Structural Analysis I
Prof Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 31
Influence Line Diagrams and Moving Loads (Continued)

Hello Everyone! Let us continue what we have been doing since last few classes we have been discussing Influence Line Diagrams and Moving Loads. What we have done so far, let us quickly review that. There are majorly three problems we may encounter. One is Influence Line Diagrams is essentially required to understand to deal with loads that are not at rest condition they are the loads are moving on the structure for trains on bridges vehicles on flyovers and so on.

Now there are mainly three problems one is the drawing of Influence Line Diagrams so that is essentially how if a unique load moves on the structure then how the internal forces and support reaction changes with the location of the unique load. So that is what that is changed the graphical representation of that change is the Influence Line Diagram, ok.

So on aspect is the drawing the Influence Line Diagram for the given problem for the given structure. Now once the Influence Line Diagram is drawn then the next case suppose we have load that could be a concentrated load that could be uniformly distributed load several concentrated load moving a train of concentrated load.

Then the next problem is for a given location of that externally applied load what would be the internal forces. That is also very straight forward what we need to do is we know the location of the load and corresponding value we can get from the Influence Line Diagrams and that value will give us what would be internal forces. We have seen that in the last class.

And then the third problem is as the load changes its position then there will be a critical position for which the internal forces or the support reactions will be maximum. What we need to find out what would be that critical position for which the structure is subjected to mode the internal forces in the structure is maximum.

We have discussed that in the case of uniformly distributed load in the last class. What we do now today is if it is not uniformly distributed load if the load is not ideal it cannot be idealize and uniformly distributed load. If it is a train of concentrated load moving on the structure then what could be the critical location for which the internal forces will be maximum in the beam.

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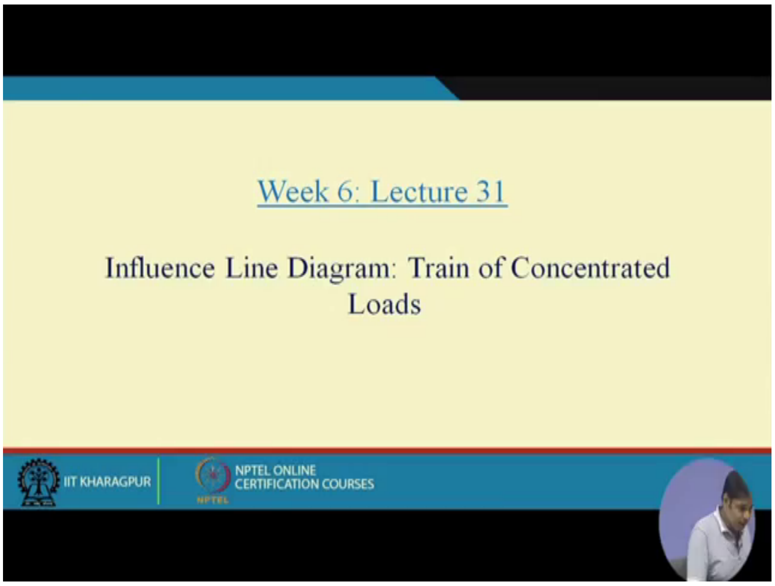

But so today's is topic is Influence Line Diagram and train of concentrated loads.

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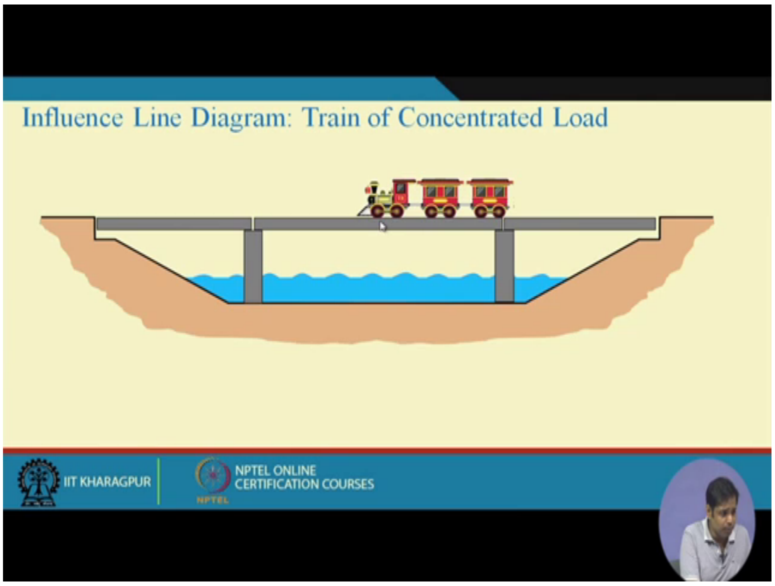

