Structural Analysis I
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Lecture 33
Analysis of Statically Intermediate Structures

Hello everyone! Welcome to the second part of our of our of our course I am calling it second part because what you have what we learnt in last 6 week is we several methods to to analyse that say to analyse statically determinate structures right. But many structures are not determinate because the number of constant in those structures are more than the number equations statically equilibrium equations available. So whatever concept we have learned those concept cannot be directly applied to the structures which are not determinate which are indeterminate structures.

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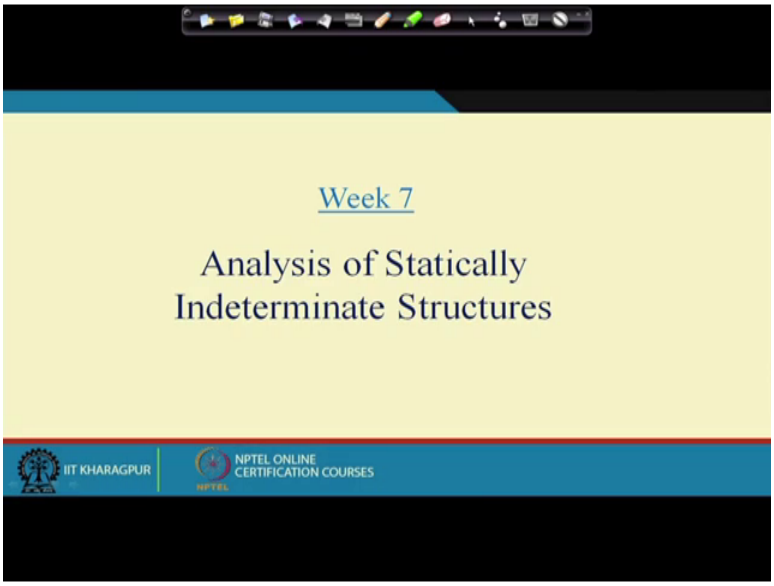

What will be what we're going to do this week and next 4 5 week is we will try to understand what is statically indeterminate structure when it becomes indeterminate and if it becomes indeterminate then what are the what are the methods available to analyse them to find out the internal forces and and displacements. Ok! So since this is the first lecture what we will be doing is we will understand the will understand the underlying philosophy of all these methods and also give you some of the basic concepts that we learnt in previous classes. And those concepts will be useful for subsequent classes as well.

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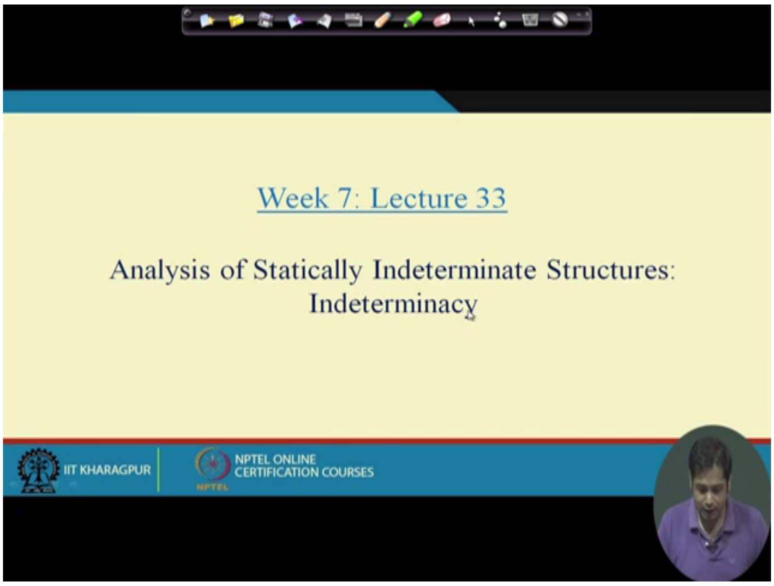

Ok! So this week we're going to start analysis of statically indeterminate structures and today what we will doing is we will just understand what is indeterminacy and what are the possible way out we have when a structure becomes indeterminate ok.

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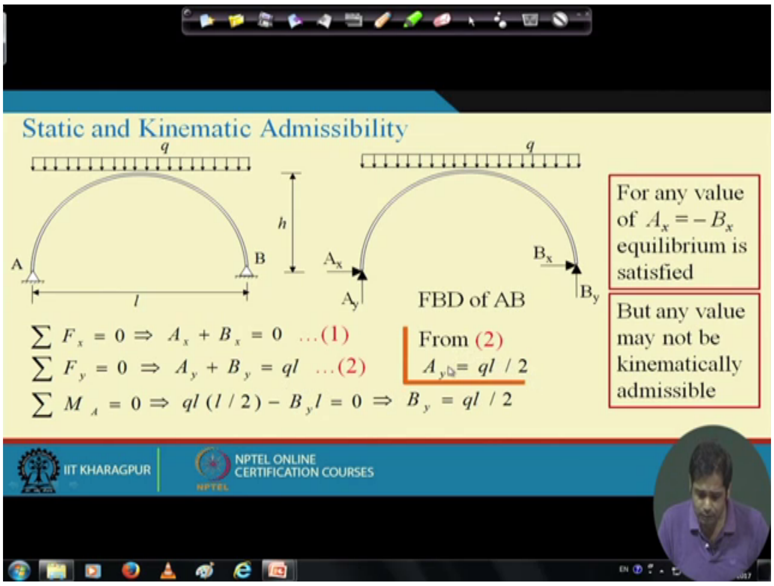

Ok! You see you I believe that you are familiar with this sliide because we discussed this slide in one of our earlier earlier lectures. What it is? We have an arch here which is subjected to uniformly distributed load and the support conditions at A and B are we have hinge at a and b. And this is this is an indeterminate structure because the number of reactions support reaction are 4- 2 here and 2 here. So essentially we have total 4 numbers of 4 equations, 4 reactions and number of equations available is 3.

