obtained for this if we add then we get this, ok.

So this is principle of Super position Now interesting thing is look at this this two problems individually if we look at then they are determinant This is a Cantilever beam subjected to a concentrated load at the mid span and this is also Cantilever beam subjected to a deep load But the load is in opposite direction the direction of support reaction.

So this beam and this beam both individually they are statically determinate ok. Now if they are statically determinant then we can solve them individually let us solve them, ok. Let us find out what is delta C in this case what is delta C and here also we can find out what is delta C suppose this is system 1 this is system 2 So this delta C 1 and this is delta C 2, ok.

(Refer Slide Time: 08:09)



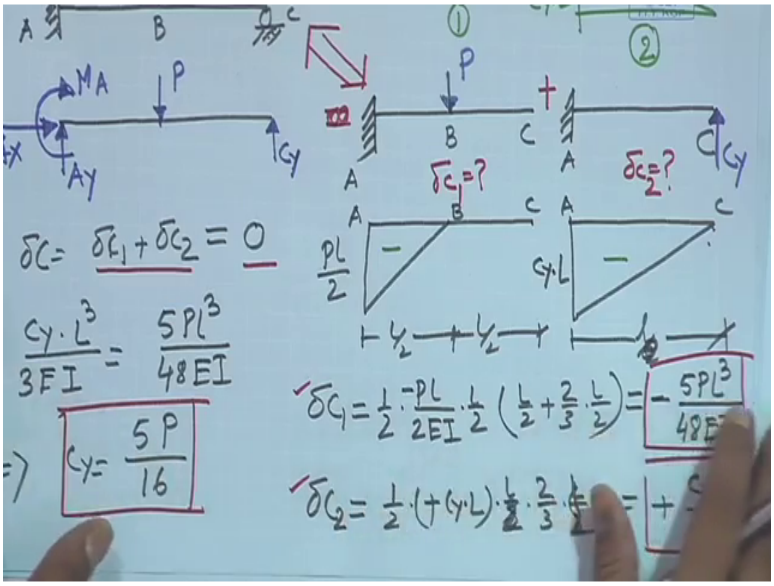
We can find out what is delta C 1 and delta C 2 let us apply moment area method So the moment area method will be the this will be the moment diagram for this and this will be C y into l this is the moment this is negative moment, ok. And in this case the moment diagram will be this and this will be PL by 2 right This is A this is B this is C AC and this distance is L by 2 and this L by 2 and this distance is L, ok.

Now if we apply moment area method in this case what will be delta C 1 delta C 1 will be this is the M by E I diagram of a moment of this diagram about C divided by E I Similarly if we apply moment area method in this case it will be moment of this diagram bending moment diagram about C divided by E I.

So what would be delta C 1 delta C 1 will be this is half P L by 2 E I because it is M by E I diagram we need to take and let us see this is negative say take negative sign minus PL by 2 E I into L by 2 into distance is L by 2 plus 2 by 3 into L by 2ok. This is delta C 1 and the delta C 1 will be minus 5 P L cube by 48 E I Please verify this you just multiply you will get this ok.

Now similarly calculate delta C 2 what will be delta C 2 Delta C 2 will be moment of this diagram about C divided by E I So moment of this diagram is half into minus C Y into L because it is negative moment this is negative moment and into L by 2 and then into 2 by 3 of L by 2 ok And not L by 2 sorry this is L this distance is L, so this distance is L so this becomes minus P L C y L cube by 3 E I ok This is 3 I.. This is delta C 1 and this is delta C 2 then once we have delta C 1 and delta C 2 then what would be the total deflection at C.

(Refer Slide Time: 11:20)



Total deflection at C will be delta C will be delta C 1 plus delta C 2 isn't it that is compatibility condition not that is compatible that is Principle of superposition. We will get delta C 1 from this system delta C 2 from this system So total deflection at C will be delta C 1 plus delta C 2 This is total deflection.

Now what compatibility condition tells us Look at this it is supported here at C we have a roller support here. So vertical displacement is there is no (())(11:55) is restricted. There should not be any vertical displacement so whatever delta C we get that has to be equal to 0 So this is principle of Super position and this is compatibility condition or kinematic admissibility condition ok.