But before I do that let us quickly see what is the problem describe the problem first again let us consider the same bridge that we started with our discussion in this with. Now suppose this is a vehicle is moving on this bridge like this now if you see there are say here we have 6 wheels in this vehicle, ok.

Now suppose this is a given location of the vehicle now whatever load on the vehicle that the (())(03:39) of the vehicle that will be transferred to the bridge through these wheels, ok. Now as the load moves the relative position of the wheels remain same, they are not changing they won't change, ok.

Now the total load will be transferred to the bridge through these wheels and that each will apply the concentrated load on this structure. So this can be idealized as a concentrated load on this structure. So these are wheel position. Now this is a train of concentrated load now this train of concentrated load is moving on the structure, but as they move their relative position between these loads they are same, ok. So what we want to do now today is as this load moves the internal forces in this member change and what would be the criticial location of this load for which the internal forces will be maximum.

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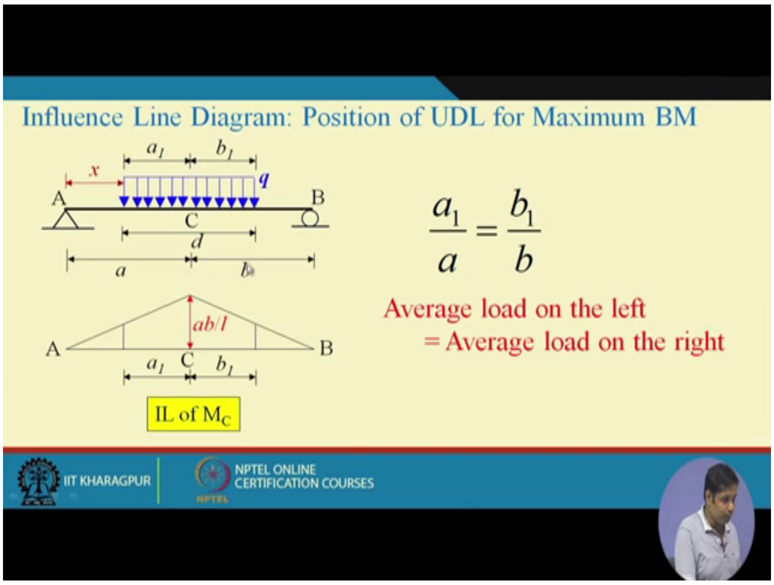

We have done it for uniformly distributed load. This is the we have already prove in the last week that if the load is uniformly distributed load and subjected on a simply supported beam because this is mean structure can be idealized in the simply supported beam.

So if a uniformly distributed load you apply on a simply supported beam then this the Influence Line Diagram for bending moment. Then for suppose C is any C is the location of the at any given location at which we want to determine what would be the bending moment.

Now the bending moment at C will be maximum if the load is placed in such a way that A 1 by A is equal to B 1 by B where A and B are the point C divides the beam AB by in A by B ratio and the same point C divides the load in A 1 by B 1 ratio. So if AB and A 1 B i they satisfies certain criteria and this is the criteria if they satisfy that then the bending moment at C will be maximum.

Now what is exactly this criteria if you if see A 1 by if you multiply both side by Q then Q into A 1 will be essentially be load on this part. Total load will be Q into D and Q into A 1 will be the externally uniformly total load AC and similarly Q into B 1 will be total load on CB.