Now if we apply equilibrium equation sation of f x is equal to zero sation f o is equal to the sation M a is equal to zero, Moment about a a and finally what we have is we can determine the you see if the structure indeterminate it doesn't mean that we cannot find out support reactions or internal forces in any number we can probly partially partially some of the unknown (())(03:09) we can determine but if the structure is indeterminate then just we are applying the equilibrium conditions we cannot determine all the unknowns ok.

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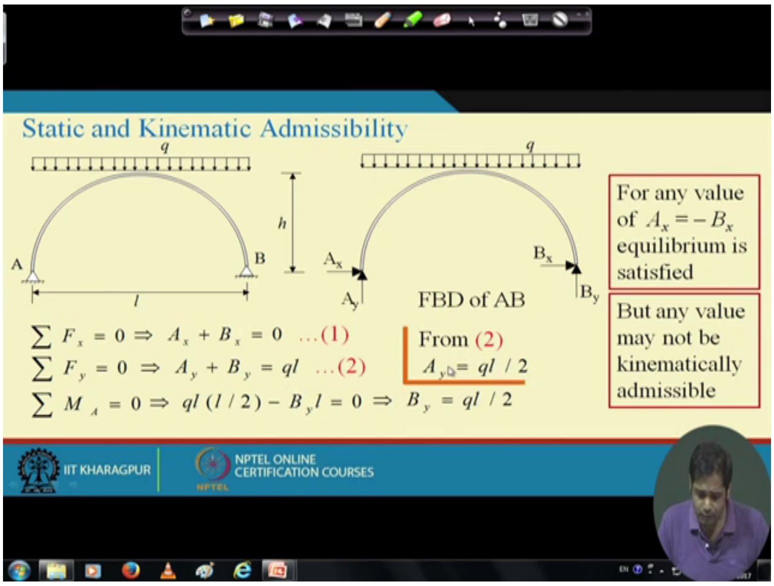

For instance in this case we can determine what is the value of what is the support reaction in vertical direction A y is equal to ql by 2 and B y is equal to ql by 2 that is obvious from the symmetry but what we have is we cannot determine whatever information available here we cannot determine what is the value of horizontal reaction A x and B x atmost what information we have that A x has to be is equal to minus B x means A x plus B x has to be is equal to 0, right?

So if AX and if whatever may be reaction whatever may be the value of AX and BX if they satisfy this condition then they satisfy the equilibrium condition equilibrium equation and this system is called for any 4 system which is we satisfy this, this is statically admissible but then again we mention that it is statically admissible but it is not kinematically admissible but that time we did not discuss what is kinematic admissibility.

Now we are going to discuss that so this is an indeterminate structure where the solutions are called partial solutions are ql EY is equal to ql by 2 B y is equal to ql by 2 and another condition is horizontal reactions at A and B they are equal and opposite. That is if any force system satisfy these condition means they are statically admissible.

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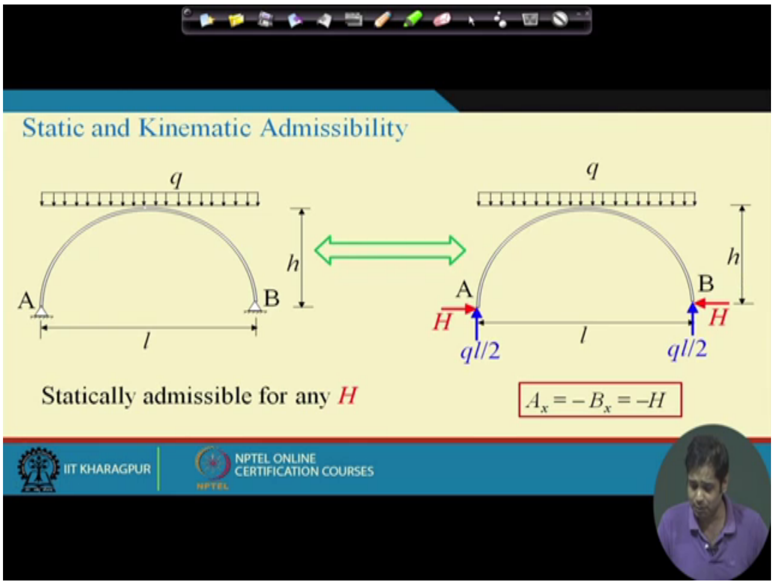

Now suppose now these let us find out an equivalent system this is our actual system right? Let us find out an equivalent system Now here equivalent system is only the support are replaced by there characteristic reactions. So we know the value of A y and value of B y is ql by 2 it is ql by 2 ql by 2 and value of only thing another thing we know that it is value of A x and B x they are equal and oppsosite but they can have any values but as per static admissibility is concerned but they should satisfy these conditions.

So let us horizontal reaction at A is equal to H corresponding horizontal reaction at B is equal to minus H ok and this is statically admissible for any H. Reason you apply the equilibrium condition here draw the free body it is the free body diagram you apply the equilibrium condition here you can see that these four systems satisfies the equilibrium condition sation of f x sation of f o and sation of moment ok.

Now let us what is the solution of these problems ok solution of these problem is if we solve it I do we have not yet discussed how to solve indeterminate structure but if we can solve it probably as we understand some of the methods in one of the class we will probably address this problem and try and solve what is the value of it. But till now take it for granted that if we solve this problem for the given value given value means if we take q is equal to 1.

