Structural Analysis 1
By Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharaghpur
Lecture 30: Influence Line Diagram
And Moving Loads (Continued)

Hello everyone! Ok welcome to lecture 30 what we have been discussing since last two classes is the influence line diagram, in the first class of this week we introduce the influence line diagram, then in the second class we did some examples and then learned how to draw influence line diagrams for different problems, the application of influence line diagram we will see definitely towards the end of this week.

But so far what we have done is when a concentrated load is moving through the length of the beam but now in most of the practical purposes the load may not be just one single load, it may be distributed load or it may be a train of concentrated load, so what we will do today is see how to deal with those distributed load ok, so today's topic is this,

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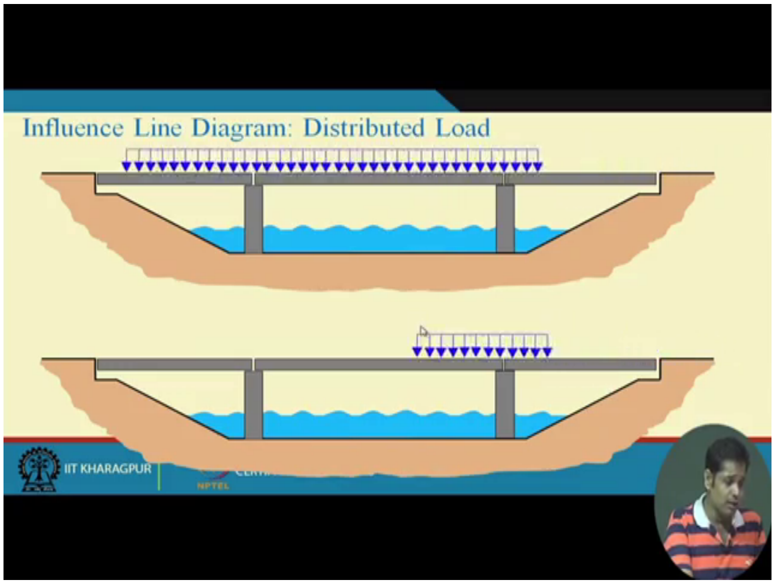

You see again consider the same problem now in the previous example it was just one concentrated load.

They was moving through the length of the garden now instead now what we have, a distributed load ok, now we have two situation one is ok let me show you the second situation in the second situation, now you see the difference between these two situation for the distributed loads, now in the first case we are concentrating on the central guarder right, in the first case what happens is, the length of the distributed load is more than the length of the guarder.

But in the second case the length of the distributed force the segment of distributed load that is less than the length of the guarder ok, what we do now is we'll consider this case an arbitrary length of distributed load and then see what happens when the load is less than the length of the guarder and more than the length of the guarder and also see how the concept of influence line diagram can be used to find out the internal forces and support reactions.

For given position of the distributed load and also is for about what will be the most critical position of the distributed load so that the internal forces or support reactions in a structure is maximum ok,

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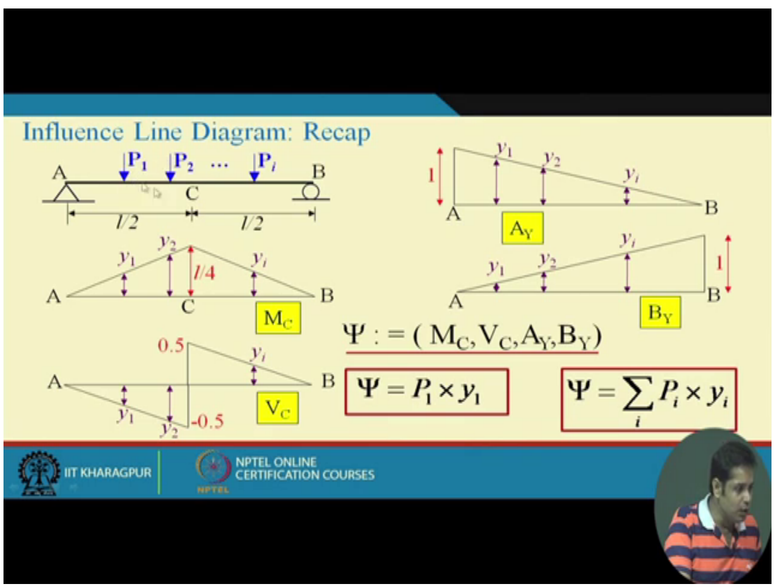

Now before that quickly recap what we have done so far you see if this is a beam and we are interested in support reactions and the bending and shear force at mid point C, now we know this is the influence line diagram for bending moment.