Now Q into A 1 by A is essentially the average load on AC and Q into B 1 by B is the average load on CB. So what it says the criteria says that you place the uniformly distributed load in such a way that the average load on the left side of the beam equal to the average load of the right side of the beam.

Then that is the critical position and in for that position you get the maximum bending moment at C. And what would be the bending moment at C this is the Influence Line Diagram, so bending moment at C will be area of this area will be the bending moment diagram. We know that this value is AB by L so we can determine this value and this value and get this area.

So this area will give you the bending moment at C. This is a critical location for uniformly distributed load. Now the same thing same exercise we are going to do it for train of concentrated load.

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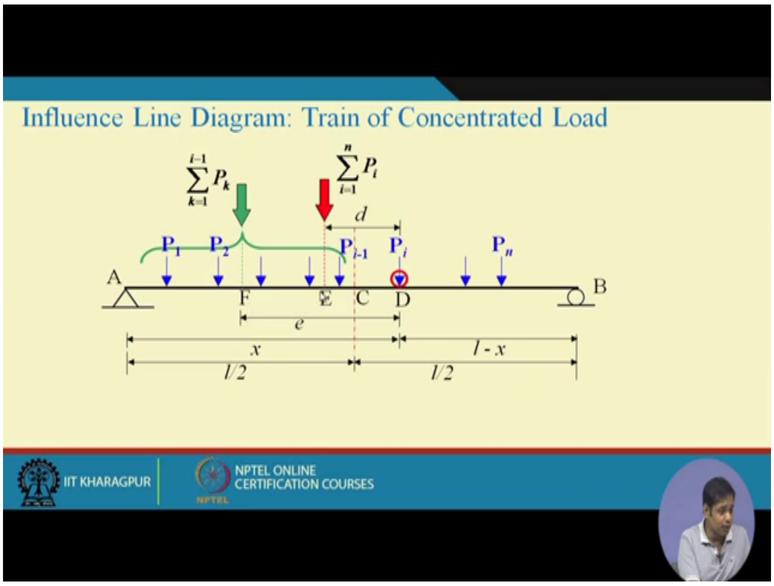

Now suppose this is a same beam AB simply supported beam and this beam is subjected to and C suppose is the center point of the mid span of the beam. And so if the total length L so AC is L by 2 and CB is L by 2 ok. Now suppose this is subjected to a train of concentrated load. And this distances between two consecutive concentrated loads are same not same they remain same they do not change as the load moves on AB, ok.

Their distances this distance this distance may be different but if you take any two consecutive forces and their distance will remain constant as the load moves on AB, ok. Now suppose this loads are suppose we take any point at D at which we want to find out what would be the bending moment.

So we need to find we are going to find out what would be the critical location of this concentrated load the load system for which bending moment at D will be maximum D could be any arbitrary location between A and B, ok. Now suppose this point D is at a distance x from A, ok. Now what is and naturally the total length is L so this length becomes L minus X, ok. Now suppose this load at suppose we have total N such loads, in this load system we have N concentrated loads.

And suppose the loads are denoted as P 1, P 2, P 3, P 4 and so on and then Pi minus 1 P i and P i plus 1 and upto P n. So total we have n concentrated load, ok. Now next is so what would be the resultant of this N concentrated load they all are vertically downward they are all parallel load but their line of actions are different.

So total result the resultant of this N concentrated load will be there sation. So resultant will be sation of Pi where i sation over i, i is equal to 1 to N. So total the sum of all this load will be the resultant of this load. Now suppose this resultant of the entire of N concentrated load that resultant acts at a point E means the line of action of the resultant suppose is this. And that line of action intersect the beam AB at point E, ok.

And now suppose this point E the line of action of the resultant is at a distance D from point D at which we want to determine the bending moment, ok. Now you see this is the resultant of total load, ok, total load acting on AB. Now upto the left side of the beam left side of the point D how many loads we have? We have P 1, P 2, P i minus 1, suppose these are loads acting on AB but on the left side of D.

