Structural Analysis 1
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Lecture 32: Influence Line Diagram
And Moving Loads (Continued)

Hello everyone! This is the last lecture this week, means week 6, what we've been doing since last 4 5 lectures, we have been discussing influence line diagram for statically determinate structures right, we learned what is influence line diagram, then we saw how to draw influence line diagram for different internal forces and then also we saw that for different force systems how that influence line diagram can be used to find out internal forces.

For instant if the external load is a concentrated load or external load is uniformly distributed load or external load is a train of concentrated load, we've seen that how to find out the internal forces for a given location of those external forces in a member, what we'll do today is we'll learn a very interesting way of drawing influence line diagram without actually solving the problem, without actually completing the or writing the equations.

Of the internal forces as a function of x, you see as of now we have not yet discussed statically indeterminate structure but solving statically indeterminate structure is not as usual as determinate structure which is of use because it is not only the equilibrium equations there are other conditions that need to be applied, in many cases we need to solve those equations iteratively, so you see for each internal force system and for each location.

Finding the influence line diagram for statically indeterminate structure is a very dangerous job, so the prosecute that's we are going to learn today is though we demonstrate through statically determinate structure but that can be a very advantages for finding influence line diagram indeterminate structures ok, the method is called Muller-Breslau's principle,

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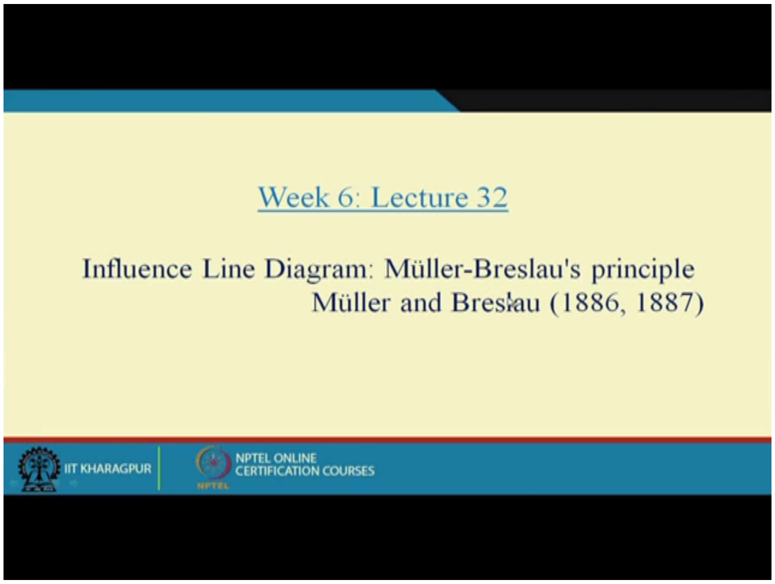

This is the method simultaneously almost they proposed by Muller and Breslau during 1886 and 1887. Before we state the principle, let us understand what the underlined philosophy behind this principle is ok, but before that let us recall if you remember when we discussed about virtual walk method, we discussed Betti's law, Betti's theorem and this theorem says

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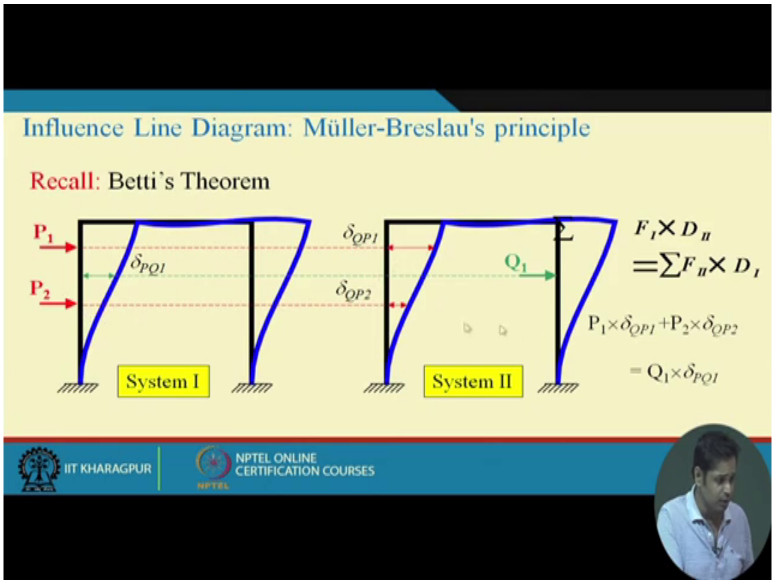

Just a quick recall of the theorem, that if you have system one and system two and in system one, there were force like P one and P two and system two is Q one.

And this is the deflected shape in system one and system two and then now if this delta QP one is the deflection in system two, in the direction where P one is applied in system one delta QP two is the deflection due to Q one but it is in the direction and the location at P when P two is being applied and similarly, delta P Q one is the deflection in the structure in system one due to P one and P two forces but in the direction of Q one.

And in the position of Q one, and if we have a system one and system two then what Betti's theorem says that there will be a summation here says that force in system one multiplied by the deflection filed in system two is equal to force field in system two multiplied by the deflection field in system and summation, this is the summation but it should somewhere here ok, so at least if we apply this theorem here.