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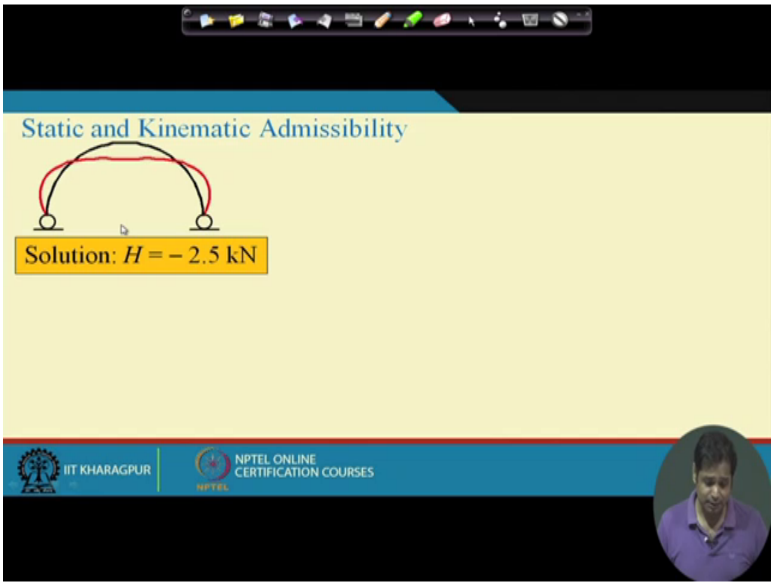

And then h reaction is h is equal to minus 2.5 ok? And the deflected shape of this is shown here. So this is the solution of the problem how you arrive at this solution for the time being you take this solution for granted we will come to this point as we proceed Now what point I want to make it this is as I said just into previous slide the four systems are the is at any value of H the equilibrium conditions are satisfied. Now let us give some arbitrary value of H and then say what happens, ok.

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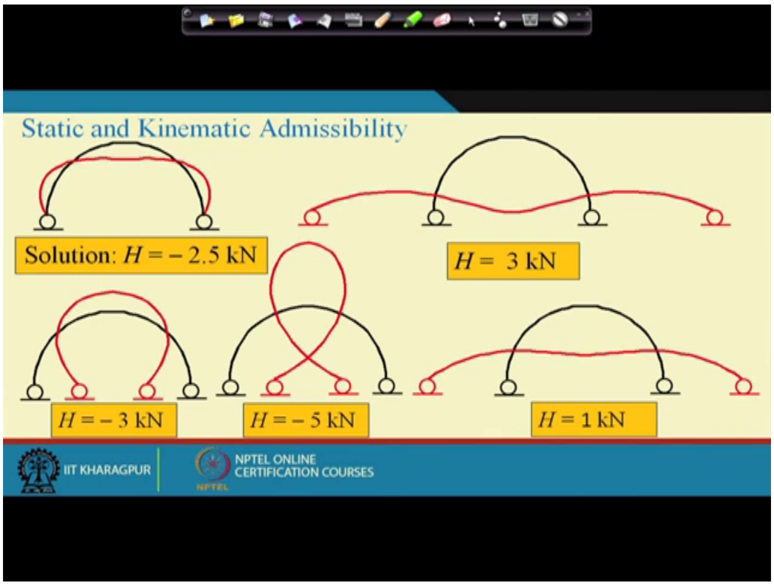

Now you see these are some solutions that we have obtained by giving some arbitrary value of H, but we satisfy that equilibrium condition, ok. Now here H is taken as 3 kilo Newton in this case H is taken as minus 3, H is minus 0.5 and H is 1. In all these cases equilibrium equations are satisfied, ok in all these cases. But just by looking at the (())(07:36) we can easily say they cannot be the solution of this problem.

One simple reason they cannot be solution of this problem because if you see this problem it was in support so there should not be any deflection here and there should not be any deflection both the deflection in both the directions are restrained and not allowed but if you see in all these cases the beam the support they deflects like this they change their position like this

So these solutions are not the solution, now it is for some value of H you can have you can put any value of H and they satisfy the equilibrium conditions but the deformation you get did not satisfy these support condition that we have in real structure, right. And we can have such infinite so what is the point we can make here we can have infinitely many possibilities, possibilities of four system which is statically admissible which satisfy the equilibrium condition, but among all these infinite four system only one system one force system will be the solution of this problem.

And that solution will satisfy these boundary conditions these conditions ok. Now this is called kinematically kinematic admissibility means all these problems if we look at this slide all these result solutions they the force system that we have here they are statically admissible they satisfy the equilibrium conditions but they are not kinematically admissible solution because they do not satisfy the kinematics that structure with given boundary condition and given load must follow ok.

So then what but at least the information that we have in this slide we cannot arrived at this solution we can keep on trying substituting different values of either by doing for different values of H we will get different kinds of solution but we shall not kinematically admissible. Now so you don't need to find out we have study equilibrium equations five they give some information.

But based on that information we cannot find out the solution uniquely we can put some value of H and get the solution where those that we can get the different shape and the force system. But that will not be the solution of the real structure, the structure what we have in this case.