And resultant of these loads are suppose again sation of all this resultant of this load is P k where k varies from 1 to i minus 1 becuase i minus 1, P 1 to P i minus 1 loads concentrated load forces are acting on one side of the point D, ok. Now again suppose this line of action of this resultant is f so E is the line of action of the resultant of the entire load and F is the line of action or F is the point at which the resultant of load P 1 to P i minus 1 acts.

And P 1 to P i minus 1 are the forces which are on AB but one side on the left side of D, ok. Now and suppose this distance is small e. So capital small d is the distance from D distance of the resultant of entire load and small e is the distance of the resultant of the loads which are on the left side of capital D, ok.

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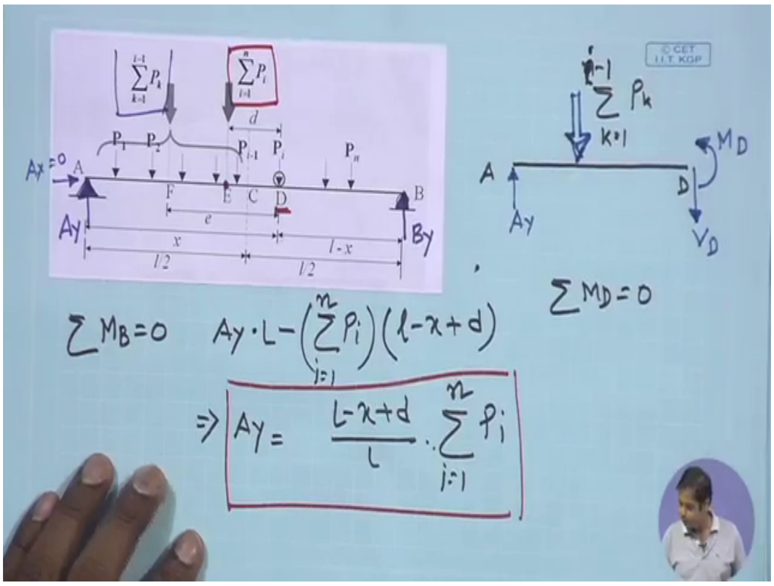

Now so now come here let us see what is the critical position, ok. Now this is the same figure just now we discussed, ok. Now let us first draw the now what we want to do is as I said we what would be the location of this entire load system for which the bending moment at point D is maximum, ok.

Now let us draw the free body diagram of the entire system, now if we draw the free body diagram so this will be BY and this will be AY ok there is no horizontal force so AX will be anywhere 0,ok. Now let us take moment about we need to find out what is the reaction at AY let us take moment about B.

If we take moment about B, so sation of moment about B is equal to 0. Now what are the forces that will contribute AY will contribute and contribution of AY will be clockwise so it will be AY into L positive. Now you see all this for BY will not continue but because we are taking moment about B all these forces will contribute all the vertical forces or concentrated load you can see they will contribute.

Now taking moment of individual forces is same as taking moment of their resultant, the resultant of the entire forces. So instead of taking moment of the individual forces what we do is we take the moment of their resultant. What is the resultant of all the forces? Resultant of all the forces is this is the resultant of all the forces.

And at what point it acts? It acts at point E and what is the distance of E from B? This distance is L minus x and then this distance is D. So this is L minus x plus D. So and all these forces will act anti clockwise moment. So this is minus resultant is sation over i is equal to 1 to n P i this is the total resultant and it is acting at a distance L minus x plus D.

So A y will be L minus x plus D by L into sation of P i, i is equal to 1 to n. So this is the support reaction at A, ok. Now once we know the support reaction let us find out what is the we want to determine what is the condition for which moment at D is maximum. So let us find out what is the moment at D.

So in order to do that let us take AD and draw the free body diagram of AD. Now what is the free body diagram at AD? At A we have A y the value of A y is this, this is A and this is D at D we have shear force VD and then moment MD and then we have all these between A to D all the concentrated forces from P i to P 1 to P i minus 1, ok.

Now we want to take moment about D, so one way is we take moment of each individual forces or instead of that take the moment of their resultant. Their resultant is this so what we do is we take the moment of their resultants from D. And their resultant acts this is the resultant and this value is sation of k where k is equal to 1 to i minus 1.