Then it says that P one into this deflection plus P two into this is equal to Q one into this, so this is Betti's theorem, we'll see the reason why we just quickly viewed Betti's theorem because the underlying principle that will be used into prove the Muller-Breslau principle is Betti's theorem ok,

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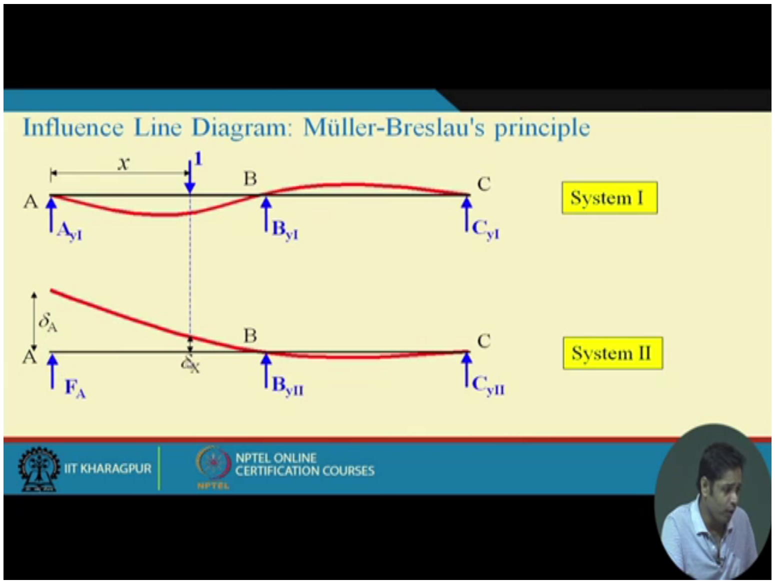

Now let us understand what this principle says ok, suppose this is a beam A B C, you see as I said this is applied for statically indeterminate structure as well. So the proof and the introduction of this principle will be done through statically indeterminate structure but rest of this examples problem we'll see only for determinate structure, ok?

So as you can this is a statically indeterminate structure because the number constraint we have is two here, one here and one here, total number of constraint is four and then equilibrium equations available is three so it is an indeterminate structure.

Now suppose we apply an unit load at a distance x from A and the corresponding deflection of this beam is this ok, now let us take the same beam once again but what we do suppose we are here, we want to find out what is the influence line diagram for say reaction at A ok, so this is the same beam, now next we'll remove the constraint A, remove the constraint at A means since we want to first determine what is the influence line diagram for vertical reactions.

So removing constraint means removing the vertical constraint ok, so this point A can move in vertical direction but still it cannot move in horizontal direction ok, now then apply a force say F A at this point, now if we apply a force like this then the beam may deflect like this ok and suppose these distances delta A, so delta A is the deflection at point A due to force F A and what is the direction of the force.

Direction of the force is in the direction of the constraint for which we want to find out the influence line ok, in this case it is A Y ok, now this is the unit load acting on the beam and suppose the corresponding deflection here when the load is F Y is delta x, so delta x essentially is the deflection at this point due to the external load, due to a load F A at point A but the position and direction of delta x is according to the unit load at this point ok.

So it is in the direction of unit load and at the location of the unit load ok, now let us just replace all those supports by their equivalent reaction process, so the support A is replaced by A Y, B is replaced by B Y and C is replaced by C Y, now please note that here it is not A Y it is A Y one, B Y one and C Y one, why it is one just we'll see that shortly and similarly if we do that same thing for this it is reaction at B Y, C Y it is B Y two and C Y two.

Now let us call this system one and this as system two ok and since it is system one, that's why it is denoted as A Y one, B Y one, C Y one, so when I say A Y one, B Y one, C Y one, these are the support reactions at A B C in system one and similarly B Y two and C Y two these are the support reactions at B and C in system two, now in what way system one and system two are different, system one is we have a beam.

And this beam is subjected to an unit load at a distance x and it deflects like this ok, so this is the deformation field or deflection field, the red deflected line that we can see, that gives you the displacement field of the system one and the force field in system one is unit load acting at a distance x, now system two is what, system two is the same beam but this constraint is removed we have a force F A, we don't know what is the value of force F A.

But for some force F A we have a deflection like this ok, now what is the force field here, force field is F A, B Y 2 and C Y 2, this constitutes the force field in system two and deflected displacement field is this one, this rate deflected shape ok, so which has deflection delta A at point A delta x at this point ok, now this is system one and system two, now let us apply the Betti's theorem in this two system.

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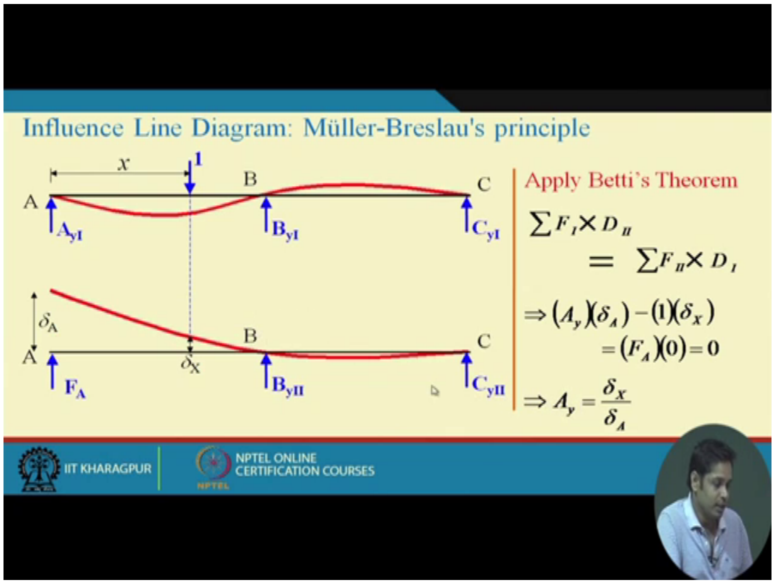

Now what it says that applying Betti's theorem it says that summation of force, field in system one multiplied by displacement field in system two is equal to summation of force field in system two multiplied by displacement in system one, just now we just review the Betti's theorem with the same thing here, now let us see what are the force field here in system one and force field in system one is A Y and then one and B Y and C Y.

And Corresponding displacement field in system two is delta A which corresponds to A one delta x which corresponds to one, this is zero which corresponds to B Y one and this is again zero which corresponds to C Y one ok, so in system one we have delta in first field in A Y and then delta A, delta A is this that is positive because the displacement is in the direction of A Y and then minus one and then first displacement (11:06 series at) system two.

So one multiplied by D X negative because it is in opposite direction, so this is first step, this is for force field in system one multiplied by displacement field in system two, you see B Y one and B C Y one are not considered here because anyway this deflection at B and deflection at C is zero, so their contribution will be again zero, now let us see what is force field in system two and corresponding displacement field in system one.