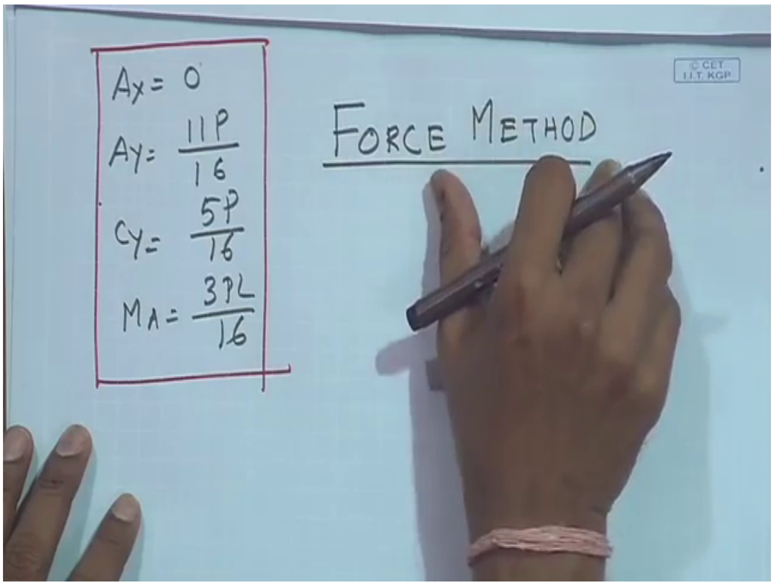
So if it is 0 then just substitute it we get 5 one thing please check since this U I is upward this will not be negative this bending moment diagram will actually like this please correct that bending moment diagram will be actually like this ok. This is U I into L so this will not be negative this will be positive ok Please correct that.

Now this is the principle of this is the condition we have now substitute that what we have C y into L cube divided by 3 E I that has to be equal to 5 PL cube by 48 E I ok. That is the condition we have now if we substitute that we have C y is equal to 5 P divided by 16 You see now what we have done here is when we get delta C 1 is equal to 0 delta C 2 is equal to 0 that is based on this bending moment diagram Ok.

This bending moment diagram we obtain by applying the equilibrium condition So essentially delta C 1 and delta C 2 they are from equilibrium condition ok Now in addition to that what we did is we applied another condition which is compatibility condition and what compatibility condition is at C is equal to deflection is equal to 0 So 2 equilibrium condition and one compatibility condition gives us the unknown reaction C y.

Now once we know C y then rest of the things become easy Now once we know C y in this expression we know C y then apply take moment about a get M a apply sation of force in Y direction is equal to 0 and calculate A y as well.

(Refer Slide Time: 14:40)



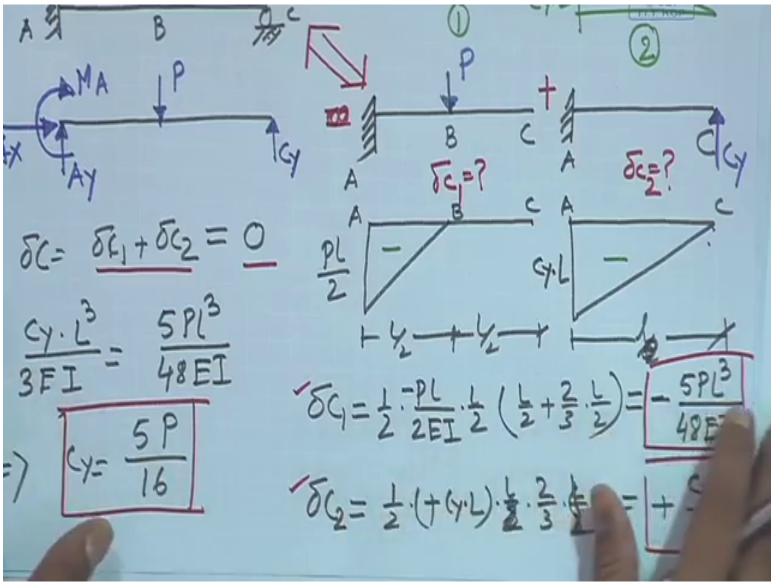
So the final solution will be for this problem is this x is equal to 0 any where x is equal to 0 A y will be is equal to 11 P by 16 C y will be 5P by 16 and M a is equal to 3 PL by 16 This is the final solution.

Now you see even if we have 4 unknowns and equilibrium conditions are 3 we use one more condition and this condition is compatibility condition and using that compatibility condition we can determine this unknown forces unknown reactions, ok?