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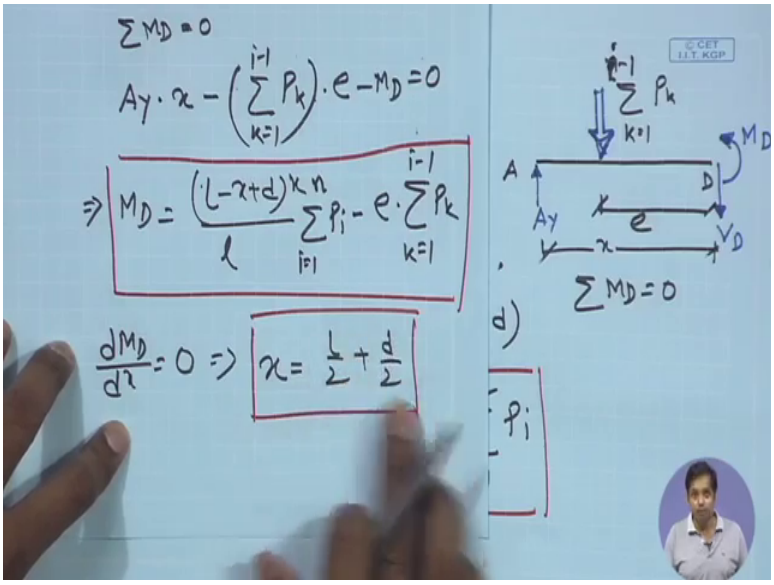

Now take moment about D if we take MD is equal to 0 then what expression what we have let us determine that we have AY into this distance, ok. AY is this distance is x we know this distance is x ok and this distance is E. So determine the forces sation of MD is equal to 0 what we have AY into AY will contribute AY into x and then minus sation over k pk k is equal to 1 to i minus 1 this is the total load, ok into E and then minus MD, that is equal to 0.

And if we do that and we get MD is equal to the expression for MD is equal to AY is AY already we obtained if we substitute that value this will be L minus x plus D into x divided by L sation of P i, i is equal to 1 to n minus this minus E into sation of k P k k is equal to 1 to i minus 1, this is the expression for MD, ok.

Now our condition as for maximum MD DMD DX has to be 0 if we do that this part will any way 0 and if we do that what we get this will give us x is equal to L by 2 plus D by 2. This is very important. So if x is equal to L by 2 plus D by 2 then MD is maximum. Now what is x is equal to L by 2 and D by 2 let us see in this case.

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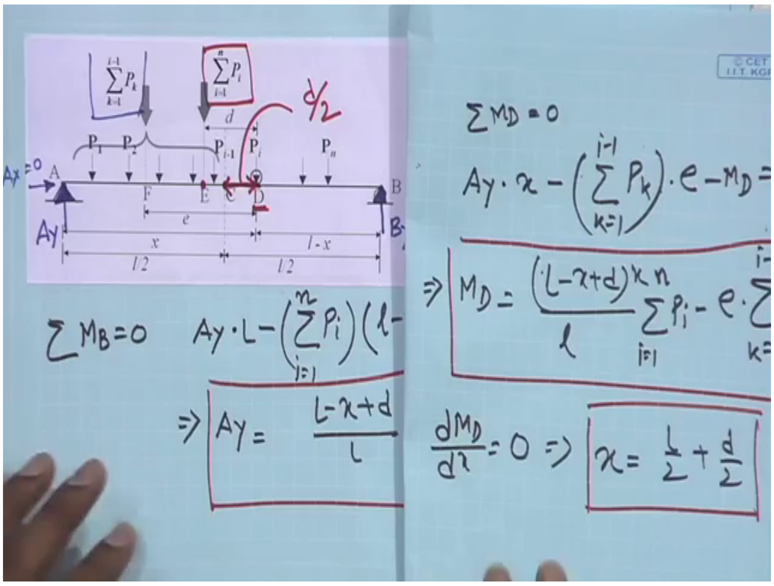

What is you see x is equal to L by 2 means at point C, ok this x. And then D by 2 means D is the distance of D from the resultant of the entire load so this distance so CD if CD if this distance becomes D by 2 then the moment at D will be maximum, ok.