The force field in system one is F A and corresponding displacement in system one is zero because at this point the displacement is zero, see other force field at this and this, they will also contribute nothing because the corresponding deflection at B and C in system one is zero, so this gives you this introduce zero means this is zero, so this gives you A Y is equal to delta x by delta a, so A Y it is actually A Y one but just to make it general. I do not explicitly wrote this one here, so A Y becomes delta x by delta A, delta x is this and delta A is this ok,

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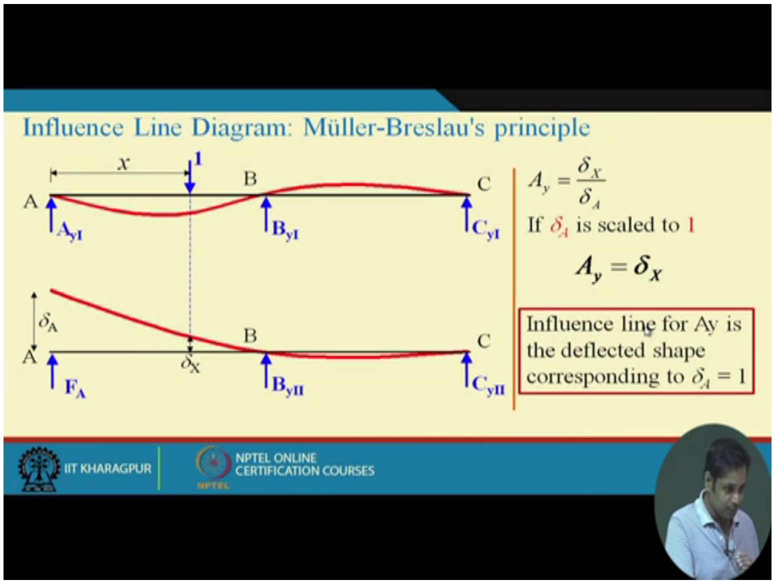

So we have let's see what it says you then A Y is equal to delta x by delta A, now this is too for any value of F A and any value of delta A right, now let the force is such that delta A is equal to one ok, so the force is the deflection at point A is reduced to one, is scale to one if we do that if delta A scale to one then what is this A Y become delta x right.

Now what is delta x, delta x gives you the deflection at delta x essentially gives you what delta x gives you how the deflection varies with x right, so it is essentially gives us the restrict carve or deflected shape of this beam ok, now then what this says, this says that influence line for A Y is the deflected shape, this deflected shape is given by delta x, this deflected shape corresponding to delta A is equal to one so if we make delta A is equal to one. Then whatever deflected shape we get that deflected shape will give us the influence line for support reaction at A.

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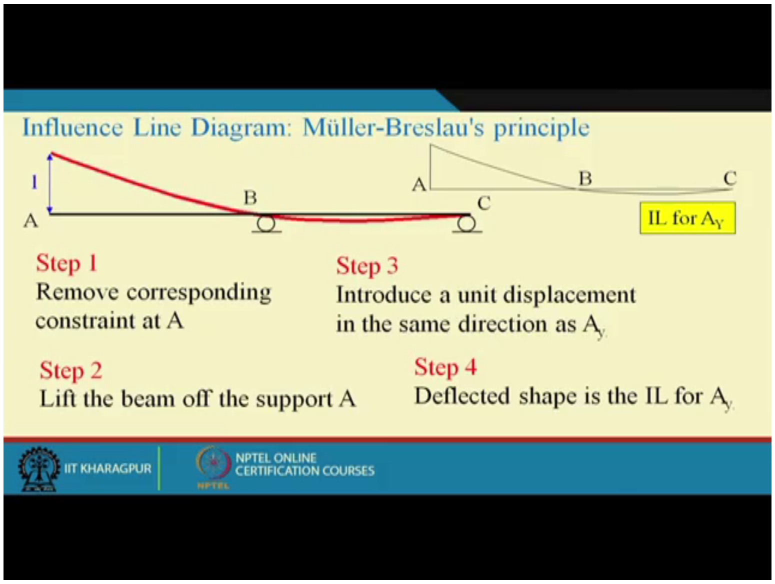

So let us see the steps then, so we have the beam here the continuous same beam, the first step is remove corresponding constraint at A, suppose we want to find out the influence line diagram for vertical reaction, so remove the corresponding constraints this is step one and then step two is lift the beam of support A.

Means lift the beam of support A and that has to be done in the direction of A Y ok and then introduce the deflection when you are lifting it, lift up to a point when the deflection at point is one, so introduce a unit displacement in the same direction as A Y, so this is one and then step four is deflected shape, whatever deflected shape what you get that deflected shape will give you the influence line for A Y.

So influence line for A Y for this beam is, this is the influence line for A Y ok, now this value is one, now here one point two is what you remember last time I mention you have also seen that for statically determinate structure the influence lines whatever we had drawn so far they were essentially piece wise linear function means they are essentially a collection of some lines but its true for statically determinate structure but if the structure is statically indeterminate.

As you can see here then the influence line may not be may not necessarily be piece wise linear and in this case it is a non linear curve you can sew, so now what is the advantage of this, the advantage of this just to draw the influence diagram, you did not have to solve the problem, you did not have to find out the support reactions, you did not have to find out the expression for support reactions, similar things you have to see for other internal forces as well. But without actually solving the problem we could draw the influence line diagram ok,