And this orbit is L by 4 and we also know that this is the influence line for shear force at C and this is the influence line for reaction at A and this is the influence line for reaction at B and this we have already learned, now what is application of this influence line, now suppose if we have any concentrated load P one at any arbitrary location ok, now for that particular location suppose these values are this is Y one.

This orbits we can calculate from this influence line diagram and if you know that then suppose define any general parameter Psi which could be moment at C, shear at C, reactions at Y and reaction at B, depending on the context then Psi can be written as P one into Y one right, means P one into this Y one will give you the moment at C, P one into this Y one will give us moment at C, P one into this Y one reaction at A and P one into this Y one will give us reaction at B ok.

So this is the application of influence line diagram as far as we have a concentrated load ok, this we have learned so far right, now suppose it is not just a one load, suppose we have two load or three load or say N I number of loads it is series of load ok, now then what we have to do is we have to find out corresponding ordinates for say P two then P three and then corresponding ordinate for I'th load ok, then what will be the reactions or moment at C.

Then this will be sation of P I into Y I, now Psi I is equal to Psi I could be anything of this from this depending on the context means if Psi I is M C then M C will be that P one into Y one plus P two into this and P three into corresponding ordinates similarly plus P I into corresponding ordinate, this will give us moment, similarly if we have to find out shear force or support reactions same concept can be applied.

So what we know is we know that if we know the what we had done so far we have the influence line diagram, now structure of beam as of now we have demonstrated through beam only so let's take beam only the if you know the influence diagram for a view then what is the use of this influence diagram if any load is applied at any location then we need to compute, we need to find out the ordinate for the influence line diagram.

At that particular location multiplied by the applied with that will give us the corresponding internal forces or reactions, if we have several such loads then just linear combination of this give us the total affect ok, so now suppose this load is now replaced by and it is just P one P two several concentrated load right, now suppose they are replaced by this concentrated loads are replaced by a distributed load like this ok.

Now remember one thing whether load is distributed or the load is concentrated at load is a train of concentrated load influence line remains same, when we talk about influence line it is just one unit concentrated load moving through the length of the beam ok, so irrespective of the actual load influence line is always drawn for unit concentrated load ok,

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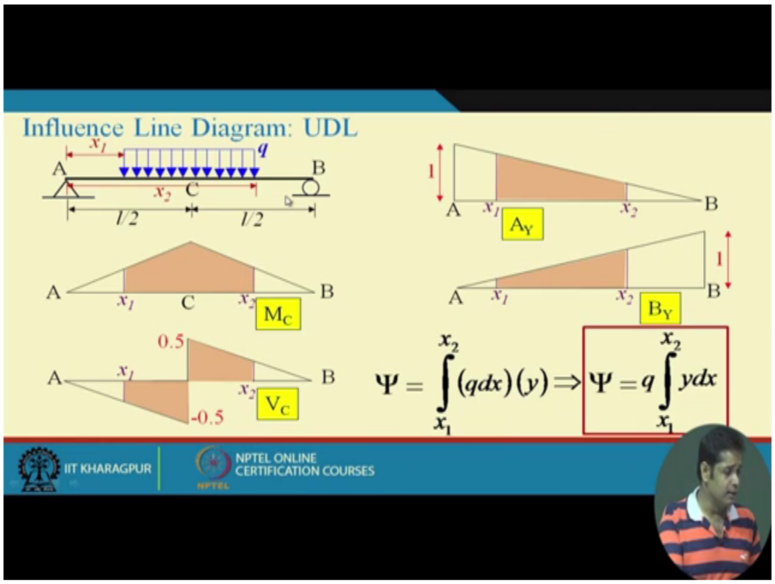

So now when the load is applied by a distributed load, the influence line remains same right. These are the influence line that remain same, what we are going to do here now is in the previous slides we discussed that if we know the influence line then for a series of concentrated load acting on the structure how to get the internal forces and reactions, now what we are going to demonstrate here now is, now if you know the influence line diagram then on that beam if the beam is subjected to uniformly distributed load of given length.