Now this method this approach is called force More descriptive name would be method of consistent deformation because we used the consistence deformation so the method of consistent deformation.

But force method is a very general noun very general term we use force method the term force method is frequently used to distinguish a method based on displacement which is displacement method ok.

(Refer Slide Time: 16:20)

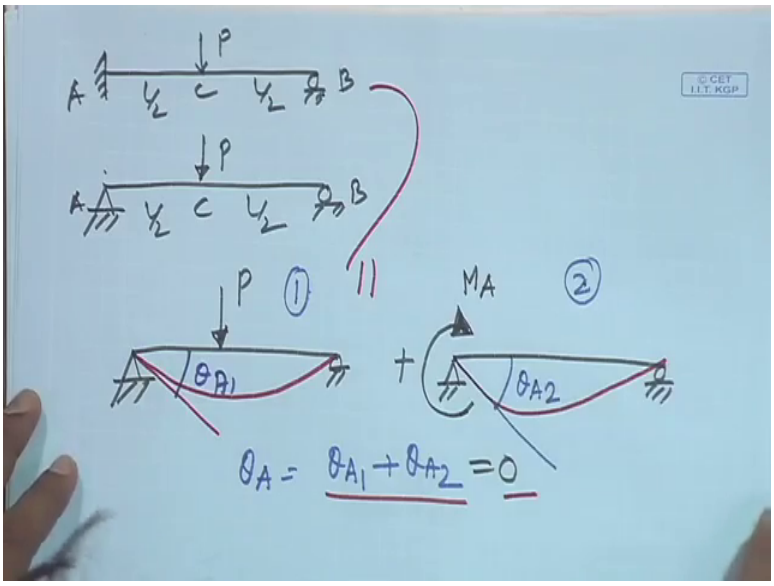


So this is force method, ok? Now in this case these what we actually did is we looked at the problem first and implicitly without actually unknowingly we determine the degree of indeterminacy in this case degree of indeterminacy is 1 and this indeterminacy external indeterminacy ok.

Now if we remove this support at C then the structure become determinant ok So it is the support at C which is making this structure indeterminate So this support reaction is redundant because for stability point of view we really do not need this support So we consider this reaction at C as redundant and then we remove this redundant and divide the problem into two part in one case this redundant is removed and the second case only this redundant force is applied here in both the cases the structure is determinate.

Then we obtain the deflection from this obtain the deflection from this and apply the compatibility condition to get the and finally we obtain the redundant support redundant reaction Once we have this reaction then rest of the things from this free body diagram we can obtain other reactions as well.

(Refer Slide Time: 17:52)



Now you may ask that ok Now suppose again consider the same problem ok. Now if we make it instead of suppose this suppose if we consider problem like this a simply supported beam subjected to load P like this this is again L by 2 L by 2. Now what is the difference between this problem and this problem this is also stable problem it is statically determinate problem simply supported beam subjected to the concentrated load of the make span.

What is the difference between this and this The difference between this and this this is roller support in both the cases here it was hinge support but here it is fixed support and what is the difference between hinge and fixed support it also restrict the translation in both the direction it also restrict the translation in both the direction but in addition it also restricts rotation Rotation is also not allowed.

So what actually we have done is In this problem in this structure if we somehow if we restrict the rotation at point A then this structure becomes this isn't it. So this structure is becoming indeterminate to here by restricting the rotation how can you restrict the rotation A. We are giving a constraint we are constraint against rotation and by providing that constraint we are making the structure indeterminate.

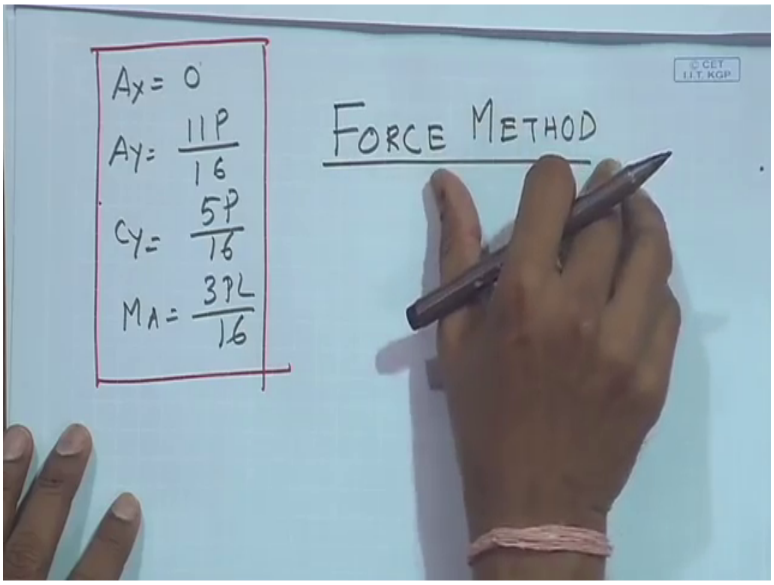
Now in this case we can take that constraint that moment constraint means in this case its moment So that moment as well we can can be taken as redundant force. So this structure again this structure can be divided into two part like this one is this it is roller support again hinge support and then applied force P then plus this additional redun this is again hinge support roller support and redundant suppose a moment like this ok.