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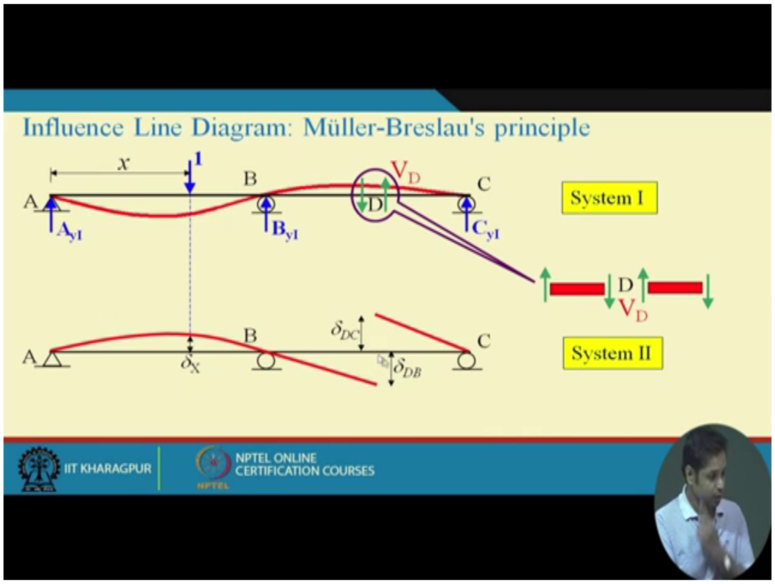

Now let’s see for shear force suppose again this is the system one and the same thing system one remain same, now suppose you want to find out what is the influence line diagram for shear force at D, at any intermediate point D between D and C ok, now you remember the last test, if you want to find out the influence line diagram for A Y we had to remove the corresponding constraint.

Now where we need to find out the influence line diagram for shear, so again we need to remove the corresponding constraint and what is the corresponding constraint we need to cut this beam at this point ok, now you see this is the shear and if you remember the sign convention what we use is if we take any segment of beam, if the shear force produces clockwise couple then this is positive, so now this is point D and this portion is this portion and this portion is this portion.

So at point D the shear force is of like this ok, so what we need to do is we need to cut this beam here and apply corresponding shear force V D here and then if we do that then what will be the deflection, how the beam may deflect you see if we consider only this path means A B D D’s part which is subjected to of shear force at point D, then this part may deflect like this and if we consider part D C, now D C is hinge and applying a force like this.

And then this will deflect like this ok, now just say this is delta D B, delta D B means delta at point D but this is a part of B D, this beam ok, and then delta D C is again delta D but this point is a part of D C ok, now then what happens then this is not system two, now we apply Betti's theorem, Betti's theorem says is again force in system one multiplied by displacement field in system two and then is equal to force in system two multiplied by the displacement field.

In system one, now force field in system one is one corresponding displacement in system two is delta x, force field in system one is V D and corresponding displacement in system two is this ok, delta D C and delta D B,

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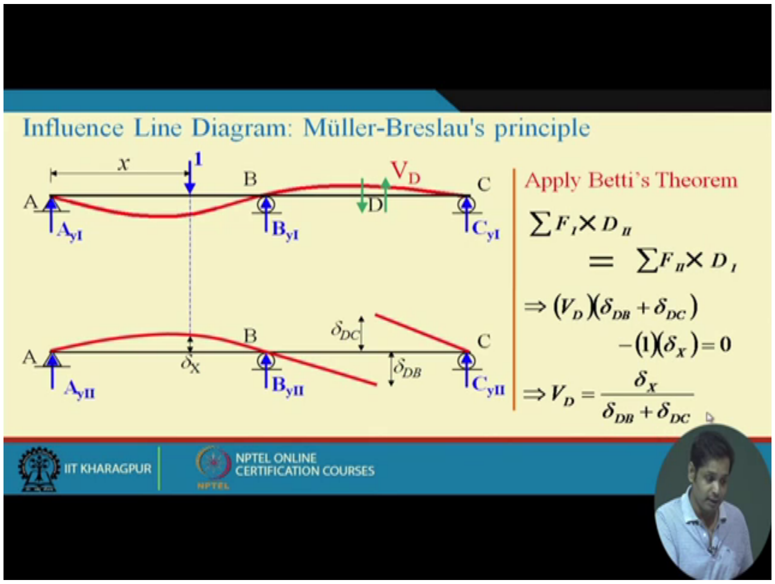

So then these are the support reactions, now let us apply Betti's theorem, Betti's theorem says this we have already seen it just now, now what it says, V D multiplied by D C into V D multiplied by D B means V D multiplied by D B.

Delta D B plus delta D C and then D C is positive because D C is acting for this segment means D C, V D is upward and the deflection is also upward and for this segment this V D is downward, deflection is also downward so this is positive and this will be negative because again this force is acting downward but this deflection is upward, so this is negative, and the rest is zero because what are the force field we have in system two.

Force field in system two is A Y two, B Y two and C Y two and corresponding deflection in system one is zero zero zero, so this force will not contribute anything, so this will be zero right, and it says that V D is equal to delta x by this.

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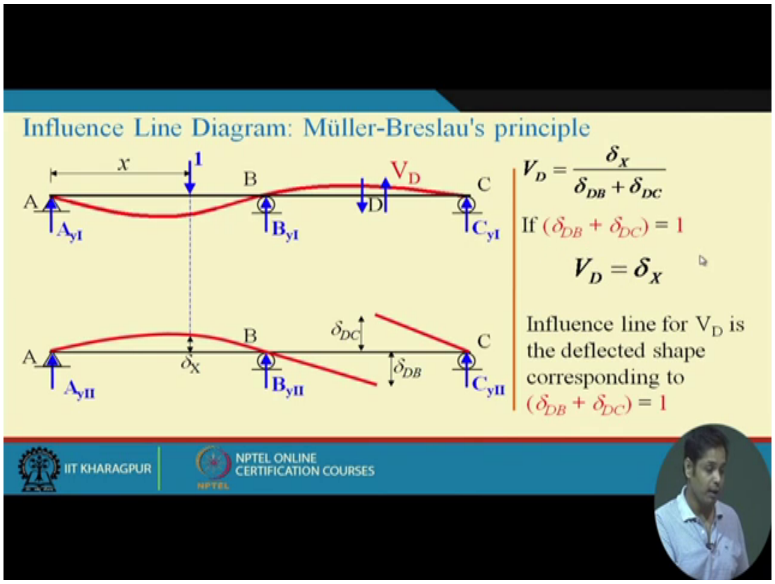

And then we have V D is equal to delta x by this, Now again similar to the previous case, support reaction if we scale this delta D B, delta D B plus delta D C gives you the this in their part field, if you scale this to one then what is V D.