Shorter than the length of the beam then how to compute, how to use the influence line diagrams to compute corresponding support reactions and the internal forces ok, now suppose intensity of this distributed load is Q and which is acting at a distance x one right, now so load starts at x one and ends here ok, suppose this distance is x two ok, so load is between x one and x two, x one varies from zero to L and x two varies from zero L ok.

When x one is equal to zero we'll see then we putting x one is equal to zero and x two is equal to L, it means the distributed load is on the entire structure ok, now suppose corresponding ordinate, ordinate means at x one and ordinate at x two of the influence lines are as this ok, this is at x one and this is at x two, x one x two x one x two x one and x two ok, now then Psi is again a general parameter it could be moment at C, shear at C or support reaction ok.

Now suppose what we have when the loads were discrete concentrated load then (09:51 we use sation) sign ok, now instead of discrete load it has a continuous load, so sation will be replaced by an integration isn't it, now integration of what, if we take a small segment here with length D X then on that segment the load will be Q into D X ok, and suppose corresponding ordinate here is Y, then what will be the Psi.

Psi will be that Q D X into Y and then integration between x one to x two ok, it is the same expression the previous expression when we have a discrete concentrated load but since it is now continuous load, continuously distributed, continuously varying load in this case this variation is constant uniform load so that sation has to be replaced by an integration sign right, and then the limit of integration is from x one to x two.

Now since at least with this case if Q is not constant then you need to consider the variation of Q here for at least this problem since Q is constant what we can do is we can take Q outside of this, now if we take outside of this and what we are left with is integration of x one x two Y into D X, now I believe you are familiar with this term, what did this, this gives you this is essentially the area of this influence lines of area between x one of this diagram between x one to x two isn't it.

So essentially this is the area of this, x one to x two, so then what happens then psi will be intensity of load multiplied by the corresponding area in the influence line diagram ok, now in the previous cases it was just a sation and now in this case the sation is replaced by integration ok, and final results that we have arrived at is if we need to find out suppose a beam is subjected to uniformly distributed load, now we need to find out what is the moment.

Shear or support reaction or any other internal forces at any particular location, what we can do is, we know the influence line diagrams, influence line diagrams is depends on the structure, depends on their length, depends on the boundary condition, it doesn't depend on what is the actual load on the structure, so the influence line diagram remains same we know the influence line diagram, now we need to find out what is the forces at a given location.

Just place the, we know the location of the uniformly distributed load, now the area of that diagram between the starting of the load and the end of the load will give you the corresponding internal forces, we'll demonstrate through one example, up to this it was very simple ok, now next is, you see what we know is if we consider a beam which is subjected to some uniformly distributed load, now before that if you see the distributed load is over the entire span.

Then the entire influence line diagram will be the internal forces ok, now what we have done so far is we have a given beam which is subjected to a distributed load we know the influence line diagram and find out, now using the influence line diagram we can determine what are the internal forces and support reactions right, now consider situation we know that at the load changes its position this internal forces and support reactions they also change.

Now naturally the question comes what would be the most critical position at what position of this distributed load your internal forces are maximum and that is were important to know because when you design a structure you design for worst condition so what position of the load gives you that worst condition that is important to know, what we'll do next is we'll find out what would be the critical placement of a distributed load.

So that bending moment at a given location is maximum, we'll demonstrate that through a simply beam,

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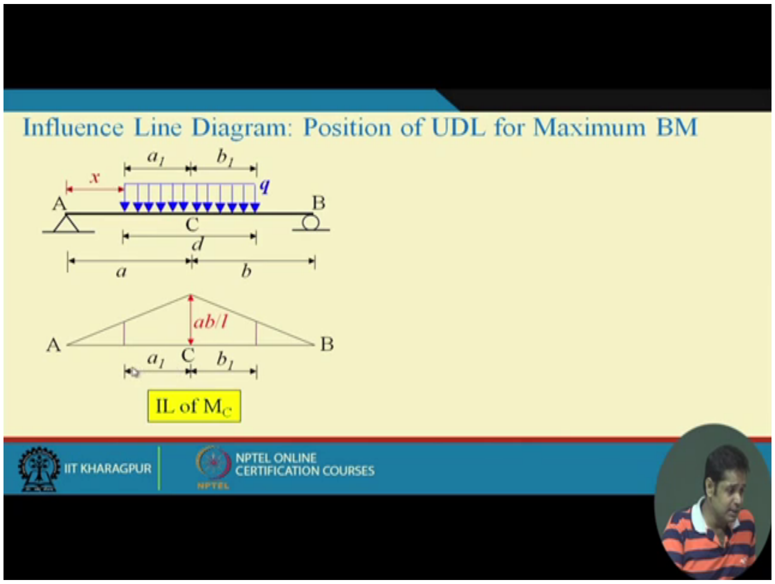