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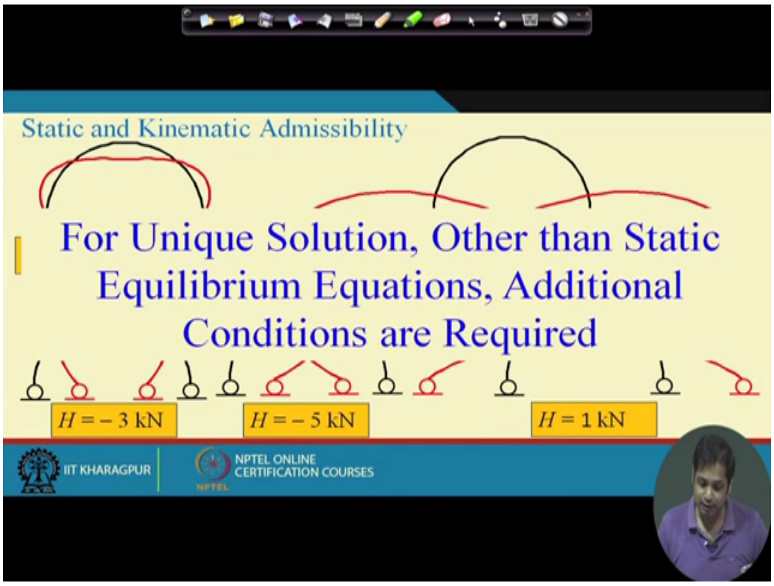

So in addition to the equilibrium equation we need some more condition in order to find out the solution uniquely, ok. So among all these infinite possibilities only one case will be the solution of the system and in order to pin point that in order to uniquely identify that equilibrium conditions along are not enough we need some additional condition which will help us to find out the solution uniquely, ok.

Now entire all the process that we have all the methods we have to solve statically indeterminate structure those methods are different because how these additional conditions are formed that is different in different methods ok.

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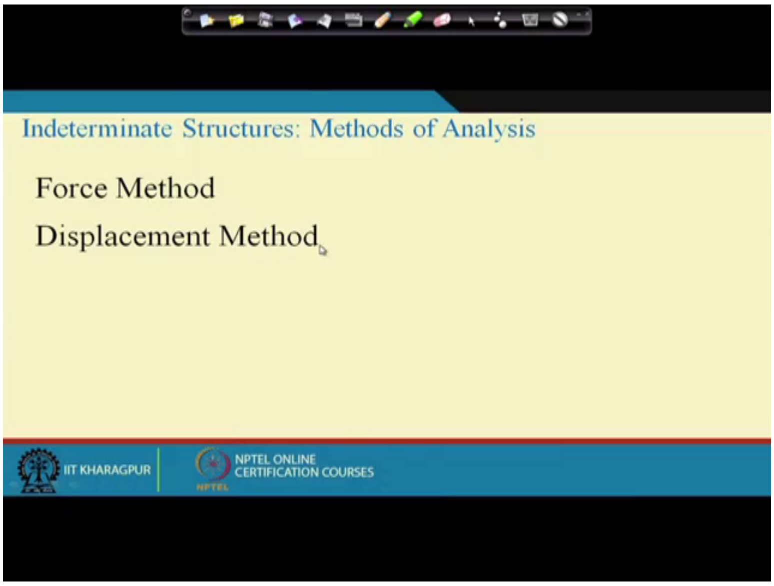

Now we have broadly two kinds of methods one is force methods one is displacement methods ok. Now these methods are different as I say it in a sense that the additional conditions we have equilibrium equations but we need some additional conditions in order to find out solutions uniquely and those additional conditions how those conditions are formulated what conditions are taken in analysis based on that we have broadly two methods one is force method one is displacement method.

We are not going to tell you right now we are not going to discuss what is force method and what is displacement method as we proceed we will understand those but there are two methods available one is force method one is displacement method. Well in this week we will introduce this method force method and displacement method. And rest of the weeks we will apply those methods in different structural problems ok.

Again we will consider three idealizations as as before and those idealizations are being plain frame and plane process ok. But before we do so before we actually formally introduce these methods force method and displacement method some of the concept that we need throughout the journey let us review them, let us understand them.

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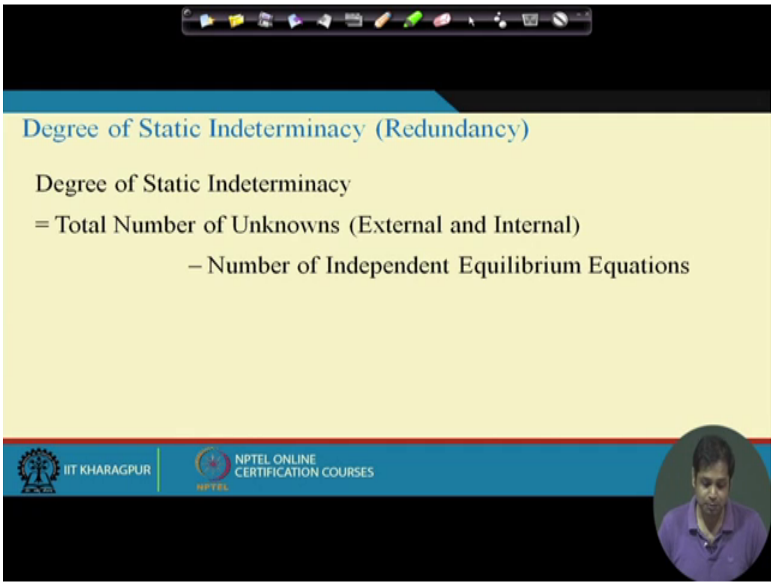

Ok! First is degree of static indeterminacy ok we know what is static indeterminacy? Statically indeterminate structure and what is the degree of that indeterminacy? Degree of indeterminacy is the total number of unknowns that unknowns could be external or internal what is external and what is internal unknown we are coming to that point shortly.

But we have total number of unknowns and we have certain equilibrium equations available some equilibrium equations available independent equations available then the static indeterminacy will be the total number of unknowns minus total number of independent equilibrium equations available ok. So as I said we need some additional conditions additional equations, how many equations are required, the number of equations required equal to the in the static indeterminacy of that problem.

There is another kind of indeterminacy if remember in one class we mentioned that is called kinematic indeterminacy will also come to that point, what is kinematic indeterminacy but for the time being what we need is static indeterminacy will discuss kinematic indeterminacy as we proceed, Ok. So this is degree of static indeterminacy ok.