V D is equal to delta x and what is delta x gives us the deflected shape of the beam ok, and then what this equation tells us, this equation tells us that V D will be same as the deflected shape in system two, if we know the deflected shape in system two we know the V D of system one, so what this equation tells us, that influence line for V D is a deflected shape corresponding to delta D B plus delta D C is equal to one. So if we cut this section at D and introduce a deflection such that this plus this is equal to one, then whatever deflected shape we get that will be the influence line diagram for V D, ok?

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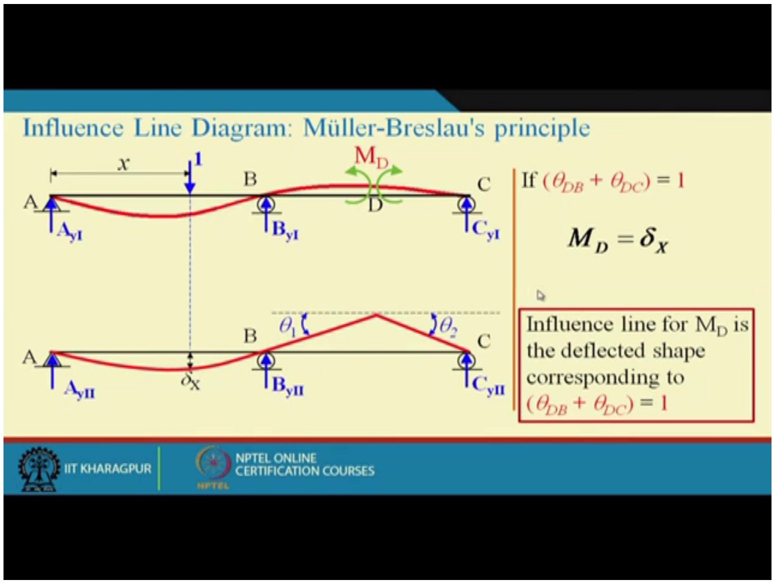

Now similarly if we do for moment, now the proof of this I will leave it to you the same way you can prove that if we make, suppose this is the moment, this is our sign convention, now due to this moment M D what will be the deflected shape of A B D will be this.

And for this moment M D deflected shape of D C will be this ok, now this angle is theta one and this angle is theta two or theta D B and theta D C, now if we make this theta D B plus theta D C is equal to one then we get M D is equal to delta x,

So what it says, it says that if you want to find out the influence line diagram for any particular point then again like the previous cases we need to release the constraint, then what is the associated constraint for moment.

The associated constant for moment is rotation right, like associated constraint for shear is the relative displacement and associated constraint for moment is rotation, so if we release the constraint and rotation is allowed means if we introduce a hinge here, if we release this constraint means introducing an hinge here and then after introducing the hinge if we rotate this beam such that if we allow rotation such that this plus this is equal to one.

Then whatever deflected shape we get that will be the influence line diagram for delta x, so influence line diagram for M D, influence line diagram for M D is the deflected shape corresponding to theta D B plus theta D C is equal to one ok, now again if you see here we haven't really derived the expression for how the bending moment and shear force or support reaction they vary as a function of A X, still it is indeterminate problem.

Even in indeterminate problem we could see that without actually solving the problem, finding the expression for internal forces just by having a sense of deflected shape we can draw the influence line diagram for this beams and that is too for statically determinate structure as well we'll see some examples for that, so now in order to apply this principle what is very important is to having a sense of deflected shape.

So I have told you many times and probably the rest of the course I'll be telling you many more times as well that solving examples problem is fine, writing expression, applying equilibrium conditions is fine but at the end of the day what is very important is to develop a sense so through that you can visualize the response of the structure right, you can visualize the deflected shape of this structure you can visualize.

The bending moment shear force diagram of the structure, action force of the structure, until and unless that visualization come we have not then understood structure analysis, that is very important, now in order to apply this principle that's visualization that sense is very much important ok,

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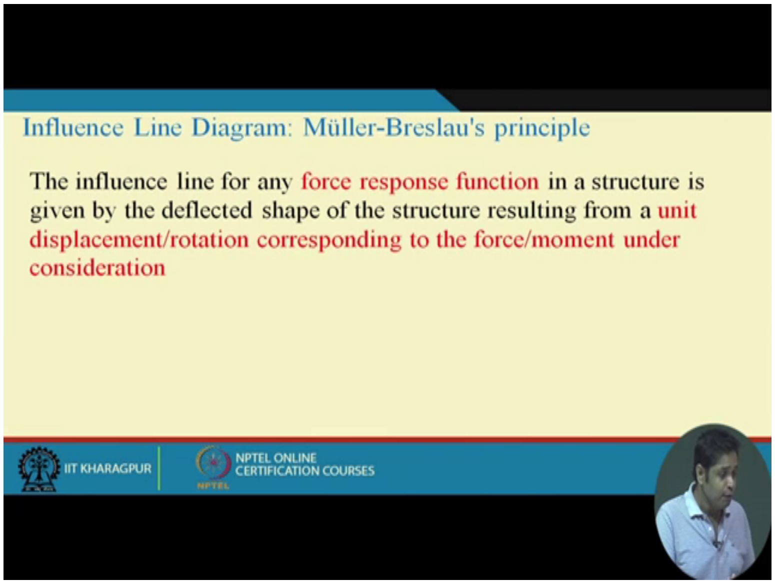

Now let me demonstrate this through one example, before that what Muller-Breslau's principle says, It says the influence line for any force response function. You please note it, force when I say force response function it could be support reaction, it could be shear force at any particular point, it could be bending moment at any particular point,

So in general it is written as a force response function in a structure is given by the deflected shape of the structure resulting from a unit displacement or rotation corresponding to the force and moment under consideration so if it is force then it is correspond to unit displacement.

And if it is moment then correspond to unit rotation, so if you want to find out the deflected, if you want to find out the influence line diagram what we have to do is we have to release that particular constraint and apply the associated displacement or rotation and get the deflected shape and that deflected shape will give you the influence line, ok?