Now this plus this will give us this ok. Now so in this case what we can do is we can in this case instead of taking in the previous case what we did is previous case we considered C y as redundant and we divided the entire problem into two sub problems like this but in this case we considered the moment at A as redundant force and then we divided the problem into two cases like this.

But in both the cases these are determinate structure you can see now we can determine this structure may deform like this this structure may deform like this and this angle is theta 1 theta A 1 and again this structure again deform like this and this angle is theta A 2 because this is system 1 and this is system 2.

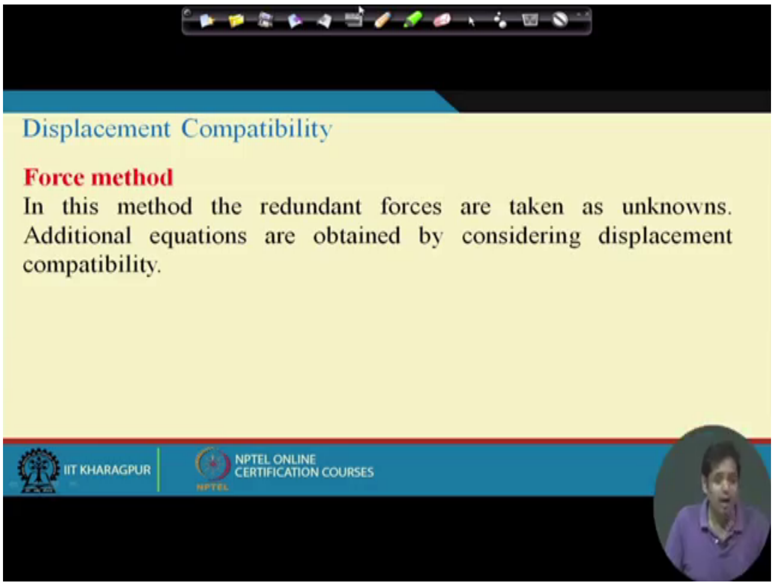
And what is the compatibility condition now what would be the theta A then theta A will be theta A 1 plus theta A 2. This is the principle of Super position what is the compatibility condition what is the support here fixed support so rotation of the support is restricted so theta A has to be 0. So this has to be 0 So this will give us additional equation. This is principle of Super position and when we are making it 0 this is compatibility condition ok.

(Refer Slide Time: 22:04)



So in this way also you can find out the support reactions and you will arrive at the same results ok And please try this and convince yourself Now this method is called force method.

(Refer Slide Time: 22:21)



Now then we have to formally introduce force method. What is force method Force method is in this method the redundant forces are taken as unknowns and if you remember the redundant force means we for the previous problem the prop Cantilever problem we took support reactions at prop or moment at the fixed length are unknown because they are the redundant forces ok.

So force method is in this method the redundant forces are taken as unknowns and then additional equations are obtained by considering displacement compatibility. Now in the first case the displacement compatibility is obtained by making deflection at C is equal to 0 because it was supported there was a roller support there. And the second approach we made rotation at theta is equal to 0 so this was compatibility equation so first we identify the redundant forces are taken as unknown and then apply the compatibility condition to find out those unknown redundant forces once we known the unknown redundant forces then rest of the other forces we can determine ok.

This is just the introduction of the force method the method force we will discuss in detail as we proceed So next class what we will do is we will discuss the force method in detail and see some more examples and so that if we do that you can appreciate the force method in a better way ok So we will stop here today and in the next class we can again continue with force method and see some of the implementation aspects and examples.

Thank You!