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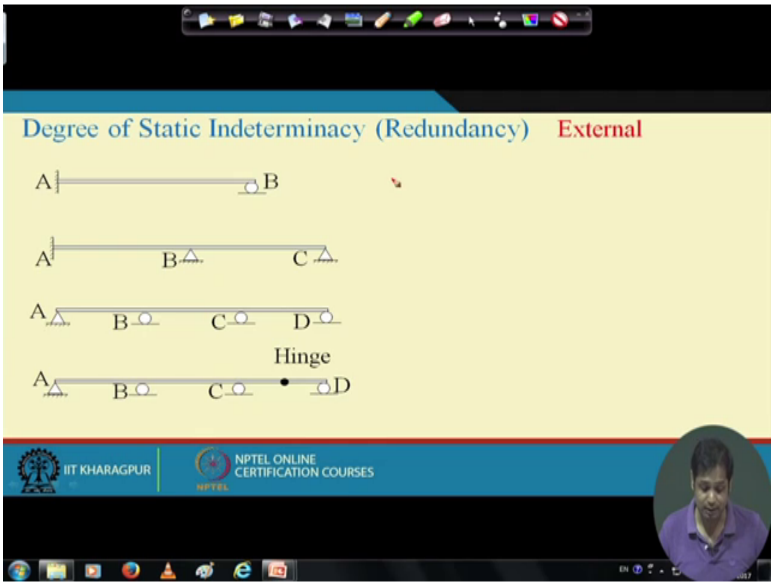

Now let us see some example for instance the first example it is [prop cantilever beam]() it is very obvious we have considered this problem many times in earlier classes, you see how many equations are available number of equations available is 3, ok. And equilibrium equations available is 3. So equations number of equations is available is 3. One is sation of f x is equal to 0 sation of f y and sation of moment is equal to 0.

And how many reactions we have? We have one is vertical reaction b and then we have A y horizontal here we have horizontal reaction and then a moment. So total unknowns are 4, so static indeterminacy which is sometime you know data is n s is equal to 1. So in this problem the degree of static indeterminacy is 4 minus 3 is equal to 1.

Let us see for this case again number of equations available is 3 and how many unknowns we have? We have here 2 unknowns horizontal and vertical because it is hinge support horizontal and vertical and then we have here 3 because it is fixed support so total 3 plus 2 plus 2 total 7 unknown is 7.

So in this case n s becomes 7 minus 3 4. And similarly in this case again equations available is 3 and unknown available is, it is roller support so only vertical reaction in this case vertical reaction and horizontal reaction so total 5, total 1, 2, 3, 4, 5 total 5 and then n s become 5 minus 2,ok. In this case now equation equilibrium equation available is 3 but you see there is a hinge here, ok.

And hinge will provide you one more condition and what is that condition at big point moment is 0. So total equations available in this case is 4 3 equilibrium conditions sation of f x sation of y and sation of moments is 0 from any point and additional equation is at the hinge point moment is 0. So equation becomes 4 and number of unknown becomes 5 in this case n s becomes 1, ok. So for a beam we can find out for a continuous beam or any statically what is the degree of static indeterminacy, ok.

Now let us see some more example for frame. Now see all these indeterminacy were external indeterminacy because these structures are indeterminate because of the external support system. For instance if in this problem if we remove this support then it becomes the cantilever beam and it is statically indeterminate structure in this problem if we remove this support and remove this again it becomes simply supported beam statically determinate.

In this case if we remove one support say one support if we remove keeping one support as it is then it becomes statically determinate. So in all these problems indeterminacy created due to the external due to the reactions, reactions from the support. So these kind of indeterminacy is called external indeterminacy.

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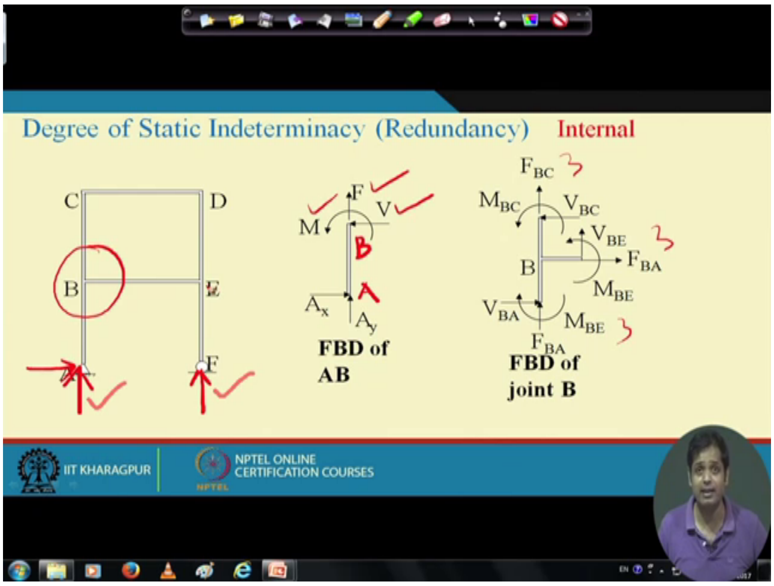

Now there could be another kind of indeterminacy, what is internal indeterminacy? Now for consider this problem this is a frame, two story frame now you see if we see the support condition this is how many how many reactions we have?

We have one reaction here and one reaction and then another reaction here and then one reaction this is roller this is hinge. What should be 3 reactions and we can apply 3 equilibrium conditions to find out these reactions so these reactions can be determined.

But so externally this structure is determinate as long as our intention is to determine the support reactions. But now in addition to support reaction we need to find out internal forces in different members as well. If we try to do that in this case let us see suppose we have already completed the support reaction, let us draw the free body diagram of AB and then this will be the free body diagram of AB at point A this is A and this is B.