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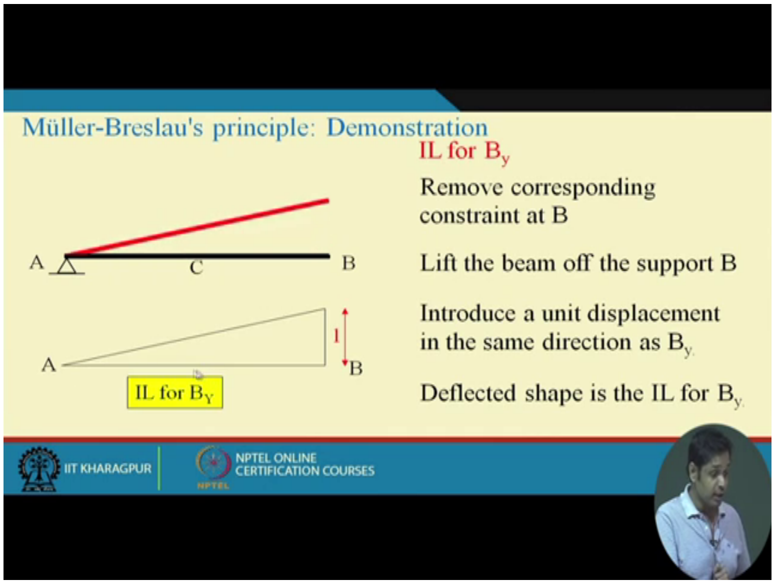

Now quickly give you one example demonstration suppose this is a statically determinate structure. A simply supported beam, we have to find out, we have seen this example before, we want to draw the influence line for support reaction at A, support reaction at B and bending and shear force at C ok, now let's what is the step remove corresponding constraint at A, constraint is removed then lift the beam off, the support A so it is lifted off the support A, now you see since it is only, it is hinge here, so there is no intermediate support.

So if you lift it, it will rotate about this point B, so it remains linear and then introduce the unit displacement and this is the deflected shape and this is unit displacement and this will be the influence line for A Y, we have seen it, let's do it for B Y again remove the corresponding constraint at B, corresponding constraint is removed and then lift the beam of the support, now this is lifted off from support B and then it moves like this and we need to lift it.

Until this deflection becomes one and introduce the displacement is one at B Y is one and then this displacement is one and then this will be the influence that deflected shape, deflected shape will be this line like this and this deflected shape will be the influence line diagram for B Y,

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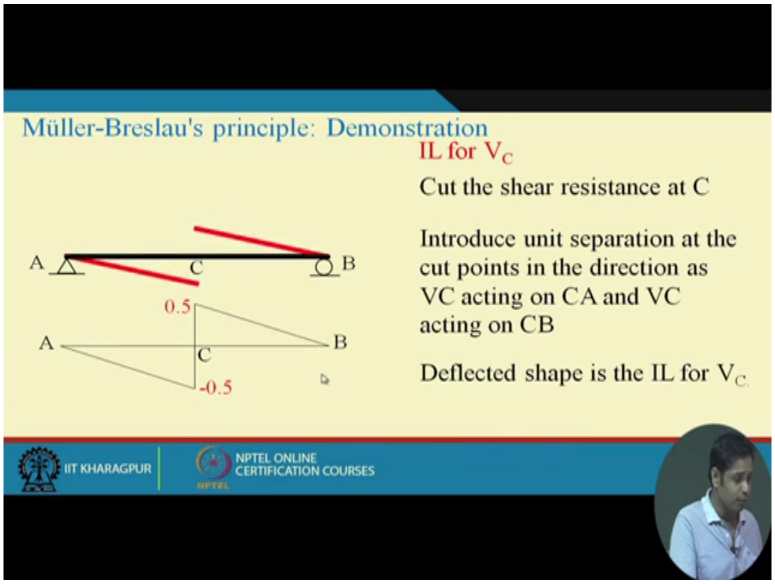

Let's see for C again cut the shear resistance at C so at C we need to cut the beam and apply corresponding displacements so introduce unit separation at the cut points in the direction. This is very important because you see in which direction you want to introduce the deflection that has to be consistent with the sign of shear force we have taken ok, so we know the what is the sign convention for shear force and the separation has to be made consistent with that sign convention and that's too for moment as well, now this is the separation and we need to do it until and unless this separation becomes one.

And this will be the influence line diagram for B C and again we've seen it, you see this separation is one, this is point five, this is minus point five, minus is written because it is negative, the absolute value should be one.

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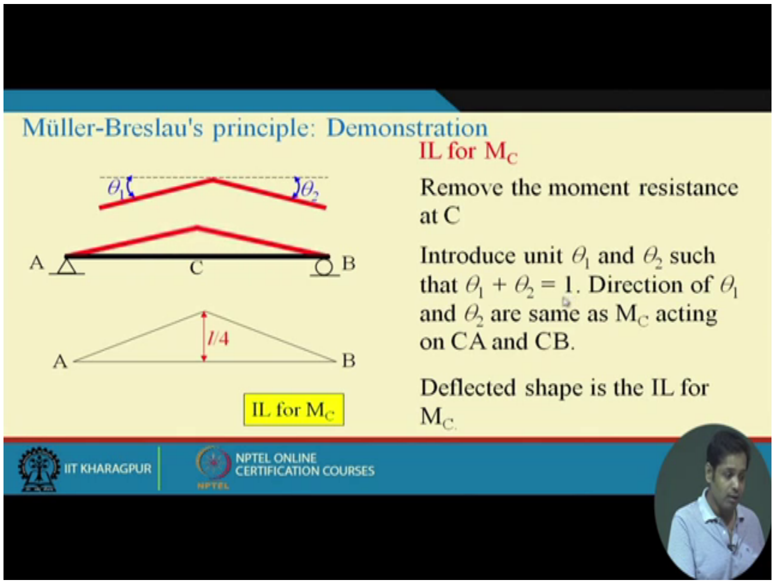

Same for moment so remove the moment resistance, when you remove the moment resistance means we need to introduce a hinge here then introduce a unit rotation theta one and theta two, theta one means.