Now suppose a simply supported beam which is influence line diagrams we know, this is the influence line diagram just to make it general, we have taken a very general dimension A and B, suppose this is subjected to a uniformly distributed load Q and we don't know really the placing of this load that we need to find out ok.

But we know this length of the distributed load is D ok, now suppose this distributed load is placed in such a way that C is the point where we need to find out the bending moment ok, now suppose the load is placed in such a way that the portion A one, the length A one of this entire length is this side of C and length B one is this side of C means we can say that the point C divide the load in A one by B one ratio ok.

So this part is this side and this part will be this side, we still yet don't know what is the value of A one and B one how they depends on A and B that we need to find out ok, now suppose the load is placed at a distance x from A ok, now this is the corresponding ordinate on the influence line diagram ok and this distance is A one and this distance is B one ok, now what is the problem we can find out here.

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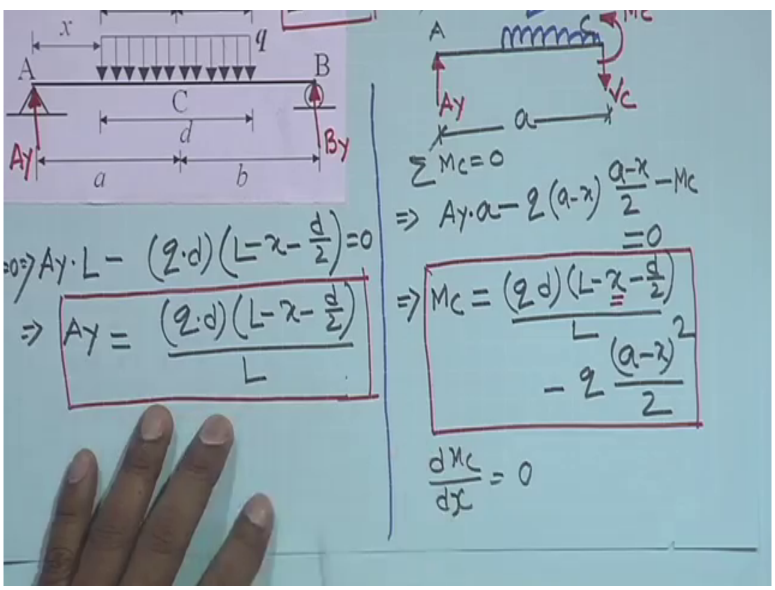

Now the problem is you see this is the problem that is the diagram that I showed you ok, what we need to find out this load, now this load can change its position ok and if it change its position accordingly the values of A one B one also changes ok, but A B they remain constant because they decide the location of C ok, now what we are interested is how this load to be placed such that the bending moment at C is maximum ok.

Now let us find out that, now suppose the load is placed in this way ok, now we need to find out what will be the value of A one and B one and how this values of A one and B one related to A B so that the position of this load can be uniquely determined which gives us maximum bending moment at C ok, let’s first draw the figure diagram for this position of this load, let us draw the figure diagram of the entire structure, the figure diagram will be, this will B Y and this is A Y.

There will be x also but since x is equal to zero so not shown here but please note that I am just to avoid drawing another same thing, I am showing the support reaction here itself but when you actually show the support reaction with figure diagram then the body has to be free the support and the support reactions cannot be shown at the same place, this is a common mistake please be careful about this, so this is the figure diagram ok.

Now once we know the figure diagram let us find out the support reaction, now if we take moment about B then what we get is A Y into this distance is L, L is equal to A plus B suppose L is equal to A plus B, so total length is L, so A Y into L then minus, now what is this entire, the load due to this distribution this will be Q into this distances, so Q into D will be the load Q into D will be the total load acting downward and at what distance the center.