So this says that the total load should be placed in such a way that D should be located at a distance D by 2 from the mid span, C is the mid span and D is the point at which we want to determine the bending moment if this distance is D by 2 then the bending moment at D will be maximum and D is the distance of the resultant of the entire load from point D, ok. So this is the this is the condition, the condition will be clear if we just quickly do one example,

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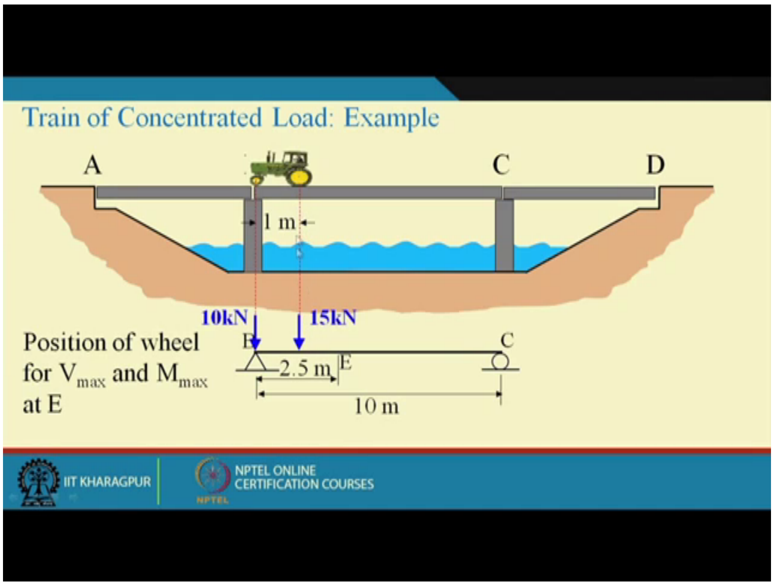

Take this example this example is again the same problem where we have ok you see this is a vehicle moving on this bridge and there are two wheels which is one meter distance apart.

And the total load of this vehicle is 25 kilo Newton and then the wheels are placed in such a way that this wheel will apply concentrated load of 10 Kilo Newton and this will apply concentrated load of 15 Kilo Newton. And this will move as this vehicle moves and this two loads also moves on BC, ok.

And this is moving in this direction and the first is 10 Kilo Newton which is followed by 15 Kilo Newton load at a distance 1 meter. So what we need to find out consider any point E at a distance 2.5 meter from B. What you need to find out what would be the position of this wheel for maximum of shear force and maximum bending moment at B.

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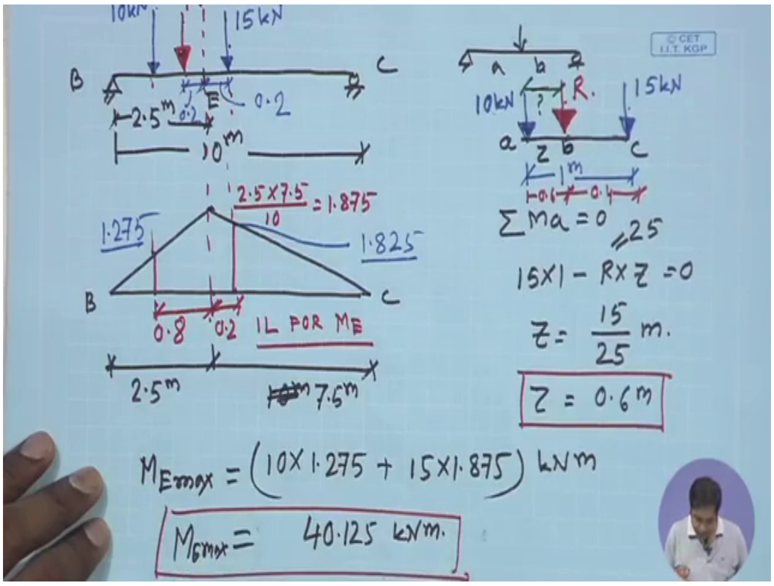

So let us try this problem so quickly it is simply supported beam and this length is total length is 10 meter, ok and then we have a point E which is 2.5 meter distance, ok. We are going to find out what is the let us first do it for maximum bending moment.