You see when the constraint is removed moment constraint is removed means now we introduce a hinge here, now it is free to rotate, there is no constraint against rotation and suppose this rotations are like this, this is theta one and this is theta two and we need to do it until and unless then theta one plus theta two becomes one and once then theta one and theta two becomes one the deflected shape what you get that deflected shape will be the influence line diagram for M C.

Now you see in this case C is the midpoint ok, so theta one and theta two will be same if this distance is L then this distance become L by two, now total theta one theta two is equal to one so theta one has to be half and theta two has to be half, so if theta one is half and this distance is L by 2, so this becomes half into L by 2 means L by 4 ok, so the values we can obtain by this condition, so this is how the influence line diagram can be drawn.

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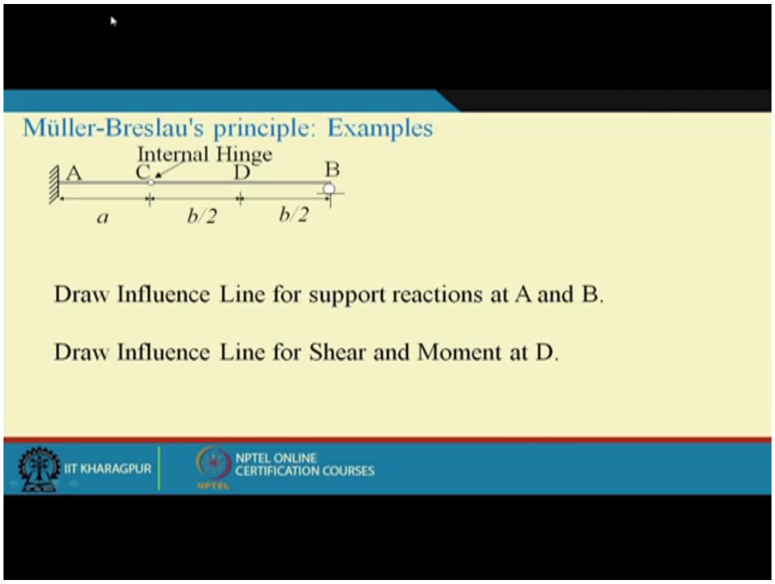

Now quickly one more example suppose this is the propped cantilever beam with one internal hinge at point C what we want to find out, we want to do draw the influence line diagram for reactions at A and B and then at any intermediate point D what is the shear and moment ok, now before I draw you let me demonstrate that through one simple model, ok?

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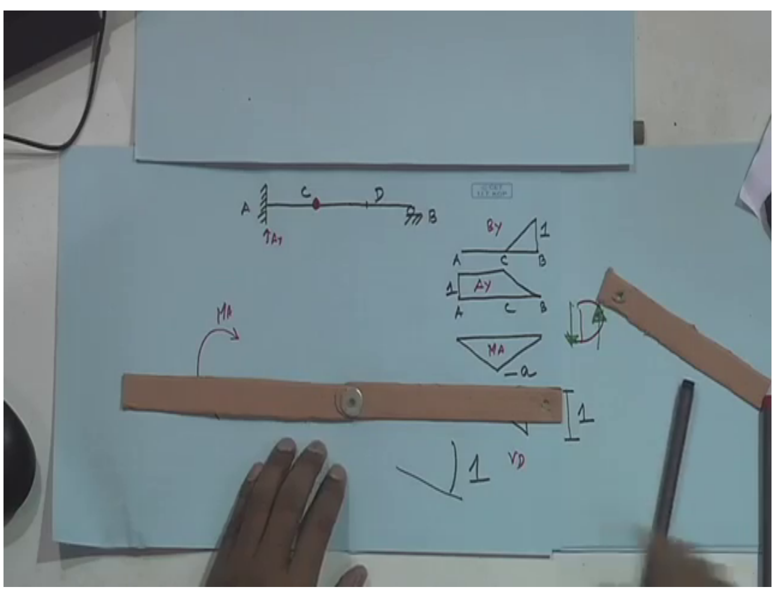

Now suppose this is the model ok. This is the beam ok, let us first draw the beam; the beam is a propped cantilever beam. We have a propped here and then in addition to that we have an internal hinge here ok and this is A, this is B, this is C and this is D, what we want is we want to find out what is the influence lien diagram at support reaction at A,

Support reaction at B and at any intermediate point D, what will be the force and moment, let us do it for support reaction first ok, now this is the beam ok suppose this point is A, this point is C which is the hinge here and this point is B.

Let us first draw the influence line diagram for B, first thing is we need to release the constraint ok, this point is now fixed we release the constraint but point A is still fixed and then we have a hinge at C, we need to release the constraint once the constraint is released then we need to apply unit displacement here, so release the constraint here and apply unit displacement ok, so this is the deflected shape of the beam, now if this is the deflected shape.

What will be the influence line diagram, influence line diagram for support reaction will be this, this is 1, this is A and this is B, you see this is A, this is C and this is B, this is C, so this is the influence line diagram for C ok and please go back to your previous classes and see we have address this problem and influence line diagram for this problem were drawn ok and you compare this with your previously obtained influence line diagram ok.

Now let us draw the influence line diagram for support reaction at A, now this point is B which is supported here, it cannot move, now support reaction A means we need to find out, let us find out for A Y, A Y means this constraint needs to be released, this constraint released means this point of the beam can move in this direction but this point cannot rotate and cannot move in this direction, so only constraint need to be release is associated with A Y.

Which is the particle direction of this point, so this cannot move this is hinge, whatever it wants and then this has to be moved vertically upward, there will be no rotation for this part A C, now if we do that this will be the deflected shape so this point which was initially here it now comes here so this is the deflected shape ok, then what will be the influence line diagram, influence line diagram will be, this is A, C B, this is for B Y, this is for A Y ok.

Now this is the influence line diagram for A Y, now let's find out influence line diagram for M A, influence line diagram for M A means then we need to release this constraint and what is this constraint now point A can be, the rotation at point A is allowed and what is our sign convention, our sign convention is this that is sagging moment right, that is our sign convention this is M A, now rotation at point A is allowed but no translation right.