So at point A we have support reaction A y and A x and point B this is frame so we have three forces horizontal vertical shear force action force and the moment ok. Now here we can apply three equilibrium condition and determine with G F M so you can determine that fine. Now let us draw the free body diagram of joint of this part join B so this is B this is member BE this is member BC and this is member BE.

Now you see here we have 3 unknown here we have 3 unknown here we have 3 unknown, so total 9 unknowns we have. You see number of equations available in this case are 3 so we cannot determine all these unknowns. And if you take any other beam any other joint or any other beam you can see that the determination of member 4 is not possible in this case just by applying the equilibrium conditions whereas as far as support reactions are concerned we can determine the support reactions.

So this kind of indeterminacy is called internal indeterminacy. This indeterminacy is due to the fact that the number of members provided in this structure is more than required for the stability of the structure. So this is statically indeterminate structure but it is internal indeterminacy, ok.

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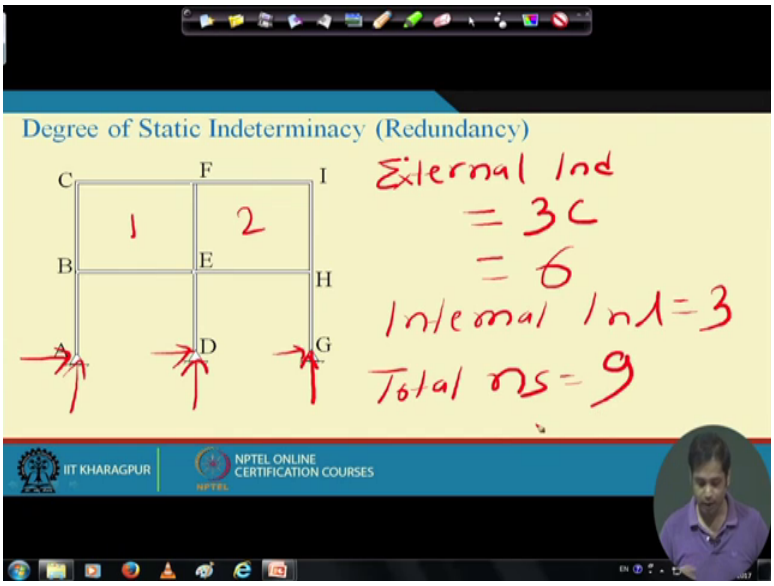

Now see the next problem now for a frame there is a rule that external indeterminacy can be computed at 3 into C , ok, where c is the number of close loop you have in the structure. For instance in the structure let us go back to the previous structure, ok! Let us see firstly for this problem this is one loop complete loop this is second loop. This cannot be taken as loop because these are not close this is open loop.

So external indeterminacy for these become 3 into 2 this is 6 and what is the internal indeterminacy we have? Internal indeterminacy here we have two reactions then two reactions these are hinge two reactions so total 6 reactions equations available is 3 so it is internal indeterminacy is 3. So total indeterminacy, total n s is external plus internal means 9, ok.So you can verify this for different frame as well.

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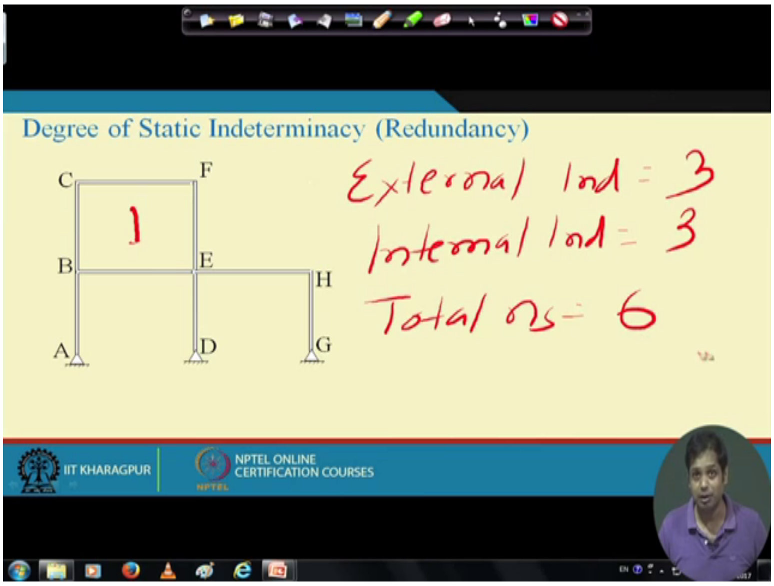

So let us see for next another problem. For this again apply the same concept we have this is one close loop this is not close so external indeterminacy is 3 into 1 3 and what is internal again internal indeterminacy in this case internal becomes 3. So total n s is 6. So this is a total indeterminacy of the structure.

Now if you want to find out this only the support reactions we need only if you want to completely analyze the structure we need in addition to equilibrium equation we need atleast we need 6 additional conditions, ok. Now these are 4 frame if you take any book of structure analysis there are different kind of problems given and their static internal and external indeterminacy are given you can do some exercise for that, ok.