Now what is the Influence Line Diagram for bending moment? Influence Line Diagram will be this is the IL for ME Influence Line Diagram for ME this is B this is C and what is this value if we know that if it is A this B then this distance will be AB by L, where A is equal to 2.5 B is equal to 2.5 and then L is equal to 10 meter and then this value becomes 2.5 A into B by L is 10 this is 1.875, ok.

And this distance is 2.5 meter this is 2.5 meter and this is 7.5 meter, ok. Now what is the load we have the load we have first is 10 kilo Newton and then this is 15 kilo Newton and this distance is 1 meter, this is 1 meter, ok. Suppose this is the resultant of the load R.

Now let us find out what is the value of R what is this distance? At what point this R acts suppose what is this distance. Now if we take load if we take suppose this is small a b c if we take moment from a then we have sation of M a is equal to 0 what are the forces we contribute 15 will contribute that is 15 into 1 it is clock wise minus R, R into say this distance is Z, R into Z this is equal to 0 And R is 25 so Z becomes 15 by 25 meter. So Z becomes 0.6 meter, ok.

So this is 0.6 and therefore this becomes 0.4 meter, ok. So this is 0.6 and this is 0.4 meter, ok. Now what is the what was the condition? Condition is that we need to place the load in such a way that resultant of R should be at a distance D by 2 from E, ok. And what is then what would be the position of this load if this point is E let us apply this load, this is the position at which we want to determine the this is the position E.

And then the resultant acts somewhere here this is the resultant of the load this is the R which is the resultant of the load and then this is 15 Kilo Newton load and this is 10 kilo Newton load, ok. And this distance is 0.4 so D is equal to 0.4, so if this distance is 0.4 E has to be E should be the mid point of this and this, ok.

So this distance will be then 0.2 and similarly this distance will be 0.2, ok. So if this distance is 0.2 draw the corresponding line here and again draw the corresponding line here again draw the corresponding line here and this distance is we know that this distance is 0.2 and then this distance will be 0.8, total distance between two load is 1 meter, ok.

Now next is what we have to find out what would be the moment? Moment will be 10 kilo Newton multiplied by this distance and then 15 kilo Newton multiplied by this distance. And if you find out this distances this values will be I leave it to you this value will be 1.275 because we know all this, this is 2.5 meter this is 0.8 meter and this is 0.2 and similarly this value will be 1.825, ok.

So bending moment MD ME ME max rather will be 10 into 1.275 plus 15 into 1.875 meter and this will be 40.125 Kilo Newton meter this is ME Max all these calculations please you do it yourself and check whether you are getting these values or not, ok. This is for maximum bending moment.

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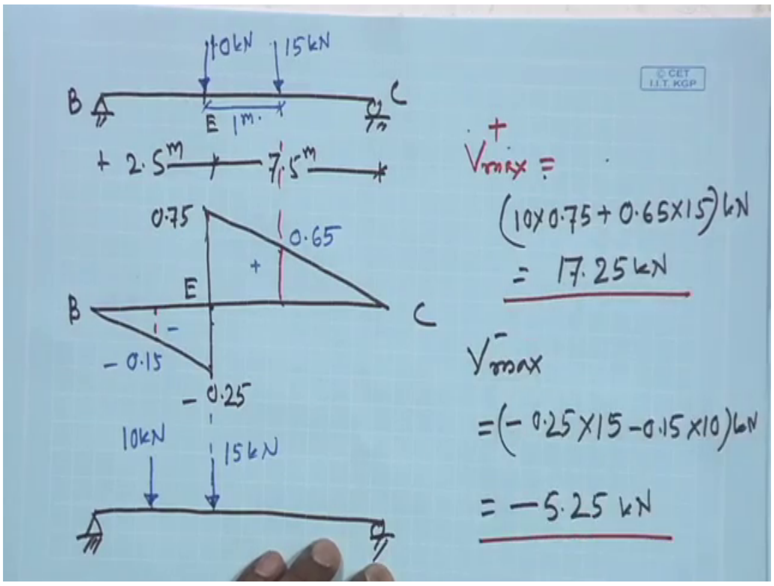