Now this point is fixed here, this point cannot move but this point and this point they can rotate and if they can rotate then this will be the deformation in the beam right, now this is the deflected shape then what will be the influence line diagram, influence line diagram will be, now what would be this value this is one, and then what would be this value, you see this angle has to be one ok, this angle ahs to be one ok, if this total angle is one and now if this length is A.

You will see that this distance will be minus A this is for M A ok, now let us do it for last two thing what is for V D and for M D now for V D we need to cut it here right, and if we need to cut it here suppose I have another segment here this is another segment ok so this point is A, this point is C, this point is D and this point is B ok, now at point D this point is D, so we need draw the influence line for shear force so we need to cut it ok, now it has to be separated.

How it has to be separated now as per our sign convention the shear force at this point will be in this direction and shear force at this point will be in this direction ok and this so if we apply this then this will move like this, this end is fixed and if we apply a force like this, this will move like this so this is the deflected shape ok and then what would be the influence line diagram, influence line diagram will be.

You see this is up to this zero then this and then this so influence line diagram will be up to point C it is zero, then from here it is this and from here it is this ok, this is the influence line diagram for V D, so this is V D so this total distance has to be one, this total separation has to be one and you please compare this influence line diagram you have already obtained for the same problem by deriving the expression ok.

The last thing is let's draw the influence line diagram for bending moment at D, we need to release the corresponding constraint and corresponding constraint is we need to introduce a hinge here right, so that the rotation is allowed, now let us introduce a hinge here, suppose we introduce a hinge here ok, this hinge is already there it was there in the original structure and this hinge we introduce at point D,

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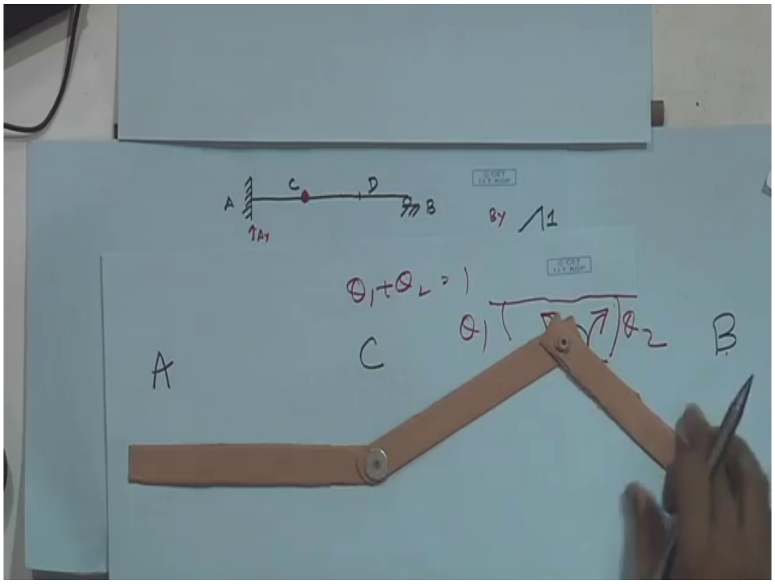

So this point is A, this point is C, this point is D for we introduce this hinge and finally this point is B ok, now then we need to apply the moment ok, now if we apply the moment, we need to apply the moment here and apply moment here that is as per our sign convention and if you do that then this will move like this, this will deflect like this ok, so this angle now this if you take this, this angle is theta one and this angle is theta two and this theta one plus theta two has to be one ok.

Then what will be the influence line diagram, from A to C it is zero and then C to B and C to D and D to B it varies like a tent function,

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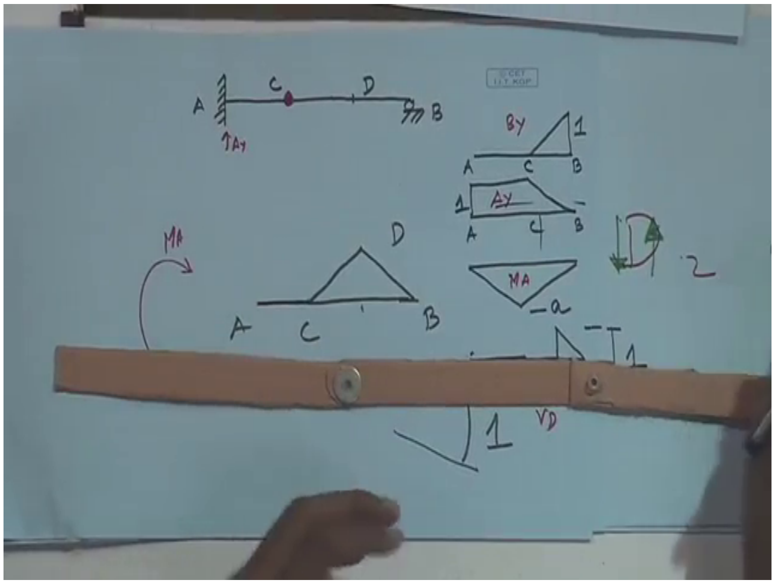

So influence line diagram will be for this is this and then from A to C it is zero and then it varies like this, so this is A C B D you can get these value by checking this angle and this angle their summation is one ok, so you see and you can do it, you can take it as puzzle, you can take it as fun, take any example.

Any structure and try to draw the influence line diagram but interesting thing is to draw that you really don't have to solve the problem, only you need is to have a sense of deflected shape of the beam, so we'll stop here today, there are many examples given in the book please do that and check whether the intuition what intuition you have, what you have been developing that is in a right direction or not ok,

So this was the end of our first part of this entire course where we started various methods to analyze statically determinate structure finding response means internal forces and then displacement of the structure, but in most of the practical cases the structures are not statically determinate so at least our structure is statically indeterminate structure, what we'll do from next week first we'll see what is statically indeterminate structure, how to characterize the indeterminacy of a structure and then see various methods to analyze or to find the internal forces and displacement in statically indeterminate structures, see you next week.

Thank you.