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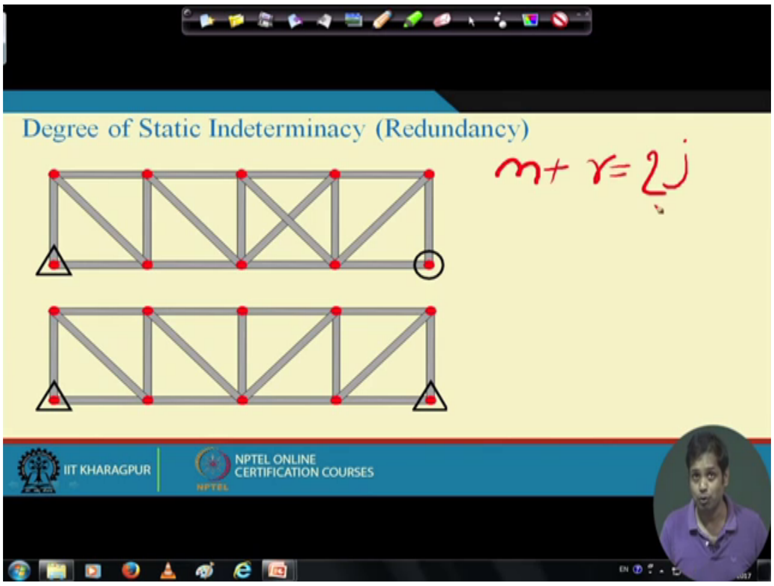

For trus also you can have external or internal indeterminacy for instance if you remember for trus the condition was if we have number of member is m and support reactions are and joint number of joint is j then total number of equations total number of unknowns are m plus r because m member forces and r reactions and total number of equations available is 2 into j because per joint you have two equations sation of f x and sation of f y is equal to 0 and for a statically determinate structure this has to be equal to 1.

And if it less than 2 j then the structure become unstable and if it is greater than 2 j then structure is statically indeterminate but having said that please note when we discuss about class we said that this is there are some examples where we can show that as per this condition structure is indeterminate and stable.

And sometimes stable and determinate but still for some loading conditions structure becomes unstable so this you cannot take just as for granted this is a genuine this is necessary condition. But whether the stability of the structure needs to be assessed must be assessed by visual inspection as well, ok.

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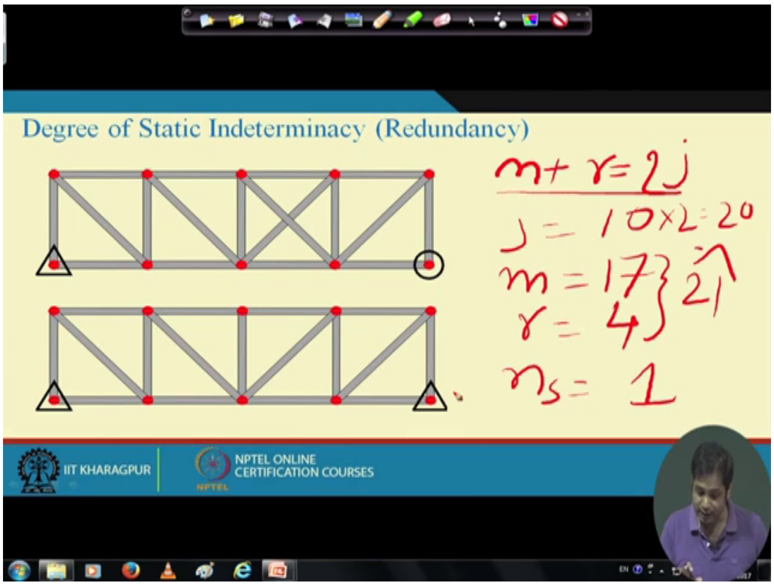

Now in this case let us see for this problem let us consider for second one, second one you see number of joints in both the cases number of joints is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 number of joint is 10 and for second case number of member is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11, 12, 13, 14, 15, 16, 17 and number of support reactions are 2 here and 2 here because both are hinge support total 4.

Now n plus this is 21 and into 2 this becomes 20 so this is greater than 20 so number of unknowns are more than number of equations so this is statically indeterminate structure. And what is n s? N s is 1 21 minus 21. Now you see in this case what causes the indeterminacy it is the support condition if we remove the hinge support instead provide a roller support then the structure becomes determinate.

So this is the structure is determinate but indeterminacy is external indeterminacy. Now take the first one again number of joints are 10 and number of members in this case we have one additional member this one, so number of members are 18 and number of supports are 2 here an 1 here so number of reactions are 3 so again this becomes 21 and this into 2 become 20.

So number of unknowns are more than number of equations available it is again indeterminate structure but what causes this indeterminacy the additional member this additional member causes the indeterminacy if we remove this member the structure becomes indeterminate. So this is an indeterminate structure but this kind of indeterminacy is internal indeterminacy, ok.

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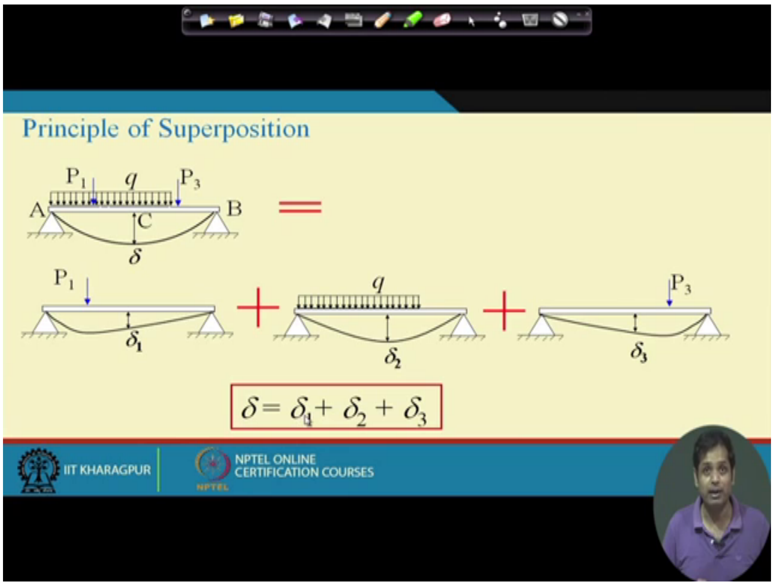

Now some more concept one is principle of super position. What is Principle of Super position? Principle of Super position is you see in one of the classes probably we discussed what is Principle of Super position since in this all the problems that we are doing here it is linear problem means force and displacement relations are linear, ok space change relations are linear.