Now just quickly we will check what would be the position for maximum shear force if this is this point is E this is 2.5 meter and this is 7.5 meter we know what is the Influence Line Diagram for shear force, Influence Line Diagram for shear force is this. This value will be 0.25 or minus 0.25 and this is 0.75 this isB C B C this value is E, ok.

Now we need to find out what would be the this is positive this is negative now first case if the load positions are like this first 10 kilo Newton load is this this is 10 kilo Newton load and then after 1 meter we have 15 Kilo Newton load and this distance is 1 meter 1 meter then what happen your shear force will be 10 kilo Newton into this plus 15 into this.

In this case both are positive so you get maximum positive shear. Now if the load slightly moves in this direction and takes a position like this here it is 15 kilo Newton load and here it is 10 Kilo Newton load in this case what happens shear force will be 15 into this distance plus 10 into this distance.

So your shear force will be maximum negative shear so for V maximum positive will be this into this, this value will be 0.65 and this value is 0.15 you can check it by applying similar triangle so V max will be 0.75 into 10 so 10 into 0.75 plus 0.65 into 15. This is 17.25 Kilo Newton and similarly V negative max this will be 0.25 minus 0.25 into 15 and this is minus 0.15 minus 0.15 into 10.

And this becomes minus 5.25 Kilo Newton , ok. So this is for maximum positive shear this is negative shear, ok. So this will be the position of the shear force, ok. Now it is a small very simple demonstration of the concept that we have learned today again I have been telling in almost all classes that this is not the all in whole you have to really see some more examples from the book to understand the concept and apply it in a better way for different kinds of problem, ok.

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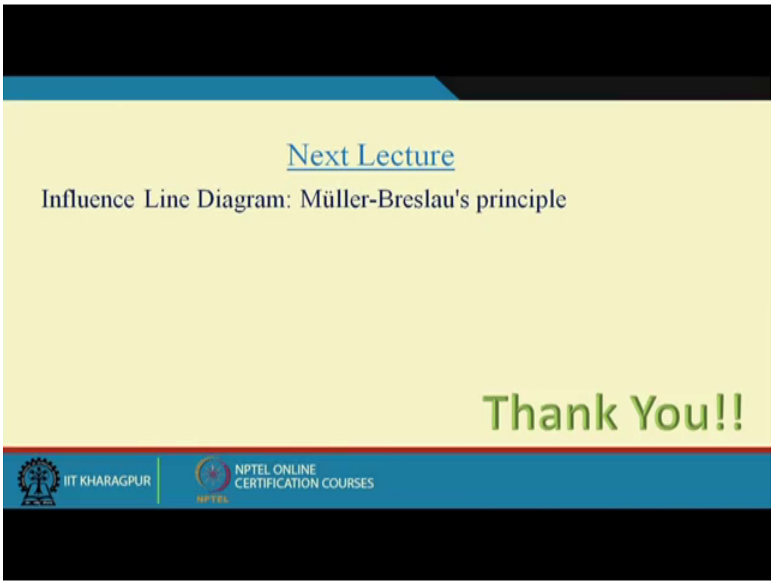

Thank you very much now we stop here in next class what we do is you see almost whatever discussion we have so far is you need to draw the Influence Line Diagram we actually need to solve the problem, we do not need to find out the support reactions get the expressions for moment shear and draw the and essentially Influence Line Diagram where is at the graphical representation of those expressions.

But there is a very interesting way to find out Influence Line Diagram without actually solving the problem and that is very much important for indeterminate structure because solving indeterminate structure is a very miserably straight forward thing like determinate structure. So what you learn next class is molar resler principle which tells you that if we have some understanding about the deflected shape of the structure then based on that we can determine we can draw the Influence Line Diagram, ok. With this Thank you very much see you in next class.