Since it is distributed load the center will be D by 2 from here, so this value will be you check this total length is L, this distance is x so minus x and from here it is D by 2, the centeroid of this load so this is again minus D by two so this is equal to zero that is the, this is actually M B is equal to zero ok, now this gives us A Y is equal to Q D into L minus x, D by 2 divided by L, this will be support reaction ok.

So is a function of x if a load is change its position so naturally support reaction will change and this will change as per this, again you see this variation is linear variation ok, now let us take the figure diagram of this part A C ok, because we need to find out the bending moment at C, so let us take figure diagram of A C, so what would be the figure diagram of A C, this point is A and say this point is C ok and what are the forces we have at A.

We have the vertical reactions, vertical reaction at A Y which already determined as this ok, then we have shear force this is V C, this is as per our sign convention and then moment M C ok, now anything else, yes we have this distributed load also, this distributed load which intensity is Q ok and this distance is A, this distance is x, ok and this distance is A one ok, now let us now take sation of M C is equal to zero, we need to find out what is the moment at C.

Now what are the forces that will contribute to this moment, the forces will be this A Y which will cause anti clock clockwise moment the M C itself which is anti clockwise and the moment due to Q which is anti clockwise ok, now this give us, I am just draw a line here separate, now this gives us what is the moment A Y into A which is clockwise and then minus this force, what is this force this is Q, and this distance this is A, this is x, so this distance x.

A minus x so force will be Q into A minus x and what is at a distance, distance again with uniformly distributed load half of A minus, so A minus x by two ok, this is the contribution from the distributed load and then which is negative because it is anti clockwise and then minus M C that is equal to zero ok, so M C is again minus because it is anti clockwise ok, now this gives us M C is equal to Q D, L minus x minus D by two by L minus this part.

This will be Q into A minus x whole square by two, so this is the expression for M C ok, what we want is that here what defines the position of this unit load, position of the unit load, position of this distributed load, the position of the distributed load is defined by this value x ok, because if x changes this position also changes, so what problem was if you remember we want what would be the critical position of the load such that your moment at C is maximum ok.

So this has to what would be the value of x for which the M C is maximum and how we can determine that, we can determine this by D M C by D x is equal to zero, so let us do that now if we have this, we have this expression just now we derived ok.

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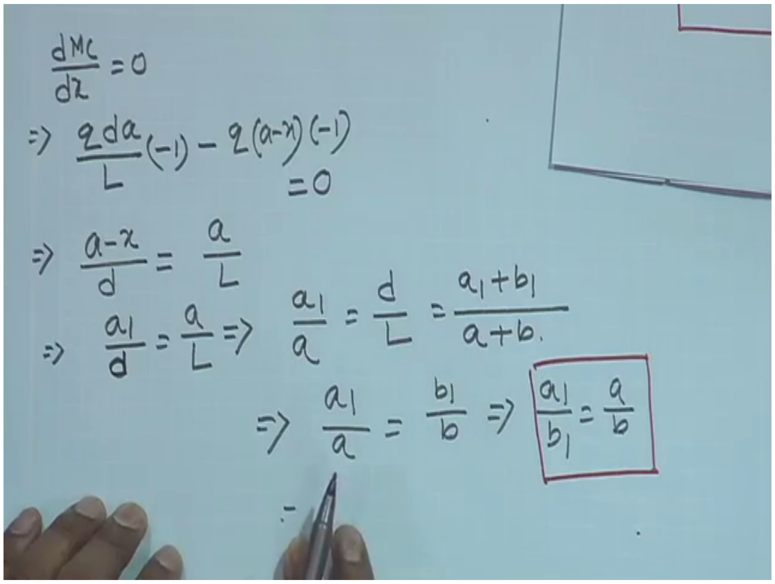

Now let us write this one so we take D M C, D X is equal to zero and if we do that then what we have is Q D, there will be A as well you please correct there will be A ok.

So because it is A Y into A, A Y is this into A now this is Q D A by L into minus one because it is minus x and then minus Q into A minus x again minus one because of minus x that is equal to zero right, now if we have this then we can determine what is the value of x and this we get A minus x by D please check that D is equal to A by L ok, now what is A minus x by D, once you say that this is A minus x, A minus x is essentially A one that's how we define right.

This is A one, so what we can write is here A one by D is equal to A by L ok, now what is A one by D ok, now this gives us A one by A is equal to D by L ok, now check here what is D, D is equal to A one plus B one right and what is L, L is equal to A plus B, so if you write here this is equal to A one plus B one and L is equal to A plus B, so this gives us A one by A is equal to B one by B or finally A one by B one is equal to A by B, this is important ok.