So what we can do suppose this is th actual structure that we need to solve we need to find out the we need to analyze find out the support reaction, suppose in this case the displacement at point C we need to find out for this force system. What we can do is we can break this structure into several small small parts.

First things suppose this is the structure and this structure same structure only consider load P 1 and second case consider the same structure but only consider the distributed load and say third case same structure but only consider the P 3 Case. Now suppose in this case it is delta 1 delta 2 and this delta 3 so delta 1 is the displacement at the same point causes by P 1 delta 2 is the displacement at the same point causes by Q and delta 3 is the displacement at the dame point causes by P 3.

Then we can say that the actual this structure is this. So total delta, delta for this structure can be obtained as delta 1 plus delta 2 plus delta 3. This is called Principle of Super position means we have we obtain the solution for small small part and then super impose for those solution to get the actual solution.

It is a very very effective way because again in advanced structure analysis or in different method of analysis we will see that this is the underlined philosophy of most of the method that you have a structure needs to be analyzed or you have a problem that needs to be analyzed divide the problem into small small sub problems and then assemble them together.

And how you assemble them that depends on whether the problem is linear or non linear but in this case since we are the linearity is one of the important assumption so we can just add them to get the final deflection at this. Whether that is true for bending moment, shear force support reactions in internal forces ok so this method this is called principle or super position we will be using frequently this principle concept of Principle Super position.

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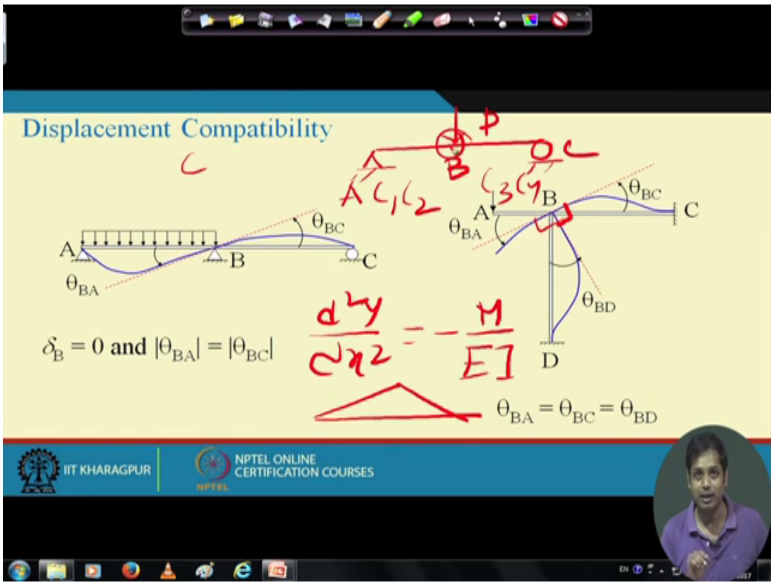

And other important concept is compatibility what compatibility say just to demonstrate take suppose this is a simply supported beam subjected to not simply supported beam it is a continuous beam indeterminate beam subjected to some loading system and this blue line that you can see that is the deflected shape of this beam.

Now if we draw a slope at b then suppose this slope is theta BA and this slope is theata BC, ok. What compatibility says that this theta BA has to be equal to a theta BA this is one condition and the second condition is at B, since it is supported at the hill supported at B the deflection of B should be equal to 0. So delta B should be equal to 0 and this angle and this angle this slope obtained from this segment and slope obtained from this segment at the same point has to be same. Ok.

Just to one more example suppose this is another example and these blue lines that you can see that is the deflected shape ok. And at this joint B we have three members member AB BC and BD. Now suppose theta BC is the slope at B but obtain from segment BC similarly BA is from BA and theta BD is from BD what compatibility condition says that because this is this is a rigid joint and initially it was 90 degree so it remains 90 degree there will be no rotation at this joint.

So these angles are 90 degree these angles are all 90 degree so it says that this angle this angle and this angle should be same, ok. And we have used compatibility conditions earlier also but probably without referring to the term compatibility for instance if you remember we solved this problem right and we have a simply supported beam wich is subjected to a concentrated load P, ok. And this is A this is B and this is C.

And if we want to solve it this is P if we want to solve it by direct integration method then the direct integration equations that we had was d 2 y d x 2 is equal to minus M by E I, right? So but what is the bending moment diagram for these beam? The bending moment diagram for this beam is like this, like this ok. So bending moment in part A B and part B C they are different. So we need to apply these we have to integrate this equation over B over AB and over BC separately. Therefore each case we will get two constants because the second order equation.

So for AB we get four constants C 1 and C 2 and for BC also we get two constants C 3 and C 4 what are conditions we have to determine this constant boundary conditions we have that A is equal to deflection is equal to 0 at C is equal to deflection B is equal to 0 but that will give us two constants but other two constants we need two equations but what are those two conditions if you remember we use that slope at B is equal to 0 and deflection and slope is continuous at B and deflection is continuous at B.

Means slope obtained from AB at B should be equal to slope obtained from BC at B similarly deflection at B obtained from AB and deflection from deflection from B obtained at BC should be same. This was compatibility equation ok and this figure and this figure exactly tells you that so we are used compatibility equation even in even while solving statically determinate structure ok.

So compatibility equation is another thing that will be used very frequently while studying indeterminate structure ok. So what we do is stop here today next class as I said there are two kinds of method one is broadly one force method one is displacement method what we do next lecture we just introduce the force method.

Ok Thank you see you in the next class.