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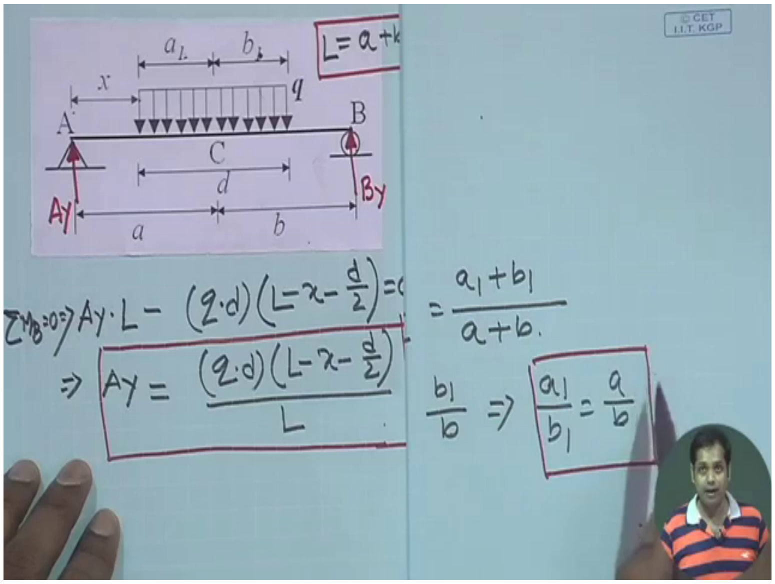

So for this problem condition for which the bending moment at C will maximum is the load needs to be placed in such a way that this condition satisfied, what this condition says that the point C divide this beam in A by B ratio, now what it says that the ratio at which the point C divides the beam, the point C should divide the uniformly this distributed the segment of this load at the same ratio, if it is done then the bending moment at C will be maximum ok.

Now again if you take any problem we'll find out the problem, to get the maximum bending moment at a particular location, first what you need to do is first draw the influence line diagram for the beam and then you place the uniformly distributed load in such a way that this condition is satisfied once that placing is done then rest is you take the area of the influence line diagram between those, starting and end point of the load. To get the bending moment at that particular location ok, so that's all for uniformly distributed load,

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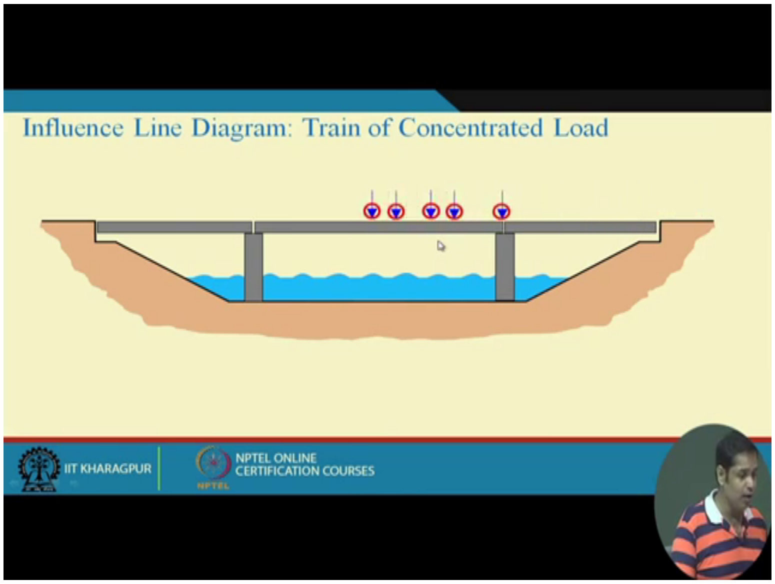

Now you see there are many cases where that cannot be idealize as uniformly distributed load which are a train of concentrated load ok, the train of concentrated load is moving and the distance of this concentrated load is maintained so what we need to find out.

What would be the position of this concentrated load, if we know the position of this concentrated load then we can find out the corresponding internal forces but now what problem is we need to find out what would be the position of this train of concentrated load for which the beam will be most critical, the same way just now we have discussed what would be the position of distributed load for which the beam is most critical in bending ok, that's all for today. See you in the next class. Thank you.